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A generalized class of difference type estimators for population median in survey sampling

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Abstract

In this paper, we propose a generalized class of difference type estimators of finite population median in simple and stratified random sampling. The expressions for bias and mean square error are derived up to first order of approximation. Numerical comparisons reveal that the proposed class of estimators performs better than the unbiased sample median estimator, ratio estimator, exponential estimator, usual difference estimator, Rao [10] estimator and other difference type estimators.

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1. Introduction

Several authors have developed some estimators for the finite population mean under different sampling schemes. However lesser degree of attention has been paid to estimation of median. Kuk and Mak [8] introduced a median estimator that makes use of the auxiliary information. Gupta et al. [5] have suggested a class of estimators for population median using two auxiliary variables. Other important contributions in this area include Al and Cingi [2], Singh and Solanki [14], Jhajj et al. [7], Sharma and Singh [12], Solanki and Singh [15] and Aladag and Cingi [3].

In this paper we consider the problem of median estimation for finite population and propose a generalized class of difference type estimators that makes use of the auxiliary information in simple and stratified random sampling.

Consider a finite population with N units. Let y_i and x_i $(i = 1, 2, ..., N)$ be the values on the ith unit for the study variable $(Y)$and the auxiliary variable (X) respectively. Let

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us draw a sample of size n from this population by using simple random sampling without replacement. Let M_y and M_x respectively be the population medians and $\hat{M}_y$ and $\hat{M}_x$ respectively be the sample medians for Y and X. Let the correlation coefficient between $(\hat{M}_y, \hat{M}_y)$ be $\rho_{(\hat{M}_y,\hat{M}_y)} = \rho_c = 4P_{11}(y,x) - 1$, where $P_{11}(y,x) = P(Y \leq M_y \cap X \leq M_x)$. It is assumed that the limiting distribution of (Y, X) is a continuous distribution with marginal densities $f_y(y)$ and $f_x(x)$ for Y and X respectively. It is further assumed that $f_y(M_y)$ and $f_x(M_x)$ are positive.

To obtain the properties of the proposed median estimator, we define the following error terms. Let $e_0 = (\hat{M}_y - M_y)/M_y$ and $e_1 = (\hat{M}_x - M_x)/M_x$ such that $E(e_0) = E(e_1) = 0$. To first degree of approximation, we have $E(e_0^2) = \lambda C_{My}^2$, $E(e_1^2) = \lambda C_{Mx}^2$, $E(e_0e_1) = \lambda C_{Myx}$, where $C_{My} = 1/[M_y f_y(M_y)]$,
$C_{Mx} = 1/[M_x f_x(M_x)]$, $C_{Myx} = \rho_c C_{My} C_{Mx}$ and $\lambda = \frac{1}{4}\left(\frac{1}{n} - \frac{1}{N}\right)$.

2. Some existing median estimators in simple random sampling

In this section, we discuss some of the existing estimators of population median (M_y). All expressions are given to first degree approximation.
The most common median estimator is the sample median $(\hat{M}_y)$ whose variance is, given by

$$Var(\hat{M}_y) = \lambda M_y^2 C_{My}^2 = MSE(\hat{M}_y). \tag{2.1}$$

Kuk and Mak [8] have introduced the following ratio estimator:

$$\hat{M}_R = \hat{M}_y \left(\frac{M_x}{\hat{M}_x}\right), \tag{2.2}$$

where M_x is known.
The bias and MSE of $\hat{M}_R$, are given by

$$Bias(\hat{M}_R) \cong \lambda M_y \left(C_{Mx}^2 - C_{Myx}\right) \tag{2.3}$$

and

$$MSE(\hat{M}_R) \cong \lambda M_y^2 \left(C_{My}^2 + C_{Mx}^2 - 2C_{Myx}\right). \tag{2.4}$$

The exponential ratio type estimator is given by

$$\hat{M}_{EX} = \hat{M}_y exp\left(\frac{M_x - \hat{M}_x}{M_x + \hat{M}_x}\right). \tag{2.5}$$

The bias and MSE of $\hat{M}_{EX}$, are given by

$$Bias(\hat{M}_{EX}) \cong \lambda M_y \left(\frac{3}{8}C_{Mx}^2 - \frac{1}{2}C_{Myx}\right) \tag{2.6}$$

and

$$MSE(\hat{M}_{EX}) \cong \lambda M_y^2 \left(C_{My}^2 + \frac{1}{4}C_{Mx}^2 - C_{Myx}\right). \tag{2.7}$$

An unbiased difference estimator $(\hat{M}_D)$, is given by

$$\hat{M}_D = \hat{M}_y + d\left(M_x - \hat{M}_x\right), \tag{2.8}$$

where d is an unknown constant.
The minimum MSE of $\hat{M}_D$, at optimum value of d i.e. $d_{opt} = \frac{M_y \rho_c C_{My}}{M_x C_{Mx}}$ is given by

$$MSE(\hat{M}_D)_{min} = \lambda M_y^2 C_{My}^2 \left(1 - \rho_c^2\right). \tag{2.9}$$

The minimum MSE of $\hat{M}_D$ is always smaller than the sample median estimator $(\hat{M}_y)$, ratio estimator $(\hat{M}_R)$ and exponential type estimator $(\hat{M}_{EX})$.
Some more difference type estimators $\hat{M}_{Di}(i = 1, 2, 3)$ which are similar to Rao [10], Gupta et al. [5] and Shabbir and Gupta [11] estimators respectively, are given by

$$\hat{M}_{D1} = d_1\hat{M}_y + d_2\left(M_x - \hat{M}_x\right), \tag{2.10}$$

$$\hat{M}_{D2} = \left[d_3\hat{M}_y + d_4(M_x - \hat{M}_x)\right]\left(\frac{M_x}{\hat{M}_x}\right), \tag{2.11}$$

$$\hat{M}_{D3} = \left[d_5\hat{M}_y + d_6(M_x - \hat{M}_x)\right] exp\left(\frac{M_x - \hat{M}_x}{M_x + \hat{M}_x}\right), \tag{2.12}$$

where $d_i(i = 1, 2, \ldots, 6)$ are unknown constants whose optimal values are to be determined.
The biases and minimum $MSEs$ of $\hat{M}_{Di}(i = 1, 2, 3)$, are given by

$$Bias(\hat{M}_{D1}) \cong M_y(d_1 - 1), \tag{2.13}$$

$$Bias(\hat{M}_{D2}) \cong (d_3 - 1)M_y + d_3\lambda M_y(C^2_{Mx} - C_{Myx}) + d_4M_x\lambda C^2_{Mx}, \tag{2.14}$$

$$Bias(\hat{M}_{D3}) \cong (d_5 - 1)M_y + d_5\lambda M_y\left\{\frac{3}{8}C^2_{Mx} - \frac{1}{2}C_{Myx}\right\} + \frac{1}{2}d_6M_x\lambda C^2_{Mx}, \tag{2.15}$$

$$MSE(\hat{M}_{D1})_{min} \cong \frac{M_y\lambda C^2_{My}(1 - \rho^2_c)}{1 + \lambda C^2_{My}(1 - \rho^2_c)}, \tag{2.16}$$

$$MSE(\hat{M}_{D2})_{min} \cong M^2_y\left[(1 - \lambda C^2_{Mx}) - \frac{(1 - \lambda C^2_{Mx})^2}{(1 - \lambda C^2_{Mx}) + \lambda C^2_{My}(1 - \rho^2_c)}\right]$$

or

$$MSE(\hat{M}_{D2})_{min} \cong M^2_y\left[\frac{(1 - \lambda C^2_{Mx})\lambda C^2_{My}(1 - \rho^2_c)}{(1 - \lambda C^2_{Mx}) + \lambda C^2_{My}(1 - \rho^2_c)}\right], \tag{2.17}$$

$$MSE(\hat{M}_{D3})_{min} \cong M^2_y\left[(1 - \frac{1}{4}\lambda C^2_{Mx}) - \frac{(1 - \frac{1}{8}\lambda C^2_{Mx})^2}{1 + \lambda C^2_{My}(1 - \rho^2_c)}\right]$$

or

$$MSE(\hat{M}_{D3})_{min} \cong M^2_y\left[\frac{\lambda C^2_{My}(1 - \rho^2_c) - \frac{1}{64}\lambda^2C^4_{Mx} - \frac{1}{4}\lambda^2C^2_{My}C^2_{Mx}(1 - \rho^2_c)}{1 + \lambda C^2_{My}(1 - \rho^2_c)}\right], \tag{2.18}$$

where optimum values of $d_i(i = 1, 2, \ldots, 6)$ are given by:
$d_{1(opt)} = \frac{1}{1+\lambda C^2_{My}(1-\rho^2_c)}$, $d_{2(opt)} = \frac{M_y}{M_x}\left[\frac{\rho_c C_{My}/C_{Mx}}{1+\lambda C^2_{My}(1-\rho^2_c)}\right]$, $d_{3(opt)} = \frac{1-\lambda C^2_{Mx}}{1-\lambda C^2_{Mx}+\lambda C^2_{My}(1-\rho^2_c)}$,
$d_{4(opt)} = \frac{M_y}{M_x}\left[1 + d_{3(opt)}\left(\frac{\rho_c C_{My}}{C_{Mx}} - 2\right)\right]$, $d_{5(opt)} = \frac{1-(\lambda C^2_{Mx}/8)}{1+\lambda C^2_{My}(1-\rho^2_c)}$,
$d_{6(opt)} = \frac{M_y}{M_x}\left[\frac{1}{2} + d_{5(opt)}\left(\frac{\rho_c C_{My}}{C_{Mx}} - 1\right)\right]$.
We can get the corresponding bias of $\hat{M}_{Di}(i = 1, 2, 3)$ by substituting the optimum values of $d_i(i = 1, 2, \ldots, 6)$ in Eqs (2.13)–(2.15).

3. Proposed median estimator in simple random sampling

Motivated by Singh and Solanki [14], Jhajj et al. [7], Sharma and Singh [12] and Solanki and Singh [15], we propose the following generalized difference type estimator of M_y.

$$\hat{M}_P^G = \left[m_1\hat{M}_y + m_2(M_x - \hat{M}_x)\right]$$

$$\left[\left(\frac{aM_x + b}{a\hat{M}_x + b}\right)^{\alpha_1} exp\left\{\frac{\alpha_2 a(M_x - \hat{M}_x)}{a\{(\gamma-1)M_x + \hat{M}_x\} + 2b}\right\}\right], \tag{3.1}$$

where a and b are the known population parameters.; m_1 and m_2 are unknown constants whose values are to be determined and α_1 , α_2 and γ are scalar quantities which can take different values.

Note: By substituting different values of α_1, α_2, γ, a, b, we can obtain many estimators as described earliar.

Let substitute $\alpha_1 = b = 0$, $\alpha_2 = \gamma = a = 1$ in Eq.(3.1), the class of generalized type estimators becomes

$$\hat{M}_{PP}^G = \left[m_1\hat{M}_y + m_2(M_x - \hat{M}_x)\right]\left[exp\left(\frac{M_x}{\hat{M}_x} - 1\right)\right]. \tag{3.2}$$

Solving Eq.(3.2), the bias and minimum MSE of $\hat{M}_{PP}^G$ at optimum values: $m_{1(opt)} = \frac{1-\frac{1}{2}\lambda C_{Mx}^2}{1+\lambda C_{My}^2(1-\rho_c^2)}$ and $m_{2(opt)} = \frac{M_y}{M_x}\left[1 + m_{1(opt)}\{\frac{\rho_c C_{My}}{C_{Mx}} - 2\}\right]$, are given by

$$Bias(\hat{M}_{PP}^G) \cong (m_1 - 1)M_y + m_2 M_y\lambda\left\{\frac{3}{2}C_{Mx}^2 - C_{My}\right\} + m_2 M_x \lambda C_{Mx}^2 \tag{3.3}$$

and

$$MSE(\hat{M}_{PP}^G) \cong M_y^2\left[(1 - \lambda C_{Mx}^2) - \frac{\left\{1 - \frac{1}{2}\lambda C_{Mx}^2\right\}^2}{1 + \lambda C_{My}^2(1-\rho_c^2)}\right] \tag{3.4}$$

or

$$MSE(\hat{M}_{PP}^G) \cong M_y^2\left[\frac{\lambda C_{My}^2(1-\rho_c^2) - \frac{1}{4}\lambda^2 C_{Mx}^4 - \lambda^2 C_{Mx}^4 C_{Mx}^2(1-\rho_c^2)}{1 + \lambda C_{My}^2(1-\rho_c^2)}\right]. \tag{3.5}$$

4. Comparison of estimators in simple random sampling

In this section, we compare the mean square error of the new class of generalized difference type estimators $\hat{M}_{PP}^G$ at optimum condition with other existing estimators.

Condition (i)

By (2.1) and (3.5), $MSE(\hat{M}_{PP}^G)_{min} < MSE(\hat{M}_y)$ if

$\frac{1}{\theta_2}\left[\lambda C_{My}^2\rho_c^2 + \lambda^2\theta_1\right] > 0$,

where $\theta_1 = \frac{1}{4}C_{Mx}^4 + C_{My}^4(1-\rho_c^2) + C_{Mx}^2 C_{My}^2(1-\rho_c^2)$ and $\theta_2 = 1 + \lambda C_{My}^2(1-\rho_c^2)$.

Condition (ii)

By (2.4) and (3.5), $MSE(\hat{M}_{PP}^G)_{min} < MSE(\hat{M}_R)$ if

$\frac{1}{\theta_2}\left[\lambda(C_{Mx} - \rho_c C_{My})^2 + \lambda C_{My}^2(1-\rho_c^2)\frac{MSE(\hat{M}_R)}{M_y^2} + \lambda^2\theta_1\right] > 0.$

Condition (iii)

By (2.7) and (3.5), $MSE(\hat{M}_{PP}^G)_{min} < MSE(\hat{M}_{EX})$ if

$\frac{1}{\theta_2}\left[\lambda(\frac{1}{2}C_{Mx} - \rho_c C_{My})^2 + \lambda C_{My}^2(1-\rho_c^2)\frac{MSE(\hat{M}_{EX})}{M_y^2} + \lambda^2\theta_1\right] > 0.$

Condition (iv)
By (2.9) and (3.5), $MSE(\hat{M}_{PP}^{G})_{min} < MSE(\hat{M}_{D})_{min}$ if
$\frac{1}{\theta_2}\left[\left\{\lambda C_{My}^2(1-\rho_c^2)\right\}^2 + \lambda^2\theta_1\right] > 0.$

Condition (v)
By (2.16) and (3.5), $MSE(\hat{M}_{PP}^{G})_{min} < MSE(\hat{M}_{D1})_{min}$ if
$\frac{1}{\theta_2}\left[\lambda^2\theta_1\right] > 0.$

Condition (vi)
By (2.17) and (3.5), $MSE(\hat{M}_{PP}^{G})_{min} < MSE(\hat{M}_{D2})_{min}$ if
$\frac{1}{\theta_2\theta_3}\left[\lambda^2 C_{My}^2 C_{Mx}^2(1-\rho_c^2)(1-\frac{3}{4}\lambda C_{My}^2 C_{Mx}^2) + \frac{1}{4}\lambda^2 C_{Mx}^4(1-\lambda C_{Mx}^2)\right] > 0,$
where $\theta_3 = 1 - \lambda C_{Mx}^2 + \lambda C_{My}^2(1-\rho_c^2)$.

Condition (vii)
By (2.18) and (3.5), $MSE(\hat{M}_{PP}^{G})_{min} < MSE(\hat{M}_{D3})_{min}$ if
$\frac{1}{\theta_2}\left[(\frac{3}{4}\lambda^2 C_{Mx}^2)\left\{C_{My}^2(1-\rho_c^2) + \frac{5}{16}C_{Mx}^2\right\}\right] > 0.$

5. Numerical study in simple random sampling

In this section, we consider seven natural populations to perform a numerical comparison of different estimators.
Population I: Source: [PDS [6], Pages 114-116]
Let y be the number of teaching staff and x be the number of students in 4 different types of schools under 36 districts in Punjab province of Pakistan.
Population II: Source: [Singh [13]]
Let y be the number of fish caught in the year 1995 and x be the number of fish caught by the marine recreational fishermen in the previous year 1994 in USA.
Population III: Source: [Singh [13]]
Let y be the number of fish caught in the year 1995 and x be the number of fish caught by the marine recreational fishermen in the previous year 1993 in USA.
Population IV: Source: [Aladag and Cingi [3]]
Let y be the number of teachers and x be the number of students in elementary schools for 340 medium-developed districts in Turkey in 2007.
Population V: Source: [Chen et al. [4]; Al and Cingi [2]]
Let y be the entire height of conifer trees in feet and x be the diameter of conifer trees in centimeters at breast height.
Population VI: Source: [Aczel and Sounderpandian [1]]
Let y be the U.S. exports to Singapore in billions of Singapore dollars and x be the money supply figures in billions of Singapore dollars.
Population VII: Source: MFA [9]
Let y be the district-wise tomato production in tons in Pakistan in the year 2003 and x be the district-wise tomato production in tons in Pakistan in the year 2002.
The summary data of Populations (I−VII) are given in Table 1. We use the following expression to obtain the percent relative efficiency (PRE) of various estimators relative to the sample median i.e.

$$PRE = \frac{MSE(\hat{M}_y)}{MSE(.) \text{ or } MSE(.)_{min}} \times 100.$$

The Bias, MSE and PRE results are given in Tables 2-4 respectively. Based on the results in Tables 2-4, it is observed that the proposed estimator $\hat{M}_{PP}^{G}$ outperforms other competing estimators. Also in Table 2, the absolute bias of $\hat{M}_{PP}^{G}$ is smaller in most cases. The ratio estimator ($\hat{M}_R$) and exponential estimator ($\hat{M}_{EX}$) show poorest $PREs$ probably because of weaker correlation between the study variable and the auxiliary variable. The performance of the proposed estimator $\hat{M}_{PP}^G$ is not affected by this weak correlation.

Table 1. Summary statistics for seven populations.

Estimator	*Pop. I*	*Pop. II*	*Pop. III*	*Pop. IV*	*Pop. V*	*Pop. VI*	*Pop. VII*
N	144	69	69	340	396	67	97
n	10	17	17	150	65	23	46
M_y	2023	2068	2068	178	30	4.8	1242
M_x	64659	2011	2307	3526	14.6	7.0	1233
$f_y(M_y)$	0.00024	0.00014	0.00014	0.00182	0.01178	0.07630	0.00021
$f_x(M_x)$	0.00001	0.00014	0.00014	0.00008	0.02194	0.05260	0.00022
ρ_c	0.8611	0.1505	0.3136	0.92	0.84	0.6624	0.2096

Table 2. Bias of different estimators.

Estimator	*Pop. I*	*Pop. II*	*Pop. III*	*Pop. IV*	*Pop. V*	*Pop. VI*	*Pop. VII*
$\hat{M}_R$	-16.522	246.828	171.241	0.414	0.224	0.084	37.718
$\hat{M}_{EX}$	-22.332	87.271	53.769	-0.053	-0.005	0.011	12.829
$\hat{M}_{D1}$	-50.326	-236.651	-219.863	-0.242	-0.226	-0.139	-47.952
$\hat{M}_{D2}$	-50.253	-232.330	-216.626	-0.242	-0.226	-0.139	-47.878
$\hat{M}_{D3}$	-49.531	-227.820	-212.652	-0.241	-0.223	-0.137	-47.459
$\hat{M}_{PP}^G$	-45.999	-149.613	-185.758	-0.233	-0.211	-0.129	-45.640

6. Some median estimator in stratified random sampling

Stratified random sampling is commonly used when population is heterogeneous. Recently Aladag and Cingi [3] suggested some median estimators in stratified random sampling. We give below some notations and some of the existing median estimators in stratified sampling.

Consider a finite population $U = (1, 2, \ldots, N)$ of N identifiable units divided into L strata with the hth stratum $(h = 1, 2, ..., L)$ having N_h units such that $\sum_{h=1}^{L} N_h = N$. Let y_{hi} and x_{hi} be the values of the study variable Y_h and the auxiliary variable X_h respectively for the ith$(i = 1, 2, ..., N)$ population element of the hth

Table 3. MSE values of different estimators.

$Estimator$	$Pop.\ I$	$Pop.\ II$	$Pop.\ III$	$Pop.\ IV$	$Pop.\ V$	$Pop.\ VI$	$Pop.\ VII$
$\hat{M}_y$	403886.96	565443.57	565443.57	281.18	23.17	1.23	64795.08
$\hat{M}_R$	109312.95	988372.76	746752.56	57.87	8.43	0.82	98581.89
$\hat{M}_{EX}$	199668.00	627420.21	524362.05	76.80	8.75	0.72	66712.60
$\hat{M}_D$	104407.52	552636.13	508766.02	43.19	6.82	0.69	61948.49
$\hat{M}_{D1}$	101810.17	489395.24	454675.78	43.13	6.77	0.67	59556.73
$\hat{M}_{D2}$	101661.15	480458.29	447982.61	43.13	6.76	0.67	59463.97
$\hat{M}_{D3}$	100200.79	471131.76	439763.44	42.94	6.70	0.66	58943.58
$\hat{M}_{PP}^G$	93055.81	402459.28	384146.79	41.78	6.34	0.62	56684.80

Table 4. PRE of different estimators.

$Estimator$	$Pop.\ I$	$Pop.\ II$	$Pop.\ III$	$Pop.\ IV$	$Pop.\ V$	$Pop.\ VI$	$Pop.\ VII$
$\hat{M}_y$	100.00	100.00	100.00	100.00	100.00	100.00	100.00
$\hat{M}_R$	369.478	57.210	75.720	485.914	274.799	148.891	65.727
$\hat{M}_{EX}$	202.279	90.122	107.835	366.094	264.860	169.932	97.126
$\hat{M}_D$	386.837	102.318	111.140	651.042	339.674	178.181	104.595
$\hat{M}_{D1}$	396.608	115.539	124.362	651.929	342.246	183.503	108.796
$\hat{M}_{D2}$	397.287	117.688	126.220	651.940	342.331	183.799	108.965
$\hat{M}_{D3}$	403.078	120.018	128.579	654.512	345.654	186.224	109.927
$\hat{M}_{PP}^G$	434.027	140.497	147.195	676.860	365.545	198.566	114.308

stratum. Let $\hat{M}_{yh}$ and $\hat{M}_{xh}$ be the sample medians respectively corresponding to population medians $\hat{M}_{yh}$ and $\hat{M}_{xh}$ in the hth stratum. Let $\hat{M}_{yst} = \sum_{h=1}^{L} W_h \hat{M}_{yh}$ and $\hat{M}_{xst} = \sum_{h=1}^{L} W_h \hat{M}_{xh}$ be weighted sample medians respectively corresponding to population medians $M_y = M_{yst} = \sum_{h=1}^{L} W_h M_{yh}$ and $M_x = M_{xst} = \sum_{h=1}^{L} W_h M_{xh}$, where $W_h = N_h/N$ is the known stratum weight. Let the correlation coefficient between$(\hat{M}_{yh}, \hat{M}_{xh})$ be $\rho_{(\hat{M}_{yh}, \hat{M}_{xh})} = \rho_{ch} = 4P_{11h}(y_h, x_h) - 1$, where $P_{11h}(y_h, x_h) = P(Y_h \leq M_{yh} \cap X_h \leq M_{xh})$. It is assumed that the distribution of (Y_h, X_h) is a continuous distribution with marginal densities $f_{yh}(y_h)$ and $f_{xh}(x_h)$ for Y_h and X_h respectively. It is further assumed that $f_{yh}(M_{yh})$ and $f_{xh}(M_{xh})$ are positive.

To obtain expressions for the biases and $MSEs$ of different estimators, we use the following error terms. Let $e_{0h} = (\hat{M}_{yh} - M_{yh})/M_{yh}$ and $e_{1h} = (\hat{M}_{xh} - M_{xh})/M_{xh}$ such that $E(e_{0h}) = E(e_{1h}) = 0$. To first degree of approximation, we have $E(e_{0h}^2) = \lambda_h C_{Myh}^2$, $E(e_{1h}^2) = \lambda_h C_{Mxh}^2$, $E(e_{0h}e_{1h}) = \lambda_h C_{Myxh}$, where $C_{Myh} = 1/[M_{yh} f_{yh}(M_{yh})]$, $C_{Mxh} = 1/[M_{xh} f_{xh}(M_{xh})]$, $C_{Myxh} = \rho_{ch} C_{Myh} C_{Mxh}$ and $\lambda_h = \frac{1}{4}\left(\frac{1}{nh} - \frac{1}{Nh}\right)$.
The variance of the usual sample median $(\hat{M}_{yst})$ as an estimator of the population median M_y, is given by

$$(6.1) \qquad Var(\hat{M}_{yst}) = \sum_{h=1}^{L} W_h^2 \lambda_h M_{yh}^2 C_{Myh}^2 = MSE(\hat{M}_{yst}).$$

The traditional ratio estimator in stratified sampling, is given by

$$(6.2) \qquad \hat{M}_{Rs} = \sum_{h=1}^{L} W_h^2 M_{yh} \left(\frac{M_{xh}}{\hat{M}_{xh}}\right),$$

where M_{xh} is known for all strata.
The bias and MSE of $\hat{M}_{Rs}$, are given by

$$(6.3) \qquad Bias(\hat{M}_{Rs}) \cong \sum_{h=1}^{L} W_h \lambda_h M_{yh} \left\{C_{Mxh}^2 - C_{Myxh}\right\},$$

and

$$(6.4) \qquad MSE(\hat{M}_{Rs}) \cong \sum_{h=1}^{L} W_h^2 \lambda_h M_{yh}^2 \left\{C_{Myh}^2 + C_{Mxh}^2 - 2C_{Myxh}\right\}.$$

The exponential ratio type estimator in stratified sampling, is given by

$$(6.5) \qquad \hat{M}_{EXs} = \sum_{h=1}^{L} W_h^2 \hat{M}_{yh} exp\left(\frac{M_{xh} - \hat{M}_{xh}}{\hat{M}_{xh} + \hat{M}_{xh}}\right).$$

The bias and MSE of $\hat{M}_{EXs}$, are given by

$$(6.6) \qquad Bias(\hat{M}_{EXs}) \cong \sum_{h=1}^{L} W_h \lambda_h M_{yh} \left\{\frac{3}{8} C_{Mxh}^2 - \frac{1}{2} C_{Myxh}\right\},$$

and

$$(6.7) \qquad MSE(\hat{M}_{EXs}) \cong \sum_{h=1}^{L} W_h^2 \lambda_h M_{yh}^2 \left\{C_{Myh}^2 + \frac{1}{4} C_{Mxh}^2 - C_{Myxh}\right\}.$$

The unbiased difference estimator $\hat{M}_{Ds}$, is given by

$$(6.8) \qquad \hat{M}_{Ds} = \sum_{h=1}^{L} W_h \left[\hat{M}_{yh} + d_h \left(M_{xh} - \hat{M}_{xh}\right)\right],$$

where d_h is an unknown constant.
The minimum MSE of $\hat{M}_{Ds}$, at optimum values of d_h i.e. $d_{h(opt)} = \frac{M_{yh}\rho_{ch}C_{Myh}}{M_{xh}C_{Mxh}}$, is given by

$$(6.9) \qquad MSE(\hat{M}_{Ds})_{min} \cong \sum_{h=1}^{L} W_h^2 \lambda_h M_{yh}^2 C_{Myh}^2 \left(1 - \rho_{ch}^2\right).$$

The minimum MSE of $\hat{M}_{Ds}$ is always smaller than the sample median estimator $(\hat{M}_{yst})$, ratio estimator $(\hat{M}_{Rs})$ and exponential type estimator $(\hat{M}_{EXs})$.

Some more difference type estimators $\hat{M}_{Dis}(i=1,2,3)$ in strtified random sampling, are given by:

$$\hat{M}_{D1s}=\sum_{h=1}^{L}W_h\left[d_{1h}\hat{M}_{yh}+d_{2h}\left(M_{xh}-\hat{M}_{xh}\right)\right], \tag{6.10}$$

$$\hat{M}_{D2s}=\sum_{h=1}^{L}W_h\left[d_{3h}\hat{M}_{yh}+d_{4h}(M_{xh}-\hat{M}_{xh})\right]\left(\frac{M_{xh}}{\hat{M}_{xh}}\right), \tag{6.11}$$

$$\hat{M}_{D3s}=\sum_{h=1}^{L}W_h\left[d_{5h}\hat{M}_{yh}+d_{6h}(M_{xh}-\hat{M}_{xh})\right]exp\left(\frac{M_{xh}-\hat{M}_{xh}}{M_{xh}+\hat{M}_{xh}}\right), \tag{6.12}$$

where $d_{ih}(i=1,2,\ldots,6)$ are unknown constants whose values are to be determined. The biases and minimum $MSEs$ of $\hat{M}_{Dis}(i=1,2,3)$, are given by

$$Bias(\hat{M}_{D1s})\cong\sum_{h=1}^{L}W_h\hat{M}_{yh}\left(d_{1h}-1\right), \tag{6.13}$$

$$Bias(\hat{M}_{D2s})\cong$$

$$\sum_{h=1}^{L}W_h\left[(d_{3h}-1)\hat{M}_{yh}+d_{3h}\lambda_hM_{yh}\left\{C^2_{Mxh}-C_{Myxh}\right\}+d_{4h}M_{xh}\lambda_hC^2_{Mxh}\right], \tag{6.14}$$

$$Bias(\hat{M}_{D3s})\cong \tag{6.15}$$

$$\sum_{h=1}^{L}W_h\left[(d_{5h}-1)\hat{M}_{yh}+d_{5h}\lambda_hM_{yh}\left\{\frac{3}{8}C^2_{Mxh}-\frac{1}{2}C_{Myxh}\right\}+\frac{1}{2}d_{6h}M_{xh}\lambda_hC^2_{Mxh}\right]$$

$$MSE(\hat{M}_{D1s})_{min}\cong\sum_{h=1}^{L}W_h^2M_{yh}^2\lambda_h\left(\frac{C^2_{Myh}(1-\rho^2_{ch})}{1+\lambda_hC^2_{Myh}(1-\rho^2_{ch})}\right), \tag{6.16}$$

$$MSE(\hat{M}_{D2s})_{min}$$

$$\cong\sum_{h=1}^{L}W_h^2M_{yh}^2\left[(1-\lambda_hC^2_{Mxh})-\frac{(1-\lambda_hC^2_{Mxh})^2}{(1-\lambda_hC^2_{Mxh})+\lambda_hC^2_{Myh}(1-\rho^2_{ch})}\right],$$

or

$$MSE(\hat{M}_{D2s})_{min}\cong\sum_{h=1}^{L}W_h^2M_{yh}^2\left[\frac{(1-\lambda_hC^2_{Mxh})\lambda_hC^2_{My}(1-\rho^2_{ch})}{(1-\lambda_hC^2_{Mxh})+\lambda_hC^2_{Myh}(1-\rho^2_{ch})}\right], \tag{6.17}$$

$$MSE(\hat{M}_{D3s})_{min}\cong\sum_{h=1}^{L}W_h^2M_{yh}^2\left[(1-\frac{1}{4}\lambda_hC^2_{Mxh})-\frac{(1-\frac{1}{8}\lambda_hC^2_{Mxh})^2}{1+\lambda_hC^2_{Myh}(1-\rho^2_{ch})}\right],$$

or

$$MSE(\hat{M}_{D3s})_{min} \tag{6.18}$$

$$\cong\sum_{h=1}^{L}W_h^2M_{yh}^2\left[\frac{\lambda_hC^2_{Myh}(1-\rho^2_{ch})-\frac{1}{64}\lambda_h^2C^4_{Mxh}-\frac{1}{4}\lambda_h^2C^2_{Myh}C^2_{Mxh}(1-\rho^2_{ch})}{1+\lambda_hC^2_{Myh}(1-\rho^2_{ch})}\right].$$

The optimum values of $d_{ih}(i=1,2,\ldots,6)$, are given by:

$d_{1h(opt)}=\frac{1}{1+\lambda_hC^2_{Myh}(1-\rho^2_{ch})}$, $d_{2h(opt)}=\frac{M_{yh}}{M_{xh}}\left[\frac{\rho_cC_{Myh}/C_{Mxh}}{1+\lambda_hC^2_{Myh}(1-\rho^2_{ch})}\right]$,

$d_{3h(opt)} = \frac{1-\lambda_h C^2_{Mxh}}{1-\lambda_h C^2_{Mxh}+\lambda_h C^2_{Myh}(1-\rho^2_{ch})}$, $d_{4h(opt)} = \frac{M_{yh}}{M_{xh}}\left[1 + d_{3h(opt)}\{\frac{\rho_{ch}C_{Myh}}{C_{Mxh}} - 2\}\right]$,
$d_{5h(opt)} = \frac{1-(\lambda_h C^2_{Mxh}/8)}{1+\lambda_h C^2_{Myh}(1-\rho^2_{ch})}$, $d_{6h(opt)} = \frac{M_{yh}}{M_{xh}}\left[\frac{1}{2} + d_{5h(opt)}\left(\frac{\rho_{ch}C_{Myh}}{C_{Mxh}} - 1\right)\right]$.

7. Proposed median estimator in stratified random sampling

Motivated by Singh and Solanki [14], Jhajj et al. [7], Sharma and Singh [12], Aladag and Cingi [3] and Solanki and Singh [15], we propose the following general class of difference type estimators for M_y in stratified random sampling:

$$\hat{M}^G_{Ps} = \sum_{h=1}^{L} W_h\left[m_{1h}\hat{M}_{yh} + m_2(M_{xh} - \hat{M}_{xh})\right]$$

$$\times\left[\left(\frac{a_h M_{xh} + b_h}{a_h\hat{M}_{xh} + b_h}\right)^{\alpha_{1h}} exp\left\{\frac{\alpha_{2h}a_h(M_{xh} - \hat{M}_{xh})}{a_h\{(\gamma_h - 1)M_{xh} + \hat{M}_{xh}\} + 2b_h}\right\}\right] \tag{7.1}$$

where a_h and b_h are the known population parameters.; m_{1h} and m_{2h} are unknown constants whose values are to be determined and α_{1h} , α_{2h} and γ_h are scalar quantities which can take different values.
Note: By substituting different values of α_{1h}, α_2, γ, a, b, we can generate many estimators.
Putting $\alpha_{1h} = b_h = 0$, $\alpha_{2h} = \gamma_h = a_h = 1$ in Eq.(7.1), a new class of generalized difference type estimators in stratified sampling becomes:

$$\hat{M}^G_{PPs} = \sum_{h=1}^{L} W_h\left[m_{1h}\hat{M}_{yh} + m_{2h}(M_{xh} - \hat{M}_{xh})\right]\left[exp\left(\frac{M_{xh}}{\hat{M}_{xh}} - 1\right)\right]. \tag{7.2}$$

Solving Eq.(7.2), the bias and minimum MSE of $\hat{M}^G_{PPs}$ at optimum values: $m_{1h(opt)} = \frac{1-\frac{1}{2}\lambda_h C^2_{Mxh}}{1+\lambda C^2_{Myh}(1-\rho^2_{ch})}$ and $m_{2h(opt)} = \frac{M_{yh}}{M_{xh}}\left[1 + m_{1h(opt)}\{\frac{\rho_{ch}C_{Myh}}{C_{Mxh}} - 2\}\right]$, are given by

$$Bias(\hat{M}^G_{PPs})$$

$$\cong \sum_{h=1}^{L} W_h\left[(m_{1h} - 1)M_{yh} + m_2 M_{yh}\lambda_h\left\{\frac{3}{2}C^2_{Mxh} - C_{Myh}\right\} + m_{2h}M_{xh}\lambda C^2_{Mxh}\right] \tag{7.3}$$

and

$$MSE(\hat{M}^G_{PPs}) \cong \sum_{h=1}^{L} W_h^2 M_{yh}^2\left[(1 - \lambda_h C^2_{Mxh}) - \frac{(1 - \frac{1}{2}\lambda_h C^2_{Mxh})^2}{1 + \lambda_h C^2_{Myh}(1 - \rho^2_{ch})}\right]$$

or

$$MSE(\hat{M}^G_{PPs})$$

$$\cong \sum_{h=1}^{L} W_h^2 M_{yh}^2\left[\frac{\lambda_h C^2_{Myh}(1 - \rho^2_{ch}) - \frac{1}{4}\lambda_h^2 C^4_{Mxh} - \lambda_h^2 C^4_{Mxh}C^2_{Mxh}(1 - \rho^2_{ch})}{1 + \lambda_h C^2_{Myh}(1 - \rho^2_{ch})}\right]. \tag{7.4}$$

8. Comparison of estimators in stratified random sampling

In this section, we compare the proposed class of generalized difference type estimators $\hat{M}^G_{PPs}$ with other existing estimators.

Condition (i)
By (6.1) and (7.4), $MSE(\hat{M}^G_{PPs})_{min} < MSE(\hat{M}_{yst})$ if
$\sum_{h=1}^{L} W_h^2 M_{yh}^2 \frac{1}{\theta_{2h}}\left[\lambda_h C^2_{Myh}\rho^2_{ch} + \lambda_h^2\theta_{1h}\right] > 0$,
where $\theta_{1h} = C^2_{Myh}(1 - \rho^2_{ch}) + C^2_{Mxh}C^2_{Myh}(1 - \rho^2_{ch}) + \frac{1}{4}C^4_{Mxh}$ and

$\theta_{2h} = 1 + \lambda_h C^2_{Myh}(1 - \rho^2_{ch})$.

Condition (ii)
By (6.4) and (7.4), $MSE(\hat{M}^G_{PPs})_{min} < MSE(\hat{M}_{Rs})$ if
$\sum_{h=1}^{L} W_h^2 M_{yh}^2 \frac{1}{\theta_{2h}} \left[\lambda_h (C_{Mxh} - \rho_{ch} C_{Myh})^2 + \lambda_h^2 (\theta^*_{1h} + \theta_{3h})\right] > 0.$
where $\theta^*_{1h} = C^2_{Myh} C^2_{Mxh}(1 - \rho^2_{ch}) + \frac{1}{4} C^4_{Mxh}$ and
$\theta_{3h} = C^2_{Myh}(1 - \rho^2_{ch}) \left(C^2_{Myh} + C^2_{Mxh} - 2C_{Myxh}\right)$.

Condition (iii)
By (6.7) and (7.4), $MSE(\hat{M}^G_{PPs})_{min} < MSE(\hat{M}_{EXs})$ if
$\sum_{h=1}^{L} W_h^2 M_{yh}^2 \frac{1}{\theta_{2h}} \left[\lambda_h (\frac{1}{2} C_{Mxh} - \rho_{ch} C_{Myh})^2 + \lambda_h^2 (\theta^*_{1h} + \theta_{4h})\right] > 0.$
where $\theta_{4h} = C^2_{Myh}(1 - \rho^2_{ch}) \left(C^2_{Myh} + \frac{1}{4} C^2_{Mxh} - C_{Myxh}\right)$.

Condition (iv)
By (6.9) and (7.4), $MSE(\hat{M}^G_{PPs})_{min} < MSE(\hat{M}_{Ds})_{min}$ if
$\sum_{h=1}^{L} W_h^2 M_{yh}^2 \frac{1}{\theta_{2h}} \left[\{\lambda_h C^2_{Myh}(1 - \rho^2_{ch})\}^2 + \lambda_h^2 \theta^*_{1h}\right] > 0.$

Condition (v)
By (6.16) and (7.4), $MSE(\hat{M}^G_{PPs})_{min} < MSE(\hat{M}_{D1s})_{min}$ if
$\sum_{h=1}^{L} W_h^2 M_{yh}^2 \frac{1}{\theta_{2h}} \left(\lambda_h^2 \theta^*_{1h}\right) > 0.$

Condition (vi)
By (6.17) and (7.4), $MSE(\hat{M}^G_{PPs})_{min} < MSE(\hat{M}_{D2s})_{min}$ if
$\sum_{h=1}^{L} W_h^2 M_{yh}^2 \frac{\lambda_h^2}{\theta_{2h}\theta_{5h}} \left[C^2_{Myh} C^2_{Mxh}(1 - \rho^2_{ch})(1 - \frac{3}{4}\lambda_h C^2_{Mx}) + \theta_{6h}\right] > 0$
where $\theta_{5h} = 1 - \lambda_h C^2_{Mxh} + \lambda_h C^2_{Myh}(1 - \rho^2_{ch})$ and
$\theta_{6h} = \frac{1}{4} C^4_{Mxh}(1 - \lambda_h C^2_{Mxh})$.

Condition (vii)
By (6.18) and (7.4), $MSE(\hat{M}^G_{PPs})_{min} < MSE(\hat{M}_{D3s})_{min}$ if
$\sum_{h=1}^{L} W_h^2 M_{yh}^2 \frac{1}{\theta_{2h}} \left[(\frac{3}{4}\lambda_h^2 C^2_{Mxh}) \left\{C^2_{Myh}(1 - \rho^2_{ch}) + \frac{5}{16} C^2_{Mxh}\right\}\right] > 0.$

9. A numerical study in stratified random sampling

In this section, we consider the following two populations to perform a numerical comparison of different estimators in stratified random sampling.
Population I: [source: PDS [6]]
Let y be the number of teaching staff as the study variable and x be the number of students as the auxiliary variable in 4 different types of schools under 36 districts in Punjab (Pakistan). Equal allocation was used to obtain the sample size in each stratum.
Population II: [source: [Aladag and Cingi [3]]
Let y be the number of teachers as the study variable and x be the number of students as the auxiliary variable in both primary and secondary schools for 923 districts in 6 regions (as 1=Marmara, 2=Agean, 3=Mediterranean, 4=Central Anatolia, 5=Black Sea, 6=East and South Anatolia) in Turkey in 2007. The Neyman allocation was used for allocating the samples to different strata. The descriptive statistics are given in Tables 5 and 6. We use the following expression to obtain the percent relative efficiency (PRE) of various estimators with respect to the sample median i.e.

$$PRE = \frac{MSE(\hat{M}_{yst})}{MSE(.) \text{ or } MSE(.)_{min}} \times 100.$$

Table 5. Data statistics for population I under stratified random sampling.

$N = 144$	$N_1 = 36$	$N_2 = 36$	$N_3 = 36$	$N_4 = 36$
$n = 20$	$n_1 = 5$	$n_2 = 5$	$n_3 = 5$	$n_4 = 5$
$M_{y1} = 38$	$M_{y2} = 3056$	$M_{y3} = 2033$	$M_{y4} = 2382$	$M_{x1} = 1480$
$M_{x2} = 127289$	$M_{x3} = 54559$	$M_{x4} = 71615$	$f_{y1}(M_{y1}) = 0.007056$	$f_{y2}(M_{y2}) = 0.0003202$
$f_{y3}(M_{y3}) = 0.0004219$	$f_{y4}(M_{y4}) = 0.0003012$	$f_{x1}(M_{x1}) = 0.0001641$	$f_{x2}(M_{x2}) = 0.000007827$	$f_{x3}(M_{x3}) = 0.0000141$
$f_{x4}(M_{x4}) = 0.00001026$	$\rho_{c1} = 0.7776$	$\rho_{c2} = 0.8888$	$\rho_{c3} = 0.8888$	$\rho_{c4} = 0.8888$

Table 6. Data statistics for population II under stratified random sampling.

$N = 923$	$N_1 = 91$	$N_2 = 129$	$N_3 = 204$	$N_4 = 145$
$N_5 = 184$	$N_6 = 170$	$n = 177$	$n_1 = 18$	$n_2 = 26$
$n_3 = 41$	$n_4 = 29$	$n_5 = 29$	$n_6 = 34$	$M_{y1} = 81$
$M_{y2} = 93$	$M_{y3} = 24$	$M_{y4} = 54$	$M_{y5} = 44$	$M_{y6} = 101$
$M_{x1} = 1265$	$M_{x2} = 1139$	$M_{x3} = 614$	$M_{x4} = 763$	$M_{x5} = 533$
$M_{x6} = 911$	$f_{y1}(M_{y1}) = 0.003160$	$f_{y2}(M_{y2}) = 0.003180$	$f_{y3}(M_{y3}) = 0.011510$	$f_{y4}(M_{y4}) = 0.000299$
$f_{y5}(M_{y5}) = 0.005120$	$f_{y6}(M_{y6}) = 0.000249$	$f_{x1}(M_{x1}) = 0.000190$	$f_{x2}(M_{x2}) = 0.000240$	$f_{x3}(M_{x3}) = 0.000468$
$f_{x4}(M_{x4}) = 0.004420$	$f_{x5}(M_{x5}) = 0.000523$	$f_{x6}(M_{x6}) = 0.000087$	$\rho_{c1} = 0.84$	$\rho_{c2} = 0.96$
$\rho_{c3} = 0.84$	$\rho_{c4} = 0.88$	$\rho_{c5} = 0.88$	$\rho_{c6} = 0.96$	

The results are given in Tables 7 and 8. From Table 7, we observed that the absolute

Table 7. Bias of different estimators under stratified random sampling.

Estimator	*Population I*	*Population II*
$\hat{M}_{Rs}$	16.977	-35.781
$\hat{M}_{EXs}$	-7.079	-20.569
$\hat{M}_{D1s}$	-25.427	-15.952
$\hat{M}_{D2s}$	-24.777	-9.186
$\hat{M}_{D3s}$	-24.513	-13.930
$\hat{M}^{G}_{PPs}$	-19.541	-6.001

bias for the proposed estimators $\hat{M}^{G}_{PPs}$ is the smallest in most cases. In Table 8, $\hat{M}^{G}_{PPs}$ has the highest percent relative efficiency as compared to all other estimators.

Table 8. MSE and PRE of different estimators with respect to $\hat{M}_{yst}$.

$Estimator$	$Population\ I$		$Population\ II$	
	MSE	PRE	MSE	PRE
$\hat{M}_{ys}$	71075.12	100.000	5160.74	100.000
$\hat{M}_{Rs}$	16174.55	439.426	3477.74	148.393
$\hat{M}_{EXs}$	25510.05	278.616	4228.89	122.044
$\hat{M}_{Ds}$	14938.38	475.789	689.193	748.809
$\hat{M}_{D1s}$	14736.57	482.304	214.682	2403.900
$\hat{M}_{D2s}$	14716.18	482.973	88.916	5804.050
$\hat{M}_{D3s}$	14421.73	492.833	177.453	2908.220
$\hat{M}^G_{PPs}$	12597.10	564.218	31.969	16142.890

10. Conclusion

We have proposed a generalized class of difference type estimators for finite population median in both simple and stratified random sampling. Some well-known estimators are particular members of the proposed classes of estimators. Numerical comparisons with other estimators show that the proposed new class of estimators $\hat{M}^G_{PPs}$ is more efficient both in simple and stratified random sampling.

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