Hacettepe Üniversitesi Eğitim Fakültesi Dergisi (H. U. Journal of Education) 33(3): 705-722 [2018]
doi: 10.16986/HUJE.2018036943 [Araştırma Makalesi / Research Article]

The Role of Computer-Assisted Instruction in the Teaching of Probability

Olasılık Öğretiminde Bilgisayar Destekli Öğretimin Rolü

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• ***Received:*** *11.05.2017* • ***Accepted:*** *21.01.2018* • ***Published:*** *31.07.2018*

Kaynakça Bilgisi: Gürbüz, R., Dede, Y., & Doğan, M. F. (2018). Olasılık öğretiminde bilgisayar destekli öğretimin rolü. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 33*(3), 705-722. doi: 10.16986/HUJE.2018036943

Citation Information: Gürbüz, R., Dede, Y., & Doğan, M. F. (2018). The role of computer-assisted instruction in the teaching of probability. *Hacettepe University Journal of Education, 33*(3), 705-722. doi: 10.16986/HUJE.2018036943

ABSTRACT: This study aimed to analyze the role of computer-assisted instruction (CAI) on students' achievement concerning the subject of 'probability'. The experimental pre-and post-test with control group research design was carried out with 48 seventh grade students by conducting The Probability Achievement Test (PAT) to all groups. Data were analyzed by employing an analysis of covariance (ANCOVA) on post-test scores with a pre-test as the covariate and by calculating effect size values. The results revealed that the CAI was more effective in helping the students develop the probability concepts than traditional instruction (TI). Specifically, this study highlights that the CAI tasks that designed for teaching probability were powerful and useful for students to enhance their understanding of important concepts of mathematics and might be used as a new and better way of teaching probability.
Keywords: Mathematics education, teaching/learning strategies, pedagogical issues, probability, computer-assisted instruction.

ÖZ: Bu çalışma, bilgisayar destekli öğretimin (BDÖ) öğrencilerin olasılık kavramlarını öğrenmelerine etkisini araştırmayı amaçlamaktadır. Bu kapsamda geliştirilen Olasılık Başarı Testi kontrol gruplu araştırma tasarımı ile deney öncesi ve sonrası deney ve kontrol grubundan 48 yedinci sınıf öğrencisine uygulanmıştır. Veriler, kovaryant olarak bir ön test alındıktan sonra puanlar üzerinde bir kovaryans analizi (ANCOVA) yapılarak etki boyutu değerleri hesaplanarak analiz edilmiştir. Sonuçlar bilgisayar destekli öğretimin geleneksel öğretime kıyasla öğrencilerin olasılık kavramlarını geliştirmelerinde daha etkili olduğunu ortaya koymuştur. Bu araştırma, özellikle olasılığın öğretilmesi için tasarlanan bilgisayar destekli öğretim etkinliklerinin, öğrencilerin matematikteki kavramsal anlamalarını geliştirmeleri için güçlü ve faydalı olduğu ve etkili öğretim için daha iyi bir yol olabileceği söylenebilir.
Anahtar sözcükler: Matematik eğitimi; Öğretim/öğrenme stratejileri; pedagojik konular; olasılık; bilgisayar destekli öğretim.

1. INTRODUCTION

Probability can be generally defined as the action of estimating about what will happen in the future. Although probability is a very common concept in games of chance, it has been currently used in many areas, such as science, industry, economics, banking, and so on, as well as in decision-making processes related to uncertain situations that are encountered in daily life. It also helps individuals enhance their effective reasoning, intuitive and critical thinking, language development, and adapting effective strategies for solving any problems they

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encounter in everyday life (e.g., Jones, Langrall, Thornton & Timothy Mogill, 1997; Gürbüz, 2010).

1.1. Issues in the Teaching of the Probability

The probability requires deep and specific thinking that is a pre-requisite for many disciplines mentioned above and most school mathematics. For this reason, learning environments that develop students' previous intuitions and thinking on probabilistic reasoning should be provided, so that students not only criticize their previous intuitions and insights also create relationships between new intuitions and information. However, many studies indicate that probability is not being effectively taught in many countries (e.g., Jones et al., 1997; Gürbüz, 2010; Kim, Kil, & Shin, 2014) and student face great difficulties in probability activities. Some potential causes for such difficulties might be due to the instructional problems and the student-centered problems. The instructional problems might be because of: i) the common teacher centered classroom environments (Bulut, 2001; Gürbüz, 2006), ii) the lack of appropriate instructional materials or the abstractness of prepared materials (Gürbüz, 2006; Pijls, Dekker & Van Hout-Wolters, 2007), iii) the teaching language (i.e. students are taught in a second language in place of their mother-tongue) (Kazıma, 2006), and iv) teachers' lack of sufficient pedagogical content knowledge in teaching probability (Bulut, 2001; Fast, 1997). The students centered problems might be because of i) students' incorrect theoretical knowledge or misconceptions (Fischbein & Schnarch, 1997; Pratt, 2000; Gürbüz & Birgin, 2012), ii) students' difficulty in probabilistic reasoning (Fischbein & Schnarch, 1997; Munisamy & Doraisamy, 1998; Gürbüz & Erdem, 2014), iii) students' incorrect relations or links between their daily life knowledge and scientific knowledge (Gürbüz, 2006), and iv) students' negative attitudes towards the subject and low level of achievement in probability (Bulut, 2001). Most of these studies that identify those difficulties reveal that traditional teaching strategies are inadequate in resolving the deficiencies preventing the effective teaching of probability.

1.2. Teaching of the Probability through the CAI

The corpus of existing literature has focused on the impact of different strategies used in the teaching of probability on students' development of probability concepts (e.g. Gürbüz, 2010; Fırat, 2011; Nilsson, 2007; Baker & Chick, 2007). Most of those studies identify students' misconceptions about the concept of probability and highlight the importance of having activity-based instruction while teaching probability. Baker and Chick (2007) conducted a study in order to teach some probability concepts with the help of two spinners. The two teachers divided the students into groups and tried to teach probability concepts by allowing them to do experiments with these spinners and play games. The researchers observed that using this kind of activities in mathematics classes helped students develop new methods and the students demonstrated more practical learnings. Similarly, Gürbüz (2010) investigated the effects of activity based and traditional instructions on students' conceptual development of certain probability concepts by using a pretest-posttest control group design with 80 seventh graders. Activity based instructions was determined to be more effective than traditional instruction in the development of probability concepts. Since the traditional approaches used in the instruction of probability concepts do not give the students the opportunity to experiment with the activities, evaluate the results, and discuss the process. On the other hand, activity-based instructions allows students to experiment with the activities, evaluate its results, and discuss the process. Recently, researchers are beginning to examine the effect of more innovative teaching approaches that utilize the computer-assisted instruction (CAI) environments to improve students' understanding and to remedy students' misconceptions (see Demetriadis, Papadopoulos, Stamelos & Fischer, 2008; Lazakidou & Retalis, 2010; González, Jover, Cobo, & Muñoz, 2010; Gürbüz & Birgin, 2012; Tan & Tan, 2015). In these learning environments, students can jointly figure out a solution to a problem, increase their problem-solving skills in a relatively short period of time, improve their approach to the solution of a given mathematical problem and showing significant signs of autonomy (Pijls, Dekker & Van Hout-Wolters, 2007; Demetriadis, Papadopoulos, Stamelos & Fischer, 2008; Lazakidou & Retalis, 2010). For example, Pijls et al. (2007) investigated the effect of cooperative education conducted using computer games on secondary school students' conceptual development and found that students who actively took part in the process and actively criticized it had high levels of conceptual development in comparison to those who did not. Gürbüz & Birgin (2012) also examined the effect of the CAI on remedying misconceptions of students regarding some probability subjects. They determined that the CAI was significantly more effective than the traditional one, in terms of remedying seventh grade students' misconceptions. In a similar study, Nilsson (2007) asked 4 groups of 7th graders to perform experiments with dice designed as ((111 222), (222 444), (333 555), (1111 22), (2222 44), (3333 55)) in order to determine different solution strategies for the sum of probable outcomes. He stated that the more the number of experiments with the dice increased, the more the students remedied the errors resulting from their traditional dice perceptions. In the same vein, Fischbein and Schnarch (1997) conducted a study to determine the changes in misconceptions of 5th, 6th, 7th, 9th and 11th graders and college students who have never been taught about probability before. They found that as students' education level increased, their misconceptions about some concepts decreased, some stayed stable and some increased. All these studies suggest having students engage in activities about probability in order to enhance students understanding and to overcome students' misconceptions in probability.

1.3. Aim and Significance of the Study

The rapid advances in technology and the search of new methods in education revealed new alternative methods that allow interactive teaching environments with use of animation and simulations in teaching mathematics instead of continuing with traditional methods. Indeed, in the literature, it is suggested that the teaching of probability and statistics should be performed by presenting the subject in an interesting way (Sanders, 1995), providing ongoing experiences with experimental activities and random generators (Truran, 1994), recognizing and confronting common errors in students' probabilistic thinking (Shaughnessy, 1992), creating situations requiring probabilistic reasoning that correspond to students' views of the world, and introducing topics through activities and simulations, not only abstractions (Bezzina, 2004; Garfield & Ahlgren, 1988). If students desperately need experiences that develop their competencies in probability and statistics, then, we need to create environments that have opportunities for learners to engage at probability reasoning in a meaningful way. Considering the ineffective instructional strategies, CAI may create the opportunities that students need to enhance their understanding of probability. Thus, this study was designed to create such a teaching environment that uses computer-aided learning material to teach the concepts of probability which are difficult to teach by teachers and to comprehend by students. In this sense, it is thought that one of the effective ways of teaching probability as mentioned above is to employ CAI strategy in learning environments. Therefore, the purpose of this study is to analyze the role of CAI on Grade 7-students' achievement on probability. For this purpose, the following research question is investigated.

1. How did the designed CAI tasks influence achievement on probability, compared to the traditional instruction (TI) method?
2. Does CAI method have higher mathematics achievement for probability than TI method if differences in the CAI and TI pre-test scores are controlled?

2. METHOD

2.1. Research Design

This study is an experimental pre-and post-test with control group research design. In here, in teaching of probability, the experimental group would be taught by using the CAI and the control group would be taught by TI. The experimental design investigates to take influences and create situations in which they can be managed, that is, reduce the joint action of the elements and the possibility that one element is influencing another (Newby, 2014, p. 152).

2.2. Study Group

Students were selected based on a combination of convenience and random sampling methods. Convenience sampling method was used because the students were chosen on the basis of their willingness and accessibility to participate (Gravetter & Forzano, 2008; Denscombe, 2010). The mathematics teacher chosen for this study had ten years of teaching experience and currently was teaching two 7th grade mathematics classes that have similar mathematics achievement scores are close together in the previous year. One of these two classes was randomly assigned as the experiment group (CAI), while the other one was assigned as the control group (TI). With these sampling methods, the target group was consisted of 48 students, 7th grade, in a formal primary state school of a city in South-East Anatolia, during the second semester of 2013-2014 school year. Students generally came from families of low or middle socio-economic status and most of them did not have computers at home. The school, which is in the suburbs of the city, was opened recently and has a well-developed physical infrastructure. The reasons behind the choice of this grade are also as follows: i) 7th grade students' cognitive development levels and nature of the concepts of probability, ii) at this level, forecast, students' intuition, reasoning and probabilistic thinking reach a certain level of maturity, and iii) even though students at this level may not own any computers in their homes, they have basic computer skills that they gain from their computer course in their schools.

2.3. Instrument

In this study, the Probability Achievement Test (PAT) was developed by researchers. Some explanations about the PAT are briefly given below:

Structure of the PAT: The PAT consisted of 20 open-ended questions, 12 of which were developed by the researchers and 8 of which were developed with the help of related literature (e.g., Baker & Chick, 2007; Fischbein et al., 1991; Jones et al., 1997; Nilsson, 2007; Gürbüz , 2010; Gürbüz & Birgin, 2012). The PAT primarily focuses on the concepts of Sample Space (SS), Probability of an Event (PE), Probability Comparisons (PC), and Mutually and Not Mutually Exclusive Event (ME).

Verification of the PAT: Verifying the language and theoretical foundations of a test (or an instrument) involves an inquiry into the meaning and understandability of the items, such as what the instrument measures, and how consistently it measures the target construct (Dede, 2011). Validity and reliability of the PAT were explored using several approaches.

Content Validity: The draft the PAT was evaluated by a panel of five experts using an evaluation checklist to provide insights to the content validity of the PAT. The panel members were two mathematics teachers, two mathematics educators, and an educational measurement and evaluation expert. The 20 open-ended questions were also evaluated for expression, relevancy, openness, fluency, and appropriateness of language structure. Based on the experts' opinions, four questions were deleted and some questions were also rewritten or rearranged, for example:

17: Musa and Meryem play with a pair of dice. If the sum of the points is 3, Ali is the winner. If the sum of the points is 6, Ayşe is the winner. Which of the following answers seems to you to be the correct one? Why?

a) Musa is the favourite b) Meryem is the favourite c) Musa and Meryem have the same chance (deleted item)

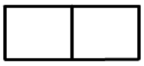
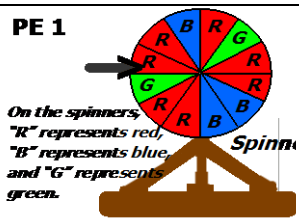
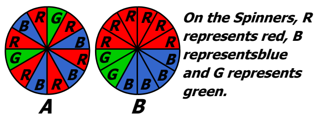
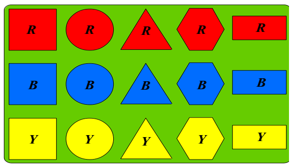
1: "We have a wall that we want to paint red and blue. Show in how many ways we can paint it and how we can paint the wall" was rewritten as "Assume that your wall is divided into two sections. Use the red and blue crayons to paint each section a different color. Show how many different ways you could paint the wall and show me how you could paint the wall." (Rearranged item).

Pilot Study: After alterations based on the experts' comments and suggestions, the PAT was performed with 28 seventh grade students who did not participate in the real study. The pilot study revealed that the students had not understood some of the statements, for example:

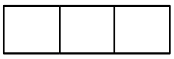
2: Assume that your wall is divided into three sections. Use different colorful pencils to paint each section a different color. Show me how many different ways you could paint the wall and show me how you could paint the wall." was rewritten as "As shown above (with figure), assume that your wall is divided into three sections. Use the red, green, and blue crayons to paint each section a different color. Show how many different ways you could paint the wall and show how you could paint the wall.

Finally, the final form of the PAT was reduced to 16 open-ended questions after being pilot study and revised by the same experts. And the overall Kuder-Richardson 20 (KR-20) coefficient of the entire test was .83. All questions of the test are presented in Table 1.

Table 1. All concepts and their questions of the PAT

Sample Space (SS)	**Probability of an Event (PE)**	**Probability Comparisons (PC)**	**Mutually and Not Mutually Exclusive Event (ME)**
SS 1  As shown above, assume that your wall is divided into two sections. Use the red and blue crayons to paint each section a <u>different</u> color. Show how many different ways you could paint the wall and show me how you could paint the wall.	**PE 1**  The spinner above is turnable spinner type. Give one example for the 'impossible', 'certain' and 'probable' concepts using this spinner.	**PC 1**  A and B are spinners that can be used in a game. If the spinner chosen in this game stops at red, you win 1 point, if it stops at another color, you lose 1 point. Which spinner do you think if you choose will increase your chance of winning? Why? Could you use numerical expressions to support your ideas?	**ME 1**  *(**<u>R</u>** represents red, **<u>B</u>** represents blue, and **<u>Y</u>** represents yellow)* What is the probability of a randomly chosen geometric shape from the board being red or blue? Could you express your ideas numerically?

SS 2

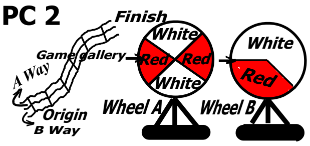


As shown above, assume that your wall is divided into three sections. Use the red, green, and blue crayons to paint each section a <u>different</u> color. Show how <u>many</u> different ways you could paint the wall and show how you could paint the wall.

PE 2

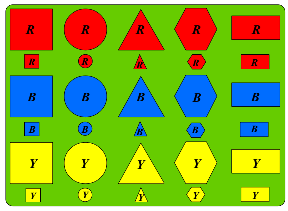
The spinner in PE 1 is used for playing the game. You and your friends choose a color and turn the spinner. If the spinner stops at the color you chose, you win 1 point, and if it stops at the one your friend chose, you lose 1 point. Which color would you choose to win the game? Why? Could you use numerical expressions to support your ideas?

PC 2



You decide to play a game with your friend by choosing spinners A and B above. Your color is red and your friend's color is white. Whose color the spinner stops at when turned, that person takes a step further. In that case, which spinner do you think if you choose will increase your chance of winning? Why?

ME 2

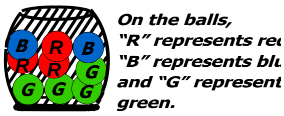


What is the probability of a randomly chosen geometric shape being blue or a rectangle? Could you express your ideas numerically?

SS 3

Faces of two dice are marked as (222 444) and (333 555). Write the sets of numbers you will get, in the experiment of tossing two cube-shaped dice? Explain how you could calculate the number of items in that sets more practically.

PE 3



There are 4 green, 3 red, and 2 blue balls, in total 9 balls, in this basket. When you close your eyes, mix the balls and choose a ball in the basket, the probability of getting which colored ball is the highest? Why? Could you use numerical expressions to support your ideas?

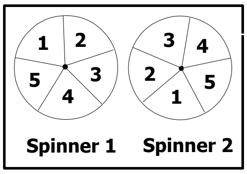
PC 3

You and your friends are playing a game using spinners in SS 4. You both are turning the spinners at the same time. When spinner stops and the total added numbers is odd you win 1 point, and if it is even your friend wins 1 point. Whoever wins 10 points first wins the game. Who will win the game? Why? Do you think this game is fair?

ME 3

What is the probability of randomly chosen geometric shape from board **ME 2** being small or red? Could you express your ideas numerically?

SS 4

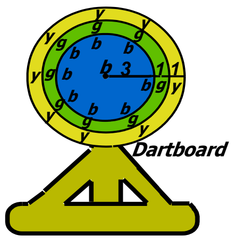


Do you think the probability of getting the same numbers or different numbers is higher when spinners above are turned together? Why? Could you express your ideas numerically?

PE 4

Faces of two dice are marked as (2222 44) and (3333 55). When these two dice are tossed together, what is the probability of getting a sum of 7 on the upper faces? Why?

PC 4



In the dartboard, whose radii are as shown above, "b" represents blue, "g" represents green, and "y" represents yellow. As each shot is aimed at any yellow, green or blue part, the probability of hitting which color is the lowest when a random throw occurs? Why? Could you use numerical expressions to support your ideas?

ME 4

What is the probability of a dart thrown randomly at the dartboard in question PC 4 hitting a blue or yellow section? Why? Could you use numerical expressions to support your ideas?

2.4. Procedure

Before the teaching intervention to the experimental group, the PAT was administered in both groups as a pre-test. Two groups were encouraged to answer all questions. The teaching in the CAI group was carried out with a two student group work using a computer. For this, it was utilized by using a framework suggested by Dekker (1994). An application of the framework for teaching probability is shown in Table 2.

Table 2. An application of the framework for teaching probability

Criterion	Description	Application
Real or meaningful	Motivate and stimulate the students	You are planning to go on a picnic tomorrow but you have just heard from meteorology experts that there is a 70% chance of rain. What do you do?
Complex	Encourage students to work together	Group work and discussion (firstly small group work and then whole-class discussion)
Construct something	Reveal the students' ideas and to promote discussion	Students were asked to write their opinions on the related concept.
Aimed at level raising	Promote students' understanding of probability	Students were asked pass to another interface on which there was a definition related to the concept in the question.

Then, based on the framework, CAI materials incorporating animations and simulations of probability were developed for the experimental group by researchers via Macromedia Dreamweaver and Flash MX 2004 and transferred into the HTML environment (see Appendix 1). The material was pilot-studied with 28 grade 7 students who did not participate in the treatment study and necessary corrections were made. For example, an applause effect was used for the "check the answer" button in the pilot study. However, the pilot study showed that this sound negatively affected in-class communication; therefore, it was later removed.

During implementations in the first class-hours, some problems appeared because most of the students could not get used to the computer environment while engaging in the activities with CAI even though they had experiences in using computer in their classes. However, in successive class-hours, they got used to the implementations and some of them even continued to deal with the materials during break times. In the CAI group, students were requested to run the computer animations and then were asked to share their thoughts about probability with their friends. Moreover, since the students asked such questions to each other as "*why are you doing that?*" and "*how did you get that" and made statements such as "... oh no, that isn't right, because ...*" and *"... but that's wrong, because ...*" during the implementations, the teacher refrained from responding to correct answers; instead he asked some follow-up questions in a Socratic dialogue as a facilitator. Thus, during this process, the teacher, instead of just lecturing, showing, giving tests and evaluating, acted as an organizer, facilitator, counselor, cooperator, and supervisor.

On the other hand, in the TI group, the lessons were taught on a teacher-centered approach and verbally according to the book and the teacher noted down the necessary points on the board. While writing on the board, the teacher framed the important parts using colored chalk. During the process the students sat on their seats silently and listened to the teacher. Then the teacher gave them some time to take notes from the board. The teacher also asked if they had any questions about the subject. Meanwhile, he walked around the class and answered their questions. However, only some bright and hard-working students asked some questions as pointed out by the teacher in an informal interview. In brief, 70-75% of the probability subject was composed of just the teacher's talk. At the end of the lesson, the teacher asked the students to answer the questions at the end of the unit. In the TI group, questions such as *"suppose that a pair of dice is rolled ...*" were generated and solved. To sum up, the experiments were done and the results were obtained by imagination without the use of any concrete materials or animations. This teaching intervention procedure was implemented within a seven class-hour period for each group in the spring of the 2013-2014 school year by the same teacher. After the applications were finished with two groups, a month later the PAT was re-administered as a post-test.

2.5. Data Analysis

In analyzing the data, students' answers were classified in regard to the levels in Table 3.

Table 3. **The description of the analysis levels and sample responses**

Level	Score	Description	Sample response
A: Completely Correct	**5**	The explanations which are accepted as scientifically true, take place in this group	**PC 3:** When we turn the spinners together, the probability of getting even numbers. Probability of getting even= P(even)=(number of even outcomes)/(total number of outcomes) =13/25 and probability of it being an odd number probability of getting odd = P(Odd)=(number of odd outcomes)/(total number of outcomes) =12/25 So my friend will win and I think it isn't fair.
B: Partially Correct	**4**	Explanations are true but when compared to the correct answers some parts are missing, so it takes place in this group.	**PE 1:** When this Spinner is turned, it's **<u>certain</u>** that it'll stop at blue or green and it's **<u>possible</u>** to stop at red. For this reason, this Spinner isn't an appropriate example for **<u>impossible</u>** event. **SS 1:** R B B R B B R R We can paint it in four different ways.
C: Incorrect Answer (1)	**3**	The explanations which contain partially correct statements but are connected to the right reasons or don't give reasons, take place in this group.	**SS 3:** {(1,1), (1,2), (1,3), (1,4),…(6,4), (6,5), (6,6)} Since there are 6 cases in each dice, it's 36. **ME 3**: Probability of getting small, 15/30 and probability of getting red, 10/30 **PC 4:** Probability of hitting is directly proportional with the radius of color's area. So the probability of hitting yellow or green section is the least.
D: Incorrect Answer (2)	**2**	Expressions that contain wholly wrong or irrelevant explanations are in this group.	**ME 4:** Since green section is in between, it targets at green section. **PC 3:** Game is a matter of luck. Whoever is lucky, wins the game. **PE 3:** Green balls are at the bottom but as they will rise to the top when mixed, green will appear. **PC 4:** Since green section is between b and y (**see Table 1**) , the probability of targeting at green is the lowest: 20 %.
E: Not-codeable	**1**	Not-understandable explanations or	**PE 2:** I would not play that game as it is not a fair one.

		explanations that have no connection to the question are in this group.	**ME 4:** As the number of b and y's is not equal, the probability of targeting at yellow section is the same. **ME 2:** The probability of having small or red is 2. **PC 3:** Whoever turns the spinner well wins the game. **PC 4:** g=8, y=8 and b=9 from here g=8/25π, y= 8/25π and b=9/25π, as 9/25π> is 8/25π=8/25π, it is at least green or yellow
F: No Answer	**0**	Those that made no explanations and those who wrote question itself in the explanation part are in this group.	**ME 3:** The probability of geometric object randomly chosen from the ME 2 board to come small or red… **PE 1:** Exemplify impossible, absolute and probable concepts.

SS: Sample Space, **PE:** Probability of an Event, **PC:** Probability Comparisons, **ME:** Mutually and Not Mutually Exclusive Event.

Since the students' answers firstly categorized by three experts mentioned above separately, they discussed the consistency of the categorization. There was high agreement, approximately 84%, in most of the categorization. All disagreements were resolved by negotiation. After each student's total score was calculated, the scores were input into a statistical program for analysis and analysis involved descriptive and inferential statistics. Descriptive statistics were performed, including frequencies, means, and standard deviations. Due to the small number of subjects and the non-normal distribution of some of the variables there was a risk of committing a Type II error. For this purpose, the Kolmogorov-Smirnov test was performed to determine whether the samples conformed to a normal distribution. And the results of the test indicated that distributions of the pre-test and post-test scores of the PAT were normal distribution, respectively ($p= 0.11$; $p= 0.86$); therefore the data met the assumption of normality (Field, 2002).

The assumption of homogeneity of the variances related to pretest scores was also verified through the Levene F test ($p > 0.05$). Therefore, the data for the post-tests scores were analyzed using an analysis of covariance (ANCOVA), with the pre-test scores as covariate. In fact, a covariance analysis was applied in order to observe any potential difference between the means of the post-test scores of the groups. Effect sizes of the PAT scores were also calculated in order to determine whether the effect was substantive or not. The effect size is small if value of d is 0.01, medium if value of d is 0.06, and large if value of d is 0.14 (Cohen, 1988).

2.6. Validity of Experimental Design

This study is a the classical type of experimental design and the advantage of this design here is the randomization, so that any differences that appear in the posttest should be the result of the experimental variable rather than possible difference between the two groups to start with. While the external validity of this experimental design is limited by the possible effect of pre-testing (to overcome this limitation, Analysis of Covariance (ANCOVA) test was used to control differences in pre-test scores), it has good internal validity (Moorhead, 2015). Nine threats to internal validity have been defined: history, maturation, testing, instrumentation, statistical regression, differential selection, selection-maturation interaction, experimental mortality, and researcher bias (see Best & Kahn, 1989; Borg, 1987). The following processes are performed to overcome the threats to internal validity in this current experimental design. a) *history*: the intervention was only implemented within a 7 class-hour period, b) *maturation*: it was very crucially hard to change students' mental, psychological, and behavioral developments within a 7 class-hour period, c) *testing*: the study provided sufficient time (four weeks) for the students to forget the test questions, d) *instrumentation*: alternative test here was not used, e) *statistical regression*: mathematics achievement scores of the groups are close together in the previous year, f) *differential selection*: the groups were selected based on random sampling method, g) *selection-maturation interaction*: students were not selected from different ages, schools, and regions etc., h) *experimental mortality*: it was very difficult to drop out of students because the treatment was only 7 class-hour period, and g) *researcher bias*: total two lessons, each of which was randomly selected from both groups, were video-typed. Total record time was about 40 minutes for each video-record. These records were analyzed by two experts who have Ph.D. in mathematics education. Also, these experts came to one of the classes to examine the one hour lesson.

3. FINDINGS

Table 4 reveals the means and standard deviations for CAI and TI methods in mathematics achievement for teaching probability, before and after controlling for the pre-test scores. As is evident from this table, virtually difference between the CAI and the TI methods remains after differences in the pre-test scores are controlled.

Table 4. Adjusted and Unadjusted Group Means and Variability for Math Achievement for Teaching Probability using the Pre-test Scores as a Covariate

		Unadjusted		**Adjusted**	
	N	M	SD	M	SE
CAI	24	3.05	0.47	3.05	0.10
TI	24	2.23	0.54	2.23	0.10

An analysis of covariance (ANCOVA) was used to assess whether CAI method has higher math achievement for probability than TI method after controlling for differences between CAI and TI pre-test scores (see Table 5). Results revealed that after controlling for the pre-test scores, there was a significant difference with significant with a large effect size between CAI and TI methods in math achievement for teaching probability ($F(1,45)= 30.969$, $p = < 0.001$). Indeed, the strength of the association between the CAI and post-test score was $\eta^2 = 0.02$ when pre-test scores were not used as a covariate versus partial $\eta^2= 0.40$ when pre-test scores were used as a covariate.

Table 5. Analysis of Covariance for Math Achievement for Teaching Probability as a Function of Group, Using Pre-test Scores as a Covariate

Source	***Df***	***Ms***	***F***	***p***	**Eta-squared (η^2)**	**Power**
Pre-tests	1	.326	1.252	.269	.02	.195
Group	1	8.075	30.969	.000	.40	1
Error	45	.261				

Furthermore, Table 6 shows some examples of students' answers and explanations (only for the Sample Space (SS)) in the pre-and post-tests. So it can be easily seen that students' answers on the sample space improved after the treatment. Improvements were also observed in all other probability concepts.

Table 6. Examples of Students' Answers for Pre-and Post-test of Groups for Sample Space (SS)

Concept	Examples of Students' Answers	
	Pre-test	**Post-test**
Sample Space (SS)	**SS 1:** We can color in 6 different ways (Fig. 1a). (Level C) **SS 2**: It would be better to paint a wall a single color. (Level E) **SS 3**: {(222 222), (222 444), (222 333), (222 555)} if done for other numbers in the same way, it would be 4 each. That is, there are 16 different shapes. (Level D) **SS 4**: As they were turned at the same time, the probability of getting the same numbers is high. (Level-D)	**SS 1**: We can color in 2 different ways (Fig. 1b). (Level A) SS 2: We can color in 3 different ways (Fig. 1c). (Level C) **SS 3**: SS=*{(2,3), (2,5), (3,2), (3,5), (4,3), (4,5), (3,4), (5,4)}*.(Level C) **SS 4**: As there are 5 cases for each spinner, the number of items of sample space is s (E) =25. While there are 5 probabilities like {(1,1),(2,2),(3,3),(4,4),(5,5)} for the same numbers, since there are 20 probabilities like {(1,2), (1,3), (1,4), (1,5), (2,1),…(5,2), (5,3), (5,4)}, the probability of getting different faces is higher (Level A).

Fig. 3.1a Fig. 3.1b Fig. 3.1c

Figure 3.1. Some examples of students' answers

When students' explanations and Table 6 are examined, it can be seen that they came up with partly correct answers through using their daily life experiences and intuitions while answering question SS 1 in the case of the small sample space. However, they were unsuccessful while answering questions SS 2, SS 3, and SS 4 in the case of large sample spaces. Students made significant errors in questions SS 1 and SS 2. For example, the reason why they answered question SS 1 as "*we can color in 2 different ways (Fig. 2a)*", "*we can color in 11 different ways (Fig. 2b)*" and in the same way the reason why they answered questions SS 2 "*we can color in 3 different ways (Fig. 2c)*" may come from their problems with language proficiency.

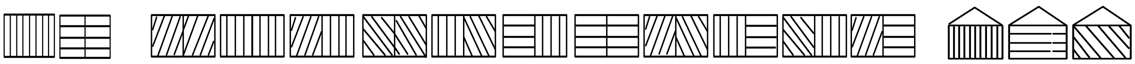


Fig. 3.2a Fig. 3.2b Fig. 3.2c

Figure 3.2. Some examples of students' answers

As it can be seen from Table 6, students did better on the post-test than on the pre-test. While students did not show any systematic technique in creating outcomes related to sample space in the pre-test (English, 1993; Gürbüz, 2010), they developed a systematic technique in creating outcomes in the post-test. Therefore, they were able to apply this technique to questions SS 1 and SS 2 in an effective way. Yet, they had difficulties in applying the same technique to questions SS 3 and SS 4.

In the pre-test, some students made errors related to questions SS 3 in specifying the sample space due to their custom dice perceptions as123 456. For example, some of these students wrote the sample space incorrectly. Some other students made errors by writing "*(3/6).(3/6).(3/6).(3/6)=1/16".* As it can be shown from Table 6, there is little improvement in TI group, but there are some improvements in CAI group. In the pre-test related to question SS 4, some students noted that, depending on chance, the probability of getting the same or different numbers was equal. The students answered this question: "*we cannot say anything* *unless we see how these tiny spinners that are turned by someone else move*” or they replied “*both are probable*”. According to this approach, outcomes of an experiment are completely random and any given conditions are not important. In the post-test, students used different methods to answer question SS 4 even though they exploited the most common method of systematic listing as sample space being (Ss)={(1,1), (1,2), (1,3), (1,4), (1,5), (2,1),…}. However, some of the students in CAI and TI groups, who made errors even in the pre-test, may have supposed that some displays such as (1,2) and (2,1) were the same. Another method used by students is shown in Figure 3a. Students in the CAI groups generally used this method and those who used this method usually came up with the right answer.

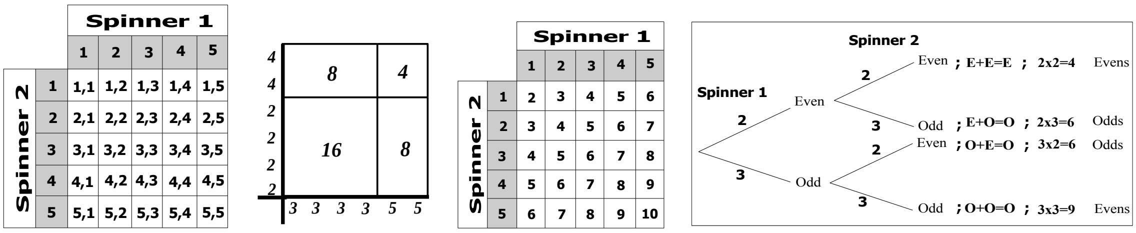


Fig. 3.3a Fig. 3.3b Fig. 3.3c Fig. 3.3d

Figure 3.3. A chart of all probable outcomes

4. DISCUSSION

Steffe and Wiegel (1994) stated that “…children will not sustain a mathematical activity unless they experience satisfaction in the process of that activity” (p. 132). Thus, it is important to create environments, like CAI, that allow students to engage with mathematical activities in a meaningful way. The results of the study showed that CAI might give students the opportunity where they can have satisfactory experiences in the process and revealed that there was a significant difference in the probability learning between the CAI and the TI groups. These results suggest that the treatment does have an effect on the probability learning score for these student groups. Indeed, traditional approaches used in the teaching of the probability concepts do not give the students the opportunity to perform experiments, to see results derived from experiments, or to discuss the process. On the other hand, since the CAI gave the students the opportunity to do experiments, to see the results derived from the experiments, and to discuss the process, learning through these processes is more meaningful and at the conceptual level. In this context, it was found that the CAI was more effective in the teaching of the probability concepts as compared to traditional approaches. This may result from the fact that the students constructed their own knowledge by completing activities using materials consisting of simulations and animations, that students felt more secure and comfortable in the CAI environment, and that the CAI process made them more enthusiastic learners and increased their responsibilities towards learning. It can be deduced that CAI may be used as an effective teaching strategy in overcoming the challenges of teaching probability concepts and providing a meaningful comprehension of these concepts. It is possible to see similar positive outcomes regarding CAI in the literature (Antoch, Cihák & Pelzla, 2007; Dewiyanti, Brand-Gruwel, Jochems & Broers, 2007; Fırat, 2011; Fırat, Gürbüz, & Doğan, 2016).

The students in CAI group were provided with learning environments in which they can collaborate more effectively in this study. Thus, it is believed that these students may make a slighter and more effective transition to work and social lives in the future. In this process, the teacher finds opportunities to intervene in order to stop pupils’ mistakes immediately (Wood, Cobb and Yackel, 1991). Chai and Tan (2009) suggested that those students who had experienced collaborative work at school will have fewer difficulties during transition to work life.

The activities played a positive role in the teaching of the CAI group, but the students were sometimes disappointed with not seeing practical evidence of what they learnt theoretically. For example, the students predicted the probability of getting a head in a coin-tossing experiment as *1/2*. However, they did not explicitly witness this ratio. This might be because of student language proficiency related to probability. This is consistent with Kazıma (2006) and Tatsis, Kafoussi, & Skoumpourdi (2008), who asserted that language development is important in understanding the probability concepts. In order to overcome these kinds of adverse situations, students should be given more time so that they will be able to carry out more trials and could be shown through various experiments that as the number of tosses increases the probability will approach *1/2*. In addition, as discussed in the results section, students had problems with identifying sample space. Shaughnessy, (1977), Fischbein et al. (1991), Batanero and Serrano (1999), Baker and Chick (2007) and Nilsson (2007) reported similar results in their studies. The reason why students had difficulties in numeric displays may result from a lack of sample space perceptions and their lack of knowledge about fractions, because when we look at the students' pre-test answers it can be inferred that they mostly tried to answer in 'percent (%)' terms. This is in agreement with the conclusions made by Jones et al. (1997). As shown at the posttest, students in CAI group had higher achievement than TI group.

As applications progressed, it was seen that students participated more actively as a result of their self-confidence, learning in a comfortable environment and the fact that they got used to the material. In this sense, Rowntree (1992), Antoch, Cihák & Pelzla, (2007) and Dewiyanti et al. (2007) pointed out those CAI environments stimulated students to explain their beliefs without the fear of punishment and being mocked. In the process, it was observed that students also gave true responses individually because of the fact that they easily shared ideas with each other and the researcher guided their discussions effectively. Therefore, students have more meaningful and permanent learning due to the fact that they construct knowledge themselves with the guidance of a researcher. To summarize, the results of this study show that CAI strategy provided a friendly, comfortable and entertaining environment that was effective in improvement students' mathematics achievement. Thus, it can be argued that the process made a contribution to students' development of mathematical language and communication skills in general**.**

5. DIRECTIONS FOR FUTURE RESEARCH

In light of the results of the current study and the related literature, it can be concluded that although the frequency of errors and misconceptions changed the errors and misconceptions are very similar to each other. This shows that various teaching methods, various learning theories, and various infrastructure equipment used in learning environments are not effective enough to completely correct the errors and misconceptions, since students' justifications were affected by their individual learning, experiences, culture, and beliefs. Some researchers reported the effect of similar factors on probability learning (see Batanero and Serrano, 1999; Fırat et al., 2016). Thus, to identify the factors that influence the students' conceptual learning, apart from learning environments or materials, further studies should be performed. Furthermore, since the present study was carried out with 7th grade students, students in the 8th grade and more advanced classes as well as the probability concepts taught in those levels were not taken into consideration. Therefore, further research is needed to understand the effect of CAI in teaching probability at those levels. Finally, in the revised middle school math program in 2013, it appears that the teaching of these concepts transferred to the 8th grade instead of 7th grade. Since the new middle school mathematics curriculum is published in 2013 after the date on which the study was conducted, this study did not assess the new curriculum. Therefore, the new curriculum should be evaluated along a similar vein to this study that examines the effects on the concept of probability.

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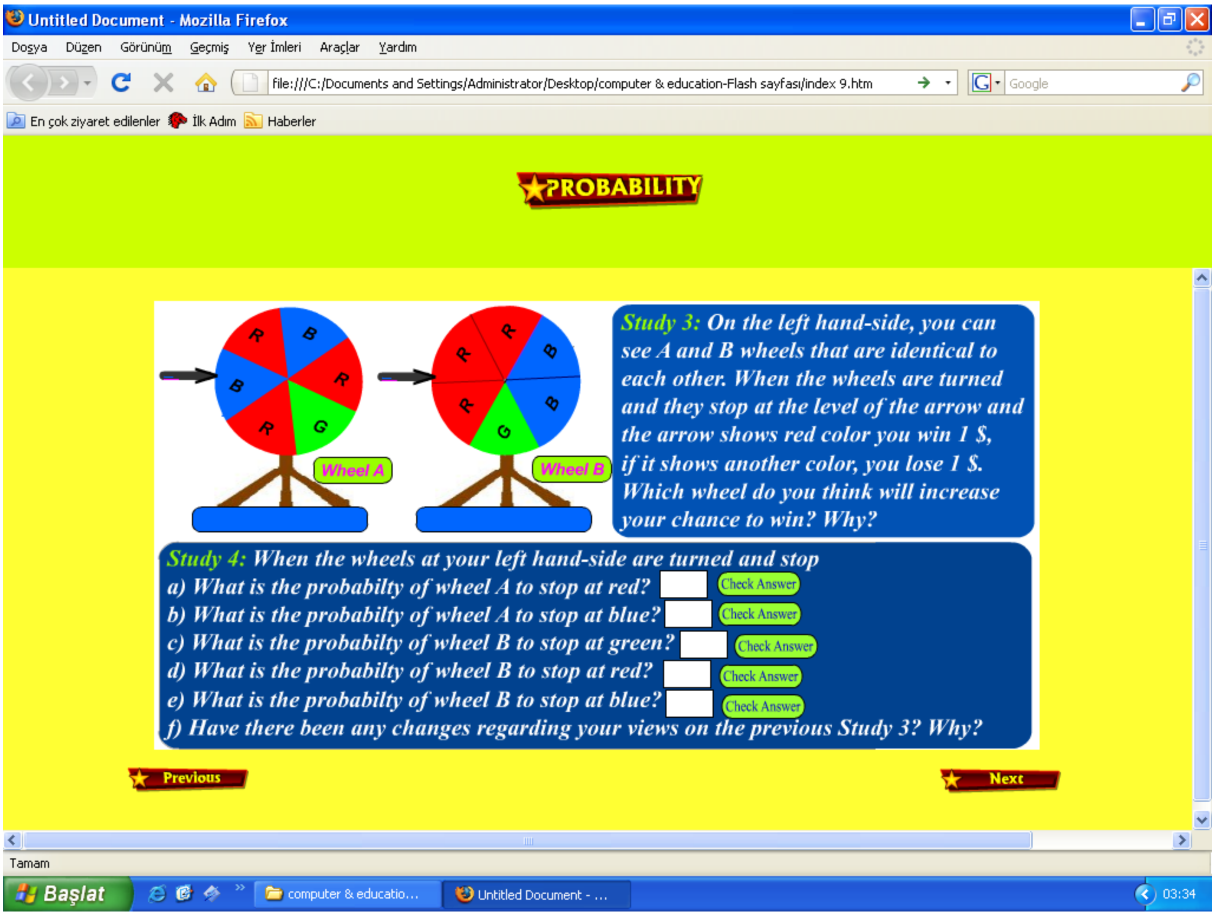
Appendix 1. A sample interface from the developed CAI material

Guideline for Using the Material: This is a sample from the material consisting of 30 interfaces presented to students in HTML medium with Dreamweaver and Flash MX 2004 software being used together in the implementation process. Students can carry out tasks they are asked to do in each interface and can check their answers by pressing the check button. If a wrong answer is given the answer check button signals “wrong answer” and if the correct answer is given then students can confirm their answers and pass to the next interface by clicking on “next” button. Also, students can return to the previous interface by clicking on the “previous” button when needed.



Note: On the wheels, **“R”** represents red; **“B”** represents blue, and **“G”** represents green.

Uzun Özet

Olasılık, genellikle gelecekte ne olacağı hakkında tahmin yapma eylemi olarak tanımlanmaktadır. Olasılık günlük yaşamın yanı sıra bilim, sanayi, ekonomi gibi birçok alanda ve şans oyunlarında çok yaygın olarak kullanılan bir kavramdır. Ayrıca, olasılıksal düşünme bireylerin etkili akıl yürütme, sezgisel ve eleştirel düşünme, dil gelişimi ve günlük yaşamda karşılaştıkları sorunların çözümünde etkili stratejiler geliştirmelerine yardımcı olur.

Olasılıksal düşünme, yukarıda bahsedilen birçok disiplin ve okul matematiği için ön koşul olan derin ve özel düşünmeyi gerektirir. Bu nedenle öğrencilerin önsezileri ve mevcut bilgilerini eleştirebilecekleri yeni fikirler geliştirerek sezgileri ile bilgileri arasında ilişki kurabilecekleri olasılıksal akıl yürütmeye dayalı bir öğrenme ortamı oluşturulmalıdır. Ancak, birçok çalışma olasılığın Türkiye’de olduğu gibi farklı birçok ülkede de etkili bir şekilde öğretilemediğini ve öğrencinin olasılık etkinliklerinde birçok zorluklarla karşılaştığını göstermektedir. Bu zorlukları belirleyen çalışmaların çoğu, olasılığın etkin öğretimini engelleyen eksiklikleri gidermede geleneksel öğretim stratejilerinin yetersiz olduğunu ortaya koymaktadır. Öğrencilerin olasılık ve istatistikte yeterliklerini geliştiren deneyimlere çokça ihtiyaç duyulduğu düşünülürse, olasılık mantığını anlamlı bir şekilde öğrenme fırsatı sunan ortamlar tasarlamamız etkili bir öğretim için oldukça önemlidir. Öğrencilerin olasılık konusundaki anlayışlarını geliştirmeleri ve anlamlı öğrenmelerinin gerçekleştirilebilmesi için tasarlanan Bilgisayar Destekli Öğretim (BDÖ) ortamıyla gerekli olanaklar sağlanabilir. Bu araştırma, olasılık kavramlarını öğretmek için bilgisayar destekli öğretim materyalinin kullanıldığı bir öğrenme ortamı tasarlanmasıyla gerçekleştirilmiştir. Bu nedenle bu araştırmanın amacı, 7. sınıf öğrencilerinin bazı olasılık kavramlarını öğrenmede BDÖ'nün rolünü analiz etmektir. Bu amaçla, araştırma soruları şu şekilde belirlenmiştir:

1. Tasarlanan BDÖ etkinlikleri geleneksel öğretim metotlarıyla karşılaştırıldığında bazı olasılık kavramlarının öğretimini nasıl etkiler?

2. Her iki grup için ön test puanlarındaki farklılıklar kontrol ediliğinde, BDÖ yönteminin olasılık kavramlarının öğretiminde geleneksel yöntemine göre daha yüksek bir matematik başarısına sahip midir?

Bu araştırma, kontrol gruplu araştırma tasarımı ile deney öncesi ve sonrası Olasılık Başarı Testi'nin uygulanmasıyla gerçekleştirilmiştir. 48 yedinci sınıf öğrencisinin katılımıyla gerçekleştirilen araştırmada genel matematik başarı düzeyleri benzer olan deney ve kontrol grupları (her grup 24 kişi) oluşturulmuştur. Deney grubunda BDÖ metotları; kontrol grubunda ise geleneksel öğretim metotları kullanılarak olasılık öğretimi gerçekleştirilmiştir. BDÖ metotlarının öğrenci başarısı üzerindeki etkisini ortaya koyabilmek için çalışmanın başında ve sonunda her iki gruba Olasılık Başarı Testi ön test ve son test olarak uygulanmıştır.

Veriler, kovaryant olarak bir ön test alınarak test sonrası puanlar üzerinde bir kovaryans analizi (ANCOVA) yapılarak etki boyutu değerleri hesaplanarak analiz edilmiştir. Öğrencilerin cevapları altı seviye olarak kodlanmıştır. Bu kodlar: Seviye 5-Tam doğru cevap, Seviye 4-Doğru ancak tam anlamıyla açıklanamamış, Seviye 3-Yanlış cevap 1 (öğrencinin cevabı kısmen doğru bilgi içermekte, ancak tam doğru sonuca ulaşılamamış), Seviye 2- Yanlış Cevap 2(öğrencinin cevabı herhangi bir doğru bilgi içermemekte, sonuç tamamen yanlış), Seviye 1-Kodlanamaz (öğrencinin cevabı yorumlanacak ve kodlanacak kadar açık değil) ve Seviye 0-Cevap yok şeklinde oluşturulmuştur.

Bulgular, ön test puanları kontrol edildikten sonra olasılık öğretimi için matematik başarısı ($F(1,45) = 30.969$, $p = 0.001$) sonuçlarında BDÖ ve geleneksel öğretim yöntemleri arasında büyük bir etki boyutu ile anlamlı bir fark olduğunu ortaya koymuştur. Burada genel anlamda olasılık konusunun anlaşılmasında daha önemli bir işlevinin olduğu değerlendirilen örnek uzay kavramını biraz daha detaylandırmak istiyoruz. Örnek uzay küçük olduğunda öğrencilerin, SS 1 sorusuna cevap verirken, günlük hayat deneyimlerini ve sezgilerini kullanarak kısmen doğru cevaplar ürettikleri görülmüştür. Ancak, örnek uzay büyüdüğünde öğrencilerin SS 2, SS 3 ve SS 4 sorularına cevap üretirken başarısız oldukları saptanmıştır. Öğrenciler, bu tür sorularda önemli hatalar yapmışlardır. Bu hataların esas olarak öğrencilerin olasılık kavramıyla ilgili dil yeterliliklerinden kaynaklandığı söylenebilir. Öğrencilerin genel başarıları son testte ön testten daha yüksek çıkmıştır. Öyle ki, öğrenciler ön testte örnek uzay kavramıyla ilgili cevaplarında herhangi bir sistematik teknik göstermezlerken, son-test cevaplarında sistematik teknikler geliştirdikleri görülmüştür. Ancak bu teknikleri SS 1 ve SS 2 sorularına etkili bir şekilde uygulayabildikleri halde, SS 3 ve SS 4 sorularına aynı tekniği uygulamakta güçlük çekmişlerdir.

Ön testte, öğrencilerin SS 3 sorusuna ilişkin hatalarının kaynağı olarak öğrencilerin özel zar algılamaları olduğu düşünülmektedir. Örneğin, bu öğrencilerin bazıları örnek uzayı hatalı yazmışken, bazıları ise "(3/6). (3/6) (3/6) (3/6) = 1/16" yazarak hatalar yapmışlardır. Son-test sonuçlarına göre, kontrol grubunda çok az gelişme olurken, BDÖ grubunda daha yüksek bir gelişme olmuştur. SS 4 sorusuna ilişkin ön testte bazı öğrenciler, tesadüfen aynı veya farklı sayıların olma ihtimalinin eşit olduğunu kaydetmiştir. Literatürdeki diğer çalışmalarda olduğu gibi, öğrencilerin sayısal hesaplamalarda zorluk çekmelerinin nedeni, örnek uzay algılamalarının eksikliği ve kesirler hakkındaki bilgi eksikliklerinin etkili olduğu söylenebilir. Son-testte, öğrenciler, $(SS) = \{(1,1), (1,2), (1,3)\}$ gibi örneklem uzay olarak sistematik listelemenin en yaygın metodunu göz ardı etseler de SS 4 sorularını cevaplamak için farklı yöntemler kullanmışlardır. Öğrencilerin bazıları, (1,2) ve (2,1) gibi bazı gösterimlerin aynı olduğunu düşünmüş olabilirler. Diğer kavramlar(PE, PC ve ME) içinde benzer süreçlerden bahsedilebilir.

Öğrencilerin matematiksel etkinliklerle uğraşırken anlamlı bir şekilde öğrenmelerine yardımcı olan BDÖ gibi ortamların tasarlanması oldukça önemlidir. Bu araştırmanın sonuçları BDÖ'nün öğrencilere süreçte tatmin edici deneyimler yaşayabilecekleri fırsatlar sunduğu ve BDÖ ile geleneksel öğretim arasında bazı olasılık kavramlarını öğrenmede önemli farklılık olduğunu ortaya koymuştur. Bu sonuçlar, BDÖ uygulamasının bu öğrenci grupları için bazı olasılık kavramlarının öğrenme puanı üzerinde pozitif bir etkisinin olduğunu göstermektedir. Olasılık kavramlarının öğretilmesinde kullanılan geleneksel yaklaşımlar, BDÖ ortamı gibi öğrencilere deney/ler yapma, deney/lerden elde edilen sonuçları görme veya süreci tartışma olanağı vermemektedir. Bu bağlamda bu araştırmayla, BDÖ'nün bazı olasılık kavramlarının öğretiminde geleneksel yaklaşımlarla karşılaştırıldığında daha etkili olduğu belirlenmiştir. Öğrenme ortamları veya materyalleri dışında öğrencilerin kavramsal öğrenimini etkileyen faktörleri tanımlamak için tek kavram odaklı daha spesifik çalışmalar yapılmalıdır.