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On fuzzy soft semi-pre-open sets and fuzzy soft semi-pre-continuous mappings

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Abstract

The main aim of this paper is to initiate and explore the properties of fuzzy soft semi-pre-open(closed) sets. We introduce and investigate fuzzy soft semi-pre-interior and fuzzy soft semi-pre-closure in fuzzy soft topological spaces. Moreover, we define and study the characterizations of fuzzy soft semi-pre-continuous and fuzzy soft semi-pre-open(closed) mappings in fuzzy soft topological spaces.

Keywords: Fuzzy soft sets, Fuzzy soft topology, Fuzzy soft semi-pre-open(closed), Fuzzy soft semi-pre-interior(closure), Fuzzy soft semi-pre-continuous, Fuzzy soft semi-pre-open(closed) mappings.

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1. Introduction

D. Molodtsov [31] presented the concept of soft set as a new mathematical tool to solve varieties of complicated problems having uncertainties in real life, economics, engineering and compute sciences, social and medical sciences etc. In [32], D. Molodtsov et. al discussed the applications of soft sets in different fields. P. K. Maji et. al [27-28] explored the fundamental concepts of soft set theory in detail and established the applications of soft sets in decision making problems.
F. Feng et.al [12] introduced the soft product in soft set theory. Moreover, they gave generalization of uni-int decision making scheme in [13]. The relation of soft sets in information systems, criteria of measuring the sound quality, normal parameter reduction and classification of natural texture have been studied in [35],[26],[39], [25] and [33]. M. Shabbir and M. Naz [36] defined and discussed the basic concepts of soft topological spaces. Later on, many researchers [1], [3-4],[9-11],[14],[17-20],[30],[41] explored different algebraic structures of soft topological spaces.
B. Chen [6-7] defined and discussed soft semi-open(closed) sets in soft topological spaces.
S. Hussain [21] continued to study the algebraic structures of soft semi-open(closed) sets.
L. A. Zadeh [40] initiated the concept of fuzzy soft sets and provided a natural base to

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handle and improve mathematically the fuzzy phenomenon found in different areas of knowledge. C. L. Chang [5] studied and discussed fuzzy topological spaces. B. Ahmad and A. Kharral [2] investigated fuzzy soft sets and established fuzzy soft functions on fuzzy soft classes. The work on fuzzy soft topology improved further in [15-16],[24],[29], [34],[37-38].

In 2015, S. Hussain [22], initiated and explored the fuzzy soft semi-open(closed) sets by combining fuzzy soft sets and soft semi-open sets. S. Hussain investigated the properties of fuzzy soft semi-interior(closure), fuzzy soft semi-exterior, fuzzy soft semi-boundary, fuzzy soft semi-continuous and fuzzy soft semi-open(closed) mapping. Recently in 2016, S. Hussain [23] introduced and examined the basic properties of fuzzy soft α-open(closed) sets, fuzzy soft pre-open(closed) sets, fuzzy soft regular-open(closed) sets and fuzzy soft neighborhood at fuzzy soft point. Moreover. S. Hussain analyzed the relationship between these notions.

2. Preliminaries

First we recall some definitions and results which will use in the sequel.

Definition 2.1[40]. A fuzzy set f on X is a mapping $f : X \to I = [0,1]$. The value $f(x)$ represents the degree of membership of $x \in X$ in the fuzzy set f, for $x \in X$.

Definition 2.2[31]. Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition. 2.3[27]. Let I^X denotes the set of all fuzzy sets on X and $A \subseteq X$. A pair (f, A) is called a fuzzy soft set over X, where $f : X \to I^X$ is a function. That is, for each $a \in A$, $f(a) = f_a : X \to I$, is a fuzzy set on X .

Definition 2.4[29]. For two fuzzy soft sets (f, A) and (g, B) over a common universe X, we say that (f, A) is a fuzzy soft subset of (g, B) if

(1) $A \subseteq B$ and

(2) for all $a \in A$, $f_a \leq g_a$; implies f_a is a fuzzy subset of g_a.

We denote it by $(f, A)\tilde{\leq}(g, B)$. (f, A) is said to be a fuzzy soft super set of (g, B), if (g, B) is a fuzzy soft subset of (f, A). We denote it by $(f, A)\tilde{\geq}(g, B)$.

Definition 2.5[29]. Two fuzzy soft sets (f, A) and (g, B) over a common universe X are said to be fuzzy soft equal, if (f, A) is a fuzzy soft subset of (g, B) and (g, B) is a fuzzy soft subset of (f, A).

Definition 2.6[29]. The union of two fuzzy soft sets of (f, A) and (g, B) over the common universe X is the fuzzy soft set (h, C), where $C = A \cup B$ and for all $c \in C$,

$$h_c = \begin{cases} f_c, & \text{if } c \in A - B \\ g_c, & \text{if } c \in B - A \\ f_c \vee g_c, & \text{if } c \in A \cap B \end{cases}$$

We write $(f, A)\tilde{\vee}(g, B) = (h, C)$.

Definition 2.7[29]. The intersection (h, C) of two fuzzy soft sets (f, A) and (g, B) over a common universe X , denoted $(f, A)\tilde{\wedge}(g, B)$, is defined as $C = A \cap B$, and $h_c = f_c \wedge g_c$, for all $c \in C$.

Definition 2.8[29]. The relative complement of a fuzzy soft set (f, A) is the fuzzy soft set (f^c, A), which is denoted by $(f, A)^c$, where $f^c : A \to F(U)$ is a fuzzy set-valued function. That is, for each $a \in A$, $f^c(A)$ is a fuzzy set in U, whose membership function is $f^c_a(x) = 1\tilde{-}f_a(x)$, for all $x \in U$. Here f^c_a is the membership function of $f^c(a)$.

Definition 2.9[16]. The difference (h, C) of two fuzzy soft sets (f, A) and (g, B) over X, denoted by $(f, A)\tilde{\setminus}(g, B)$, is defined as $(f, A)\tilde{\setminus}(g, B) = (f, A)\tilde{\wedge}(g, B)^c$.

For our convenience, we will use the notation f_A for fuzzy soft set instead of (f, A).
Definition 2.10[37]. Let τ be the collection of fuzzy soft sets over X, then τ is said to be a fuzzy soft topology on X if
(1) $\tilde{0}_A$, $\tilde{1_A}$ belong to τ.
(2) If $(f_A)_i \in \tau$, for all $i \in I$, then $\tilde{\bigvee}_{i\in I}(f_A)_i \in \tau$.
(3) $f_a, g_b \in \tau$ implies that $f_a\tilde{\bigwedge}g_b\tilde{\in}\tau$.
The triplet (X, τ, A) is called a fuzzy soft topological space over X. Every member of τ is called fuzzy soft open set. A fuzzy soft set is called fuzzy soft closed if and only if its complement is fuzzy soft open.
Definition 2.11[38]. Let (X, τ, A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. Then
(1) fuzzy soft interior of fuzzy soft set f_A over X is denoted by $(f_A)^\circ$ and is defined as the union of all fuzzy soft open sets contained in f_A. Thus $(f_A)^\circ$ is the largest fuzzy soft open set contained in f_A.
(2) fuzzy soft closure of f_A , denoted by $\overline{f_A}$ is the intersection of all fuzzy soft closed super sets of f_A. Clearly $\overline{f_A}$ is the smallest fuzzy soft closed set over X which contains f_A.
Definition 2.12 [22]. Let (X, τ, A) be a fuzzy soft topological space over X. A fuzzy soft set f_A is called fuzzy soft semi-open, if there exists a fuzzy soft open set g_A such that $g_A \leq f_A \leq \overline{g_A}$. The class of all fuzzy soft semi-open sets in X is denoted by FSSO(X). Note that every fuzzy soft open set is fuzzy soft semi-open but the converse is not true in general.
Definition 2.13[22]. A fuzzy soft set f_A in fuzzy soft topological space (X, τ, A) is fuzzy soft semi-closed if and only if its complement $(f_A)^c$ is fuzzy soft semi-open. The class of fuzzy soft semi-closed sets is denoted by FSSC(X).
Note that every fuzzy soft closed set is fuzzy soft semi-closed in fuzzy soft topological space (X, τ, A).
Proposition 2.14[22]. Let f_A be a fuzzy soft set in fuzzy soft topological space (X, τ, A). Then f_A is fuzzy soft semi-closed if and only if there exists a fuzzy soft closed set h_A such that $(h_A)^0\tilde{\leq}f_A\tilde{\leq}h_A$.
Definition 2.15[22]. Let f_A be a fuzzy soft set in fuzzy soft topological space (X, τ, A). The fuzzy soft semi-closure of f_A, denoted by $scl^{fs}(f_A)$ and is defined as the intersection of all fuzzy soft semi-closed supersets of f_A.
It is clear from the definition that $scl^{fs}(f_A)$ is the smallest fuzzy soft semi-closed set over X which contains f_A.
Definition 2.16[22]. Let f_A be a fuzzy soft set in fuzzy soft topological space (X, τ, A). The fuzzy soft semi-interior of f_A, denoted by $sint^{fs}(f_A)$ and is defined as the union of all fuzzy soft semi-open subsets of f_A.
It is clear from the definition that $sint^{fs}(f_A)$ is the largest fuzzy soft semi-open set over X contained in f_A.
Definition 2.17[23]. Let (X, τ, A) be a fuzzy soft topological space over X. Then a fuzzy soft set f_A over X is said to be a fuzzy soft pre-open, if $f_A\tilde{\leq}(\overline{f_A})^\circ$.
Definition 2.18[23]. Let (X, τ, A) be a fuzzy soft topological space over X. Then a fuzzy soft set f_A over X is said to be a fuzzy soft pre-closed, if $\overline{(f_A)^\circ}\tilde{\leq}f_A$.

3. Fuzzy soft semi-pre-open(closed) sets

Definition 3.1. Let (X, τ, A) be a fuzzy soft topological space over X, where X is a nonempty set and τ is a family of fuzzy soft sets. Then a fuzzy soft set f_A is said to be a fuzzy soft semi-pre-open, if there exists a fuzzy soft pre-open set g_A such that $g_A \tilde{\leq} f_A \tilde{\leq} \overline{(g_A)}$.
Definition 3.2. Let (X, τ, A) be a fuzzy soft topological space over X, where X is a nonempty set and τ is a family of fuzzy soft sets. Then a fuzzy soft set f_A is said to be a fuzzy soft semi-pre-closed, if there exists a fuzzy soft pre-closed set g_A such that $(g_A)^\circ \tilde{\leq} f_A \tilde{\leq} g_A$.
Note that the fuzzy soft set f_A is fuzzy soft semi-pre-open if and only if f_A^c is fuzzy soft semi-pre-closed.
Remark 3.3. It is clear that any fuzzy soft semi-open as well as fuzzy soft pre-open set is a fuzzy soft semi-pre-open set.
The following example shows that the converse of above remark is not true in general. For this we consider the Example 3.3[22] as:
Example 3.4. Let $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$ and $\tau = \{\tilde{0}, \tilde{1}, (f_A)_1, (f_A)_2, (f_A)_3, (f_A)_4\}$ where $(f_A)_1$, $(f_A)_2$, $(f_A)_3$, $(f_A)_4$ are fuzzy soft sets over X, defined as follows
$f_1(e_1)(h_1) = 0.5$, $f_1(e_1)(h_2) = 0.3$, $f_1(e_1)(h_3) = 0.2$,
$f_1(e_2)(h_1) = 0.3$, $f_1(e_2)(h_2) = 0.5$, $f_1(e_2)(h_3) = 0.2$,
$f_2(e_1)(h_1) = 1$, $f_2(e_1)(h_2) = 0$, $f_2(e_1)(h_3) = 0.5$,
$f_2(e_2)(h_1) = 0.5$, $f_2(e_2)(h_2) = 0.3$, $f_2(e_2)(h_3) = 1$,
$f_3(e_1)(h_1) = 0.5$, $f_3(e_1)(h_2) = 0$, $f_3(e_1)(h_3) = 0.2$,
$f_3(e_2)(h_1) = 0.3$, $f_3(e_2)(h_2) = 0.3$, $f_3(e_2)(h_3) = 0.2$,
$f_4(e_1)(h_1) = 1$, $f_4(e_1)(h_2) = 0.3$, $f_4(e_1)(h_3) = 0.5$,
$f_4(e_2)(h_1) = 0.5$, $f_4(e_2)(h_2) = 0.5$, $f_4(e_2)(h_3) = 1$.
Then τ is a fuzzy soft topology on X and hence (X, τ, A) is a fuzzy soft topological space over X.
Note that the fuzzy soft closed sets are
$\{\{h_{0.5}, h_{0.7}, h_{0.8}\}, \{h_{0.7}, h_{0.5}, h_{0.8}\}\}$, $\{\{h_0, h_1, h_{0.5}\}, \{h_{0.5}, h_{0.7}, h_0\}\}$,
$\{\{h_{0.5}, h_1, h_{0.8}\}, \{h_{0.7}, h_{0.7}, h_{0.8}\}\}$, $\{\{h_0, h_{0.7}, h_{0.5}\}, \{h_{0.5}, h_{0.5}, h_0\}\}$, $\tilde{1}$ and $\tilde{0}$.
Let us take fuzzy soft set f_A over X defined by
$f(e_1)(h_1) = 0.3$, $f(e_1)(h_2) = 0.2$, $f(e_1)(h_3) = 0.1$,
$f(e_2)(h_1) = 0.2$, $f(e_2)(h_2) = 0.4$, $f(e_2)(h_3) = 0.1$.
That is, $f_A = \{\{h_{0.3}, h_{0.2}, h_{0.1}\}, \{h_{0.2}, h_{0.4}, h_{0.1}\}\}$. Then there exists fuzzy soft set g_A over X defined by
$g(e_1)(h_1) = 0.2$, $g(e_1)(h_2) = 0.1$, $g(e_1)(h_3) = 0.1$,
$g(e_2)(h_1) = 0.2$, $g(e_2)(h_2) = 0.3$, $g(e_2)(h_3) = 0.1$.
That is, $g_A = \{\{h_{0.2}, h_{0.1}, h_{0.1}\}, \{h_{0.2}, h_{0.3}, h_{0.1}\}\}$.
Since $g_A \tilde{\leq} (\overline{g_A})^\circ \tilde{=} \{\{h_{0.5}, h_{0.3}, h_{0.2}\}, \{h_{0.3}, h_{0.5}, h_{0.2}\}\}$, then g_A is fuzzy soft pre-open set over X. Moreover, calculations show that
$g_A \tilde{\leq} f_A \tilde{\leq} \overline{g_A} \tilde{=} \{\{h_{0.5}, h_{0.7}, h_{0.8}\}, \{h_{0.7}, h_{0.5}, h_{0.8}\}\}$. This implies that f_A is fuzzy soft semi-pre-open set. But f_A is not fuzzy soft semi-open set, since there does not exist any fuzzy soft open set h_A such that $h_A \tilde{\leq} f_A \tilde{\leq} \overline{h_A}$. Moreover, f_A is not a fuzzy soft pre-open set.
Lemma 3.5. Let (X, τ, A) be fuzzy soft topological space over X and f_A be a fuzzy soft set over X. Then $f_A \tilde{\leq} \overline{(f_A)^\circ}$ if and only if $\overline{f_A} \tilde{=} \overline{(f_A)^\circ}$.
Proof. $f_A \tilde{\leq} \overline{(f_A)^\circ}$ follows that $\overline{f_A} \tilde{\leq} \overline{(\overline{(f_A)^\circ})} \tilde{=} \overline{(f_A)^\circ} \tilde{\leq} \overline{f_A}$. Therefore, $\overline{f_A} \tilde{=} \overline{(f_A)^\circ}$. The other inclusion follows similarly.

Using Proposition 3.4[22] and above Lemma 3.5, we have
Theorem 3.6. Let (X, τ, A) be a fuzzy soft topological space over X and f_A be a fuzzy

soft set over X. Then the following statements are equivalent.
(1) f_A is fuzzy soft semi-open.
(2) $f_A\tilde{\leq}\overline{(f_A)^\circ}$.
(3) $\overline{f_A}\tilde{=}\overline{(f_A)^\circ}$.

The proof of the following theorem follows directly by taking complements in above Theorem 3.6.
Theorem 3.7. Let (X,τ,A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X.Then the following statements are equivalent.
(1) f_A is fuzzy soft semi-closed.
(2) $(\overline{f_A})^\circ\tilde{\leq}f_A$.
(3) $(\overline{f_A})^\circ\tilde{=}(f_A)^\circ$.
Theorem 3.8. Let (X,τ,A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. Then the following hold.
(1) If f_A is fuzzy soft semi-pre-open, then $f_A\tilde{\leq}\overline{(\overline{f_A})^\circ}$.
(2) If f_A is fuzzy soft semi-pre-closed, then $(\overline{(f_A)^\circ})^\circ\tilde{\leq}f_A$.
Proof. (1) f_A is fuzzy soft semi-pre-open implies that there exists a fuzzy soft pre-open set g_A such that $g_A\tilde{\leq}f_A\tilde{\leq}\overline{(g_A)}$. This follows that $\overline{f_A}\tilde{=}\overline{g_A}$. Also g_A is fuzzy soft pre-open set implies that $f_A\tilde{\leq}\overline{g_A}\tilde{\leq}\overline{(\overline{g_A})^\circ}\tilde{\leq}\overline{(\overline{f_A})^\circ}$. Therefore, $f_A\tilde{\leq}\overline{(\overline{f_A})^\circ}$.
(2) This can be proved in the same way as (1).

The following theorem directly follows from Theorems 3.6, 3.7 and 3.8.
Theorem 3.9. Let (X,τ,A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. If f_A is fuzzy soft semi-pre-open(closed) and fuzzy soft open(closed), then f_A is fuzzy soft semi-open(closed).

The following lemma is easy to proof.
Lemma 3.10. Let (X,τ,A) be a fuzzy soft topological space over X and f_{A_i} be a family of fuzzy soft set over X. Then $\bigvee\overline{F_{A_i}}\tilde{\leq}\overline{\bigvee F_{A_i}}$. Moreover, for finite case, $\bigvee\overline{F_{A_i}}\tilde{=}\overline{\bigvee F_{A_i}}$ and $\bigvee(F_{A_i})^\circ\tilde{=}(\bigvee F_{A_i})^\circ$.
Theorem 3.11. Let (X,τ,A) be a fuzzy soft topological space over X. Then
(1) Arbitrary union of fuzzy soft semi-pre-open sets is a fuzzy soft semi-pre-open set.
(2) Arbitrary intersection of fuzzy soft semi-pre-closed sets is a fuzzy soft semi-pre-closed set.
Proof. Suppose that f_{A_i} be a family of fuzzy soft semi-pre-open set in fuzzy soft topological space over X. Then for each f_{A_i}, there exists a fuzzy soft pre-open set g_{A_i} such that $g_{A_i}\tilde{\leq}f_{A_i}\tilde{\leq}\overline{g_{A_i}}$. Using Lemma 3.10 and Theorem 3.10[23], we have
$\bigvee g_{A_i}\tilde{\leq}\bigvee f_{A_i}\tilde{\leq}\bigvee\overline{g_{A_i}}\tilde{\leq}\overline{(\bigvee g_{A_i})}$, where $\bigvee g_{A_i}$ is fuzzy soft pre-open set.
(2) This follows directly by taking complements in (1).
Theorem 3.12. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then
(1) $scl^{fs}(f_A)\tilde{=}f_A\tilde{\bigvee}(\overline{f_A})^\circ$.
(2) $sint^{fs}(f_A)\tilde{=}f_A\tilde{\bigwedge}\overline{((f_A)^\circ)}$.
(3)$sint^{fs}(scl^{fs}(f_A))\tilde{=}scl^{fs}(f_A)\tilde{\bigwedge}\overline{((\overline{f_A})^\circ)}$.
(4) $scl^{fs}(sint^{fs}(f_A))\tilde{=}sint^{fs}(f_A)\tilde{\bigvee}(\overline{((f_A)^\circ)})^\circ$.
Proof. (1)and (2) directly follows from the definitions of fuzzy soft interior(closure) and fuzzy soft semi-interior(closure).
(3) Using (2), we have, $sint^{fs}(scl^{fs}(f_A))\tilde{=}scl^{fs}(f_A)\tilde{\bigwedge}\overline{((scl^{fs}(f_A))^\circ)}$
$\tilde{=}scl^{fs}(f_A)\tilde{\bigwedge}\overline{((f_A\tilde{\bigvee}(\overline{f_A})^\circ)^\circ)}\tilde{\geq}scl^{fs}(f_A)\tilde{\bigwedge}\overline{((f_A)^\circ\tilde{\bigvee}(\overline{f_A})^\circ)}\tilde{=}scl^{fs}(f_A)\tilde{\bigwedge}\overline{(((\overline{f_A}))^\circ)}$.

Also, $sint^{fs}(scl^{fs}(f_A))\tilde{=}scl^{fs}(f_A)\tilde{\bigwedge}\overline{((scl^{fs}(f_A))^\circ)}\tilde{\leq}scl^{fs}(f_A)\tilde{\bigwedge}\overline{((\overline{f_A})^\circ)}$. This follows (3).
(4) Using (1), we have, $scl^{fs}(sint^{fs}(f_A))\tilde{=}sint^{fs}(f_A)\tilde{\bigvee}(\overline{(sint^{fs}(f_A))})^\circ\tilde{=}$
$sint^{fs}(f_A)\tilde{\bigvee}\overline{((f_A\tilde{\bigwedge}\overline{((f_A)^\circ)})^\circ)}\tilde{\leq}sint^{fs}(f_A)\tilde{\bigvee}(\overline{f_A}\tilde{\bigwedge}\overline{((f_A)^\circ)})^\circ\tilde{=}sint^{fs}(f_A)\tilde{\bigvee}(\overline{((f_A)^\circ)})^\circ$.
Also, $scl^{fs}(sint^{fs}(f_A))\tilde{=}sint^{fs}(f_A)\tilde{\bigvee}(\overline{(sint^{fs}(f_A))})^\circ\tilde{\geq}sint^{fs}(f_A)\tilde{\bigvee}(\overline{((f_A)^\circ)})^\circ$. This proves (4). Hence the proof.

Theorem 3.13. Let f_A and g_A are fuzzy soft sets in fuzzy soft topological space (X, τ, A) over X. If f_A is fuzzy soft semi-pre-open and $f_A\tilde{\leq}g_A\tilde{\leq}\overline{f_A}$, then g_A is a fuzzy soft semi-pre-open set.

Proof. f_A is fuzzy soft semi-pre-open implies that there exists a fuzzy soft pre-open set k_A in (X, τ, A) such that $k_A\tilde{\leq}f_A\tilde{\leq}\overline{k_A}$. As $f_A\tilde{\leq}g_A$, $k_A\tilde{\leq}f_A\tilde{\leq}g_A$ follows that $k_A\tilde{\leq}g_A$. Moreover, $\overline{f_A}\tilde{\leq}\overline{(\overline{k_A})}\tilde{=}\overline{k_A}$ implies that $g_A\tilde{\leq}\overline{k_A}$. Therefore, $k_A\tilde{\leq}g_A\tilde{\leq}\overline{k_A}$. Hence g_A is fuzzy soft semi-pre-open.

Definition 3.14[23]. A fuzzy soft set f_A is said to be a fuzzy soft point in fuzzy soft topological space (X, τ, A) denoted by $e(f_A)$, if for the element $e\tilde{\in}A$, $f(e)\tilde{\neq}\tilde{0}$ and $f(e^c)\tilde{=}\tilde{0}$, for all $e^c\in A\backslash\{e\}$.

Definition 3.15[23]. The fuzzy soft point $e(f_A)$ is said to be in the fuzzy soft set g_A, denoted by $e(f_A)\tilde{\in}g_A$, if for the element $e \in A$, $f(e) \leq g(e)$. Clearly, every fuzzy soft set g_A can be expressed as the union of all fuzzy soft points which belong to g_A.

Definition 3.16[23]. The fuzzy soft point $(e(f_A))^c$ is called the complement of a fuzzy soft point $e(f_A)$, if for all $e^c \in A - \{e\}$, $f(e^c)\tilde{=}\tilde{0}$ and $f(e)\tilde{\neq}\tilde{0}$, for any element $e\tilde{\in}A$.

Example 3.17[23]. Let $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$ and consider the fuzzy soft set $(f_A)_1$ over X is defined as follows:
$f_1(e_1)(h_1) = 0.5,\ f_1(e_1)(h_2) = 0.3,\ f_1(e_1)(h_3) = 0.2,$
$f_1(e_2)(h_1) = 0.3,\ f_1(e_2)(h_2) = 0.5,\ f_1(e_2)(h_3) = 0.2,$
That is $(f_A)_1 = \{\{h_{0.5}, h_{0.3}, h_{0.2}\}, \{h_{0.3}, h_{0.5}, h_{0.2}\}\}$. Then
$e((f_A)_1)\tilde{=}\{e_1 = \{h_{0.5}, h_{0.3}, h_{0.2}\}\}$ is a fuzzy soft point. Moreover the complement $(e((f_A)_1)^c$ of $e((f_A)_1)$ is $(e((f_A)_1)^c\tilde{=}\{e_1 = \{h_{0.5}, h_{0.7}, h_{0.8}\}\}$.

Remark 3.18[23]. Note that, if the soft point $e(f_A)$ is in the soft set g_A, then it is not necessary that the complement $(e(f_A))^c$ is in the soft set $(g_A)^c$.

The following example verify the above remark.

Example 3.19[23]. Let $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$. Then it is to be noted that the fuzzy soft point $e(f_A)\tilde{=}\{e_1 = \{h_{0.2}, h_{0.1}, h_{0.4}\}\}$ is contained in the fuzzy soft set $g_A = \{\{h_{0.5}, h_{0.3}, h_{0.6}\}, \{h_{0.3}, h_{0.5}, h_{0.2}\}\}$. Now we can see that the complement of fuzzy soft point $(e(f_A))^c\tilde{=}\{e_1 = \{h_{0.8}, h_{0.9}, h_{0.6}\}\}$ is not contained in the complement of fuzzy soft set $(g_A)^c = \{\{h_{0.5}, h_{0.7}, h_{0.4}\}, \{h_{0.7}, h_{0.5}, h_{0.8}\}\}$.

Theorem 3.20. Let f_A be a fuzzy soft set in fuzzy soft topological space (X, τ, A) over X. Then the following statements are equivalent:
(1) f_A is fuzzy soft semi-pre-open.
(2) For every fuzzy soft point $e(g_A)$ in f_A, there exists a fuzzy soft semi-pre-open set h_A such that $e(g_A)\tilde{\leq}h_A\tilde{\leq}f_A$.

Proof. (1) $\Rightarrow$ (2) Suppose that f_A is fuzzy soft semi-pre-open. Then for every fuzzy soft point $e(g_A)$ in f_A, take $h_A\tilde{=}f_A$. We have (2).
(2) $\Rightarrow$ (1) Consider $f_A\tilde{=}\tilde{\bigvee}_{e(g_A)\tilde{\in}f_A}\{e(g_A)\}\tilde{\leq}\tilde{\bigvee}_{e(g_A)\tilde{\in}f_A}h_A\tilde{\leq}f_A$. Hence the proof.

Theorem 3.21. Let h_A and k_A are fuzzy soft sets in fuzzy soft topological space (X, τ, A) over X. If h_A is fuzzy soft semi-pre-closed with $(h_A)^\circ\tilde{\leq}k_A\tilde{\leq}h_A$, then k_A is fuzzy soft semi-pre-closed.

Proof. This directly follows from Theorem 3.13.

Definition 3.22. Let k_A be fuzzy soft set in fuzzy soft topological space (X, τ, A) over X and $e(f_A)$ be a fuzzy soft point. If there exists a fuzzy soft semi-pre-open set g_A with

$e(f_A)\tilde{\in}g_A\tilde{\leq}k_A$, then k_A is called fuzzy soft semi-pre-neighborhood(nbd) of a fuzzy soft point $e(f_A)$.

The proof of the following theorem directly follows from the definitions of fuzzy soft point, fuzzy soft semi-pre-open and Theorem 3.11.

Theorem 3.23. Let $e(f_A)$ be a fuzzy soft point and g_A be fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then g_A is fuzzy soft semi-pre-open if and only if g_A is fuzzy soft semi-pre-nbd of $e(f_A)$, for every fuzzy soft point $e(f_A)\tilde{\in}g_A$.

Definition 3.24. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then

(1) fuzzy soft semi-pre-interior of fuzzy soft set f_A denoted by F^spint^s and is defined as $F^spint^s(f_A)\tilde{=}Sup\{g_A : g_A$ is fuzzy soft semi-pre-open and $g_A\tilde{\leq}f_A\}$.

(2) fuzzy soft semi-pre-closure of fuzzy soft set f_A denoted by F^spcl^s and is defined as $F^spcl^s(f_A)\tilde{=}Inf\{g_A : g_A$ is fuzzy soft semi-pre-closed and $f_A\tilde{\leq}g_A\}$.

Theorem 3.25. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then

(1) $F^spcl^s(f_A)^c\tilde{=}(F^spint^s(f_A))^c$.

(2) $F^spint^s(f_A)^c\tilde{=}(F^spcl^s(f_A))^c$.

Proof. The proof follows form the Definition 3.24 and rule of complements.

Theorem 3.26. Let $e(f_A)$ be a fuzzy soft point and g_A be fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then $e(f_A)\tilde{\in}F^spcl^s(g_A)$ if and only if $h_A\tilde{\leq}g_A$, for every fuzzy soft semi-pre-nbd h_A.

Proof. $(\Rightarrow)$ Contrarily suppose that there exists fuzzy soft semi-pre-nbd h_A of $e(f_A)$ such that $h_A\tilde{\nleq}g_A$. Then there exists a fuzzy soft semi-pre-open set k_A such that $e(f_A)\tilde{\in}k_A\tilde{\leq}h_A$ and $k_A\tilde{\nleq}g_A$. Now k_A^c is fuzzy soft semi-pre-closed set with $g_A\tilde{\leq}k_A$ implies that $F^spcl^s(g_A)\tilde{\leq}k_A^c$. Also $e(f_A)\tilde{\in}k_A^c$ follows that $e(f_A)\tilde{\notin}F^spcl^s(g_A)$. A contradiction.

$(\Leftarrow)$ Contrarily suppose that $e(f_A)\tilde{\notin}F^spcl^s(g_A)$. Then there exists a fuzzy soft semi-pre-closed set l_A such that $e(f_A)\tilde{\notin}l_A$ and $g_A\tilde{\leq}l_A$. Therefore, l_A^c is fuzzy soft semi-pre-open set such that $e(f_A)\tilde{\in}l_A^c$ with $g_A\tilde{\nleq}l_A^c$. A contradiction. This completes the proof.

Theorem 3.27. Let f_A be fuzzy soft set and g_A be a fuzzy soft semi-pre-open set in fuzzy soft topological space (X,τ,A) over X. If $f_A\tilde{\nleq}g_A$, then $F^spcl^s(f_A)\tilde{\nleq}g_A$.

Proof. Contrarily suppose that $F^spcl^s(f_A)\tilde{\leq}g_A$. Then there exists a fuzzy soft point $e(h_A)$ in (X,τ,A) such that $F^spcl^s(f_{A_{(e(h_A))}})\tilde{\bigvee}(g_{A_{(e(h_A))}})\tilde{\geq}\tilde{1}_A$. Take $F^spcl^s(f_{A_{(e(h_A))}})\tilde{=}e(k_A)$. Then g_A is a fuzzy soft semi-pre-nbd of $e(h_A)$ with respect to $e(k_A)$ such that $f_A\tilde{\nleq}g_A$. Therefore, $e(h_A)\tilde{\notin}F^spcl^s(f_A)$. A contradiction. Hence the proof.

Theorem 3.28. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then $F^spcl^s(f_A)\tilde{=}f_A\tilde{\bigvee}(\overline{((f_A)^\circ)})^\circ$.

Proof. Note that $\{\overline{((f_A\tilde{\bigvee}(\overline{((f_A)^\circ)})^\circ)^\circ)}\}^\circ\tilde{\leq}\{\overline{(f_A\tilde{\bigvee}(\overline{((f_A)^\circ)})^\circ)}\}^\circ\tilde{\leq}\{\overline{((f_A)^\circ\tilde{\bigvee}\overline{((f_A)^\circ)})}\}^\circ$ $\tilde{=}\{\overline{((f_A)^\circ)}\}^\circ\tilde{=}(\overline{((f_A)^\circ)})^\circ\tilde{\leq}f_A\tilde{\leq}(\overline{((f_A)^\circ)})^\circ$. Therefore, $f_A\tilde{\bigvee}(\overline{((f_A)^\circ)})^\circ$ is fuzzy soft semi-pre-closed follows that $F^spcl^s(f_A)\tilde{\leq}f_A\tilde{\bigvee}(\overline{((f_A)^\circ)})^\circ$.

Moreover, $F^spcl^s(f_A)$ is fuzzy soft semi-pre-closed implies that $(\overline{((f_A)^\circ)})^\circ\tilde{\leq}(\overline{((F^spcl^s(f_A))^\circ)})^\circ\tilde{\leq}F^spcl^s(f_A)$. Hence $f_A\tilde{\bigvee}(\overline{((f_A)^\circ)})^\circ\tilde{\leq}F^spcl^s(f_A)$. This completes the proof.

Theorem 3.29. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then $F^spint^s(f_A)\tilde{=}f_A\tilde{\bigwedge}\overline{((\overline{f_A})^\circ)}$.

Proof. Note that $f_A\tilde{\bigwedge}\overline{((\overline{f_A})^\circ)}\tilde{\leq}\overline{((\overline{f_A})^\circ)}\tilde{=}(\overline{\{\overline{(f_A)}\tilde{\bigwedge}(\overline{(f_A)})^\circ\}^\circ})$

$\tilde{\le}(\overline{\{\overline{(f_A\tilde{\bigwedge}(\overline{(f_A)})^\circ)}\}^\circ})\tilde{\le}(\overline{(\{f_A\tilde{\bigwedge}\overline{((\overline{(f_A)})^\circ)}\})})^\circ$. Therefore, $f_A\tilde{\bigwedge}(\overline{\{\overline{(f_A)}\}^\circ})$ is fuzzy soft semi-pre-open set. Hence $f_A\tilde{\bigwedge}(\overline{\{\overline{(f_A)}\}^\circ})\tilde{\in}F^s pint^s(f_A)$.
Also, since $F^s pint^s(f_A)$ is fuzzy soft semi-pre-open, then
$F^s pint^s(f_A)\tilde{\le}(\overline{\{\overline{(F^s pint^s(f_A))}\}^\circ})\tilde{\le}(\overline{((\overline{(f_A)}))^\circ})$. Thus, $F^s pint^s(f_A)\tilde{\le}f_A\tilde{\bigwedge}(\overline{\{((\overline{f_A}))^\circ\}^\circ})$. Hence the proof.

Theorem 3.30. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then $F^s pint^s(F^s pcl^s(f_A))\tilde{=}F^s pcl^s(F^s pint^s(f_A))$.

Proof. Using Theorems 3.28 and 3.29, we have
$F^s pint^s(F^s pcl^s(f_A))\tilde{=}F^s pcl^s(f_A)\tilde{\bigwedge}(\overline{((\overline{F^s pcl^s(f_A)}))^\circ})\tilde{=}(f_A\tilde{\bigvee}(\overline{((f_A)^\circ)})^\circ)\tilde{\bigwedge}(\overline{\{\overline{f_A}\}^\circ})$
$\tilde{=}(f_A\tilde{\bigwedge}(\overline{\{\overline{f_A}\}^\circ}))\tilde{\bigvee}\{\overline{((f_A)^\circ)}\}^\circ\tilde{=}F^s pint^s(f_A)\tilde{\bigvee}\{\overline{(\{F^s pint^s(f_A)\}^\circ)}\}^\circ\tilde{=}F^s pcl^s(F^s pint^s(f_A))$.
This completes the proof.

Corollary 3.31. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then
(1) $f_A\tilde{\bigvee}F^s pint^s(F^s pcl^s(f_A))\tilde{=}F^s pcl^s(f_A)$.
(2) $f_A\tilde{\bigwedge}F^s pint^s(F^s pcl^s(f_A))\tilde{=}F^s pint^s(f_A)$.

Theorem 3.32. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then, $scl^{fs}(sint^{fs}(f_A))\tilde{\le}F^s pint^s(F^s pcl^s(f_A))\tilde{\le}sint^{fs}(scl^{fs}(f_A))$.

Proof. Using Theorems 3.12(3)(4) and 3.30, we get
$scl^{fs}(sint^{fs}(f_A))\tilde{=}sint^{fs}(f_A)\tilde{\bigvee}\{\overline{((f_A)^\circ)}\}^\circ\tilde{=}(f_A\tilde{\bigwedge}\overline{((f_A)^\circ)})\tilde{\bigvee}\{\overline{((f_A)^\circ)}\}^\circ\tilde{=}$
$(f_A\tilde{\bigvee}\{\overline{((f_A)^\circ)}\}^\circ)\tilde{\bigwedge}\overline{((f_A)^\circ)}\tilde{\le}(f_A\tilde{\bigvee}\{\overline{((f_A)^\circ)}\}^\circ)\tilde{\bigwedge}(\overline{(\overline{f_A})^\circ})\tilde{=}F^s pint^s(F^s pcl^s(f_A))$
$\tilde{\le}(f_A\tilde{\bigvee}\{\overline{f_A}\}^\circ)\tilde{\bigwedge}(\overline{(\overline{f_A})^\circ})\tilde{=}sint^{fs}(scl^{fs}(f_A))$. Hence the proof.

The following theorem gives us the useful characterization of fuzzy soft semi-pre-open sets.

Theorem 3.33. Let f_A be a fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then the following statements are equivalent.
(1) f_A is fuzzy soft semi-pre-open.
(2) $f_A\tilde{\le}F^s pint^s(F^s pcl^s(f_A))$.
(3) $f_A\tilde{\le}sint^{fs}(scl^{fs}(f_A))$.

Proof. (1) $\Rightarrow$ (2) Suppose that f_A be a fuzzy soft semi-pre-open set. Then $f_A\tilde{=}F^s pint^s(f_A)\tilde{\le}F^s pint^s(F^s pcl^s(f_A))$.
(2) $\Rightarrow$ (3) This follows directly from Theorem 3.32.
(3) $\Rightarrow$ (1) Consider $f_A\tilde{\le}sint^{fs}(scl^{fs}(f_A))\tilde{=}scl^{fs}(f_A)\tilde{\bigwedge}(\overline{(\overline{f_A})^\circ})$. This follows that $f_A\tilde{\le}(\overline{(\overline{f_A})^\circ})$. Hence f_A is fuzzy soft semi-pre-open. This completes the proof.

The following theorem can be easily verified by using definitions.

Theorem 3.34 Let f_A and g_A are fuzzy soft set in fuzzy soft topological space (X,τ,A) over X. Then
(1) $F^s pcl^s(f_A)\tilde{\le}\overline{f_A}$ and $(f_A)^\circ\tilde{\le}F^s pint^s(f_A)$.
(2) $F^s pcl^s(f_A)(F^s pint^s(f_A))$ is fuzzy soft semi-pre-closed(open).
(3) f_A is fuzzy soft semi-pre-closed(open) if and only if $F^s pcl^s(f_A)\tilde{=}f_A(F^s pint^s(f_A)\tilde{=}f_A)$.
(4) $f_A\tilde{\le}g_A$ implies that $F^s pcl^s(f_A)\tilde{\le}F^s pcl^s(g_A)$ and $F^s pint^s(f_A)\tilde{\le}F^s pint^s(g_A)$.

4. Fuzzy soft semi-pre-continuous functions

In this section, we define and explore the characterizations of fuzzy soft semi-pre-continuous and fuzzy soft semi-pre-open(closed) mappings in a fuzzy soft topological spaces. First we recall some definitions.

Definition 4.1[2]. Let $F(X,A)$ and $F(Y,B)$ be families of fuzzy soft sets. $u: X \to Y$ and $p: A \to B$ are mappings. Then image and inverse image of a function f_{pu} : $F(X,A) \to F(Y,B)$ is defined as :
(1) Let f_A be a fuzzy soft set in $F(X,A)$. The image of f_A under f_{pu} , written as $f_{pu}(f_A)$, is a fuzzy soft set in $F(Y,B)$ such that for $\beta \in p(A) \subseteq B$ and $y \in Y$,

$$f_{pu}(f_A)(\beta)(y) = \begin{cases} \bigvee_{x\in u^{-1}(y)}(\bigvee_{\alpha\in p^{-1}(\beta)\cap A}(f_A(\alpha))(y), & u^{-1}(y) \neq \phi, p^{-1}(\beta)\cap A \neq \phi \\ 0, & \text{otherwise} \end{cases},$$

for all $y \in B$. $f_{pu}(f_A)$ is known as a fuzzy soft image of a fuzzy soft set f_A.
(2) Let g_B be a fuzzy soft set in $F(Y,B)$. Then the fuzzy soft inverse image of g_B under f_{pu} , written as $f_{pu}^{-1}(g_B)$, is a fuzzy soft set in $F(X,A)$ such that

$$f_{pu}^{-1}(g_B)(\alpha)(x) = \begin{cases} g_B(p(\alpha))(u(x)), & p(\alpha) \in B \\ 0, & \text{otherwise} \end{cases},$$

for all $x \in A$. $f_{pu}^{-1}(g_B)$ is known as a fuzzy soft inverse image of a fuzzy soft set g_B.

The fuzzy soft function f_{pu} is called fuzzy soft surjective, if p and u are surjective. The fuzzy soft function f_{pu} is called fuzzy soft injective, if p and u are injective.
Definition 4.2[23]. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces.
(1) A fuzzy soft mapping $f_{pu} : F(X,A) \to F(Y,B)$ is said to be fuzzy soft semi-continuous, if for any fuzzy soft open set g_B in (Y,τ_2,B), $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-open in (X,τ_1,A).
(2) A fuzzy soft mapping $f_{pu} : F(X,A) \to F(Y,B)$ is said to be a fuzzy soft semi-open, if for any fuzzy soft open set f_A in (X,τ_1,A), $f_{pu}(f_A)$ is fuzzy soft semi-open in (Y,τ_2,B).

Now we define:
Definition 4.3. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces and $f_{pu} : F(X,A) \to F(Y,B)$ is a fuzzy soft mapping. Then $f_{pu} : F(X,A) \to F(Y,B)$ is said to be fuzzy soft semi-pre-continuous, if for every fuzzy soft open set g_B in (Y,τ_2,B), $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-pre-open in (X,τ_1,A).
Remark 4.4. Every fuzzy soft semi-continuous function is fuzzy soft semi-pre-continuous, since every fuzzy soft semi-open set is fuzzy soft semi-pre-open in fuzzy soft topological space. But the converse is not true in general.

The following theorem directly follows from Definition 4.3.
Theorem 4.5. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces and $f_{pu} : F(X,A) \to F(Y,B)$ is a fuzzy soft mapping. Then the following statements are equivalent:
(1) f_{pu} is fuzzy soft semi-pre-continuous.
(2) For every fuzzy soft closed set g_B in (Y,τ_2,B), $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-pre-closed in (X,τ_1,A).
Theorem 4.6. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces and $f_{pu} : F(X,A) \to F(Y,B)$ is a fuzzy soft mapping. Then the following statements are equivalent:
(1) f_{pu} is fuzzy soft semi-pre-continuous.
(2) For every fuzzy soft point $e(f_A)$ in (X,τ_1,A) and every fuzzy soft open set g_A with $f_{pu}(e(f_A))\tilde{\in} g_A$, there is fuzzy soft semi-pre-open set h_A with $e(f_A)\tilde{\in} h_A$ and $f_{pu}(h_A)\tilde{\leq} g_A$.
Proof. (1) $\Rightarrow$ (2) Suppose that $e(f_A)$ be a fuzzy soft point in (X,τ_1,A) and g_A be fuzzy

soft open in (Y,τ_2,B) such that $f_{pu}(e(f_A))\tilde{\in}\, g_A$. Take $h_A\tilde{=}f_{pu}^{-1}(g_A)$. Then by (1), h_A is fuzzy soft semi-pre-open in (X,τ_1,A) with $e(f_A)\tilde{\in}h_A$ and $f_{pu}(h_A)\tilde{\leq}g_A$.
(2) $\Rightarrow$ (1) Suppose that g_A be fuzzy soft open set in (Y,τ_2,B) and $e(f_A)\tilde{\in}f_{pu}^{-1}(g_A)$. Then $f_{pu}(e(f_A))\tilde{\in}g_A$. By (2), there is a fuzzy soft semi-pre-open set h_A in (X,τ_1,A) such that $e(f_A)\tilde{\in}h_A$ and $f_{pu}(h_A)\tilde{\leq}g_A$. This follows that $e(f_A)\tilde{\in}h_A\tilde{\leq}f_{pu}^{-1}(g_A)$. Therefore, by Theorem 3.20, $f_{pu}^{-1}(g_A)$ is fuzzy soft semi-pre-open. Hence f_{pu} is fuzzy soft semi-pre-continuous. This completes the proof.
Definition 4.7[21]. Let f_A be fuzzy soft set in fuzzy soft topological space (X,τ,A) over X and $e(f_A)$ be fuzzy soft point. If there exists a fuzzy soft open set g_A with $e(f_A)\tilde{\in}g_A\tilde{\leq}f_A$, then f_A is called fuzzy soft neighborhood(nbd) of a fuzzy soft point $e(f_A)$.
Theorem 4.8. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces and $f_{pu}: F(X,A)\to F(Y,B)$ is a fuzzy soft mapping. Then the following statements are equivalent:
(1) f_{pu} is fuzzy soft semi-pre-continuous.
(2) For every fuzzy soft point $e(f_A)$ in (X,τ_1,A) and every fuzzy soft nbd g_A of $f_{pu}(e(f_A))$, $f_{pu}^{-1}(g_A)$ is fuzzy soft semi-pre-nbd of $e(f_A)$.
(3) For every fuzzy soft point $e(f_A)$ in (X,τ_1,A) and every fuzzy soft nbd g_A of $f_{pu}(e(f_A))$, there is fuzzy soft semi-pre-nbd k_A of $e(f_A)$ such that $f_{pu}(k_A)\tilde{\leq}g_A$.
(4) For every fuzzy soft point $e(f_A)$ in (X,τ_1,A) and every fuzzy soft open set g_A with $f_{pu}(e(f_A))\tilde{\in}g_A$, there is fuzzy soft semi-pre-open set h_A with $e(f_A)\tilde{\in}h_A$ and $f_{pu}(h_A)\tilde{\leq}g_A$.
Proof. (1) $\Rightarrow$ (2) Suppose that $e(f_A)$ be a fuzzy soft point in (X,τ_1,A) and g_A be a fuzzy soft nbd of $f_{pu}(e(f_A))$. This follows that there is a fuzzy soft open set k_A such that $f_{pu}(e(f_A))\tilde{\leq}k_A\tilde{\leq}g_A$. Now $f_{pu}^{-1}(k_A)$ is fuzzy soft semi-pre-open and $e(f_A)\tilde{\leq}f_{pu}^{-1}(k_A)\tilde{\leq}f_{pu}^{-1}(g_A)$. Therefore $f_{pu}^{-1}(h_A)$ is fuzzy soft semi-pre-nbd of $e(f_A)$ in (X,τ_1,A).
(2) $\Rightarrow$ (3) Suppose that $e(f_A)$ be a fuzzy soft point in (X,τ_1,A) and g_A be a fuzzy soft nbd of $f_{pu}(e(f_A))$. This implies that $k_A\tilde{=}f_{pu}^{-1}(g_A)$ is fuzzy soft semi-pre-nbd of $e(f_A)$ and $f_{pu}(k_A)\tilde{=}f_{pu}(f_{pu}^{-1}(g_A))\tilde{\leq}g_A$.
(3) $\Rightarrow$ (4) Let $e(f_A)$ be a fuzzy soft point in (X,τ_1,A) and g_A be a fuzzy soft open set such that $f_{pu}(e(f_A))\tilde{\in}g_A$. Then g_A is fuzzy soft nbd of $f_{pu}(e(f_A))$. Therefore, there is fuzzy soft semi-pre-nbd k_A of $e(f_A)$ in (X,τ_1,A) such that $e(f_A)\tilde{\in}k_A$ and $f_{pu}(k_A)\tilde{\leq}g_A$. Therefore, there is a fuzzy soft semi-pre-open set h_A such that $e(f_A)\tilde{\in}h_A\tilde{\leq}k_A$. Hence $f_{pu}(h_A)\tilde{\leq}f_{pu}(k_A)\tilde{\leq}g_A$.
(4) $\Rightarrow$ (1) This follows from Theorem 4.6. Hence the proof.
Theorem 4.9. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces and $f_{pu}: F(X,A)\to F(Y,B)$ is a fuzzy soft mapping. Then the following statements are equivalent:
(1) f_{pu} is fuzzy soft semi-pre-continuous.
(2) $f_{pu}^{-1}((g_B)^\circ)\tilde{\leq}F^s pint^s(f_{pu}^{-1}(g_B))$, for every fuzzy soft set g_B in (Y,τ_2,B).
Proof. (1) $\Rightarrow$ (2) Suppose that g_B be fuzzy soft set in (Y,τ_2,B). Then $(g_B)^\circ$ is fuzzy soft open set. Since $f_{pu}: F(X,A)\to F(Y,B)$ is fuzzy soft semi-pre-continuous, then $f_{pu}^{-1}((g_B)^\circ)$ is fuzzy soft semi-pre-open in (X,τ_1,A). Therefore, $f_{pu}^{-1}((g_B)^\circ)\tilde{\leq}f_{pu}^{-1}(g_B)$ implies that $f_{pu}^{-1}((g_B)^\circ)\tilde{\leq}F^s pint^s(f_{pu}^{-1}(g_B))$.
(2) $\Rightarrow$ (1) This can be proved similarly. This completes the proof.
Theorem 4.10. Let (X,τ_1,A) and (Y,τ_2,B) be two fuzzy soft topological spaces and $f_{pu}: F(X,A)\to F(Y,B)$ be a fuzzy soft mapping. Then the following statements are equivalent:
(1) f_{pu} is fuzzy soft semi-pre-continuous.
(2) $f_{pu}(F^s pcl^s(f_A))\tilde{\leq}\overline{(f_{pu}(f_A))}$, for every fuzzy soft set f_A in (X,τ_1,A).
(3) $F^s pcl^s(f_{pu}^{-1}(g_B))\tilde{\leq}f_{pu}^{-1}(\overline{g_B})$, for every fuzzy soft set g_B in (Y,τ_2,B).
Proof. (1) $\Rightarrow$ (2) Let f_A be a fuzzy soft set in (X,τ_1,A). Then $\overline{(f_{pu}(f_A))}$ is fuzzy soft

closed. Since f_{pu} is fuzzy semi-pre-continuous, then by Theorem 4.5, $f_{pu}^{-1}(\overline{(f_{pu}(f_A))})$ is fuzzy soft semi-pre-closed. Thus $f_{pu}^{-1}(\overline{(f_{pu}(f_A))})\tilde{=}$
$F^s pcl^s(f_{pu}^{-1}(\overline{(f_{pu}(f_A))}))$. Now $f_A\tilde{\leq}f_{pu}^{-1}(f_{pu}(f_A))$ implies that $F^s pcl^s(f_A)$
$\tilde{\leq}F^s pcl^s(f_{pu}^{-1}(f_{pu}(f_A)))\tilde{\leq}F^s pcl^s(f_{pu}^{-1}(f_{pu}(\overline{f_A})))\tilde{=}f_{pu}^{-1}(\overline{(f_{pu}(f_A))})$.
(2) $\Rightarrow$ (3) Let g_B be fuzzy soft set in (Y, τ_2, B). Then by (2), we get
$f_{pu}(F^s pcl^s(f_{pu}^{-1}(g_B)))\tilde{\leq}\overline{f_{pu}(f_{pu}^{-1}(g_B))}$. Hence $F^s pcl^s(f_{pu}^{-1}(g_B))\tilde{\leq}f_{pu}^{-1}(\overline{f_{pu}(f_{pu}^{-1}(g_B))})$
$\tilde{\leq}f_{pu}^{-1}(\overline{g_B})$.
(3) $\Rightarrow$ (1) Let g_B be fuzzy soft open set in (Y, τ_2, B). Then g_B^c is fuzzy soft closed. By (3), we have $F^s pcl^s(f_{pu}^{-1}(g_B^c))\tilde{\leq}f_{pu}^{-1}(\overline{g_B^c})\tilde{=}f_{pu}^{-1}(g_B^c)$. Therefore, Theorem 3.25 follows that $f_{pu}^{-1}(g_B^c)\tilde{\geq}F^s pcl^s(f_{pu}^{-1}(g_B^c))\tilde{=}(F^s pint^s(f_{pu}^{-1}(g_B)))^c$. Hence $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-pre-open and therefore f_{pu} is fuzzy soft semi-pre-continuous. Hence the proof.

Definition 4.11. Let (X, τ_1, A) and (Y, τ_2, B) be two fuzzy soft topological spaces. Then a fuzzy soft mapping $f_{pu} : F(X, A) \to F(Y, B)$ is said to be a fuzzy soft semi-open, if for any fuzzy soft open set f_A in (X, τ_1, A), $f_{pu}(f_A)$ is fuzzy soft semi-pre-open set in (Y, τ_2, B).

Remark 4.12. Every fuzzy soft semi-open function is fuzzy soft semi-pre-open, since every fuzzy soft semi-open set is fuzzy soft semi-pre-open. But the converse is not true in general.

Theorem 4.13. Let (X, τ_1, A) and (Y, τ_2, B) be two fuzzy soft topological spaces and f_A be fuzzy soft set in (X, τ_1, A). Then a fuzzy soft mapping $f_{pu} : F(X, A) \to F(Y, B)$ is fuzzy soft semi-pre-open if and only $f_{pu}((f_A)^\circ)\tilde{\leq}sint^{fs}(scl^{fs}(f_{pu}(f_A)))$.

Proof. ($\Rightarrow$) Suppose f_{pu} is fuzzy soft semi-pre-open and also for any fuzzy soft set f_A in (X, τ_1, A), we get $f_{pu}((f_A)^\circ)\tilde{\leq}f_{pu}(f_A)$. Our hypothesis follows that $f_{pu}((f_A)^\circ)$ is fuzzy soft semi-pre-open in (Y, τ_2, B). Using Theorem 3.33, we have
$f_{pu}((f_A)^\circ)\tilde{\leq}sint^{fs}(scl^{fs}(f_{pu}(f_A)))$.
($\Leftarrow$) Suppose that $f_{pu}((f_A)^\circ)\tilde{\leq}sint^{fs}(scl^{fs}(f_{pu}(f_A)))$, for fuzzy soft set f_A in (X, τ_1, A). Then g_A be any fuzzy soft open set in (X, τ_1, A). Now take $g_A\tilde{=}(f_A)^\circ$
$f_{pu}(g_A)\tilde{=}f_{pu}((f_A)^\circ)\tilde{\leq}sint^{fs}(scl^{fs}(f_A))$. This follows that $f_{pu}(g_A)\tilde{\leq}sint^{fs}(scl^{fs}(f_{pu}(f_A)))$. Theorem 3.33 implies that $f_{pu}(g_A)$ is a fuzzy soft semi-pre-open in (Y, τ_2, B). Therefore, f_{pu} is a fuzzy soft semi-open function. This completes the proof.

Theorem 4.14. Let (X, τ_1, A) and (Y, τ_2, B) be two fuzzy soft topological spaces. Then fuzzy soft mapping $f_{pu} : F(X, A) \to F(Y, B)$ is fuzzy soft semi-pre-open if and only $f_{pu}(f_A^\circ)\tilde{\leq}F^s pint^s(f_{pu}(f_A))$, for every fuzzy soft set f_A in (X, τ_1, A).

Proof. ($\Rightarrow$) Suppose that f_{pu} is fuzzy soft semi-pre-open and f_A be fuzzy soft set in (X, τ_1, A). Then $f_{pu}(f_A^\circ)$ is fuzzy soft semi-pre-open. Thus,
$f_{pu}(f_A^\circ)\tilde{=}F^s pint^s(f_{pu}(f_A^\circ))\tilde{\leq}F^s pint^s(f_{pu}(f_A))$. Hence $f_{pu}(f_A^\circ)\tilde{\leq}F^s pint^s(f_{pu}(f_A))$.
($\Leftarrow$) Suppose that f_A be a fuzzy soft open set in (X, τ_1, A). Then our hypothesis follows that $f_{pu}(f_A)\tilde{=}f_{pu}(f_A^\circ)\tilde{\leq}F^s pint^s(f_{pu}(f_A))$. Thus $f_{pu}(f_A)$ is fuzzy soft semi-pre-open in (Y, τ_2, B). Hence the proof.

Definition 4.15. Let (X, τ_1, A) and (Y, τ_2, B) be two fuzzy soft topological spaces. Then a fuzzy soft mapping $f_{pu} : F(X, A) \to F(Y, B)$ is said to be a fuzzy soft semi-pre-closed, if for any fuzzy soft closed set f_A in (X, τ_1, A), $f_{pu}(f_A)$ is fuzzy soft semi-pre-closed in (Y, τ_2, B).

Remark 4.16. Every fuzzy soft semi-closed function is fuzzy soft semi-pre-closed but the converse is not true in general, since every fuzzy soft semi-closed set is fuzzy soft semi-pre-closed [23].

Theorem 4.17. Let (X, τ_1, A) and (Y, τ_2, B) be two fuzzy soft topological spaces and f_A be fuzzy soft set in (X, τ_1, A). Then a fuzzy soft mapping $f_{pu} : F(X, A) \to F(Y, B)$ is fuzzy soft semi-pre-closed if and only $F^s pcl^s(f_{pu}(f_A))\tilde{\leq}f_{pu}(\overline{f_A})$.

Proof. ($\Rightarrow$) Let f_{pu} be fuzzy soft semi-pre-closed and f_A be fuzzy soft set in (X, τ_1, A).

Then $f_{pu}(\overline{f_A})$ is fuzzy soft semi-pre-closed in (Y, τ_2, B). Therefore, $f_{pu}(f_A)\tilde{\leq} f_{pu}(\overline{f_A})$ follows that $F^s pcl^s(f_{pu}(f_A))\tilde{\leq} f_{pu}(\overline{f_A})$.
($\Leftarrow$) Suppose that g_A be fuzzy soft closed in (X, τ_1, A). Then
$f_{pu}(g_A)\tilde{=} f_{pu}(\overline{g_A})\tilde{\geq} F^s pcl^s(f_{pu}(g_A))$. Also by definition of fuzzy soft semi-pre-closure, $f_{pu}(g_A)\tilde{\leq} F^s pcl^s(f_{pu}(g_A))$. This implies that $f_{pu}(g_A)\tilde{=} F^s pcl^s(f_{pu}(g_A))$. Therefore, f_{pu} is a fuzzy soft semi-pre-closed mapping. This completes the proof.

Conclusion: The separation properties for fuzzy soft sets seem to be of special similarity to the problem of pattern discrimination. The fuzzification of soft set theory is very important topic in recent days. Every day new methods are developing in the literature using fuzzy soft sets from an imprecise mutiobserver data for problems of decision making. In this paper, We initiated and explored the properties of fuzzy soft semi-pre-open(closed) sets. We observed that any fuzzy soft semi-open as well as fuzzy soft pre-open set is a fuzzy soft semi-pre-open set. But the converse is not true. We introduced and investigated fuzzy soft semi-pre-interior and fuzzy soft semi-pre-closure in fuzzy soft topological spaces. Moreover, we defined and studied the characterization of fuzzy soft semi-pre-continuous and fuzzy soft semi-pre-open(closed) mappings in fuzzy soft topological spaces, which generalized fuzzy soft semi-continuous and fuzzy soft semi-open(closed) mappings. In future studies, we may develop more properties with applications of fuzzy soft sets in optimization problems as well as the problems of pattern discrimination.

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