

**AN ALTERNATIVE APPROACH TO GENERATE MAXWELL
ALGEBRAS**

Salih KİBAROĞLU¹
Mustafa ŞENAY²
Mustafa ASLAN³

¹*Naval Academy, National Defense University, Tuzla, Istanbul, Turkey,
salihkibaroglu@gmail.com*

²*Naval Academy, National Defense University, Tuzla, Istanbul, Turkey,
mustafasenay86@hotmail.com*

³*Naval Academy, National Defense University, Tuzla, Istanbul, Turkey,
maslan@dho.edu.tr*

Date of Receive: 02.04.2018

Date of Acceptance: 18.04.2018

ABSTRACT

Symmetries are so important for explaining our universe. From this idea, one can get more knowledge about the universe using wider symmetries. Last century shows us that the Group theoretical methods provide very effective mathematical background to construct extended symmetries. Recent years there are a lot of studies on the Maxwell symmetry which is obtained by extension of Poincare symmetry. In this paper, we present an alternative method to produce the Maxwell algebras. We show that $D = 4$ Maxwell algebras can be obtained by inducing from $D = 6$ Maxwell-Lorentz algebra. From this method, some Maxwell algebras are constructed.

Keywords: *Lie Algebras, Tensors, Group Extension, Maxwell Algebra.*

MAXWELL CEBİRLERİNİN OLUŞTURULMASI İÇİN ALTERNATİF BİR YAKLAŞIM

ÖZ

Simetrilerin kullanımı kainatın açıklanmasında büyük bir önem arz etmektedir. Bu fikirden hareketle denilebilir ki, daha geniş simetrilerin kullanılmasıyla kainatın işleyişi hakkında daha fazla bilgi edinebiliriz. Geçtiğimiz asır bize gösterdi ki Grup teorik metotlar, simetrilerin genişletilmesinde çok kullanışlı bir matematiksel temel hazırlamaktadır. Son yıllarda, Poincare simetrisinin genişletilmiş hali olan Maxwell simetrisi hakkında pek çok çalışma yapılmıştır. Biz bu çalışmada Maxwell cebirlerinin oluşturulması için alternatif bir yöntem sunduk. $D = 6$ Maxwell-Lorentz cebirinin indirgenmesi ile $D = 4$ Maxwell cebirlerinin elde edilebileceğini gösterdik. Bu yöntemi kullanarak bazı Maxwell cebirlerini oluşturduk.

Anahtar Kelimeler: *Lie Cebirleri, Tensörler, Grupların Genişletilmesi, Maxwell Cebri.*

1. INTRODUCTION

The main role of Lie algebras in constructing of Supersymmetry and Gauge theory shows us that the Group theory is very important in physics. Lie algebras represent symmetries in physics, and so one can say if we extend given Lie algebra, we obtain extended symmetry, in other words new interactions or new interacting terms. Supersymmetry is a clear example of this motivation which is obtained from extension of Poincare symmetry.

Maxwell algebra is a non-central extension of Poincare algebra [1-6] in which momentum operators are no longer abelian and it satisfies the relation $[P_a, P_b] = iF_{ab}$, where F_{ab} is antisymmetric tensor under Lorentz transformations and $a, b = 0, 1, 2, 3$. Gauging this symmetry and supposing F_{ab} is electromagnetic stress tensor, one can get the equation of motion of charged particle which is moving along constant electromagnetic field [1].

In 2005 Soroka [7] derived new kind of Maxwell algebra by adding new six degrees of freedom into Poincare algebra. Soroka used the generator Z_{ab} instead of F_{ab} and new generator depends on new coordinate parameter

θ^{ab} . Thus, the space-time is extended as $\Phi(x^a) \rightarrow \Phi(x^a, \theta^{ab})$. This algebra is used to produce new symmetries [7-12], to obtain generalization of (super)gravity theories [13-25] and for applying higher spin field and Landau dynamics [26-28].

There are some methods about extension of Lie algebras in literature. The expansion [29-35], S-expansion [36-37], and Chevalley-Eilenberg (CE) cohomology [38-40] methods are used to obtain new algebras with changing group dimensions. If we don't need to change group dimensions it is enough to use contraction, deformation and extension methods [32-33].

In this paper, we present $D = 4$ Maxwell algebras can be obtained from $D = 6$ Maxwell-Lorentz algebra by dimensional reduction. After these notional preparations some Maxwell algebras are obtained.

2. GENERATING $D = 4$ MAXWELL ALGEBRAS FROM $D = 6$ MAXWELL-LORENTZ ALGEBRA

Commutation relationships of Maxwell-Lorentz algebra can be shown as,

$$\begin{aligned} [J_{AB}, J_{CD}] &= i(\eta_{AD}J_{BC} + \eta_{BC}J_{AD} - \eta_{AC}J_{BD} - \eta_{BD}J_{AC}), \\ [J_{AB}, Z_{CD}] &= i(\eta_{AD}Z_{BC} + \eta_{BC}Z_{AD} - \eta_{AC}Z_{BD} - \eta_{BD}Z_{AC}), \\ [Z_{AB}, Z_{CD}] &= i(\eta_{AD}Z_{BC} + \eta_{BC}Z_{AD} - \eta_{AC}Z_{BD} - \eta_{BD}Z_{AC}), \end{aligned} \quad (1)$$

where J_{AB} is the Lorentz generator, Z_{AB} is Maxwell generator in $D = 6$, the Minkowski metric is $\eta_{AB} = (+, -, -, -, -, +)$ and capital Latin indices $A, B, \dots = 0, 1, 2, 3, 5, 6$. Then if we choose following relations,

$$M_{ab} = J_{ab}, \quad P_a = \alpha J_{5a} + \beta J_{6a} + \rho Z_{5a} + \sigma Z_{6a}, \quad Z_{ab} = Z_{ab}, \quad (2)$$

we get commutation of momentum generator as follows,

$$[P_a, P_b] = i(\alpha M_{ab} - \beta M_{ab} + \alpha \rho Z_{ab} - \beta \sigma Z_{ab}). \quad (3)$$

where M_{ab}, P_a, Z_{ab} are Lorentz, momentum and Maxwell generators respectively and $\alpha, \beta, \rho, \sigma$ are arbitrary constants. After that, choosing the

constants as $\alpha = 1, \beta = 0, \rho = -1, \sigma = 0$ then the AdS-Maxwell algebra [17] is obtained as given below,

$$\begin{aligned}
 [M_{ab}, M_{cd}] &= i(\eta_{ad}M_{bc} + \eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac}), \\
 [P_a, P_b] &= i(M_{ab} - Z_{ab}), \\
 [M_{ab}, P_c] &= i(\eta_{bc}P_a - \eta_{ac}P_b), \\
 [M_{ab}, Z_{cd}] &= i(\eta_{ad}Z_{bc} + \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac}), \\
 [Z_{ab}, Z_{cd}] &= i(\eta_{ad}Z_{bc} + \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac}),
 \end{aligned} \tag{4}$$

Also, if one changes the selection with $\alpha = 0, \beta = 1, \rho = 0, \sigma = -1$ then commutation relationship of momentum generators takes following form,

$$[P_a, P_b] = -i(M_{ab} - Z_{ab}), \tag{5}$$

thus, we get dS-Maxwell algebra. Lastly, taking the constants as $\alpha = 1, \beta = 1, \rho = \frac{1}{2}, \sigma = -\frac{1}{2}$ the semi simple Maxwell algebra presented in [21] is obtained by following relation,

$$[P_a, P_b] = iZ_{ab}. \tag{6}$$

One can generate different kind of Maxwell algebras by changing the choices given in Eq. (2) and also using simple Maxwell-Lorentz algebra,

$$\begin{aligned}
 [J_{AB}, J_{CD}] &= i(\eta_{AD}J_{BC} + \eta_{BC}J_{AD} - \eta_{AC}J_{BD} - \eta_{BD}J_{AC}), \\
 [J_{AB}, Z_{CD}] &= i(\eta_{AD}Z_{BC} + \eta_{BC}Z_{AD} - \eta_{AC}Z_{BD} - \eta_{BD}Z_{AC}), \\
 [Z_{AB}, Z_{CD}] &= 0,
 \end{aligned} \tag{7}$$

or Maxwell-General-Linear group [16] given below,

$$\begin{aligned}
 [L^A_B, L^C_D] &= i(\delta^C_B L^A_D - \delta^A_D L^C_B), \\
 [L^A_B, Z_{CD}] &= i(\delta^A_D Z_{BC} - \delta^A_C Z_{BD}).
 \end{aligned} \tag{8}$$

4. CONCLUSION

We showed that $D = 4$ Maxwell algebras can be obtained by dimensional reduction from $D = 6$ Maxwell-Lorentz algebras. The (A)ds-Maxwell and semi simple Maxwell algebras were constructed by using presented method. For generating different Maxwell algebras two possible $D = 6$ algebras were given.

REFERENCES

- [1] Schrader, R. (1972). “The Maxwell Group and the Quantum Theory of Particles in Classical Homogeneous Electromagnetic Fields”, *Fortschritte der Physik*, vol.20, pp.701-734.
- [2] Bacry, H., Combe, P., and Richard, J. L., (1970). “Group-Theoretical Analysis of Elementary Particles in an External Electromagnetic Field II. The Non Relativistic Particle In a Constant and Uniform Field”, *Il Nuovo Cimento*, vol. 70, pp.289-312.
- [3] Bacry, H., Combe, P., and Richard, J. L., (1970). “Group-Theoretical Analysis of Elementary Particles In an External Electromagnetic Field”, *Il Nuovo Cimento*, vol.67, pp.267-299.
- [4] Beckers, J., and Hussin, V. (1983). “Minimal Electromagnetic Coupling Schemes. II. Relativistic and Nonrelativistic Maxwell Groups”, *J. Math. Phys.*, vol.24, pp.1295.
- [5] Negro, J., and del Olmo, M. A. (1990). “Local Realizations of Kinematical Groups with a Constant Electromagnetic Field. I. The Relativistic Case”, *J. Math. Phys.*, vol.31, pp.568.
- [6] Negro, J., and del Olmo, M. A. (1990). “Local Realizations of Kinematical Groups with a Constant Electromagnetic Field. II. The Non Relativistic Case”, *J. Math. Phys.*, 31, pp.2811.
- [7] Soroka, D.V., and Soroka, V.A. (2005). “Tensor Extension of the Poincare' Algebra”, *Phys.Lett. B*, vol.607, pp.302-305.
- [8] Bonanos, S., and Gomis, J. (2009). “A Note on the Chevalley–Eilenberg Cohomology for the Galilei and Poincaré Algebras”, *J. Phys. A*, vol.42, pp. 145206.
- [9] Bonanos, S., and Gomis, J. (2010). “Infinite Sequence of Poincare Group Extensions: Structure and Dynamics”, *J. Phys. A*, vol.43, pp.015201.
- [10] Gibbons, G.W., Gomis, J., and Pope, C.N. (2010). “Deforming the Maxwell-Sim Algebra”, *Phys. Rev. D*, vol.82, pp.065002.

An Alternative Approach to Generate Maxwell Algebras

- [11] Gomis, J., Kamimura, K., and Lukierski, J. (2009). “Deformations of Maxwell algebra and their Dynamical Realizations”, *JHEP*, vol.08, pp.39.
- [12] Salgado, P., and Salgado S. (2013). “ $So(D-1,1) \oplus so(D-1,2)$ Algebras and Gravity”, *Physics Letters B*, vol.728, pp.5-10.
- [13] de Azcárraga, J.A., Kamimura, K., and Lukierski, J. (2011). “Generalized Cosmological Term from Maxwell Symmetries”, *Phys.Rev. D*, vol.83, pp. 124036.
- [14] de Azcárraga, J.A., Kamimura, K., and Lukierski, J. (2013). “Maxwell Symmetries and Some Applications”, *Int.J.Mod.Phys.Conf.Ser.*, vol.23, pp. 01160.
- [15] Cebecioglu, O., and Kibaroglu, S. (2014). “Gauge Theory of The Maxwell-Weyl Group”, *Phys. Rev. D*, vol.90, pp.084053.
- [16] Cebecioglu, O., and Kibaroglu, S. (2015). “Maxwell-Affine Gauge Theory of Gravity”, *Phys. Lett. B*, vol.751, pp.131-134.
- [17] Durka, R., Kowalski-Glikman, J., and Szczachor, M. (2011). “Gauged AdS-Maxwell Algebra and Gravity”, *Mod.Phys.Lett. A*, vol.26, pp.2689- 2696.
- [18] Hoseinzadeh, S., and Rezaei-Aghdam, A. (2014). “(2+1)-Dimensional Gravity from Maxwell and Semi Simple Extension of the Poincaré Gauge Symmetric Models”, *Phys. Rev. D*, vol.90, pp.084008.
- [19] Inostroza, C., Salazar, A., and Salgado, P. (2014). “Brans–Dicke Gravity Theory from Topological Gravity”, *Phys.Lett. B*, vol.734, pp.377–382.
- [20] Salgado, S., Izaurieta, F., González, N., and Rubio, G. (2014). “Gauged Wess–Zumino–Witten Actions for Generalized Poincaré Algebras”, *Physics Letters B*, vol.732, pp.255-262.

- [21] Soroka, D.V., Soroka, and V.A. (2012). “Gauge Semi-Simple Extension of the Poincaré Group”, *Phys.Lett. B*, vol.707, pp.160-162.
- [22] de Azcárraga, J. A., Izquierdo, and J. M. (2014). “Minimal D=4 Supergravity from the Supermaxwell Algebra”, *Nuclear Physics B*, vol.885, pp.34–45.
- [23] Bonanos, S., Gomis, J., Kamimura, K., and Lukierski, J. (2010). “Maxwell Superalgebra and Superparticles in Constant Gauge Backgrounds”, *Phys. Rev. Lett.*, vol.104, pp.090401.
- [24] Concha, P.K., and Rodríguez, E.K. (2014). “N=1 Supergravity and Maxwell Superalgebras”, *JHEP*, vol.1409, pp.090.
- [25] Durka, R., Kowalski-Glikman, J., and Szczachor, M. (2012). “AdS-Maxwell Superalgebra and Supergravity”, *Mod.Phys.Lett. A*, vol.27, pp.1250023.
- [26] Fedoruk, S., and Lukierski, J. (2012). “New Particle Model in Extended Space-Time and Covariantization of Planar Landau Dynamics”, *Phys.Lett. B*, vol.718, pp.646-652.
- [27] Fedoruk, S., and Lukierski, J. (2013). “Maxwell Group and HS Field Theory”, *J. Phys. Conf. Ser.*, vol.474, pp.012016.
- [28] Fedoruk, S., and Lukierski, J. (2013). “New Spinorial Particle Model in Tensorial Space-Time and Interacting Higher Spin Fields”, *JHEP*, vol.1302, pp.128.
- [29] Castellani L., and Perotto A., (1996). “Free Differential Algebras: Their Use in Field Theory and Dual Formulation”, *Lett. Math. Phys.*, vol.38, pp. 321.
- [30] Hatsuda M., and Sakaguchi M., (2003). “Wess-Zumino Term for the AdS Superstring and Generalized Inonu-Wigner Contraction. Prog. Theor”, *Phys.*, vol.109, pp.853.
- [31] de Azcárraga J. A., Izquierdo J. M., Picon M., and Varela O., (2003). “Generating Lie and Gauge Free Differential (Super)Algebras by Expanding Maurer–Cartan Forms and Chern–Simons Supergravity. Nucl”, *Phys. B*, vol. 662, pp.185.

- [32] de Azcárraga J. A., Izquierdo J. M., Picon M., and Varela O., (2004). “Extension, Expansion, Lie Algebra Cohomology and Enlarged Superspace. Class”, *Quantum Grav.*, vol.21, pp.1375.
- [33] de Azcárraga J. A., Izquierdo J. M., Picon M., and Varela O., (2007). “Expansion of Algebras and Superalgebras and Some Applications. Class. Quantum Grav”, *Int. J. Theor. Phys.*, vol.46, pp.2738.
- [34] de Azcárraga J. A., Izquierdo J. M., Lukierski J., and Woronowicz M., (2013). “Generalizations of Maxwell (Super)Algebras by the Expansion Method. Nucl”, *Phys. B*, vol.869, pp.303–314.
- [35] Gibbons G. W., Gomis J., and Pope C. N., (2007). “General Very Special Relativity is Finsler Geometry”, *Phys. Rev. D*, DOI: 10.1103/PhysRevD.76.081701.
- [36] Izaurieta F., Rodriguez E., and Salgado P., (2006). “Expanding Lie (Super) Algebras Through Abelian Semigroups”, *J. Math. Phys.*, DOI: 10.1063/1.2390659.
- [37] Diaz J., Fierro O., Izaurieta F., Merino N., Rodriguez E., Salgado P., and Valdivia O., (2012). “A Generalized Action for (2+1)-Dimensional Chern-Simons Gravity”, *J. Phys. A: Math. Theor.*, DOI: 10.1088/1751-8113/45/25/255207.
- [38] de Azcárraga J. A., and Izquierdo J. M., (1995). *Lie Groups, Lie Algebras, Cohomology and Some Applications in Physics*, Cambridge: 1st Ed, Cambridge University Press.
- [39] Bonanos S., and Gomis J., (2008). “A Note on the Chevalley–Eilenberg Cohomology for the Galilei and Poincare Algebras”, *J. Phys. A*, DOI: 10.1088/1751-8113/42/14/145206.
- [40] Bonanos S., and Gomis J., (2008). “Infinite Sequence of Poincare Group Extensions: Structure and Dynamics”, *J. Phys. A*, DOI: 10.1088/1751-8113/43/1/015201.