THE DEVELOPMENT OF THE CAPITAL ASSET PRICING MODEL AND THE EFFECT OF INTEREST RATE MOVEMENTS ON IT

ABSTRACT

Markowitz'in portföy modelinden başlayıp, Finansal Varlıkları Fiyatlama Modeline kadar olan gelişmeler kısaca izah edilmiş ve bu modelin hazırlanmasında kullanılan varsayımlar üzerinde yapılan çalışmalar dikkate alınarak, modelin eleştirisi yapılmıştır.

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I — MARKOWITZ'S MEAN - VARIANCE PORTFOLIO SELECTION MODEL

The approach for selecting securities for an investment portfolio was first stated by Markowitz in his 1952 article (1). The work of Markowitz on portfolio selection resulted in a revolution in the theory of finance. He described the mean - variance portfolio selection approach in great detail in his 1959 book (2).

Markowitz developed the two - parameter portfolio analysis model and said that the two relevant characteristics of a portfolio are its expected return and risk. He assumed that rational investors will hold efficient portfolios and he defined portfolio effiency as the portfolio with largest return for a given level of risk or as the portfolio with lowest risk for a given rate of return. In order to choose the efficient portfolios, he mentioned that the required inputs were expected return and variance of return for each security and the relationships of the return of each security to every other security. In other words, variance of the portfolio depends on the variance of each individual security and the covariance among them. In order to apply the Markowitz technique «for an analysis of N securities, the analyst must provide estimates of N expected

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(1) Markowitz, Harry M., «Portfolio Selection», **Journal of Finance**, March 1952, pp. 77-91.

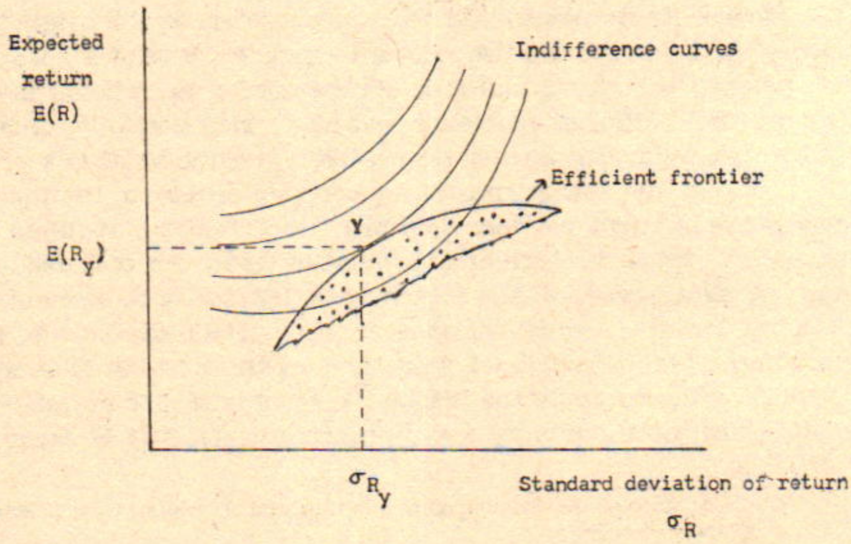
(2) Markowitz, Harry M., **Portfolio Selection : Efficient Diversification of** Investment, New York : John Wiley and Sons, Inc., 1959.

returns, N variances of return, and N (N + 1)/2 covariances of return. Thus the number of estimates required is N (N + 3)/2.» (3) For example, 5, 150 items of input data are required for an analysis of a 100 security universe.

«His (Markowitz's) treatment of investor portfolio selection as a problem of utility maximization under conditions of uncertainty is a pathbreaking contribution. Markowitz deals mainly with the special case in which investor preferences are assumed to be defined over the mean and variances of the probability distribution of single - period portfolio returns» (4). He defined the efficient frontier concept as the upper border of the available set of portfolios. The portfolios which lie on the efficient frontier are efficient because they offer the maximum return for a given level of risk or minimum risk for a given level of return. Figure I shows the graphical form of the Markowitz's mean - variance model.

An investor limited only to investments in risky assets whose indifference curves are shown in this figure maximizes his expected utility by investing in portfolio Y.

Figure 1



(3) Cohen Kalman J. and Pogue, Jerry A., «An Empirical Evaluation of Alternative Portfolio - Selection Models,» **Journal of Business**, Vol : XL, No : 2, April 1967, p. 166.

(4) Jensen, Michael C., «Capital Markets : «Theory and Evidence», **Bell** Journal, Autumn 1972, p. 358.

Comovement among securities as the basis for diversification is one of the most important elements in Markowitz analysis. If the correlation coefficient between two securities is less than the ratio of the smaller standard deviation to the larger standard deviation, portfolio variance can be minimized by diversification.

Since the application of the above model needed a great deal of input data for portfolio selection, it was difficult to use in practice. Although Markowitz first suggested the idea of single - index model, Sharpe developed the diagonal model for portfolio selection.

II — SHARPE'S SINGLE - INDEX MODEL

Sharpe proposed a simplified portfolio selection model which required less input data (5). He did not use the coveriences of each security with each other security. «The major characteristic of the single - index model is the assumption that the various sesurities are related only through common relationships with an index of general market performance» (6). Sharpe suggested the idea that the return on a security may be related to some business index or market index. Each security's rate of return is assumed to be related to the level of the index. For security i,

$$R_i = a_i + b_i I + e_i$$

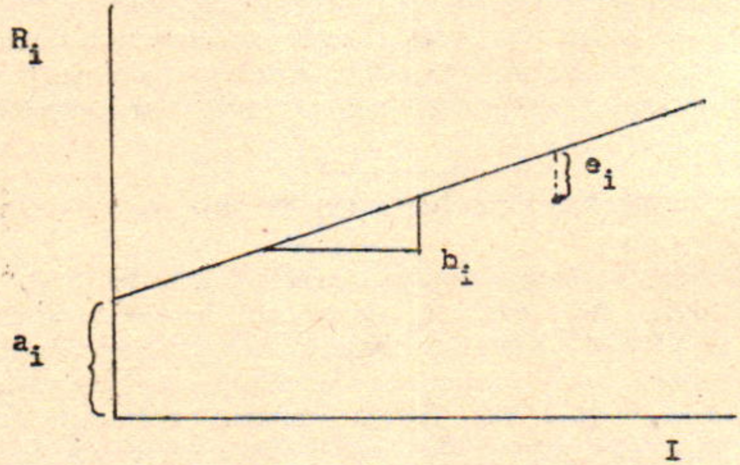
where R_i = actual return on security i
a_i = constant
b_i = constant
I = actual level of index
e_i = uncertain variable

(5) Sharpe, William, F., «A Simplified Model For Portfolio Analysis», **Management Science,** Vol : IX, No : 2, January 1963, pp. 277 - 293.

(6) **Op. cit.,** Cohen and Pogue, p. 168.

Figure 2 provides an illustration for the single - index model.

Figure 2



According to this model, return from an individual security is determined by random factors and the relationship between the security and the common index. Under the assumptions of the diagonal model, the return from a security can be broken into two factors (7). One factor is the result of the unique characteristies of the security and the other is the result of the things common to all securities.

It was Sharpe's idea that the return on a security varies with its sensitivity to changes in the market. If we take the variance of the error term (e_i) in the diagonal model, we will get the following equation :

$$Var\ (R_i) = Var\ (a_i + b_i\ I + e_i)$$

$$Var\ (R_i) = b_i^2\ Var\ (I) + Var\ (e_i)$$

(7) Sharpe, William F., **Portfolio Theory and Capital Markets,** New York : McGraw - Hill, Inc., 1970, p. 96.

Since the variance of the index is common to all securities, its effect can not be eliminated by any amount of diversification. Sharpe's beta coefficient, b_i, measures the sensitivity of R_i to variation in I. This part of the variance is called «systematic or undiversiable risk» and the second part of the total risk is called «unsystematic or diversiable risk.»

The practical application of the portfolio selection technique is greatly facilitated by the assumption of the single - index model which reduced the estimation task. Thus the number of estimates required for portfolio analysis is reduced from $N (N + 3)/2$ in the Markowitz - model to $3N + 2$ in the single - index model.

III — CAPITAL ASSET PRICING MODEL AND ITS LIMITATIONS

Tobin (8) showed that when investors place part of their funds in a risk - free asset, portfolio selection needs two separate decisions. Under these conditions, an investor first determines the set of risky securities which provide an efficient portfolio, then he distributes his funds between this efficient portfolio and the risk - free asset.

Sharpe (9) - Lintner (10) and Mossin (11) apply the principle of Tobin's separation theory to provide what is by far the most important positive extension of the mean - variance concept - The Capital Asset Pricing Model (CAPM). It describes the market equilibrium relationship between risk and return for all risky assets.

In the development of the capital asset pricing model, it is generally assumed that :

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(8) Tobin, James, «Liquidity Preference as Behavior Towards Risk» **Review of Economic Studies,** Vol : 25, February 1958, pp. 65 - 85.

(9) Sharpe, William F., «Capital Asset Prices : A Theory of Market Equilibrium under Conditions of Risk», **Journal of Finance**, Sept. 1964, pp. 425-442.

(10) Lintner, John, «Security Prices, Risk, and Maximal Gains from Diversification» **Journal of Finance,** December 1965, pp. 587 - 615.

(11) Mossin, J., «Equiliurium in a Capital Asset Market», **Econometrica, October** 1966, pp. 768 - 783.

1.. There are perfect capital markets and they are in equilibrium. This implies that information is available to all investors at no cost.

2. All investors are risk averse and they choose portfolios that maximize their expected single period utility of wealth.

3. All investors have same subjective estimates regarding the parameters of the joint probability distribution of all available security returns.

4. All investors can borrow or lend any amount at a given risk - free rate of interest.

Given these assumptions, Sharpe - Lintner and Mossin asset pricing theory states that

$$E(R_i) = R_f + B_i (E(R_m) - R_f) \qquad (1)$$

Simply rearranging this equation, it can be rewritten as

$$E(R_i) = (1 - B_i) R_f + B_i E(R_m) \qquad (2)$$

The symbols in equations (1) and (2) are defined as follows :

$E(R_i)$ = equilibrium expected return on any asset i.
R_f = riskless rate of interest
$E(R_m)$ = expected return on the market portfolio
B_i = «market sensitivity» of asset i. (the «systematic» risk of the i(th) asset). It is defined by.

$$B_i = \frac{Cov(R_i, R_m)}{Var(R_m)}$$

Although CAPM received widespread attention in the literature in recent years, it was subject to theoretical and empirical criticism. Most of the major assumptions of the model do not conform to what people observe in the real world. So, many scholars have examined the effect of relaxing some the basic assumptions of the model. Evidence presented by Lintner indicates that relaxing assumption (3) does not necessarily change the structure of CAPM in any significant way (12).

(12) Lintner, John, «The Aggregation of Investors' Diverse Judgments and Preferences in Perfectly Competitive Security Markets», **Journal of Financial and Quantitative Analysis**, December 1969, pp. 347 - 400.

In addition, recent empirical studies that challenged the adequacy of the CAPM have shown that the model does not provide a complete description of the structure of security returns. The first test of the model was done by Douglas (13) and he found out that the average realized return on common stocks was positively related to the variance of the security's returns over time. This result seems to be inconsistent with the relation given by the CAPM Another work done by Miller and Scholes suggests that there is a negative relation between risk and performance (14). The empirical work of Black, Jensen and Scholes (15) has demonstrated that the expected excess return from holding a security is not proportional to the covariance of its return with the market portfolio. They found that average return on low beta assets was higher and average return on high beta assets was lower than predicted by the model.

Although many empirical studies have tested several theories of the CAPM, there are few studies about the effect of relaxing assumption (4) on the CAPM.

The standard CAPM assumes that the interest rate on a risk - free asset is constant. But, in reality, risk - free rate is not constant and fluctuates over time. Merton asserts that since the interest rate varies over time, investors are willing to pay a premium for those assets which have no systematic risk, but hedge against rising interest rates (16).

In addition, the existence of debt instruments which have a interest - rate risk causes some problems for the CAPM. Since the effects of varying interest - rate on asset returns are not considered in the standard CAPM, two - index model is better than the single - index market model in order to deal with the problem of interest - rate effects on asset returns.

(13) Douglas, G.W., «Risk in the Equity Markets : An Empirical Appraisal of Market Efficiency», **Yale Economic Essays,** Spring 1969, pp. 3 - 45.

(14) Miller, Merton H. and Scholes, Myron, «Rates of Return in Relation to Risk : A Re - Examniation of Some Recent Findings», in **Studies in the Theory of Capital Markets,** ed. M.C. Jensen, N.Y. : Praeegr, 1972.

(15) Black, F., Jensen, M.C. and Scholes, M., «The Capital Asset Pricing Model : Some Empirical Tests», in M.C. Jensen (Ed.) **Studies in the Theory of Capital Markets,** New York : Praeger, 1972, pp. 79 - 121.

(16) Merton, R.C., «An Intertemporal Capital Asset Pricing Model», **Econometrica,** September 1973, pp. 867 - 887.

IV — A TWO - INDEX MODEL AND INTEREST - RATE RISK

Stone's (17) two - index model states that return on security i is given by :

$$R_i = A_i + B_i\,(R_e) + C_i\,(R_d) + e_i \qquad (3)$$

where : $Cov\,(R_d, e_i) = 0$
$Cov\,(R_e, e_i) = 0$
and $E\,(e_i) = 0$

In equation (3), the symbols are defined as follows :
A_i, B_i, and C_i are constants
R_e = return on equity index
R_d = return on debt index
B_i = measure the responsiveness of security i to equity market movements (systematic equity risk).
C_i = measure the responsiveness of security i to debt market movements (systematic interest - rate risk).

In Stone's two - index model, the beta of security i, S_i, is a combination of the effects of B_i and C_i.

$$S_i = B_i + C_i\,(B_d) \qquad (4)$$

where $B_i = \dfrac{Cov(R_i, R_e)}{Var\,(R_e)}$; $C_i = \dfrac{Cov(R_i, R_d)}{Var\,(R_d)}$; and $B_d = \dfrac{Cov(R_d, R_e)}{Var\,(R_e)}$

When we substitute the values of B_i, C_i and B_d into equation (4), the beta of security i is given by :

$$S_i = \frac{Var\,(R_d)\,Cov\,(R_i, R_e) + Cov\,(R_i, R_d)\,Cov\,(R_d, R_e)}{Var\,(R_e)\,Var\,(R_d)}$$

(17) Stone, Bernell K., «Systematic Interest - Rate Risk in a Two - Index Model of Returns», **Journal of Financial and Quantitative Analysis, Vol: 9**, November 1974, pp. 709 - 721.

If B_d is zero, that is Cov $(R_d, R_e) = 0$, then the beta of a security will not be affected by R_d (return on debt index). When B_d is not equal to zero, C_i must be equal to zero, in order for the beta of a security to be unaffected by R_d. Either $B_d = 0$, or $C_i = 0$, is a sufficient condition to say that R_d has no effect on the beta of a security.

In a two - index model, betas reflect a security's interest - rate sensitivity in varying degrees depending on both the values of C_i and the comovement of R_e and R_d. So, failure to treat interest - rate effects means instability in the measurement of equity responsiveness.

V — BLACK'S CAPM WITH RESTRICTED BORROWING AND PRICE LEVEL CHANGES

If it is assumed that price - level changes make all assets risky, then Black's CAPM which assumes no riskless borrowing and lending can be used to investigate the impact of price - level changes. Black (18) has shown that when no risk - free borrowing or lending is allowed, the expected real return on any asset will equal :

$$E(R_i) = E(R_z) + B_i\,(E(R_m) - E(R_z)) \qquad (5)$$

where $B_i = (\text{Cov}(R_i, R_m))/\text{Var}(R_m)$, and $E(R_z)$ is the expected return from a minimum varience portfolio which has zero covariance with return from the market portfolio, R_m.

This relationship also holds for the expected real return on the risk - free borrowing or lending and it can be stated by the following equation :

$$E(R_f) = E(R_z) + \frac{\text{Cov}(R_f, R_m)}{\text{Var}(R_m)}(E(R_m) - E(R_z)) \qquad (6)$$

If we subtract equation (6) from equation (5), we will get :

(18) Black, Fisher, «Capital Market Equilibrium with Restricted Borrowing» **Journal of Business,** Vol : 45, July 1972, pp. 444 - 454.

$$E(R_i) - E(R_f) = \frac{Cov(R_i, R_m) - Cov(R_f, R_m)}{Var(R_m)} (E(R_m) - E(R_z)) \quad (7)$$

This equation can also be rewritten as :

$$E(R_i) = (R_f) + \frac{Cov(R_i - R_f, R_m)}{Cov(R_m - R_f, R_m)} (1 - \frac{Cov(R_f, R_m)}{Var(R_m)})$$
$$(E(R_m) - E(R_z)) \quad (8)$$

However, we know from equation (6) that

$$E(R_f) - E(R_z) = \frac{Cov(R_f, R_m)}{Var(R_m)} (E(R_m - E(R_z)) \quad (9)$$

and finally substituting equation (9) into equation (8), we obtain

$$E(R_i) = E(R_f) + \frac{Cov(R_i - R_f, R_m)}{Cov(R_m - R_f, R_m)} (E(R_m) - E(R_f)) \quad (10)$$

Equation (10) shows the equilibrium relationship between risk and real return when changes in the price level are introduced into the CAPM (19). It states that the expected real return on any asset is still a linear function of its risk. But there is big difference between equation (10) and the standard CAPM equation (1), that is, the measure of risk. This difference is due to the fact that the real return on the borrowing and lending rate varies stochastically. On the other hand, if we assume a constant R_f, then the measure of risk in equation (10) will be exactly equal to the risk in the standard CAPM.

(19) Equation (10) is also derived in the appendix of the following article: Hagerman, Robert L. and Kim, E. Han, «Capital Asset Pricing With Price Level Changes», **Journal of Financial and Quantitative Analysis,** September 1976, pp. 381 - 391.

If we write equation (10) in the following way,

$$E(R_i) = E(R_f) + \left(\frac{Cov(R_i, R_m) - Cov(R_m, R_f)}{Var(R_m) - Cov(R_m, R_f)}\right)(E(R_m) - E(R_f)) \qquad (11)$$

it can be seen that the expected real return on any risky asset is not affected by the relationship between changes in the general price level and the asset itself. Furthermore if we assume that real market returns are independent of the price - level changes, that is, $Cov(R_m, R_f) = 0$, then the equilibrium condition expressed in equation (10) is identical to that of the standard CAPM except the constant risk - free rate replaces the expected real return on borrowing and lending rate.

VI — MERTON'S INTERTEMPORAL CAPM

Merton (20) demonstrates that people will hold investments in three portfolios if the riskless interest rate in the investment opportunity set varies over time. These three portfolios are : (1) the riskless asset (short - term Treasury bills), (2) the market portfolio, and (3) a portfolio N which is perfectly negatively correlated with changes in the riskless interest rate. In this case, investors will hold the shares of portfolio N in order to hedge against the effects of changes in the riskless interest rate.

Merton also demonstrates that the instantaneous expected return on a risky asset depends not only on systematic risk but also on opportunities to hedge against changes in interest rates as stated by the following equation :

$$E(R_i) = r + \lambda_1 (E(R_m) - r) + \lambda_2 (E(R_m) - r)$$

where :

r = rate of return on «risk - less asset.»

R_m, R_n = return on market and asset N (long - term bonds) respectively.

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(20) Op. Cit., Merton, R.C.

$$\lambda_1 = \frac{\delta_i (\rho_{im} - \rho_{in} \ \rho_{nm})}{\delta_m (1 - \rho^2_{mn})}$$

$$\lambda_2 = \frac{\delta_i (\rho_{in} - \rho_{im} \ \rho_{nm})}{\delta_n (1 - \rho^2_{nm})}$$

ρ_{nm} = correlation between the returns on the market portfolio and the riskless rate.

The expected return on the asset would not be equal to the riskless rate even if the systematic risk ρ_{im} were 0, because the asset returns may be systematically related to changes in the interest rate ($\rho_{in} > 0$). Thus, Merton concludes that the intertemporal model is consistent with the results found by Black - Jensen and Scholes that indicate that high beta assets earn less and low beta assets earn more than predicted by the standard CAPM.

Furthermore, by analyzing the relationship between Black's zero - beta model and Merton's intertemporal model, Hadaway (21) shows that low - beta securities provide higher returns than predicted by the standard CAPM during periods of rising interest rates. Thus low - beta stocks provide a hedge against rising interest rates. Hadaway also demonstrates that when the covariance between Treasury bill rates and market return (Cov (R_r, R_m)) is negative, high - beta stocks return less than predicted by the traditional CAPM. He concludes that these findings are consistent with Merton's hedging behavior.

VII — SUMMARY AND CONCLUSION

Because of the fact that most recent empirical evidences challenged the adequacy of the standard CAPM on the issue that it did not provide a satisfactory description of the structure of security returns, we tried to explain the reason for this fact by investigating several assumptions of the model. Especially, we analyzed the effect of relaxing the assumption of constant risk - free lending and borrowing rate on the CAPM. Although many empirical studies that tested the other assumptions of the model very strong

(21) Hadaway, Samuel C. Jr., «The Zero - Beta Portfolio and Intertemporal Asset Pricing», Sept. 1976, **Journal of Finance.**

strong and robust with regard to the assumptions under consideration, it is shown that the relaxation of the constant risk - free rate is the reason for inconsistencies found by several empirical tests.

Fluctuating risk - free interest rates have an important effect on CAPM explaining the behavior of asset returns. It is found that the return on high - beta assets are less and the return on low - beta assets are more than predicted by the standard CAPM.

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