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Formation of Matrices of S = 1, S = 3/2 Spin Systems in Quantum Information Theory Formation of Matrices Some Spin Systems

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ABSTRACT: There are many methods for designing quantum computers, which are generated by rapid progress of computer technology. In this work, it is aimed to find matrices and processors by using an algorithm for spin 1 and 3/2, which can be observed with EPR spectroscopy and used for Quantum information processing. Spin matrices or processors that can be formed using the basic properties of processors. Some of the spin processors, some of which are known, are the most well-known Pauli spin matrices, which can be found in various sources, but are computed with an algorithm for convenience in practice. Matrix representations for s= 1 and 3/2 are found in the theoretical calculations. In addition to the s = 1/2 spin operators given in the literature, matrix representations of spin processors and spin systems are found for s = 1 and s = 3/2 using an algorithm. Thus it can be used in theoretical studies and applications in quantum information theory. For other spin systems spin operators can be created.
Keywords: *Spin systems, Quantum computing, Qutrit, Quantum information theory, Spin processor*

1. Introduction

The extraordinary progress of the technology and the methods to be applied in quantum computer design are being investigated. In pulsed Nuclear Magnetic Resonance spectroscopy, which is one of these methods, the applications of quantum information theory for spin 1/2, ie qubit-based systems, make the symbolic representations of rotation processors of other spin systems necessary with the discussion of operation and spreading of applications to different spin systems. In this study, spin algorithms of spin systems for spin 1 and spin 3/2 are found.

2. Material and Methods

2.1. Spin processors for spin 1/2 systems

The electron spin represents the qubit with the s = 1/2 value and the quantum states 1/2 and -1/2. $|0\rangle$ and $|1\rangle$ base vectors, quantization z-axis direction that is along the spin up $|\uparrow\rangle$ and spin down $|\downarrow\rangle$ represents status. The geometric interpretation of the qubit in 1 unit size is given in Eq.1.

$$|\psi\rangle = cos(\theta/2)|0\rangle + e^{i\varphi}\sin(\theta/2)|1\rangle \quad (1)$$

This notation is called the Bloch sphere, Figüre 1. Here θ polar angle, φ azimuth angle are real numbers. A qubit Bloch is represented by every point on the Bloch sphere. In Figure 1, a point is defined on the three-dimensional sphere by the angles θ and φ. This sphere is useful for visualizing a single qubit. However, the Bloch sphere is not visualized for multiple qubits(Olivera et al. 2007; McMahon, 2008; Jones, 2011).

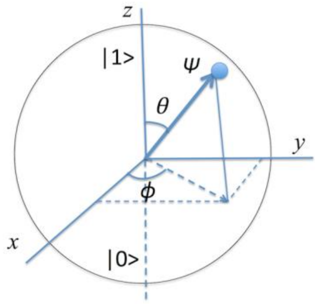

Figure 1. Bloch Sphere

Spin matrices or processors can be created using the basic properties. Some of the spin processors already available at the source are the most well-known Pauli spin matrices, which can be found in various sources, but it will be useful to show on an example that they are created for convenience in practice.

Here, for example, spin 1 or qutrit system and spin matrices for spin 3/2 are shown using Algorithm 1.

Algorithm 1. Algorithm that can be used to create spin processors or matrix representations for different spin systems.

a. Create the $\hat{S}_z$ matrix that will give the M_S eigenvalues when it affects the spin eigenstates.

b. $\hat{S}_+$ create an elevation operator, which will raise the state to a higher state when it affects the spin eigenstates $\hat{S}_+|0\rangle$ operation is undefined because it can not go out of the defined space).

c. If it applies to the spin eigenstates, the descent processor, which will reduce the state to a sub state, $\hat{S}_-$ ($\hat{S}_-|N\rangle$) operation is undefined because it can not go out of the defined space).

d. From the obtained $\hat{S}_+$ and $\hat{S}_-$ matrices, generate $\hat{S}_x = \frac{1}{2}(\hat{S}_+ + \hat{S}_-)$ and $\hat{S}_y = \frac{1}{2i}(\hat{S}_+ - \hat{S}_-)$ matrices.

3. Results and Discussion

For the Spin 1 system $M_S = 1,0,-1$ because $\hat{S}^2$ and $\hat{S}_z$ processors with solvers, $|S, M_S\rangle = |F\rangle$, (F=0,1,2) are shown in Equations 2 and 3 for eigenstates.

$$|1,1\rangle = |0\rangle = \begin{bmatrix}1\\0\\0\end{bmatrix}, |1,0\rangle = |1\rangle = \begin{bmatrix}0\\1\\0\end{bmatrix}, |1,-1\rangle = |2\rangle = \begin{bmatrix}0\\0\\1\end{bmatrix} \quad (2)$$

$$\hat{S}^2 = 2\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \quad \hat{S}_z = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 0 & -1\end{bmatrix} \tag{3}$$

The eigenvalues of the $\hat{S}_+$ operator are $M_S = 0,-1$ for the new $|1\rangle$ and $|2\rangle$ states. The $\hat{S}_+$ matrix operator is shown in Equation 4.

$$\hat{S}_+|S,M_S\rangle = \sqrt{S(S+1) - M_S(M_S+1)}|S,M_{S+1}\rangle \tag{4}$$

Here $\hat{S}_+|1,0\rangle = \sqrt{2}|1,1\rangle$ and $\hat{S}_+|1,-1\rangle = \sqrt{2}|1,0\rangle$ will be found. According to this;

$$\hat{S}_+ = \sqrt{2}\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix} \tag{5}$$

it will be. Similarly, if the eigenvalues of the $\hat{S}_-$operator are $M_S = 1,0$ and $\hat{S}_-|1,1\rangle = \sqrt{2}|1,0\rangle$, $\hat{S}_-|1,0\rangle = \sqrt{2}|1,-1\rangle$ the matrix operator with the results of is as shown in Equation 6.

$$\hat{S}_- = \sqrt{2}\begin{bmatrix}0 & 0 & 0\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix} \tag{6}$$

The $\hat{S}_x$ and $\hat{S}_y$ processors of the $\hat{S}_+$ and $\hat{S}_-$ matrices are shown in Equation 7.

$$\hat{S}_x = \frac{1}{2}(\hat{S}_+ + \hat{S}_-) = \frac{1}{\sqrt{2}}\begin{bmatrix}0 & 1 & 0\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix}$$

$$\hat{S}_y = \frac{1}{2i}(\hat{S}_+ - \hat{S}_-) = \frac{1}{i\sqrt{2}}\begin{bmatrix}0 & 1 & 0\\-1 & 0 & 1\\0 & -1 & 0\end{bmatrix} \tag{7}$$

If similar operations are repeated for the spin 3/2 system, the spin states and matrices will be as in Eq.8.

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = |0\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \quad \left|\frac{3}{2},\frac{1}{2}\right\rangle = |1\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix},$$

$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = |2\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \quad \left|\frac{3}{2},-\frac{3}{2}\right\rangle = |3\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \tag{8}$$

In equations, eigenstates are given in accordance with quantum information processing theory.

$$\hat{S}^2 = \frac{15}{4}\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} \quad \hat{S}_z = \frac{1}{2}\begin{bmatrix}3 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & -1 & 0\\0 & 0 & 0 & -3\end{bmatrix} \tag{9}$$

$$\hat{S}_+ = \begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \hat{S}_- = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix} \qquad (10)$$

$$\hat{S}_x = \frac{1}{2}\begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix}, \hat{S}_y = \frac{1}{2i}\begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ -\sqrt{3} & 0 & 2 & 0 \\ 0 & -2 & 0 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & 0 \end{bmatrix} \qquad (11)$$

The spin matrices for the spin 3/2 system will be as in Equations 9, 10 and 11.

In this work, spin processors and spin matrices for spin systems have been found for the applicability of the theory and applications of electron paramagnetic resonance (EPR) spectroscopy in different fields to quantum information processing theory. In addition to these processors it is possible to find an algorithm 1 for the formation of spin operators for all spin systems (for spin ½ ve spin 3). Thus it can be used in theoretical studies and applications in quantum information theory. For other spin systems spin operators can be created.

4. Conclusion

There are many methods for designing quantum computers that are of interest to design computers with atomic dimensions. In the quantum information processing process, mathematical operations have been performed for other spin systems in order to make their applications spread to different spin systems and to be used in the symbolic representations of spin processors of other spin systems. In this study, spin 1 or qutrit system and spin matrices for spin 3/2 were derived from the algorithm. The spin eigenstates for spin 1 and 3/2 were calculated and $\hat{S}^2, \hat{S}_x, \hat{S}_y, \hat{S}_z, \hat{S}_+, \hat{S}_-$ matrix processors are shown. It is thought that the existence of spin spin processors will be useful for many aspects such as spin systems, rotation processors, etc. in quantum information processing.

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