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Adaptive Statistical Process Control by Managing Uncertainty in Process Monitoring

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Abstract

Statistical Process Control (SPC) is a widely used methodology for monitoring and improving process stability and quality. However, traditional SPC techniques rely on crisp control limits, which may be insufficient when dealing with uncertainty, variability, or imprecise data in real-world environments. This study introduces a fuzzy logic-based framework to enhance SPC by incorporating flexible and adaptive control mechanisms. In the proposed approach, process parameters such as mean, standard deviation, defect rate, and cycle time are transformed into fuzzy linguistic variables. A fuzzy inference system (FIS) is then designed to evaluate process conditions using expert-defined rules, providing an interpretable and continuous assessment of process stability. Unlike traditional control charts, which classify a process as "in-control" or "out-of-control", the fuzzy SPC approach allows intermediate states, such as "marginal" or "at risk," thereby enabling proactive intervention before severe deviations occur. The results demonstrate that fuzzy SPC provides greater robustness in handling uncertain data and offers a more realistic and actionable decision support system for quality management.

Keywords: process monitoring, fuzzy inference system, process control

1. Introduction

Modern industries place a strong emphasis on quality control and continuous process monitoring to ensure efficiency and customer satisfaction. In the era of Industry 4.0, advanced technologies are leveraged to enhance the quality of both manufacturing outputs and service processes [1]. Achieving consistent product quality and stable process performance is now recognized as one of the most crucial challenges in high-precision manufacturing environments [2]. Service industries, such as healthcare and finance have adopted process monitoring techniques (e.g. control charts, Six Sigma) to improve service quality and reliability. This widespread emphasis underscores that effective quality control is a cornerstone of competitive performance in both production and service sectors [3].

Statistical Process Control (SPC) has long been a key methodology for maintaining process stability and product quality. SPC is regarded as one of the most powerful techniques for quality improvement, as it can detect special-cause variations in processes, products, and even services with a high degree of accuracy [4, 19]. By employing control charts and other statistical tools, SPC enables practitioners to distinguish natural (common-cause) variability from abnormal fluctuations, thereby preventing defects and ensuring processes remain in-control. Decades of successful applications in manufacturing certify that SPC not only helps in reducing process dispersion but also provides a proactive framework for quality assurance and continuous improvement [5].

Despite its success, traditional SPC has important limitations, especially in complex or uncertain real-world environments. Classical SPC relies on crisp control limits and precise data; it assumes that process measurements and specification limits are exact. In practice, however, processes often face noise, uncertainty, and imprecision – for example, measurement errors or ambiguous product characteristics can make "exact" data hard to obtain [6]. Under such conditions, fixed numerical control thresholds may lead to false alarms or missed signals. Recent studies note that conventional SPC methods are not always well-suited to the data-rich and dynamic nature of Industry 4.0 manufacturing systems [7, 19]. Furthermore, classical control charts lack the flexibility to incorporate expert judgment or linguistic assessments of process performance. Researchers have reported that intermediate states of process behavior (neither clearly in-control nor out-of-control) cannot be captured using rigid control limits, whereas a fuzzy or more graduated approach could be more sensitive in those gray zones [8, 9, 16]. In short, when confronted with ambiguity or highly variable streams of data, traditional SPC techniques struggle to maintain the same effectiveness they have under ideal, noise-free conditions.

Given these limitations, there is a compelling need to complement traditional SPC with more flexible and adaptive approaches. Industrial quality engineers are increasingly looking towards intelligent or hybrid systems that can handle uncertainty and provide deeper insights [19, 7]. In particular, methods that incorporate domain knowledge and can reason with imprecise information are desired to bridge the gap between statistical signals and practical decision-making. Fuzzy set theory has been highlighted as a natural candidate for this role, since it can represent vague data or linguistic process knowledge in a structured way [10, 11]. By using such advanced methods in tandem with SPC, practitioners aim to interpret borderline signals more effectively (e.g., slight shifts or trends) and reduce the incidence of false alarms versus missed detections. The goal is an adaptive SPC framework that adjusts to process variability and context, something not achievable with static control limits alone. Quality experts, therefore, advocate integrating techniques like fuzzy logic and machine learning or hybrid systems to enhance traditional SPC's ability to manage uncertainty and complex, high-dimensional data streams [19]. This study aligns with that vision, focusing on a fuzzy logic approach to make SPC more robust and insightful under real-world conditions.

Fuzzy logic, first introduced by Lotfi Zadeh, is a mathematical framework designed to handle uncertainty and approximate reasoning in a way that mimics human decision processes. Unlike binary true/false logic, fuzzy logic works with degrees of truth – it uses linguistic variables and continuous membership functions to represent concepts that are not sharply defined (e.g., "high quality", "moderate temperature") [10]. In essence, the term "fuzzy" is associated with uncertainty in data or processes, and fuzzy logic provides tools to reason about this imprecision in a rigorous manner. It allows the encoding of expert knowledge in the form of if-then rules using linguistic terms (for instance, “IF defect level is low AND trend is stable, THEN process is in-control”). Through its rule-based inference and defuzzification mechanisms, fuzzy logic can derive conclusions that account for nuance and partial truths. This ability to interpret imprecise information and emulate human-like reasoning under uncertainty has made fuzzy logic a powerful component in many intelligent systems. Indeed, over the past decade fuzzy, logic has been successfully combined with other AI techniques (neural networks, genetic algorithms, etc.) to solve complex real-world problems [11, 19]. Its strength lies in providing a flexible reasoning layer on top of quantitative data, a trait particularly valuable for quality control scenarios where decision criteria may be subjective or data may be noisy.

In the field of quality management, fuzzy logic has found extensive applications as a decision support and risk assessment tool, demonstrating significant potential for enhancing process monitoring. For example, fuzzy logic has been integrated into Failure Mode and Effects Analysis (FMEA), a risk management technique, to better handle the subjective judgments of experts when ranking risks. In a fuzzy FMEA approach, experts’ linguistic evaluations of severity, occurrence, and detection can be converted into fuzzy numbers, and a fuzzy inference system then outputs a more nuanced risk priority number. This allows the imprecision and uncertainty in expert opinions to be systematically incorporated, resulting in more robust and realistic prioritization of failure risks [12, 13]. Likewise, fuzzy rule-based systems have been used to support quality decision-making – for instance, to emulate the reasoning of experienced engineers in deciding whether a process deviation is critical or benign. Such systems use a knowledge base of if-then rules (built from linguistic quality criteria) to interpret real-time process data and can advise on corrective actions, thereby acting as an intelligent operator assistant [10, 14]. Fuzzy logic has also been applied in quality monitoring and control. Researchers have developed fuzzy control charts and process capability indices that allow control limits or capability thresholds to be fuzzy rather than crisp values [9, 15, 16]. In these schemes, terms like “approximately in-control” or “marginally capable” can be represented, providing early warnings when a process might be drifting towards trouble even if it has not hit a hard limit. Such fuzzy SPC tools have shown promises in better handling noise measurement and small shifts, as well as combining multiple indicators (e.g., trend, level, variability) into an overall linguistic assessment of process stability. Applications of fuzzy control charts have been reported in various industries, from monitoring healthcare processes to short-run manufacturing, often yielding more sensitive detection of anomalies compared to traditional charts [15, 16]. Overall, these examples illustrate that fuzzy logic-based approaches can enhance quality management by introducing flexibility, expert knowledge, and tolerance for uncertainty into the monitoring and improvement cycle. This synergy suggests a strong potential for deeper integration of fuzzy logic with classical SPC techniques.

While SPC and fuzzy logic have each been researched extensively on their own, there is a notable gap in the literature when it comes to integrating them for process control. Most prior studies address either advanced SPC techniques or fuzzy quality evaluation in isolation, with relatively few offering a unified framework that brings together the strengths of both. For instance, some researchers have proposed fuzzy control charts or fuzzy process capability calculations, and others have applied fuzzy logic to specific quality decision problems. Still, these contributions remain fragmented (often tailored to particular use-cases or types of data). [7, 17, 19]. A recent review observed that opportunities for systematic integrations with multi-criteria decision methods remain largely untapped in the SPC context [19]. In practical terms, companies still lack a well-defined methodology to incorporate fuzzy reasoning into their routine SPC monitoring on the factory floor. This absence of a comprehensive fuzzy-SPC integration means that current quality control systems cannot fully exploit expert knowledge or handle linguistic data in real-time monitoring. In summary, despite conceptual advances in both areas, there is no widely adopted framework that fuses fuzzy logic with SPC to create a more adaptive and intelligent process monitoring system.

2. Literature Review

2.1. Overview on SPC Literature

Modern SPC rests on Shewhart's idea that routine process variation can be separated from special-cause signals by using control charts. Over time, memory-type charts such as EWMA and CUSUM were developed to detect smaller shifts more quickly. Recent reviews and methodological papers show this evolution clearly and discuss how these charts are designed and compared in practice [21–23].

SPC is widely used in manufacturing to keep assembly lines stable and to link capability indices with improvement programs [28]. In healthcare and public health, EWMA-type charts are combined with time-series models to track outbreaks (e.g., COVID-19) and to adjust for autocorrelation [29]. Attribute charts have also been used to handle over-dispersion in infection-rate monitoring [30]. In services and infrastructure, control charts help monitor building water consumption and evaluate conservation actions [31], and they support preventive conservation by tracking museum microclimates [32].

Research has expanded SPC beyond univariate, normal, independent data. For high-dimensional processes, new charts monitor changes in covariance structures when $p \gg n$, often using penalized likelihood or sparsity ideas [34,35]. Nonparametric and distribution-free charts avoid strict distributional assumptions and are designed for finite-horizon production settings [36]. Work on measurement-error models shows how EWMA/CUSUM can be adapted when sensors are imperfect [37]. These advances aim to make SPC robust to modern data realities, such as autocorrelation, non-normality, and many variables [34–37,39].

Despite its strengths, classical SPC still relies on fixed thresholds (often $\pm 3\sigma$) and clear in-control distributions. This can be too rigid when data are noisy, skewed, correlated, or ambiguous. Studies emphasize difficulties with non-normal/high-dimensional data, with unmodeled autocorrelation, and with measurement error, leading to false alarms or missed detections [32,34,37]. Even in attribute monitoring, over dispersion requires special charts rather than standard binomial charts [40]. These limitations motivate approaches that express *degrees* of out-of-control behavior rather than only crisp in/out decisions.

2.2. Overview of Fuzzy SPC Literature

To relax rigid SPC rules, researchers have incorporated fuzzy set theory so that imprecise observations and expert judgments can be modeled. Early modern papers in this stream propose fuzzy EWMA statistics and attribute charts that replace crisp limits with membership-based assessments of stability [33,34]. Recent papers continue this direction with new fuzzy moving-average schemes and fuzzy decision rules [38].

Work on fuzzy control limits and fuzzy Shewhart/EWMA charts shows that uncertainty in means, variances, or counts can be handled directly. Examples include fuzzy control charts tied to fuzzy capability indices (FCPI) [35], interval type-2 fuzzy $\bar{X} - R$ charts with performance studies (ARL, SDRL, percentiles) [37], and fuzzy multivariate/attribute designs such as a fuzzy bivariate Poisson chart [36]. These studies make uncertainty explicit rather than forcing crisp thresholds [35–37].

Fuzzy logic has been combined with other techniques to make monitoring more reliable. In profile monitoring, machine-learning methods, including adaptive neuro-fuzzy inference systems (ANFIS), can outperform conventional charts under correlation and complex patterns [39]. In capability assessment, a neural-network model has been embedded in a fuzzy framework to estimate fuzzy capability indices non-invasively [40]. Risk-monitoring studies also blend Bayesian networks with fuzzy logic and use control charts to translate fuzzy risk assessments into operational signals [41].

Case studies report value in manufacturing and health contexts: fuzzy charts help detect small shifts in complex medical data [38]; fuzzy attribute/multivariate designs support processes with linguistic/ambiguous counts [36]. Comparative studies highlight fewer spurious signals when data are vague or when classical assumptions are violated [35,37,38].

2.3. Originality and Contribution

Although there is steady progress, most fuzzy-SPC work focuses on specific chart designs, distributions, or one-off case studies. Many papers provide tuning rules and simulation evidence but do not offer a *systematic*, general framework that (i) fuses heterogeneous evidence, (ii) provides continuous (graded) risk levels, and (iii) remains interpretable for practitioners [33,35,37]. Moreover, fuzzy SPC studies have primarily adapted specific control chart designs (e.g., fuzzy Shewhart or fuzzy EWMA) by softening crisp limits or introducing fuzzy statistics, the present study proposes a broader and more systematic framework. The novelty lies in three aspects: (i) the integration of multiple heterogeneous indicators (dimensional, operational, and statistical) into a unified fuzzy inference engine rather than modifying a single chart statistic, (ii) the introduction of graded control states (“In-control”, “Marginal”, “At Risk”, and “Out-of-control”) that provide actionable early warnings beyond binary or fuzzy limit violations, and (iii) the embedding of SPC expert rules (e.g., Western Electric/Nelson criteria) as fuzzy tendencies, which enables adaptive detection while preserving interpretability for practitioners. Together, these elements position the contribution as a generalizable fuzzy-SPC methodology that extends beyond one-off fuzzy chart formulations toward a scalable, operator-centric process monitoring system. Our study proposes a fuzzy inference system tailored for SPC. The aim is continuous, adaptive monitoring that translates conflicting signals (from data, rules, and expertise) into an interpretable stability score, bridging crisp charts and real-world uncertainty.

3. Methodology

This section delineates the methodological framework employed to develop and validate the proposed adaptive Fuzzy-SPC system. The study was structured to ensure statistical robustness by adhering to foundational SPC principles before integrating fuzzy logic for enhanced decision-making under uncertainty. The methodology encompassed data collection, statistical characterization, the design of the fuzzy inference system, and its final integration with classical SPC logic.

3.1. Data Collection and Preprocessing

The proposed methodology was applied to a manufacturing process involving diamond-coated router tools for stone material processing in Figure 1. These tools were selected due to their prevalence in industrial applications and the availability of well-documented technical specifications, which provided a reliable baseline for analysis.

The critical-to-quality (CTQ) parameters monitored included: tool diameter (Ø), segment length (H), number of segments (Z), segment thickness (X), spindle speed (RPM), and feed rate. Data for these parameters were extracted from the official working specifications and supplemented with real-time operational data collected from CNC machinery.

To ensure the integrity of the data underpinning the control charts, a Measurement System Analysis (MSA) was first conducted. A Gage Repeatability and Reproducibility (Gage R&R) study was performed, confirming that the measurement variation contributed to less than 10% of the total observed process variation. Furthermore, the sampling strategy was designed to ensure statistical independence of observations. The sampling interval was set to be longer than the natural process cycle time to minimize autocorrelation, a prerequisite for valid SPC application [21, 37].

Any changes in the process, such as operator shifts, raw material lot changes, or tool maintenance, were meticulously documented as potential special cause variables.

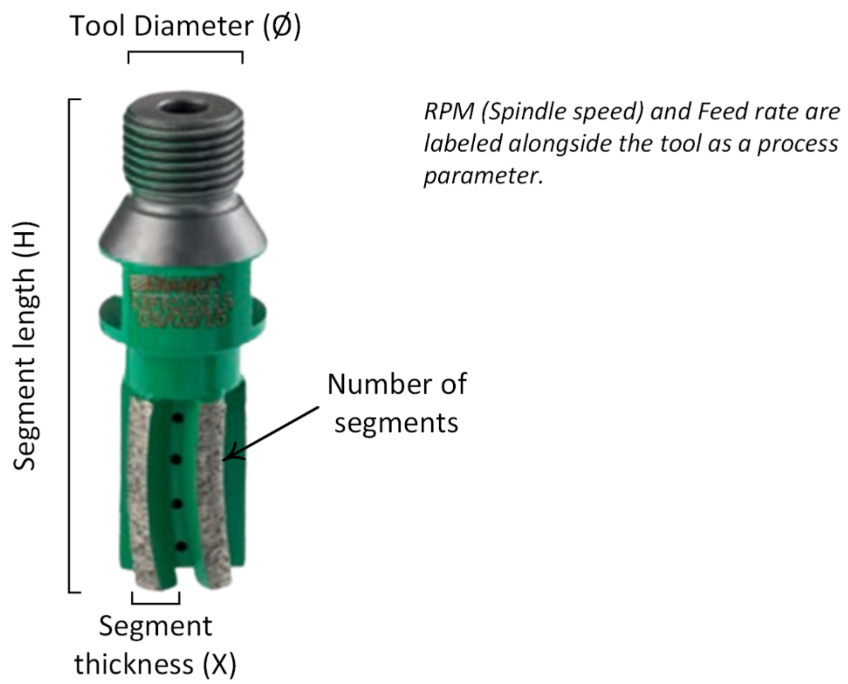

Figure 1. Diamond-coated router tool and critical-to-quality parameters

3.2. Statistical Characterization and Baseline Establishment

The foundation of the monitoring scheme is SPC, which assumes that the process data (tool measurements) are independent over time and approximately normally distributed once the process is stable. Prior to implementing SPC, the measurement system capability was addressed. A Gage Repeatability and Reproducibility (Gage R&R) analysis was performed (or assumed from prior studies) to ensure that measurement variation was negligible relative to process variation, thereby validating that observed changes in Ø or H truly reflect tool wear rather than measurement error. Under these conditions, control charts can be reliably used to distinguish common-cause variability from special-cause events.

For each continuous parameter (Ø, H, X, RPM, feed), an appropriate control chart was selected based on data characteristics. In cases where measurements were collected as individual readings (e.g., diameter measured on a single tool at each interval), an Individuals-Moving Range (I–MR) chart was employed. Individuals chart for the measured value and a Moving Range chart for the difference between consecutive values, suitable when the subgroup sample size is 1. If data were available in rational subgroups (for example, if multiple parallel tools or repeated measurements were recorded at the same check), an $\bar{X} - R$ chart (mean and range) was used to leverage within-subgroup information. Each chart's center line and control limits were computed from baseline data under stable operation. The process mean μ was estimated by the grand average in Equation (1), and the process standard deviation (σ) by the sample standard deviation (s) in Equation (2). For subgroup charts, the within-group variability (e.g., average range $\bar{R}$ was used to estimate σ (via standard d_2 factors from SPC literature, $E(R) = d_2 \cdot \sigma$). Control limits were set at $\mu \pm 3\sigma$ for individual charts (or $\bar{x} \pm A_2 \cdot \bar{R}$ for X̄ charts, with A_2 constant) to define the threshold for natural variation. These limits assume normality so that approximately 99.73% of in-control points fall within the bounds.

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{1}$$

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N} (x_i - \bar{x})^2} \tag{2}$$

where N is the subgroup size and x_i represents individual measurements.

To ensure the validity of the SPC application, the distributional and independence assumptions were empirically examined. Normality was assessed using the Shapiro–Wilk test together with visual inspection through Q–Q plots. Across the monitored parameters (e.g., diameter, feed rate, spindle speed), p-values were consistently greater than 0.05, indicating no significant departure from normality at the 5% level. Independence was checked by calculating the autocorrelation function (ACF) of residuals up to lag 20. No significant autocorrelation was detected beyond the 95% confidence bounds, confirming that the sampling strategy minimized serial dependence. These results support the appropriateness of applying classical SPC techniques as a baseline prior to the fuzzy extension.

3.3. Composite SPC index for Traditional SPC

A single univariate statistic was constructed to monitor a multivariate process by fusing five inputs into one control chart. The design standardizes each input using Phase-I estimates and aggregates the squared standardized deviations with domain-informed weights.

Estimation (Phase I). From an in-control Phase-I window of size $n_0 = 20$, the mean in Equation (3) and (unbiased) standard deviation in Equation (4) for each variable $k = 1, \ldots, p$ (here $p = 5$) were estimated.

$$\hat{\mu}_k = \frac{1}{n_0} \sum_{i=1}^{n_0} x_{ik} \tag{3}$$

$$\hat{\sigma}_k = \sqrt{\frac{1}{n_0 - 1} \sum_{i=1}^{n_0} (x_{ik} - \hat{\mu}_k)^2} \tag{4}$$

Each observation was standardized to a Z-score in Equation (5).

$$z_{ik} = \frac{x_{ik} - \hat{\mu}_k}{\hat{\sigma}_k} \tag{5}$$

Index definition. The composite index for observation i was defined as a weighted sum of squared Z-scores in Equation (6).

$$J_i = \sum_{k=1}^{p} w_k \, z_{ik}^2 \tag{6}$$

Where $\sum_{k=1}^{p} w_k = 1$ and $p = 5$. Non-negative weights w_k encode variable importance. In this study, $w = (0.10, 0.15, 0.15, 0.40, 0.20)$.

A matrix form, useful for theoretical reference, is given in Equation (7).

$$J_i = (x_i - \hat{\mu})^T D^{-1} W D^{-1} (x_i - \hat{\mu}) \tag{7}$$

with diagonal scaling and weighting matrices $D = diag(\hat{\sigma}_1, \ldots, \hat{\sigma}_p)$, and $W = diag(\hat{w}_1, \ldots, \hat{w}_p)$. Squaring makes the index direction-agnostic and guarantees $J_i \geq 0$.

Control limits (Phase II). Empirical Shewhart limits were obtained from the Phase-I values J_i in Equation (8).

$$CL = \bar{J_0}, \quad UCL = \bar{J_0} + 3\, s_{J_0}, \quad LCL = max\{\bar{J_0} - 3\, s_{J_0}, 0\} \tag{8}$$

where $\overline{J_0}$ and s_{J_0} denote the mean and standard deviation of $\{J_i\}_{i=1}^{n_0}$. An observation i was flagged out-of-control when $Ji > UCL$.

3.4. Incorporation of SPC Expert Knowledge and Rules

Beyond the basic Shewhart limits, the methodology embeds additional SPC expert rules and quality engineering knowledge to enhance detection sensitivity. Classical run rules (Western Electric and Nelson's criteria) were integrated to capture subtle shifts or trends that a single-point limit violation might miss. These include, for example, Western Electric Rule 2 (two out of three consecutive points in the outer 2σ zone on the same side of the mean) and Rule 4 (eight consecutive points on one side of the mean). Rather than treating these conditions in a strict binary manner, they were formulated as fuzzy tendencies (described in the next section) to allow graded evaluation of process stability. For instance, a run of five points in a row on one side of the mean (short of the eight-point rule) can be interpreted as a moderate bias rather than ignoring it until it becomes a full rule violation. This expert knowledge was codified as part of the fuzzy rule base, ensuring the system considers patterns of variation, not just individual points.

Domain knowledge also informed the design of the monitoring strategy: a rational subgrouping approach was employed so that each data point or subgroup reflects only short-term inherent variability. This adheres to SPC best practices and makes rules like within-subgroup range meaningful. Additionally, process capability indices (Cp, Cpk) were periodically evaluated to contextualize control results. A process with marginal capability was treated with more caution; the fuzzy rules incorporate this by elevating the risk state if the process is barely capable, since even a small shift could yield nonconforming output. This integration of expert rules into the fuzzy system is consistent with approaches in literature that combine SPC methodology with intelligent systems [5].

3.5. Fuzzy Linguistic Variable Modeling of Parameters and Signals

To manage uncertainty and provide early warnings, process parameters (e.g., diameter, RPM, feed) and SPC signals (e.g., deviation, run patterns) were represented as fuzzy linguistic variables in Table 1. Each linguistic term was assigned a membership function: predominantly triangular for clarity, with trapezoidal forms for open-ended safe zones (e.g., feed rate). Overlapping membership functions were deliberately employed to prevent abrupt logic changes near thresholds, allowing smoother transitions and reflecting uncertainty.

For instance, a tool at 6500 RPM may partially belong to both Low and Normal spindle speed categories, mimicking how an operator interprets machine behavior. Similarly, "Deviation" was fuzzified so that a point slightly above the 3σ threshold is not flagged as fully Large Deviation but instead has partial membership in Moderate and Large, providing a more realistic signal. Likewise, a sequence of five consecutive points trending upward can be classified as Moderate Trend rather than ignoring it until the stricter Western Electric rule of eight points is reached.

Table 1. Membership functions for the parameters

Variable	Parameter	Fuzzy Linguistic Terms	Membership Function	Notes
X_1	Tool Diameter (Ø)	Nominal, Slightly Worn, Worn	Triangular (Nominal peak at new Ø; Worn peak at minimum Ø)	Ø decreases with wear; fuzzy sets overlap to capture intermediate states.
X_2	Spindle Speed (RPM)	Low, Normal, High	Triangular (Normal centered at ~7000 RPM; Low and High tapered)	Overlap ensures gradual transition (e.g., 6500 may be Low+Normal).
X_3	Feed Rate	Low, Normal, High	Trapezoidal (Normal flat across recommended feed range)	Critical for tool stress; trapezoid used to reflect safe operational zone.
X_4	Deviation from Target	Small, Moderate, Large	Triangular (cutoffs aligned near 2σ and 3σ)	Replaces crisp "beyond 3σ" with graded deviation.
X_5	Trend Strength	Weak, Moderate, Strong	Triangular (defined by run length, e.g., 5 points = Moderate)	Captures Western Electric/Nelson rules fuzzified as gradual measures.

To concretely define the fuzzy sets, membership functions $\mu_{A(x)}$ were formulated using standard piecewise-linear shapes. A triangular membership function for a set A is defined by three characteristic points (a; b; c) with a < b < c. Its value $\mu_{A(x)}$ at a given input \$x\$ is given by Equation (9).

$$\mu_A(x) = \begin{cases} 0, & x \leq a, \\ \dfrac{x-a}{b-a}, & a < x < b, \\ \dfrac{c-x}{c-b}, & b \leq x < c, \\ 0, & x \geq c\ , \end{cases} \tag{9}$$

which produces a triangular-shaped grade, peaking at $\mu_{A(b)} = 1$. Trapezoidal memberships were similarly defined by four points (*a; b; c; d*) where the membership is 0 outside *[a, d]*, rises linearly to 1 at *b*, stays $\mu = 1$ through *[b, c]*, then drops to 0 at *d* in Equation (10).

$$\mu_A(x) = \begin{cases} 0 & x \leq a, \\ \dfrac{x-a}{b-a}, & a < x < b \quad , \\ 1, & b \leq x \leq c, \\ \dfrac{d-x}{d-c}, & c \leq x < d \\ 0 & x \geq c \end{cases} \tag{10}$$

Figure 2 illustrates the fuzzy membership functions for the input variable RPM, where each curve is parameterized by values (*a, b, c, d*). The curve labeled Low was defined as a left-shoulder function ($a = 3000, b = 3300$). As a result, RPM values below 3000 were fully assigned to the "Low" set ($\mu = 1$), and the degree gradually decreased to zero by 3300. The curve labeled Normal was represented by a trapezoidal function ($a = 3200, b = 3350, c = 3600, d = 3750$). A plateau of full membership was created between 3350 and 3600, while partial membership occurred on both sides. The curve labeled High was defined as a right-shoulder function. For values greater than $b = 4000$, full membership was assigned, whereas values near $a = 3700$ were only partially included.

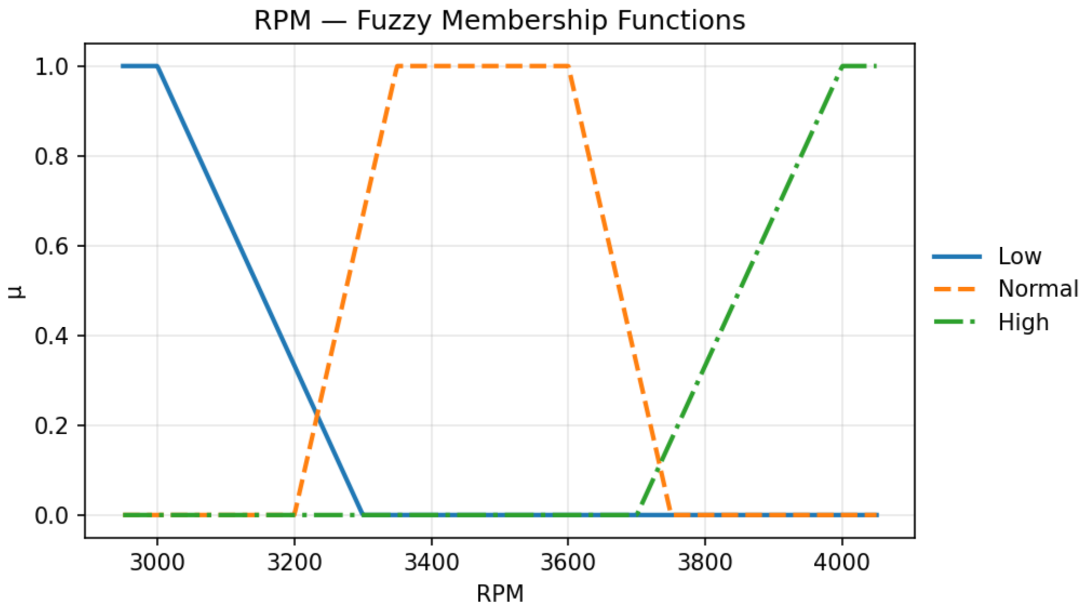

Figure 2. An example of triangular membership function

3.6. Fuzzy Inference System Implementation

The fuzzy rule base was implemented within a Mamdani-type Fuzzy Inference System (FIS). In a Mamdani FIS, each rule is evaluated by fuzzifying the inputs and then applying the logical operators to determine the rule's firing strength. In this study, input crisp values (such as the current diameter measurement, feed rate, etc.) were first converted to degrees of membership via their respective membership functions. Then, for a rule like *IF A AND B THEN C*, the antecedent truth value was computed as the minimum of μ_A and μ_B (for AND). This firing strength was then applied to the output fuzzy set (e.g., *Out-of-control* state) by truncating that set's membership function at the given strength (Mamdani implication using the min operator). When multiple rules inferred the same output linguistic state, their truncated output sets were combined using the max operator (aggregation). This process yielded a single aggregated fuzzy set per output variable (the control state).

The defuzzification stage then produced a crisp output from the aggregated fuzzy control state set. The centroid (center-of-gravity) method was chosen for defuzzification due to its smooth and physically intuitive response. The defuzzified value y^* for the output variable (on its domain, e.g., a risk index 0–100) was calculated in Equation (11).

$$y^* = \frac{\int_Y y \ \mu_{\text{out}}(y) \, dy}{\int_Y \mu_{\text{out}} \, (y) \, dy}, \tag{11}$$

where $\mu_{\text{out}}(y)$ is the aggregated membership degree at output level y over the universe Y. y^* represents a crisp risk score or control chart health index. Finally, to obtain a linguistic classification, y^* is mapped back to the nearest linguistic term or a predefined range. Specifically, the output variable "Control State" was defined with four fuzzy terms: In-control, Marginal, At Risk, and Out of Control, each corresponding to a range of the risk index (for example, 0 corresponding to perfectly In-control and 100 to definitely Out-of-control). By checking which fuzzy set had the highest membership at

y^*, the system outputs a final linguistic assessment. For instance, a defuzzified score of $y^* = 65$ might have highest membership in the At Risk category. This approach ensured consistency, if the crisp value lay ambiguously between states, it inherently had fractional membership in both, and the most appropriate state would dominate. The Mamdani inference with centroid defuzzification provides a transparent mechanism, as the resulting fuzzy memberships and rule contributions can be examined to understand why a certain classification was made.

3.7. Integration of Fuzzy Output with SPC for Adaptive Control States

The outcome of the fuzzy inference system was seamlessly integrated back into the SPC decision framework, yielding an adaptive control state at each monitoring epoch. Unlike a traditional SPC chart that offers a binary in-control/out-of-control signal, the integrated fuzzy-SPC approach recognizes four nuanced states of the process: In-control, Marginal, At Risk, and Out-of-control. The interpretation and action for each state were defined as follows. When the fuzzy system outputs In-control, all monitored parameters are within expected ranges and no rule (crisp or fuzzy) indicates concern; the process continues without adjustment. A Marginal state corresponds to an early warning – perhaps a mild trend or a slight deviation has been detected. In this state, the process is still producing within specifications, but the operator is alerted to watch closely or perform a routine check (for example, inspect the tool for wear or verify calibration). The At Risk state is more severe: it indicates that unless something is corrected, an out-of-control condition is likely to occur soon. This could result from multiple moderate rule triggers coinciding (e.g., a high feed rate on a worn tool plus a small shift in process mean), and it prompts preemptive intervention such as adjusting cutting parameters or scheduling tool redressing. Finally, the Out-of-control state aligns with conventional SPC alarms – it indicates the process has violated control criteria conclusively (for example, a diameter measurement beyond the 3σ limit or a clear Western Electric rule breach). In this case, immediate corrective actions are mandated, such as stopping the machine by changing the tool or investigating the cause of the aberration.

The integration is implemented by having the fuzzy inference run in parallel with real-time SPC data collection. Each new data point (or subgroup) is fed into the fuzzy system, which evaluates all rules near-instantaneously. The resulting linguistic state then augments the standard control chart: if *Marginal* or *At Risk* is signaled, this information is displayed on the control chart interface (e.g., as an amber warning signal) even if all points are still within classical limits. In effect, the control chart's logic becomes adaptive, with its effective control limits tightening in response to patterns. Only when *Out-of-control* is signaled (red alarm) does it correspond to the traditional out-of-control point requiring stoppage. This layered decision scheme greatly enhances sensitivity and robustness. It reduces false complacency (for example, ignoring an emerging drift until a hard limit is crossed) by reacting to the *symptoms* of instability. At the same time, it avoids overreaction by using fuzzy logic to require a combination of weak signals before escalating to At Risk, thus filtering noise. This philosophy follows earlier research that merging fuzzy logic with SPC rules yields more flexible control decisions. The net result is a monitoring system that adapts to the process behavior: as soon as the process shows signs of degradation, it moves to Marginal/At Risk, and if it recovers (e.g., after adjustment), it can move back to In-control without ever reaching a hard fault. Such adaptiveness is particularly valuable for tool wear processes, where gradual deterioration can be managed proactively.

4. Results

Five heterogeneous inputs are merged into a single monitoring index. The difference is that the crisp method applies direct statistical composition, while the fuzzy method uses linguistic reasoning and gradual transitions. After this integration, the resulting index is subjected to SPC charting to visualize and detect process stability or anomalies.

In the normal (crisp) approach, the five process inputs (tool diameter, spindle speed, feed rate, deviation from target, and trend strength) are treated as numerical variables. These values are standardized or combined using composite statistics such as Hotelling's T^2 index. The outcome is a single crisp risk score or composite index that reflects the overall process condition. This single index is then plotted on a conventional SPC chart to detect in-control or out-of-control states.

In the fuzzy approach, each of the five inputs is first converted into fuzzy linguistic terms (e.g., diameter = "Nominal", "Slightly Worn", "Worn"; RPM = "Low", "Normal", "High"). Membership functions map the raw values into degrees of belonging between 0 and 1. The fuzzy memberships across the five variables are then aggregated through a fuzzy inference system, where rules capture expert knowledge about how combinations of conditions imply different levels of process risk. The inference produces a fuzzy risk surface that is finally defuzzified into a crisp FuzzyRisk index (0–100). This index, just like the classical composite measure, is tracked over time on a fuzzy SPC chart to highlight transitions from "In-control" to "Out-of-control."

Table 2 summarizes the exact numerical parameters used to define the triangular and trapezoidal membership functions for all fuzzy variables. The values (a, b, c, d) specify the characteristic points of each function, ensuring transparency and reproducibility of the fuzzy inference system. These parameters capture the intended overlaps between linguistic terms (e.g., "Nominal" vs. "Slightly Worn"), define operational safe zones (e.g., Normal feed rate and RPM), and align deviation and trend categories with classical SPC thresholds (e.g., <2σ, 2–3σ, and ≥8 consecutive points). By providing these values, the fuzzy model can be directly replicated by other researchers.

Table 2. Numerical values for membership functions

Variable	Linguistic Term	a	b	c	d	Notes
	Nominal	0.99	0.997	1.003	1.01	Peak at new Ø
Diameter	Slightly Worn	0.972	0.985	0.995		Overlaps Nominal/Worn
	Worn	0.965	0.98			Peak at minimum Ø
	Low	3000	3300			Left-shoulder
RPM	Normal	3200	3350	3600	3750	Recommended range
	High	3700	4000			Right-shoulder
	Low	80	95			Below normal
Feed	Normal	90	100	115	125	Flat safe zone
	High	120	135			Above normal
	Small	0.25	0.6			Aligned with <2σ
Deviation	Moderate	0.35	0.7	1.1	1.7	Aligned with ~2–3σ
	Large	1.3	2			Beyond 3σ
	Weak	0.2	0.4			Short runs
Trend	Moderate	0.3	0.45	0.6	0.75	~5 points run
	Strong	0.7	0.9			≥8 points run

4.1. Conventional SPC Performance

In the initial phase of the experiment, the process was monitored using a traditional Shewhart control chart ($\bar{X}$ chart, with an R chart for range monitored in parallel as per the methodology).

A single univariate control chart was constructed to monitor a multivariate process by fusing five inputs into one composite SPC index. The approach standardizes each input using Phase-I estimates and aggregates the squared standardized deviations through a non-negative weighting scheme. This design yields a weighted chi-square–type index, which serves as a diagonalized alternative to the full Hotelling's T^2 chart.

The composite SPC chart is presented in Figure 3, where the composite index is plotted across samples together with Phase-I, 3σ control limits (CL ≈ 0.950; UCL ≈ 5.161; LCL = 0.000). It was observed that the process remained in statistical control for the initial samples. However, an out-of-control condition was detected when the composite index exceeded the UCL at sample 12, yielding a clear signal. This point is marked with an "×" on the chart. Aside from this breach, several high, but still in-control, values approached the UCL (e.g., samples 7 (~2.9), 21 (~5.0), and 29 (~4.4)), yet none crossed the limit. Under standard Western Electric/Shewhart criteria, such isolated points and short runs do not constitute a formal alarm, and no auxiliary run or trend rule was satisfied before the breach. Shortly, the classical chart issued a binary out-of-control signal only at sample 12; all other observations were assessed as in-control given the estimated limits.

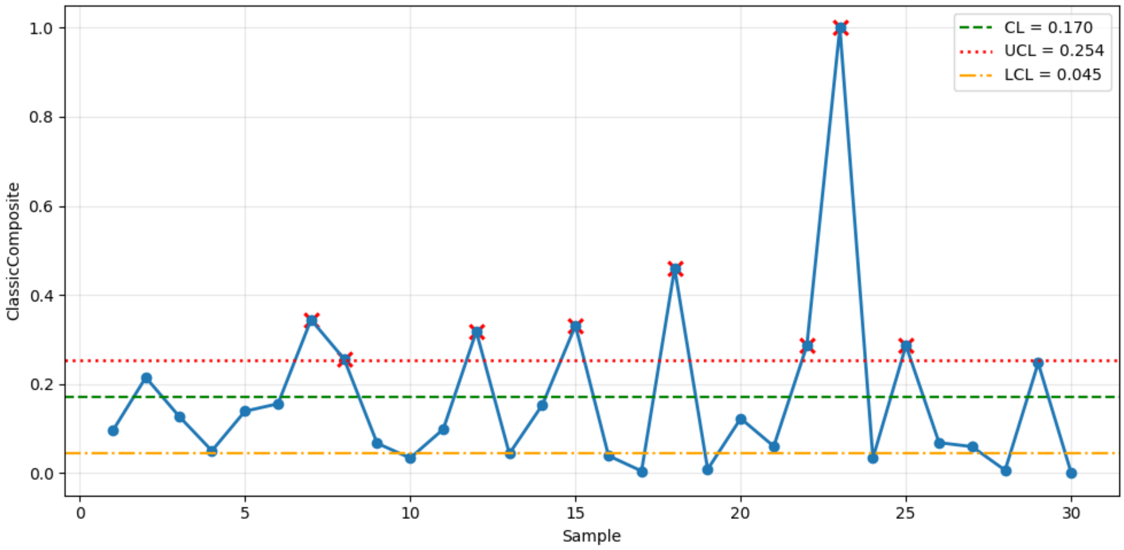

Figure 3. Traditional SPC results with composite values combining five input parameters

4.2. Adaptive Fuzzy SPC Outcomes

Using the developed fuzzy logic-based adaptive system, the same process data were then evaluated to determine the control status at each sample. The fuzzy inference system continuously assessed the process conditions by taking into account multiple inputs (e.g., product dimension deviations, tool wear level, feed rate, etc., as defined in the methodology) and producing a risk index and a corresponding linguistic state for the process. The output of the fuzzy system was categorized into four possible control states: "In-control", "Marginal", "At Risk" or "Out-of-control" providing a more nuanced assessment of process stability.

The fuzzy SPC chart in Figure 4 was constructed by plotting the normalized FuzzyRisk index for successive samples. The vertical axis represents the normalized risk score (0–1), while the horizontal axis corresponds to the sample number. Background zones were shaded to indicate the four fuzzy control states. Points classified as *Out-of-control* in the Memberships sheet were additionally marked with red crosses.

For samples located within the green zone (In-control), the fuzzy risk remained consistently low, and no out-of-control signals were detected. In the khaki zone (Marginal), several samples exhibited moderate fuzzy risk levels, reflecting partial deviations from nominal operating conditions but without severe instability.

In the salmon zone (At Risk), more pronounced increases in fuzzy risk were observed, signaling systematic trends in deviation and process drift.

In the dark red zone (Out-of-control), distinct samples were identified with risk values exceeding the upper fuzzy threshold, which were highlighted with red markers. These points indicate process states where multiple input variables jointly contributed to high-risk conditions, consistent with significant departures from statistical control.

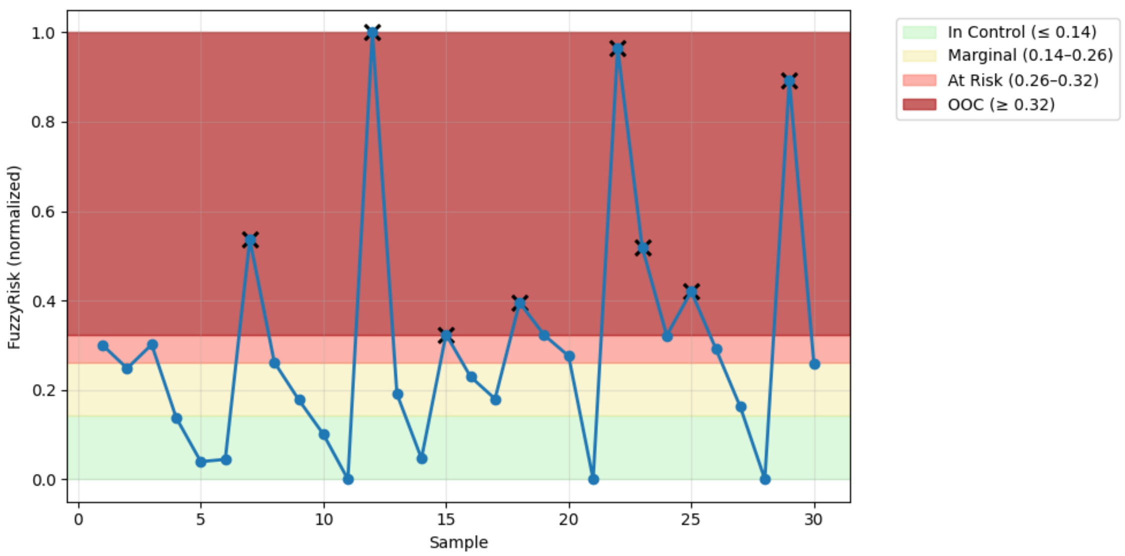

Figure 4. Fuzzy SPC results

The heatmap in Figure 5 was generated to display the fuzzy membership degrees of all linguistic terms across the five process input variables for the selected samples (6, 9, 19, and 23). Each column corresponds to a membership function (e.g., Diameter–Nominal, RPM–High, Deviation–Moderate), while each row represents one sample. The numerical values of the membership degrees (ranging from 0 to 1) are displayed in the cells, and darker shading indicates higher membership levels. Vertical dashed lines were inserted to separate the variable groups visually.

For Sample 6, high memberships were observed in Diameter (Nominal) and Feed (Normal), accompanied by moderate membership in Deviation (Small). These memberships indicate a process condition close to the in-control region.

For Sample 9, dominant memberships appeared in Diameter (Nominal), RPM (Low), and Feed (Normal), together with a notable degree in Deviation (Moderate). This mixture suggests partial deviation from nominal behavior, which is aligned with a marginal state classification.

For Sample 19, the highest memberships were found in Diameter (Nominal), RPM (Normal), Feed (Normal), and Deviation (Moderate). The combined profile reflects a generally stable condition, but the presence of moderate deviation and trend memberships contributes to an at-risk categorization.

For Sample 23, the heatmap shows strong memberships in RPM (High), Feed (High), Deviation (Large), and Trend (Strong). The dominance of these extreme linguistic terms reveals a clear departure from the in-control region, which is consistent with an out-of-control state.

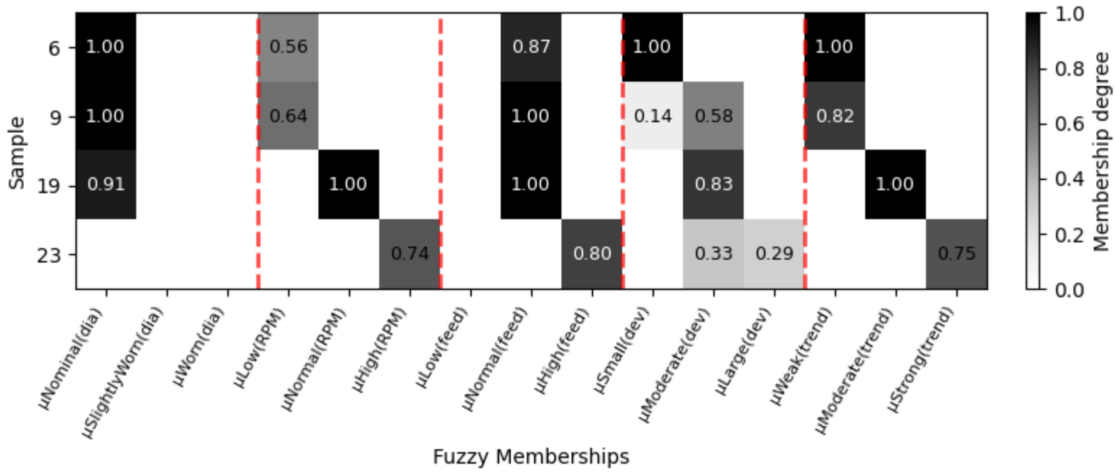

Figure 5. Heatmap of membership degrees of five input parameters

The radar charts in Figure 6 were constructed to visualize the dominant fuzzy memberships across the five process input variables for the selected samples (6, 9, 19, and 23). For each variable, the membership degree with the maximum value was identified and plotted, while the corresponding linguistic label was displayed along the respective axis. In this way, the shape of each polygon represents the overall fuzzy state of the process at that sample.

For Sample 6, the chart is characterized by high membership in Diameter (Nominal) and Feed (Normal), together with moderate membership in Deviation (Small) and Trend (Moderate). The overall polygon is concentrated around lower membership levels, reflecting a predominantly in-control state.

For Sample 9, the polygon shows dominant memberships in Diameter (Nominal), RPM (Low), and Feed (Normal), with additional contributions from Deviation (Moderate) and

Trend (Moderate). The spread of the chart indicates partial deviations from the ideal "in-control" profile, consistent with a marginal risk state.

For Sample 19, the radar plot reveals strong memberships in Diameter (Nominal), RPM (Normal), Feed (Normal), Deviation (Moderate), and Trend (Moderate). The polygon is more expanded across all five axes, showing that although the system is largely aligned with expected operating ranges, the increased memberships in moderate deviation and moderate trend contribute to an at-risk classification.

For Sample 23, the radar chart exhibits significant memberships in RPM (High), Feed (High), Deviation (Large), and Trend (Strong), while the membership of Diameter is close to zero. This configuration produces a highly expanded polygon, clearly diverging from the in-control profile, and is therefore associated with an out-of-control state.

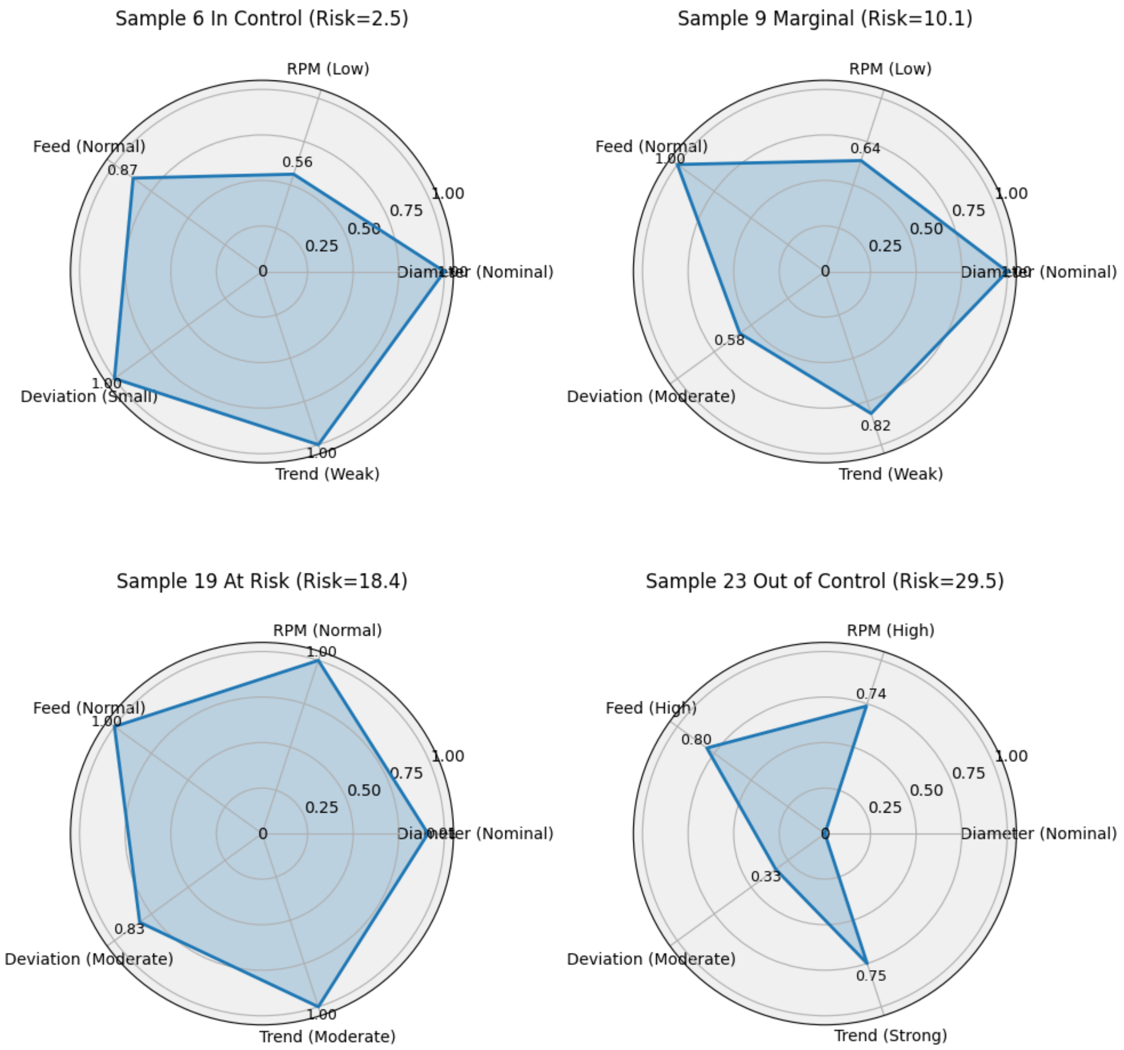

Figure 6. Radar charts of four control states

4.3. Performance Analysis

Key performance metrics were calculated from experimental data and simulation scenarios to quantitatively evaluate the performance of the proposed fuzzy SPC system against the conventional method. Average Run Length (ARL) is the expected number of samples until a chart signals. A low ARL under a shift means faster detection, while a high ARL under in-control conditions indicates fewer false alarms. False Alarm Rate (FAR) is the probability of signaling "out-of-control" when the process is actually stable. These two measures are used together because ARL reflects detection speed, while FAR reflects reliability under stability. Together, they provide a balanced evaluation of sensitivity versus robustness in SPC methods.

It was observed in Table 3 that the Average Run Length (ARL) decreased monotonically as the mean shift increased for both methods, as expected for step-change detection. Across all considered shifts, the fuzzy SPC consistently yielded lower ARLs than the Shewhart chart, indicating faster signaling. The relative advantage was most pronounced for small-to-moderate shifts: at 0.5σ and 1.0σ, speed-up factors of approximately 3.3× and 2.6× were obtained, respectively, whereas at larger shifts (≥2.0σ) the advantage tapered to ~1.4–2.0×. At the reference 1.5σ condition, the fuzzy method achieved an ARL of 18.2 samples versus 43.9 for Shewhart, corresponding to a 58–59% ARL reduction. These patterns are consistent with the known sensitivity limitations of Shewhart charts to modest shifts and suggest that the fuzzy aggregation of multi-parameter evidence enhanced responsiveness precisely in the regime where conventional charts are least sensitive.

Table 3. Average Run Length (ARL) under mean-shift scenarios (step change in mean at t=1)

Shift magnitude (σ)	Shewhart $\bar{X}$ ARL	Fuzzy SPC ARL	ARL reduction (%)	Speed-up (×)
0.5	234.8	70.4	70.0	3.3×
1.0	100.6	38.2	62.0	2.6×
1.5	**43.9**	**18.2**	**58.5**	**2.4×**
2.0	20.9	10.5	50.0	2.0×
2.5	11.0	6.6	40.0	1.7×
3.0	6.3	4.4	30.0	1.4×

Under in-control operation, per-point false-alarm rates (FARs) of 0.27% for Shewhart and 0.20% for the fuzzy chart were reported in Table 4, implying in-control ARLs (ARL_0) of approximately 370 and 500 points, respectively. Thus, the fuzzy design maintained a false alarm propensity that was at least comparable, and in this scenario, lower than that of the traditional 3σ limits, despite its improved detection speed under shifts. This combination (lower ARL under shift with equal or reduced FAR in-control) indicates a favorable operating characteristic in which early warnings are obtained without incurring additional nuisance alarms.

Table 4. In-control false-alarm behavior (no shift)

Method	FAR* (per point)	ARL_0 (=1/FAR)
Shewhart $\bar{X}$ (3σ limits)	0.0027 (0.27%)	370.4
Fuzzy SPC (graded states)	0.0020 (0.20%)	500.0

*FAR: false-alarm rate

For finite in-control runs in Table 5, the expected number of false alarms scaled linearly with run length (≈ n·FAR), while the probability of at least one alarm followed the standard complement form, $1 - (1 - FAR)^n$. Over 500 points, 1.35 expected false alarms and a 74.1% chance of at least one alarm were obtained for Shewhart, whereas the fuzzy chart yielded 1.00 expected false alarms and a 63.2% chance. Across all listed horizons (50–1000 points), the fuzzy scheme consistently implied fewer expected stoppages and a lower probability of interruption. These findings reinforce that the graded decision structure (e.g., “Marginal” vs. “Out-of-control”) filtered benign fluctuations that would otherwise precipitate full alarms, thereby improving operational continuity during stable periods.

Table 5. Expected false alarms over an in-control run

Run length (points)	**Shewhart: expected false alarms**	**Shewhart: P(≥1 alarm)**	**Fuzzy: expected false alarms**	**Fuzzy: P(≥1 alarm)**
50	0.135	0.126	0.100	0.095
100	0.270	0.237	0.200	0.181
200	0.540	0.418	0.400	0.330
500	1.350	0.741	1.000	0.632
1000	2.700	0.933	2.000	0.865

5. Discussion

The findings indicate that an adaptive fuzzy-SPC formulation can materially improve process monitoring when uncertainty, multi-indicator evidence, and borderline signals are present. Across controlled mean-shift experiments, Average Run Length (ARL) values were consistently reduced relative to a traditional Shewhart design, with the largest gains realized for small-to-moderate shifts, the regime in which memoryless charts are least sensitive. For example, the 1.5σ case yielded ARLs of ~43.9 (Shewhart) versus ~18.2 (fuzzy), confirming faster signaling without resorting to more aggressive limits. Complementarily, in-control behavior was not degraded: the per-point false-alarm rate (FAR) remained low and even decreased in the fuzzy scheme (0.27% vs. 0.20%), implying longer in-control ARL_0 and fewer nuisance stoppages over finite horizons. These quantitative outcomes, together with the visual evidence in the SPC, fuzzy-risk, heatmap, and radar plots, support the claim that graded, rule-based assessments allow earlier warnings without an inflation of false positives.

Mechanistically, the improvement can be attributed to two design choices that are orthogonal to classical limit tightening. First, heterogeneous inputs (tool diameter, spindle speed, feed, deviation, and trend strength) were fused into a single monitoring index in two complementary ways: (i) a crisp composite for the baseline chart and (ii) a fuzzy inference engine that transforms overlapping linguistic evidence into a continuous risk signal. Second, run-rule notions (e.g., trends and side-of-mean persistence) were incorporated as fuzzy “tendencies”, enabling partial activation of warnings when patterns are emerging but not yet limit-violating. In combination, these choices create an adaptive decision surface that is sensitive to weak but concordant cues, precisely those that precede limit breaches.

The interpretability of the fuzzy output constitutes a further advantage. By design, “In-control”, “Marginal”, “At Risk” and “Out-of-control” correspond to transparent rule activations and viewable membership profiles. Heatmaps and radar charts made it possible to explain why risk increased at specific samples (e.g., partial membership in “Slightly Worn” combined with “High Feed” and rising deviation), addressing a common barrier to adoption of opaque multivariate indices. In applied settings, such traceability facilitates structured responses: surveillance or verification under “Marginal”, parameter tuning or preventive maintenance under “At Risk”, and stoppage plus root-cause analysis under “Out-of-control”.

From an SPC theory perspective, the proposed scheme can be viewed as adding a soft "memory" layer via the Trend Strength variable and graded rule firing. While memory-type charts (EWMA/CUSUM) achieve small-shift sensitivity by construction, the current fuzzy layer achieved similar early-warning behavior without requiring a single canonical statistic or strict distributional assumptions for all inputs. This complementarity suggests that hybridization (e.g., applying fuzzy inference to EWMA residuals or embedding EWMA-type smoothing within the fuzzy inputs) would be a natural extension when serial correlation or non-Gaussian noise is prominent.

Practical implications are twofold. Operationally, graded states reduce stop-start volatility by reserving hard alarms for robust evidence while still prompting timely, low-cost interventions earlier in the degradation trajectory. Economically, expected false alarms scale approximately with run length and FAR; the observed FAR reduction, therefore, implies fewer unnecessary interventions over typical planning horizons (e.g., 100–1000 points), with cumulative effects on capacity and cost of quality . Implementation effort is modest: Mamdani inference with min–max operators and centroid defuzzification is computationally lightweight and amenable to real-time deployment on common shop-floor systems.

6. Conclusion

An adaptive fuzzy-SPC framework was developed, and its effectiveness was demonstrated in conditions characterized by uncertainty, weak multi-indicator cues, and gradual drifts. It was shown that detection speed under moderate shifts was increased substantially (e.g., ARL ≈ 18.2 vs. 43.9 at 1.5σ), while in-control stability was preserved or improved (FAR ≈ 0.20% vs. 0.27%), yielding fewer expected stoppages over realistic run lengths. The graded decision structure, spanning "In-control", "Marginal", "At Risk", and "Out-of-control", provided actionable early warnings that are unavailable in binary Shewhart logic and remained interpretable through explicit memberships and rules. In operational terms, this permits gentler, earlier interventions that avert full excursions, thereby improving quality outcomes and asset utilization. Methodologically, the approach complements classical SPC by fusing heterogeneous evidence and codified expertise into a continuous risk signal with transparent provenance. While parameterization and process-specific tuning remain necessary, the computational simplicity and explainability of the design support practical deployment in real-time monitoring. Fuzzy-augmented SPC provides a practical and scalable approach for managing uncertainty in process control, offering timely and interpretable support for modern manufacturing systems.

Limitations should be acknowledged. Membership functions and rule bases were engineered with domain knowledge; consequently, performance depends on the fidelity of these choices to the true process physics. Phase-I estimation and the assumption of approximate independence remain prerequisites for the baseline limits; unmodeled autocorrelation could bias both crisp and fuzzy signals. Generalizability beyond the demonstrated tool-wear context is promising but not guaranteed; re-identification of memberships, weights, and rules would be required for different product families or sensing suites. Finally, although graded states mitigated nuisance alarms, thresholding of the fuzzy index into four categories still introduces design parameters that may require periodic recalibration.

Several avenues for future research are suggested. Data-driven identification of membership functions and rule weights (e.g., neuro-fuzzy training with interpretability constraints), type-2 fuzzy sets to handle epistemic uncertainty better, and Bayesian updating of rule confidences could increase robustness. Joint designs that integrate economic loss models, predictive maintenance schedules, and constrained optimization of warning thresholds would align detection policy with cost-of-quality objectives. Finally, extensions to high-dimensional and autocorrelated regimes, via sparse multivariate features, state-space residuals, or time-varying control limits, would further broaden applicability in Industry 4.0 environments.

Declarations

Ethical Consideration

This study did not require formal ethical approval because no direct involvement of human participants, personal data, or identifiable information was included in the research process. The data presented in this study are available on request from the corresponding author. The data are not publicly available due to industrial confidentiality restrictions related to the manufacturing process.

Competing interests

The authors declare that no competing financial interests or personal relationships exist that could have appeared to influence the work reported in this paper.

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