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PSEUDO-SLANT LIGHTLIKE SUBMANIFOLDS OF INDEFINITE KAEHLER MANIFOLDS

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Abstract. In this paper, we introduce the notion of pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds giving characterization theorem with some non-trivial examples of such submanifolds. Integrability conditions of distributions D_1, D_2 and $RadTM$ on pseudo-slant lightlike submanifolds of an indefinite Kaehler manifold have been obtained. We also obtain necessary and sufficient conditions for foliations determined by above distributions to be totally geodesic.

1. Introduction

In 1990, B.Y. Chen defined slant immersions in complex geometry as a natural generalization of both holomorphic immersions and totally real immersions ([4], [5]). Further, A. Carriazo defined and studied bi-slant submanifolds of almost Hermitian and almost contact metric manifolds and further gave the notion of pseudo-slant submanifolds ([3]). The theory of lightlike submanifolds of a semi-Riemannian manifold was introduced by Duggal and Bejancu ([7]). Various classes of lightlike submanifolds of indefinite Kaehler manifolds are defined according to the behaviour of distributions on these submanifolds with respect to the action of (1,1) tensor field $\overline{J}$ in Kaehler structure of the ambient manifolds. Such submanifolds have been studied by Duggal and Sahin in ([8]). The geometry of slant and screen-slant lightlike submanifolds of indefinite Hermitian manifolds was studied by Sahin in ([14], [15]). The theory of slant, Cauchy-Riemann lightlike submanifolds of indefinite Kaehler manifolds has been studied in ([7], [8]).

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The objective of this paper is to introduce the notion of pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds. This new class of lightlike submanifolds of an indefinite Kaehler manifold includes slant, Cauchy-Riemann lightlike submanifolds as its sub-cases. The paper is arranged as follows. There are some basic results in section 2. In section 3, we study pseudo-slant lightlike submanifolds of an indefinite Kaehler manifold, giving some examples. Section 4 is devoted to the study of foliations determined by distributions on pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds.

2. Preliminaries

A submanifold (M^m, g) immersed in a semi-Riemannian manifold $(\overline{M}^{m+n}, \overline{g})$ is called a lightlike submanifold ([7]) if the metric g induced from $\overline{g}$ is degenerate and the radical distribution $RadTM$ is of rank r, where $1 \leq r \leq m$. Let $S(TM)$ be a screen distribution which is a semi-Riemannian complementary distribution of $RadTM$ in TM, that is

$$TM = RadTM \oplus_{orth} S(TM). \tag{2.1}$$

Now consider a screen transversal vector bundle $S(TM^{\perp})$, which is a semi-Riemannian complementary vector bundle of $RadTM$ in $TM^{\perp}$. Since for any local basis $\{\xi_i\}$ of $RadTM$, there exists a local null frame $\{N_i\}$ of sections with values in the orthogonal complement of $S(TM^{\perp})$ in $[S(TM)]^{\perp}$ such that $\overline{g}(\xi_i, N_j) = \delta_{ij}$ and $\overline{g}(N_i, N_j) = 0$, it follows that there exists a lightlike transversal vector bundle $ltr(TM)$ locally spanned by $\{N_i\}$. Let $tr(TM)$ be complementary (but not orthogonal) vector bundle to TM in $T\overline{M}|_M$. Then

$$tr(TM) = ltr(TM) \oplus_{orth} S(TM^{\perp}), \tag{2.2}$$

$$T\overline{M}|_M = TM \oplus tr(TM), \tag{2.3}$$

$$T\overline{M}|_M = S(TM) \oplus_{orth} [RadTM \oplus ltr(TM)] \oplus_{orth} S(TM^{\perp}). \tag{2.4}$$

Following are four cases of a lightlike submanifold $\big(M, g, S(TM), S(TM^{\perp})\big)$:

Case.1 r-lightlike if $r < min\,(m, n)$,

Case.2 co-isotropic if $r = n < m$, $S\left(TM^{\perp}\right) = \{0\}$,

Case.3 isotropic if $r = m < n$, $S\,(TM) = \{0\}$,

Case.4 totally lightlike if $r = m = n$, $S(TM) = S(TM^{\perp}) = \{0\}$.

The Gauss and Weingarten formulae are given as

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \tag{2.5}$$

$$\overline{\nabla}_X V = -A_V X + \nabla^t_X V, \tag{2.6}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(tr(TM))$, where $\nabla_X Y, A_V X$ belong to $\Gamma(TM)$ and $h(X, Y), \nabla^t_X V$ belong to $\Gamma(tr(TM))$. ∇ and ∇^t are linear connections on M and on the vector bundle $tr(TM)$, respectively. The second fundamental form h is a symmetric $F(M)$-bilinear form on $\Gamma(TM)$ with values in $\Gamma(tr(TM))$ and the shape operator A_V is a linear endomorphism of $\Gamma(TM)$. From (2.5) and (2.6), for any $X, Y \in \Gamma(TM)$, $N \in \Gamma(ltr(TM))$ and $W \in \Gamma(S(TM^\perp))$, we have

$$\overline{\nabla}_X Y = \nabla_X Y + h^l(X, Y) + h^s(X, Y), \tag{2.7}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^l_X N + D^s(X, N), \tag{2.8}$$

$$\overline{\nabla}_X W = -A_W X + \nabla^s_X W + D^l(X, W), \tag{2.9}$$

where $h^l(X,Y) = L(h(X,Y))$, $h^s(X,Y) = S(h(X,Y))$, $D^l(X,W) = L(\nabla^t_X W)$, $D^s(X,N) = S(\nabla^t_X N)$. L and S are the projection morphisms of $tr(TM)$ on $ltr(TM)$ and $S(TM^\perp)$ respectively. ∇^l and ∇^s are linear connections on $ltr(TM)$ and $S(TM^\perp)$ called the lightlike connection and screen transversal connection on M respectively.

Now by using (2.5), (2.7)-(2.9) and metric connection $\overline{\nabla}$, we obtain

$$\overline{g}(h^s(X,Y), W) + \overline{g}(Y, D^l(X,W)) = g(A_W X, Y), \tag{2.10}$$

$$\overline{g}(D^s(X,N), W) = \overline{g}(N, A_W X). \tag{2.11}$$

Denote the projection of TM on $S(TM)$ by $\overline{P}$. Then from the decomposition of the tangent bundle of a lightlike submanifold, for any $X, Y \in \Gamma(TM)$ and $\xi \in \Gamma(RadTM)$, we have

$$\nabla_X \overline{P}Y = \nabla^*_X \overline{P}Y + h^*(X, \overline{P}Y), \tag{2.12}$$

$$\nabla_X \xi = -A^*_\xi X + \nabla^{*t}_X \xi, \tag{2.13}$$

By using above equations, we obtain

$$\overline{g}(h^l(X, \overline{P}Y), \xi) = g(A^*_\xi X, \overline{P}Y), \tag{2.14}$$

$$\overline{g}(h^*(X, \overline{P}Y), N) = g(A_N X, \overline{P}Y), \tag{2.15}$$

$$\overline{g}(h^l(X, \xi), \xi) = 0, \quad A^*_\xi \xi = 0. \tag{2.16}$$

It is important to note that in general ∇ is not a metric connection. Since $\overline{\nabla}$ is metric connection, by using (2.7), we get

$$(\nabla_X g)(Y, Z) = \overline{g}(h^l(X,Y), Z) + \overline{g}(h^l(X,Z), Y). \tag{2.17}$$

An indefinite almost Hermitian manifold $(\overline{M}, \overline{g}, \overline{J})$ is a 2m-dimensional semi-Riemannian manifold $\overline{M}$ with semi-Riemannian metric $\overline{g}$ of constant index q, $0 < q < 2m$ and a (1, 1) tensor field $\overline{J}$ on $\overline{M}$ such that following conditions are satisfied:

$$\overline{J}^2 X = -X, \tag{2.18}$$

$$\overline{g}(\overline{J}X, \overline{J}Y) = \overline{g}(X, Y), \tag{2.19}$$

for all $X, Y \in \Gamma(T\overline{M})$.

An indefinite almost Hermitian manifold $(\overline{M}, \overline{g}, \overline{J})$ is called an indefinite Kaehler manifold if $\overline{J}$ is parallel with respect to $\overline{\nabla}$, i.e.,

$$(\overline{\nabla}_X \overline{J})Y = 0, \tag{2.20}$$

for all $X, Y \in \Gamma(T\overline{M})$, where $\overline{\nabla}$ is Levi-Civita connection with respect to $\overline{g}$.

3. Pseudo-Slant Lightlike Submanifolds

In this section, we introduce the notion of pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds. At first, we state the following Lemmas for later use:

Lemma 3.1. *Let M be a r-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$. Suppose that $\overline{J}RadTM$ is a distribution on M such that $RadTM \cap \overline{J}RadTM = \{0\}$. Then $\overline{J}ltr(TM)$ is a subbundle of the screen distribution $S(TM)$ and $\overline{J}RadTM \cap \overline{J}ltr(TM) = \{0\}$.*

Lemma 3.2. *Let M be a q-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$. Suppose $\overline{J}RadTM$ is a distribution on M such that $RadTM \cap \overline{J}RadTM = \{0\}$. Then any complementary distribution to $\overline{J}RadTM \oplus \overline{J}ltr(TM)$ in $S(TM)$ is Riemannian.*

The proofs of Lemma 3.1 and Lemma 3.2 follow as in Lemma 3.1 and Lemma 3.2 of [15], respectively, so we omit them.

Definition 3.1. Let M be a q-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$ such that $q < dim(M)$. Then we say that M is a pseudo-slant lightlike submanifold of $\overline{M}$ if following conditions are satisfied:
(i) $\overline{J}RadTM$ is a distribution on M such that $RadTM \cap \overline{J}RadTM = \{0\}$,
(ii) there exists non-degenerate orthogonal distributions D_1 and D_2 on M such that $S(TM) = (\overline{J}RadTM \oplus \overline{J}ltr(TM)) \oplus_{orth} D_1 \oplus_{orth} D_2$,
(iii) the distribution D_1 is anti-invariant, i.e. $\overline{J}D_1 \subset S(TM^{\perp})$,
(iv) the distribution D_2 is slant with angle $\theta(\neq \pi/2)$, i.e. for each $x \in M$ and each non-zero vector $X \in (D_2)_x$, the angle θ between $\overline{J}X$ and the vector subspace $(D_2)_x$ is a constant$(\neq \pi/2)$, which is independent of the choice of $x \in M$ and $X \in (D_2)_x$. This constant angle θ is called slant angle of distribution D_2. A screen pseudo-slant lightlike submanifold is said to be proper if $D_1 \neq \{0\}$, $D_2 \neq \{0\}$ and $\theta \neq 0$.
From the above definition, we have the following decomposition

$$TM = RadTM \oplus_{orth} (\overline{J}RadTM \oplus \overline{J}ltr(TM)) \oplus_{orth} D_1 \oplus_{orth} D_2. \tag{3.1}$$

In particular, we have
(i) if $D_1 = 0$, then M is a slant lightlike submanifold,
(ii) if $D_1 \neq 0$ and $\theta = 0$, then M is a CR-lightlike submanifold.
Thus above new class of lightlike submanifolds of an indefinite Kaehler manifold includes slant, Cauchy-Riemann lightlike submanifolds as its sub-cases which have been studied in ([7],[8]).
Let $(\mathbb{R}^{2m}_{2q}, \overline{g}, \overline{J})$ denote the manifold $\mathbb{R}^{2m}_{2q}$ with its usual Kaehler structure given by
$\overline{g} = \frac{1}{4}(-\sum_{i=1}^{q} dx^i \otimes dx^i + dy^i \otimes dy^i + \sum_{i=q+1}^{m} dx^i \otimes dx^i + dy^i \otimes dy^i)$,
$\overline{J}(\sum_{i=1}^{m}(X_i\partial x_i + Y_i\partial y_i)) = \sum_{i=1}^{m}(Y_i\partial x_i - X_i\partial y_i)$,
where (x^i, y^i) are the Cartesian coordinates on $\mathbb{R}^{2m}_{2q}$. Now, we construct some examples of pseudo-slant lightlike submanifolds of an indefinite Kaehler manifold.

Example 1. Let $(\mathbb{R}^{12}_2, \overline{g}, \overline{J})$ be an indefinite Kaehler manifold, where $\overline{g}$ is of signature $(-,+,+,+,+,+,-,+,+,+,+,+)$ with respect to the canonical basis $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6\}$.

Suppose M is a submanifold of $\mathbb{R}^{12}_2$ given by $x^1 = y^2 = u_1$, $x^2 = u_2$, $y^1 = u_3$, $x^3 = y^4 = u_4$, $x^4 = y^3 = u_5$, $x^5 = u_6 \cos u_7$, $y^5 = u_6 \sin u_7$, $x^6 = \cos u_6$, $y^6 = \sin u_6$, where u_i are real parameters and $u_6 \neq 0$.

The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\}$, where

$Z_1 = 2(\partial x_1 + \partial y_2), \quad Z_2 = 2\partial x_2, \quad Z_3 = 2\partial y_1,$

$Z_4 = 2(\partial x_3 + \partial y_4), \quad Z_5 = 2(\partial x_4 + \partial y_3),$

$Z_6 = 2(\cos u_7 \partial x_5 + \sin u_7 \partial y_5 - \sin u_6 \partial x_6 + \cos u_6 \partial y_6),$

$Z_7 = 2(-u_6 \sin u_7 \partial x_5 + u_6 \cos u_7 \partial y_5).$

Hence $RadTM = Span\{Z_1\}$ and $S(TM) = Span\{Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\}$.

Now $ltr(TM)$ is spanned by $N_1 = -\partial x_1 + \partial y_2$ and $S(TM^\perp)$ is spanned by

$W_1 = 2(\partial x_3 - \partial y_4), \quad W_2 = 2(\partial x_4 - \partial y_3),$

$W_3 = 2(\cos u_7 \partial x_5 + \sin u_7 \partial y_5 + \sin u_6 \partial x_6 - \cos u_6 \partial y_6),$

$W_4 = 2(u_6 \cos u_6 \partial x_6 + u_6 \sin u_6 \partial y_6).$

It follows that $\overline{J} Z_1 = Z_2 - Z_3$, which implies that $\overline{J} RadTM$ is a distribution on M. On the other hand, we can see that $D_1 = span\{Z_4, Z_5\}$ such that $\overline{J} Z_4 = W_2$, $\overline{J} Z_5 = W_1$, which implies that D_1 is anti-invariant with respect to $\overline{J}$ and $D_2 = span\{Z_6, Z_7\}$ is a slant distribution with slant angle $\pi/4$. Hence M is a pseudo-slant 2-lightlike submanifold of $\mathbb{R}^{12}_2$.

Example 2. Let $(\mathbb{R}^{12}_2, \overline{g}, \overline{J})$ be an indefinite Kaehler manifold, where $\overline{g}$ is of signature $(-,+,+,+,+,+,-,+,+,+,+,+)$ with respect to the canonical basis $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6\}$.

Suppose M is a submanifold of $\mathbb{R}^{12}_2$ given by $-x^1 = y^2 = u_1$, $x^2 = u_2$, $y^1 = u_3$, $x^3 = u_4 \cos\beta$, $y^3 = u_4 \sin\beta$, $x^4 = u_5 \sin\beta$, $y^4 = u_5 \cos\beta$, $x^5 = u_6 \cos\theta$, $y^5 = u_7 \cos\theta$, $x^6 = u_7 \sin\theta$, $y^6 = u_6 \sin\theta$, where u_i are real parameters.

The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\}$, where

$Z_1 = 2(-\partial x_1 + \partial y_2), \quad Z_2 = 2\partial x_2, \quad Z_3 = 2\partial y_1,$

$Z_4 = 2(\cos\beta \partial x_3 + \sin\beta \partial y_3)$, $Z_5 = 2(\sin\beta \partial x_4 + \cos\beta \partial y_4)$,

$Z_6 = 2(\cos\theta \partial x_5 + \sin\theta \partial y_6)$, $Z_7 = 2(\sin\theta \partial x_6 + \cos\theta \partial y_5)$.

Hence $RadTM = Span\{Z_1\}$ and $S(TM) = Span\{Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\}$.

Now $ltr(TM)$ is spanned by $N_1 = \partial x_1 + \partial y_2$ and $S(TM^\perp)$ is spanned by

$W_1 = 2(\sin\beta \partial x_3 - \cos\beta \partial y_3)$, $W_2 = 2(\cos\beta \partial x_4 - \sin\beta \partial y_4)$,

$W_3 = 2(\sin\theta \partial x_5 - \cos\theta \partial y_6)$, $W_4 = 2(\cos\theta \partial x_6 - \sin\theta \partial y_5)$.

It follows that $\overline{J} Z_1 = Z_2 + Z_3$, which implies that $\overline{J} RadTM$ is a distribution on M. On the other hand, we can see that $D_1 = span\{Z_4, Z_5\}$ such that $\overline{J} Z_4 = W_1$, $\overline{J} Z_5 = W_2$, which implies that D_1 is anti-invariant with respect to $\overline{J}$ and $D_2 = span\{Z_6, Z_7\}$ is a slant distribution with slant angle 2θ. Hence M is a pseudo-slant 2-lightlike submanifold of $\mathbb{R}^{12}_2$.

Now, for any vector field X tangent to M, we put $\overline{J} X = PX + FX$, where PX and FX are tangential and transversal parts of $\overline{J} X$ respectively. We denote the projections on $RadTM$, $\overline{J}RadTM$, $\overline{J}ltr(TM)$, D_1 and D_2 in TM by P_1, P_2, P_3, P_4, and P_5 respectively. Similarly, we denote the projections of $tr(TM)$ on $ltr(TM)$, $\overline{J}(D_1)$ and D' by Q_1, Q_2 and Q_3 respectively, where D' is non-degenerate orthogonal complementary subbundle of $\overline{J}(D_1)$ in $S(TM^{\perp})$. Then, for any $X \in \Gamma(TM)$, we get

$$X = P_1X + P_2X + P_3X + P_4X + P_5X. \tag{3.2}$$

Now applying $\overline{J}$ to (3.2), we have

$$\overline{J}X = \overline{J}P_1X + \overline{J}P_2X + \overline{J}P_3X + \overline{J}P_4X + \overline{J}P_5X, \tag{3.3}$$

which gives

$$\overline{J}X = \overline{J}P_1X + \overline{J}P_2X + \overline{J}P_3X + \overline{J}P_4X + fP_5X + FP_5X, \tag{3.4}$$

where fP_5X (resp. FP_5X) denotes the tangential (resp. transversal) component of $\overline{J}P_5X$. Thus we get $\overline{J}P_1X \in \Gamma(\overline{J}RadTM), \overline{J}P_2X \in \Gamma(RadTM), \overline{J}P_3X \in \Gamma(ltr(TM)), \overline{J}P_4X \in \Gamma(\overline{J}D_1) \subseteq \Gamma(S(TM^{\perp})), fP_5X \in \Gamma(D_2)$ and $FP_5X \in \Gamma(D')$. Also, for any $W \in \Gamma(tr(TM))$, we have

$$W = Q_1W + Q_2W + Q_3W. \tag{3.5}$$

Applying $\overline{J}$ to (3.5), we obtain

$$\overline{J}W = \overline{J}Q_1W + \overline{J}Q_2W + \overline{J}Q_3W, \tag{3.6}$$

which gives

$$\overline{J}W = \overline{J}Q_1W + \overline{J}Q_2W + BQ_3W + CQ_3W, \tag{3.7}$$

where BQ_3W (resp. CQ_3W) denotes the tangential (resp. transversal) component of $\overline{J}Q_3W$. Thus we get $\overline{J}Q_1W \in \Gamma(\overline{J}ltr(TM)), \overline{J}Q_2W \in \Gamma(D_1)$, $BQ_3W \in \Gamma(D_2)$ and $CQ_3W \in \Gamma(D')$.

Now, by using (2.20), (3.4), (3.7) and (2.7)-(2.9) and identifying the components on $RadTM$, $\overline{J}RadTM$, $\overline{J}ltr(TM)$, D_1, D_2, $ltr(TM)$, $\overline{J}(D_1)$ and D', we obtain

$$\begin{aligned} &P_1(\nabla_X\overline{J}P_1Y) + P_1(\nabla_X\overline{J}P_2Y) - P_1(A_{\overline{J}P_4Y}X) + P_1(\nabla_X fP_5Y) \\ &= P_1(A_{FP_5Y}X) + P_1(A_{\overline{J}P_3Y}X) + \overline{J}P_2\nabla_XY, \end{aligned} \tag{3.8}$$

$$\begin{aligned} &P_2(\nabla_X\overline{J}P_1Y) + P_2(\nabla_X\overline{J}P_2Y) - P_2(A_{\overline{J}P_4Y}X) + P_2(\nabla_X fP_5Y) \\ &= P_2(A_{FP_5Y}X) + P_2(A_{\overline{J}P_3Y}X) + \overline{J}P_1\nabla_XY, \end{aligned} \tag{3.9}$$

$$\begin{aligned} &P_3(\nabla_X\overline{J}P_1Y) + P_3(\nabla_X\overline{J}P_2Y) - P_3(A_{\overline{J}P_4Y}X) + P_3(\nabla_X fP_5Y) \\ &= P_3(A_{FP_5Y}X) + P_3(A_{\overline{J}P_3Y}X) + \overline{J}h^l(X,Y), \end{aligned} \tag{3.10}$$

$$\begin{aligned} &P_4(\nabla_X\overline{J}P_1Y) + P_4(\nabla_X\overline{J}P_2Y) - P_4(A_{\overline{J}P_4Y}X) + P_4(\nabla_X fP_5Y) \\ &= P_4(A_{FP_5Y}X) + P_4(A_{\overline{J}P_3Y}X) + \overline{J}Q_2h^s(X,Y), \end{aligned} \tag{3.11}$$

$$\begin{aligned} &P_5(\nabla_X\overline{J}P_1Y) + P_5(\nabla_X\overline{J}P_2Y) - P_5(A_{\overline{J}P_4Y}X) + P_5(\nabla_X fP_5Y) \\ &= P_5(A_{FP_5Y}X) + P_5(A_{\overline{J}P_3Y}X) + fP_5\nabla_XY + BQ_3h^s(X,Y), \end{aligned} \tag{3.12}$$

$$\begin{aligned} &h^l(X,\overline{J}P_1Y)+h^l(X,\overline{J}P_2Y)+D^l(X,\overline{J}P_4Y)+h^l(X,fP_5Y)\\ &=\overline{J}P_3\nabla_XY-\nabla^l_X\overline{J}P_3Y-D^l(X,FP_5Y), \end{aligned} \tag{3.13}$$

$$\begin{aligned} &Q_2h^s(X,\overline{J}P_1Y)+Q_2h^s(X,\overline{J}P_2Y)+Q_2\nabla^s_X\overline{J}P_4Y+Q_2h^s(X,fP_5Y)\\ &=Q_2\nabla^s_XFP_5Y-Q_2D^s(X,\overline{J}P_3Y)+\overline{J}P_4\nabla_XY, \end{aligned} \tag{3.14}$$

$$\begin{aligned} &Q_3h^s(X,\overline{J}P_1Y)+Q_3h^s(X,\overline{J}P_2Y)+Q_3\nabla^s_X\overline{J}P_4Y+Q_3h^s(X,fP_5Y)\\ &=CQ_3h^s(X,Y)-Q_3\nabla^s_XFP_5Y-Q_3D^s(X,\overline{J}P_3Y)+FP_5\nabla_XY. \end{aligned} \tag{3.15}$$

Theorem 3.3. *Let M be a q-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$. Then M is a pseudo-slant lightlike submanifold of $\overline{M}$ if and only if*
(i) $\overline{J}RadTM$ is a distribution on M such that $RadTM\cap\overline{J}RadTM=\{0\}$,
(ii) the distribution D_1 is an anti-invariant, i.e. $\overline{J}D_1\subset S(TM^\perp)$,
(iii) there exists a constant $\lambda\in(0,1]$ such that $P^2X=-\lambda X$.
Moreover, there also exists a constant $\mu\in[0,1)$ such that $BFX=-\mu X$, for all $X\in\Gamma(D_2)$, where D_1 and D_2 are non-degenerate orthogonal distributions on M such that $S(TM)=(\overline{J}RadTM\oplus\overline{J}ltr(TM))\oplus_{orth}D_1\oplus_{orth}D_2$ and $\lambda=\cos^2\theta$, θ is slant angle of D_2.

Proof. Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then distribution D_1 is anti-invariant with respect to $\overline{J}$ and $\overline{J}RadTM$ is a distribution on M such that $RadTM\cap\overline{J}RadTM=\{0\}$.
Now for any $X\in\Gamma(D_2)$, we have $|PX|=|\overline{J}X|\cos\theta$, which implies

$$\cos\theta=\frac{|PX|}{|\overline{J}X|}. \tag{3.16}$$

In view of (3.16), we get $\cos^2\theta=\frac{|PX|^2}{|\overline{J}X|^2}=\frac{g(PX,PX)}{g(\overline{J}X,\overline{J}X)}=\frac{g(X,P^2X)}{g(X,\overline{J}^2X)}$, which gives

$$g(X,P^2X)=\cos^2\theta\, g(X,\overline{J}^2X). \tag{3.17}$$

Since M is pseudo-slant lightlike submanifold, $\cos^2\theta=\lambda(constant)\in(0,1]$ therefore from (3.17), we get $g(X,P^2X)=\lambda g(X,\overline{J}^2X)=g(X,\lambda\overline{J}^2X)$, which implies

$$g(X,(P^2-\lambda\overline{J}^2)X)=0. \tag{3.18}$$

Since X is non-null vector, we have $(P^2-\lambda\overline{J}^2)X=0$, which implies

$$P^2X=\lambda\overline{J}^2X=-\lambda X. \tag{3.19}$$

Now, for any vector field $X\in\Gamma(D_2)$, we have

$$\overline{J}X=PX+FX, \tag{3.20}$$

where PX and FX are tangential and transversal parts of $\overline{J}X$ respectively.
Applying $\overline{J}$ to (3.20) and taking tangential component, we get

$$-X=P^2X+BFX. \tag{3.21}$$

From (3.19) and (3.21), we get

$$BFX = -\sin^2\theta\, X, \quad \forall X \in \Gamma(D_2), \tag{3.22}$$

where $\sin^2\theta = 1 - \lambda = \mu(constant) \in [0, 1)$.
This proves (iii).
Conversely suppose that conditions (i), (ii) and (iii) are satisfied. From (3.21), for any $X \in \Gamma(D_2)$, we get

$$-X = P^2X - \mu X, \tag{3.23}$$

which implies

$$P^2X = -\cos^2\theta\, X, \tag{3.24}$$

where $\cos^2\theta = 1 - \mu = \lambda(constant) \in (0, 1]$.
Now $\cos\theta = \frac{g(\overline{J}X,PX)}{|\overline{J}X||PX|} = -\frac{g(X,\overline{J}PX)}{|\overline{J}X||PX|} = -\frac{g(X,P^2X)}{|\overline{J}X||PX|} = -\lambda\frac{g(X,\overline{J}^2X)}{|\overline{J}X||PX|} = \lambda\frac{g(\overline{J}X,\overline{J}X)}{|\overline{J}X||PX|}$.
From above equation, we get

$$\cos\theta = \lambda\frac{|\overline{J}X|}{|PX|}. \tag{3.25}$$

Therefore (3.16) and (3.25) give $\cos^2\theta = \lambda(constant)$.
Hence M is a pseudo-slant lightlike submanifold.

Corollary 3.1. *Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ with slant angle θ, then for any $X, Y \in \Gamma(D_2)$, we have*
(i) $g(PX, PY) = \cos^2\theta\, g(X, Y)$,
(ii) $g(FX, FY) = \sin^2\theta\, g(X, Y)$.

The proof of above Corollary follows by using similar steps as in proof of Corollary 3.1 of [15].

Theorem 3.4. *Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $RadTM$ is integrable if and only if*
(i) $P_1(\nabla_X\overline{J}P_1Y) = P_1(\nabla_Y\overline{J}P_1X)$ *and* $P_5(\nabla_X\overline{J}P_1Y) = P_5(\nabla_Y\overline{J}P_1X)$,
(ii) $Q_2h^s(Y, \overline{J}P_1X) = Q_2h^s(X, \overline{J}P_1Y)$ *and* $h^l(Y, \overline{J}P_1X) = h^l(X, \overline{J}P_1Y)$,
(iii) $Q_3h^s(Y, \overline{J}P_1X) = Q_3h^s(X, \overline{J}P_1Y)$, *for all* $X, Y \in \Gamma(RadTM)$.

Proof. Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Let $X, Y \in \Gamma(RadTM)$. From (3.8), we have $P_1(\nabla_X\overline{J}P_1Y) = \overline{J}P_2\nabla_XY$, which gives $P_1(\nabla_X\overline{J}P_1Y) - P_1(\nabla_Y\overline{J}P_1X) = \overline{J}P_2[X, Y]$. From (3.12), we get $P_5(\nabla_X\overline{J}P_1Y) = fP_5\nabla_XY + Bh^s(X, Y)$, which gives $P_5(\nabla_X\overline{J}P_1Y) - P_5(\nabla_Y\overline{J}P_1X) = fP_5[X, Y]$. In view of (3.13), we obtain $h^l(X, \overline{J}P_1Y) = \overline{J}P_3\nabla_XY$, which implies $h^l(X, \overline{J}P_1Y) - h^l(Y, \overline{J}P_1X) = \overline{J}P_3[X, Y]$. From (3.14), we have $Q_2h^s(X, \overline{J}P_1Y) = \overline{J}P_4\nabla_XY$, which gives $Q_2h^s(X, \overline{J}P_1Y) - Q_2h^s(Y, \overline{J}P_1X) = \overline{J}P_4[X, Y]$. Also from (3.15), we get $Q_3h^s(X, \overline{J}P_1Y) = Ch^s(X, Y) + FP_5\nabla_XY$, which implies $Q_3h^s(X, \overline{J}P_1Y) - Q_3h^s(Y, \overline{J}P_1X) = FP_5[X, Y]$. This concludes the theorem.

Theorem 3.5. *Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 is integrable if and only if*

(i) $P_1(A_{\overline{J}P_4Y}X) = P_1(A_{\overline{J}P_4X}Y)$ *and* $P_2(A_{\overline{J}P_4Y}X) = P_2(A_{\overline{J}P_4X}Y)$,

(ii) $D^l(Y, \overline{J}P_4X) = D^l(X, \overline{J}P_4Y)$ *and* $Q_3\nabla^s_Y\overline{J}P_4X = Q_3\nabla^s_X\overline{J}P_4Y$,

(iii) $P_5(A_{\overline{J}P_4Y}X) = P_5(A_{\overline{J}P_4X}Y)$, *for all* $X, Y \in \Gamma(D_1)$.

Proof. Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Let $X, Y \in \Gamma(D_1)$. From (3.8), we have $P_1(A_{\overline{J}P_4Y}X) + \overline{J}P_2\nabla_XY = 0$, which gives $P_1(A_{\overline{J}P_4X}Y) - P_1(A_{\overline{J}P_4Y}X) = \overline{J}P_2[X, Y]$. From (3.9), we get $P_2(A_{\overline{J}P_4Y}X) + \overline{J}P_1\nabla_XY = 0$, which gives $P_2(A_{\overline{J}P_4X}Y) - P_2(A_{\overline{J}P_4Y}X) = \overline{J}P_1[X, Y]$. In view of (3.12), we obtain $P_5(A_{\overline{J}P_4Y}X) + fP_5\nabla_XY + BQ_3h^s(X, Y) = 0$, which implies $P_5(A_{\overline{J}P_4X}Y) - P_5(A_{\overline{J}P_4Y}X) = fP_5[X, Y]$. From (3.13), we have $D^l(X, \overline{J}P_4Y) = \overline{J}P_3\nabla_XY$, which gives $D^l(X, \overline{J}P_4Y) - D^l(Y, \overline{J}P_4X) = \overline{J}P_3[X, Y]$. Also from (3.15), we obtain $Q_3\nabla^s_X\overline{J}P_4Y = CQ_3h^s(X, Y) + FP_5\nabla_XY$, which implies $Q_3\nabla^s_X\overline{J}P_4Y - Q_3\nabla^s_Y\overline{J}P_4X = FP_5[X, Y]$. Thus, we obtain the required results.

Theorem 3.6. *Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_2 is integrable if and only if*

(i) $P_1(\nabla_XfP_5Y - \nabla_YfP_5X) = P_1(A_{FP_5Y}X - A_{FP_5X}Y)$,

(ii) $P_2(\nabla_XfP_5Y - \nabla_YfP_5X) = P_2(A_{FP_5Y}X - A_{FP_5X}Y)$,

(iii) $h^l(X, fP_5Y) - h^l(Y, fP_5X) = D^l(Y, FP_5X) - D^l(X, FP_5Y)$,

(iv) $Q_2(\nabla^s_XFP_5Y - \nabla^s_YFP_5X) = Q_2(h^s(X, fP_5Y) - h^s(Y, fP_5X))$,

for all $X, Y \in \Gamma(D_2)$.

Proof. Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Let $X, Y \in \Gamma(D_2)$. From (3.8), we have $P_1(\nabla_XfP_5Y) - P_1(A_{FP_5Y}X) = \overline{J}P_2\nabla_XY$, which gives $P_1(\nabla_XfP_5Y - \nabla_YfP_5X) - P_1(A_{FP_5Y}X - A_{FP_5X}Y) = \overline{J}P_2[X, Y]$. From (3.9), we get $P_2(\nabla_XfP_5Y) - P_2(A_{FP_5Y}X) = \overline{J}P_1\nabla_XY$, which gives $P_2(\nabla_XfP_5Y - \nabla_YfP_5X) - P_2(A_{FP_5Y}X - A_{FP_5X}Y) = \overline{J}P_1[X, Y]$. In view of (3.13), we obtain $h^l(X, fP_5Y) + D^l(X, FP_5Y) = \overline{J}P_3\nabla_XY$, which implies $h^l(X, fP_5Y) - h^l(Y, fP_5X) + D^l(X, FP_5Y) - D^l(Y, FP_5X) = \overline{J}P_3[X, Y]$. From (3.14), we have $Q_2h^s(X, fP_5Y) - Q_2\nabla^s_XFP_5Y = \overline{J}P_4\nabla_XY$, which gives $Q_2(\nabla^s_YFP_5X - \nabla^s_XFP_5Y) + Q_2(h^s(X, fP_5Y) - Q_2h^s(Y, fP_5X)) = \overline{J}P_4[X, Y]$. This proves the theorem.

4. Foliations Determined by Distributions

In this section, we obtain necessary and sufficient conditions for foliations determined by distributions on a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold to be totally geodesic.

Definition 4.1. A pseudo-slant lightlike submanifold M of an indefinite Kaehler manifold $\overline{M}$ is said to be mixed geodesic if its second fundamental form h satisfies $h(X, Y) = 0$, for all $X \in \Gamma(D_1)$ and $Y \in \Gamma(D_2)$. Thus M is mixed geodesic pseudo-slant lightlike submanifold if $h^l(X, Y) = 0$ and $h^s(X, Y) = 0$, for all $X \in \Gamma(D_1)$ and $Y \in \Gamma(D_2)$.

Theorem 4.1. *Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $RadTM$ defines a totally geodesic foliation if and only if* $\overline{g}(\nabla_X \overline{J}P_2Z + \nabla_X fP_5Z, \overline{J}Y) = \overline{g}(A_{\overline{J}P_3Z}X + A_{\overline{J}P_4Z}X + A_{FP_5Z}X, \overline{J}Y)$, *for all* $X, Y \in \Gamma(RadTM)$ *and* $Z \in \Gamma(S(TM))$.

Proof. Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. It is easy to see that $RadTM$ defines a totally geodesic foliation if and only if $\nabla_X Y \in \Gamma(RadTM)$, for all $X, Y \in \Gamma(RadTM)$. Since $\overline{\nabla}$ is metric connection, using (2.7), (2.19), (2.20) and (3.4), for any $X, Y \in \Gamma(RadTM)$ and $Z \in \Gamma(S(TM))$, we get $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X(\overline{J}P_2Z + \overline{J}P_3Z + \overline{J}P_4Z + fP_5Z + FP_5Z), \overline{J}Y)$, which gives $\overline{g}(\nabla_X Y, Z) = \overline{g}(A_{\overline{J}P_3Z}X + A_{FP_5Z}X + A_{\overline{J}P_4Z}X - \nabla_X \overline{J}P_2Z - \nabla_X fP_5Z, \overline{J}Y)$. This completes the proof.

Theorem 4.2. *Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 defines a totally geodesic foliation if and only if*

(i) $\overline{g}(\nabla^s_X FZ, \overline{J}Y) = -\overline{g}(h^s(X, fZ), \overline{J}Y)$,

(ii) $h^s(X, \overline{J}N)$ *and* $D^s(X, \overline{J}W)$ *have no components in* $\overline{J}(D_1)$,

for all $X, Y \in \Gamma(D_1)$, $Z \in \Gamma(D_2)$, $N \in \Gamma(ltr(TM))$, $W \in \Gamma(\overline{J}ltr(TM))$.

Proof. Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. The distribution D_1 defines a totally geodesic foliation if and only if $\nabla_X Y \in \Gamma(D_1)$, for all $X, Y \in \Gamma(D_1)$. Since $\overline{\nabla}$ is metric connection, using (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $Z \in \Gamma(D_2)$, we obtain $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X \overline{J}Z, \overline{J}Y)$, which implies $\overline{g}(\nabla_X Y, Z) = \overline{g}(\nabla^s_X FZ + h^s(X, fZ), \overline{J}Y)$. In view of (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $N \in \Gamma(ltr(TM))$, we have $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}N)$, which gives $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, h^s(X, \overline{J}N))$. Now, from (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $W \in \Gamma(\overline{J}ltr(TM))$, we get $\overline{g}(\nabla_X Y, W) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}W)$, which implies $\overline{g}(\nabla_X Y, W) = \overline{g}(\overline{J}Y, D^s(X, \overline{J}W))$. This concludes the theorem.

Theorem 4.3. *Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_2 defines a totally geodesic foliation if and only if*

(i) $\overline{g}(A_{\overline{J}Z}X, fY) = \overline{g}(\nabla^s_X \overline{J}Z, FY)$,

(ii) $\overline{g}(fY, \nabla_X \overline{J}N) = -\overline{g}(FY, h^s(X, \overline{J}N))$,

(iii) $\overline{g}(fY, A_{\overline{J}W}X) = \overline{g}(FY, D^s(X, \overline{J}W))$,

for all $X, Y \in \Gamma(D_2)$, $Z \in \Gamma(D_1)$, $N \in \Gamma(ltr(TM))$, $W \in \Gamma(\overline{J}ltr(TM))$.

Proof. Let M be a pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. The distribution D_2 defines a totally geodesic foliation if and only if $\nabla_X Y \in \Gamma(D_2)$, for all $X, Y \in \Gamma(D_2)$. Since $\overline{\nabla}$ is metric connection, using (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_2)$ and $Z \in \Gamma(D_1)$, we get $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X \overline{J}Z, \overline{J}Y)$, which gives $\overline{g}(\nabla_X Y, Z) = \overline{g}(A_{\overline{J}Z}X, fY) - \overline{g}(\nabla^s_X \overline{J}Z, FY)$. In view of (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_2)$ and $N \in \Gamma(ltr(TM))$, we have $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}N)$, which implies $\overline{g}(\nabla_X Y, N) = -\overline{g}(fY, \nabla_X \overline{J}N) - \overline{g}(FY, h^s(X, \overline{J}N))$. Now, from (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_2)$

and $W \in \Gamma(\overline{J}ltr(TM))$, we have $\overline{g}(\nabla_X Y, W) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}W)$, which gives $\overline{g}(\nabla_X Y, W) = \overline{g}(fY, A_{\overline{J}W}X) - \overline{g}(FY, D^s(X, \overline{J}W))$. Thus, we obtain the required results.

Theorem 4.4. *Let M be a mixed geodesic pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 defines a totally geodesic foliation if and only if $\nabla^s_X FZ$, $h^s(X, \overline{J}N)$ and $D^s(X, \overline{J}W)$ have no components in $\overline{J}(D_1)$, for all $X \in \Gamma(D_1)$, $Z \in \Gamma(D_2)$, $N \in \Gamma(ltr(TM))$ and $W \in \Gamma(\overline{J}ltr(TM))$.*

Proof. Let M be a mixed geodesic pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $h(X, Y) = 0$, for all $X \in \Gamma(D_1)$ and for all $Y \in \Gamma(D_2)$. The distribution D_1 defines a totally geodesic foliation if and only if $\nabla_X Y \in \Gamma(D_1)$, for all $X, Y \in \Gamma(D_1)$. Since $\overline{\nabla}$ is metric connection, using (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $Z \in \Gamma(D_2)$, we get $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X \overline{J}Z, \overline{J}Y)$, which gives $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\nabla^s_X FZ + h^s(X, fZ), \overline{J}Y)$. In view of (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $N \in \Gamma(ltr(TM))$, we obtain $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}N)$, which implies $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, h^s(X, \overline{J}N))$. Now, from (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $W \in \Gamma(\overline{J}ltr(TM))$, we have $\overline{g}(\nabla_X Y, W) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}W)$, which gives $\overline{g}(\nabla_X Y, W) = \overline{g}(\overline{J}Y, D^s(X, \overline{J}W))$. This proves the theorem.

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