AN IMPORTANCE OF GROWTH POTENTIAL IN VALUATION MODELS

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Ö Z E T

Bu makalede klâsik değerleme modellerinde kullanılan büyüme faktörünün önemi dikkate alınmış ve bu modeller değişik büyüme varsayımları altında kısaca izah edilmiştir.

I— INTRODUCTION

Most of the early valuation models (1) have been restricted to firms with no growth potential. It is now time to relax this assumption and to see what effect if any the existence of growth potential may have on the market value of a firm's equity.

Besides current earning power of a firm, there are other factors which play a crucial role in determining the value of a firm. The most important one is the growth potential. This potential exists as long as the firm has the opportunity to invest in projects in the future at rates of return greater than the cost of capital.

Miller and Modigliani (2) used an infinite - growth valuation model to estimate the cost of capital to the electric utility industry and to test their controversial propositions on the role of capital structure in the valuation of shares. By using the investment opportunity approach, they derived the valuation formula based on the assumption that a company will grow at a fixed rate forever.

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(1) Myron J. Gordon, «Optimal Investment and Financing Policy», **Journal of Finance,** Vol : 18, No : 2, (May 1963), pp. 264 - 272.
James E. Walter, «Dividend Policies and Common Stock Prices», **Journal of Finance,** Vol : 11, No : 1, (March 1966), pp. 29 - 41.

(2) Merton H. Miller and Franco Modigliani, «Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954 - 57», **American Economic Review,** Vol : 56, No : 3, (June 1966) pp. 333 - 391.

Instead of a constant growth, Mao (3) used a three - stage growth in his valuation model. Then Taylor (4) derived the closed form expression for Mao's valuation formula.

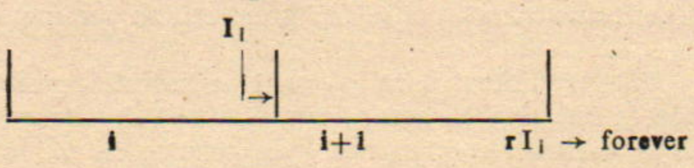
The purpose of this article is to analyze the effect of growth potential in valuation analysis by explaining each model briefly.

II — INVESTMENT OPPORTUNITY APPROACH

Suppose a firm's existing assets are expected to earn a constant annual income of E dollars per share and these earnings are capitalized at a rate of k percent. This firm also invests a constant amount each year at a rate of r percent, where r is greater than k. Assume this constant amount of investment is equal to bE, where b is the percent of earnings retained and E is the constant earnings per share expected from current assets.

Under these assumptions, Investment Opportunity Approach states that the price of the firm's shares, P_0, is equal to $P' + P''$ where P' is the value attributable to present operations of the firm and P'' is the value due to future assets of the firm. Thus the present value of constant and perpetual earnings stream expected from current assets of the firm is equal to $\frac{E}{k}$, (that is, $P' = \frac{E}{k}$)

Figure 1



Since the earnings from the investment in period i, I_i starts in the next period, the present value of these earnings from the investment at the beginning of period $i + 1$ is given by the following formula.

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(3) James T.C. Mao, «The Valuation of Growth Stocks: The Investment Opportunity Approach», **Journal of Finance**, Vol: 21, No: 1 (March 1966) pp. 95 - 102.

(4) Walton Taylor, «A Note on Mao's Growth Stock - Investment Opportunities Approach», **Journal of Finance**, Vol: 29, No: 5, (December 1974), pp. 1573 - 1576.

$$PV = \frac{r I_i}{k} \quad (1)$$

and the net present value of these earnings is equal to

$$NPV = \frac{r I_i}{k} - I_i \quad (2)$$

or

$$NPV = I_i \left(\frac{r - k}{k}\right) \quad (3)$$

In this case if $r > k$, then the investment is desirable. But the NPV of these earnings at the beginning of period i is equal to

$$NPV = \frac{I_i}{1 + k} \left(\frac{r - k}{k}\right) \quad (4)$$

Thus the capitalized value of the expected investment opportunities due to future assets is given by the following equation :

$$P'' = \sum_{i=1}^{\infty} \frac{I_i}{(1 + k)^i} \left(\frac{r - k}{k}\right) \quad (5)$$

Since $P_0 = P' + P''$,

$$P_0 = \frac{E}{k} + \left(\frac{r - k}{k}\right) \sum_{i=1}^{\infty} \frac{I_i}{(1 + k)^i} \quad (6)$$

Eq. (6) is known as the investment opportunity approach formula in the literature of finance. Note that this formula is based on the assumption that investments yield a return of r percent per year and the amount invested annually is constant. But, if the amount of investment increases as a function of time, Miller and Modigliani (5) show that the value of the firm's growth potential, P″, can be derived by using a similar approach.

III — MILLER AND MODIGLIANI'S CONTANT GROWTH VALUATION MODEL

If it is assumed that the firm increases its annual investments at the rate «g», where g = br, then the amount of investments are given by the following table :

TABLE : 1

Time	Investments
$i = 1$	$I_1 = I_0 (1 + g)$
$i = 2$	$I_2 = I_1 (1 + g)$
$i = 3$	$I_3 = I_2 (1 + g) = I_1 (1 + g)^2$
⋮	⋮
$i = n$	$I_n = I_1 (1 + g)^{n-1}$

If the value of the amount of investments from the Table is substituted in Eq. (6), the following formula will be obtained :

$$P_0 = \frac{E}{k} + \frac{(r - k)}{k} \left(\frac{I_1}{(1 + k)} + \frac{I_1 (1 + g)}{(1 + k)^2} + \dots\dots\dots \right) \quad (7)$$

If investment in any given year is always equal to b percent of the earnings of that year, (e.g. $I_1 = bE$), then the Miller and Modigliani's valuation formula is given by the following equation : (6)

$$P_0 = \frac{E}{k} + \left(\frac{r - k}{k}\right) bE \sum_{i=1}^{\infty} \frac{(1 + g)^{i-1}}{(1 + k)^i} \quad (8)$$

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(5) Merton H. Miller and Franco Modigliani, «Dividend Policy, Growth and the Valuation of Shares», **Journal of Business**, Vol : 34, No : 4, (October 1961) pp. 411 - 432.

(6) If $g < k$, then the closed form expression for P_0 can be derived by using the sum of an infinite geometric series formula.

$$P_0 = \frac{E}{k} + \left(\frac{r - k}{k}\right) \left(\frac{bE}{k - g}\right)$$

The first term is the present value of constant and perpetual earnings stream expected from existing assets of the firm and the second term is the capitalized value of the expected investment opportunities. The rate of return, r, on these investment opportunities is assumed to be greater than the cost of capital, k, in perpetuity. In other words, excess profits that arise from the firm's ability to invest funds at above the cost of capital are capitalized at k and $(r - k)/k$ is called an index of profitability for these future investments. The second component of Eq. (8) is simply equal to the present value of future investments multiplied by the index of profitability.

Since a firm's opportunity for growth, realistically, is both a function of the amount of internal funds that can be invested and the amount of time over which such investment opportunities are available, growth potential should not be projected indefinitely into the future (7). Miller and Modigliani recognize the fact that a firm may experience vigorous growth initially, but this growth is likely to slow down as maturity is reached, because these investment opportunities and the amount of internal funds that can be invested are actually limited (8).

IV — MAO'S THREE - STAGE GROWTH MODEL

In real world, since the growth experienced by most firms follows a different pattern, Mao extends the constant growth valuation model of Miller and Modigliani to a three - stage growth model. In his analysis, he assumes that the firm invests b percent of its earnings in each of n_1 years at a rate of r percent, where $r > k$ (9). Such opportunities vanish and the firm invests the same amount, bE, at a rate $r > k$ for n_2 years. Finally, the firm invests an amount that declines by $1/n_3$ per year in perpetual investment projects at a rate r (where $r > k$) for n_3 years. Under these assumptions, the valuation formula derived by Mao is : (10).

(7) Ezra Solomon, **The Theory of Financial Management,** New York : Columbia University Press, 1963.

(8) Merton H. Miller and Franco Modigliani, «Corporate Income Taxes and the Cost of Capital : A Correction», **American Economic Review,** Vol : 53, No : 3 (June 1963) pp. 433 - 443.

(9) Op. cit., James T.C. Mao, p. 96.

(10) For the derivation of this formula, see James T.C. Mao, «The Valuation of Growth Stocks : The Investment Opportuity Approach», in **Elements of Investments : Selected Readings,** Edited by Hsıu - Kwang Wu and Alan J. Zakon, 2 nd Ed. Holt, Rinehart and Winston, Inc., 1972, p. 255.

$$Po = \frac{E}{k} + \left(\frac{r-k}{k}\right) bE \left(A + \frac{(1+g)^{n1-1}}{(1+k)^{n1}} B + \frac{(1+g)^{n1-1}}{(1+k)^{n1+n2}} C\right) \quad (9)$$

In this formula, A is the present value of the investments made in periods 1 ...., n_1 that corresponds to first stage of growth and given by the following equation :

$$A = \sum_{i=1}^{n_1} \frac{(1+g)^{i-1}}{(1+k)^{i}} \qquad (10)$$

B is the present value of the constant amount of investments made in periods $n_1 + 1, n_1 + 2,$ .... $n_1 + n_2$, as of the beginning of period $n_1 + 1$. It corresponds to second stage and simply expressed as

$$B = \sum_{i=1}^{n_2} \frac{1}{(1+k)^{i}} \qquad (11)$$

Finally, C is the present value of the declining amount of investments made in periods $n_1 + n_2 + 1, n_1 + n_2 + 2,$ ...., $n_1 + n_2 + n_3$ as of the beginning of period $n_1 + n_2 + 1$. C which corresponds to the case of declining growth is shown by the following formula :

$$C = \sum_{i=1}^{n_3} \frac{(n_3 - i + 1)}{n_3 (1+k)^{i}} \qquad (12)$$

V — TAYLOR'S CLOSED FORM VALUATION FORMULA

As an extension of Mao's three - stage growth model, Taylor derives the closed form expression for the formula (11). He shows that the generalized model can be used to derive other models which require more restrictive assumptions.

(11) Op. cit., Walton Taylor, p. 1575.

By finding the closed form expressions for A, B, and C in Mao's formula, Taylor derives his closed form equation for P_0 which is the following :

$$P_0 = \frac{E}{k} + \frac{r-k}{k} bE \left[\frac{1-(dc)^{n_1}}{k-rb} + \frac{d^{n_1-1} c^{n_1}}{k} \left\{ 1 - c^{n_2} + \frac{c^{n_2}}{n_2} \left\{ \frac{n_3 c^{n_3} (c-1) - 2c^{n_3+1} + c^{n_3} + 2c - 1}{c-1} \right\} \right\} \right] \quad (13)$$

where $d = 1 + rb$ and $c = (1 + k)^{-1}$

If it is assumed that there are no n_2 and n_3 periods and n_1 period approaches infinity, he shows that the result is the Miller and Modigliani's constant growth model. It is also illustrated that the Gordon and Walter Model can be derived from the generalized model by using certain assumptions. Thus, the closed form Mao model provides an useful explanation for other valuation models.

VI — CONCLUSION

If it is assumed that there exists a growth potential for most firms, the growth factor must be taken into consideration in valuing a firm. Since these models clearly show the importance of growth potential in the valuation process, it is unwise for a financial investor to ignore the growth factor in valuing a firm's share. By studying the growth behavior of firms more rigorously, it is possible to derive more realistic valuation models.