Hacettepe Journal of Mathematics and Statistics
Volume 46 (3) (2017), 511 – 523

Evaluation of life quality by integrated method of AHP and TOPSIS based on interval type-2 fuzzy sets

Aynur SAHIN [*†] and Nimet YAPICI PEHLIVAN [‡]

Abstract

In this study, we present an integrated method of Analytical Hierarchy Process based on interval type-2 fuzzy sets (IT2FAHP) and Technique for Order Performance by Similarity to Ideal Solution based on on interval type-2 fuzzy sets (IT2FTOPSIS). The applicability of the proposed integrated method was performed on a case study of evaluation of life quality for 28 European Union Countries and other six countries, Iceland, Macedonia, Montenegro, Serbia, Turkey and Kosovo, in terms of 30 subjective criteria. The weights of the criteria were computed by IT2FAHP and the ranking of life quality for considered countries was constituted by IT2FTOPSIS. Furthermore, the rankings were obtained by using AHP and TOPSIS methods under crisp/ fuzzy environment to show performance of the proposed method.

Keywords: Multi criteria decision making, AHP, TOPSIS, Interval type-2 fuzzy sets, Life Quality.

2000 AMS Classification: 90-08, 90B50, 90C70, 03E72

Received : 23.06.2016 *Accepted* : 01.09.2016 *Doi* : 10.15672/HJMS.2017.420

[*]Selcuk University, Faculty of Science, Department of Statistics
Email: aynursahin@selcuk.edu.tr
[†]Corresponding Author.
[‡]Selcuk University, Faculty of Science, Department of Statistics
Email: nimet@selcuk.edu.tr

1. Introduction

Multi criteria decision making (MCDM) is an evaluation process and it has been widely used by several researchers and practitioners. MCDM methods have been applied for evaluating and ranking of alternatives under conflicting criteria with respect to subjective judgements of decision makers (DMs). Since classical MCDM methods cannot deal with imprecise and vagueness information in the decision making process, many fuzzy MCDM methods based on fuzzy sets (also known as type-1 fuzzy sets) proposed by Zadeh [30], have been presented: Fuzzy AHP (Van Laarhoven and Pedrycz [26], Buckley [4], Chang [5]), Fuzzy TOPSIS (Triantaphyllou and Lin [25], Hwang and Yoon [15], Chen [6], Wang and Elhag [28], Ashtiani *et al.* [2]), Fuzzy VIKOR (Park *et al.* [22], Wan *et al.* [27]), Fuzzy COPRAS (Zavadskas and Antucheviciene [32], Yazdani *et al.* [29]), Fuzzy PROMETHEE (Chen *et al.* [12], Goumas and Lygerou [14]), and etc.

However, mentioned Fuzzy MCDM methods are unable to handle high complexity and vagueness. Type-2 fuzzy sets (T2Fs), characterized by a fuzzy membership function, are introduced by Zadeh [31] as an extension of type-1 fuzzy sets (T1Fs) to better represent the uncertainty of the real world. In practical applications, usage of T2Fs are limited because of the computational complexity. Thus, interval type-2 fuzzy sets (IT2Fs), as a special case of type-2 fuzzy sets, have been widely used. Recently, many fuzzy methods based on IT2Fs have been presented for MCDM problem in literature. Chen and Lee [7] presented an IT2FTOPSIS method to handle fuzzy multiple attributes group decision-making problems based on IT2Fs. Nasab and Malkhalifeh [20] introduced an extension of fuzzy TOPSIS based on IT2FSs to handle fuzzy MCDM problems. Chen *et al.* [11] developed an extended QUALIFLEX method to deal multiple criteria decision-making problems in the context of IT2Fs. Chen [8] developed new methods based on PROMETHEE that use a signed distance-based approach within the environment of IT2Fs for multiple criteria decision analysis. Kahraman *et al.* [16] developed an IT2FAHP method together with a new ranking method for type-2 fuzzy sets. Abdullah and Najib [1] proposed a new fuzzy AHP characterized by IT2FS for linguistic terms. Chen [9] developed a novel IT2FTOPSIS method for multiple criteria decision analysis that is based on interval type-2 trapezoidal fuzzy numbers. Kilic and Kaya [17] proposed a model composed of type-2 fuzzy AHP and type-2 fuzzy TOPSIS methods for the investment project evaluation problem.

In this study, an integrated method of Analytical Hierarchy Process based on IT2Fs (IT2FAHP) and Technique for Order Performance by Similarity to Ideal Solution based on IT2Fs (IT2FTOPSIS) is presented. The applicability of the proposed method is performed on a case study of evaluation of life quality for 28 EU countries and other six countries: Iceland, Macedonia, Montenegro, Serbia, Turkey and Kosovo, in terms of 30 criteria.

The rest of this study is organized as follows. In Section 2, some basic concepts of T2Fs and IT2FSs and arithmetic operations on IT2FSs are given. In Section 3, IT2FAHP, IT2FTOPSIS and the proposed method are presented respectively. A case study on evaluation of life quality is performed by using proposed method and ranking results of AHP and TOPSIS methods under crisp/ fuzzy environment are obtained to demonstrate performance of the proposed method in Section 4. Finally, conclusions are drawn in Section 5.

2. Interval Type-2 fuzzy sets

In this section, we briefly give some basic definitions of T2Fs and IT2Fs and arithmetic operations on IT2Fs.

2.1. Definition. [7, 19] A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X which can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$ is given as,

$$\tilde{\tilde{A}} = \{((x,u), \mu_{\tilde{\tilde{A}}}(x,u)) | \forall x \in X, \quad \forall u \in J_x \subseteq [0,1], \quad 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1\} \tag{2.1}$$

where J_x denotes an interval [0,1]. Moreover, $\tilde{\tilde{A}}$ represented by:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u) \tag{2.2}$$

where $\int\int$denotes union over all admissible x and u.

2.2. Definition. [7, 19] Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, then $\tilde{\tilde{A}}$ is called an Interval type-2 fuzzy set (IT2Fs) as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1/(x,u), \quad J_x \subseteq [0,1] \tag{2.3}$$

2.3. Definition. [7, 19] The upper and lower membership functions of an IT2Fs are type-1 membership functions. The reference points and the heights of the upper and the lower membership functions of IT2Fs are used to characterize IT2Fs. Fig.2.1 shows a trapezoidal IT2Fs $\tilde{\tilde{A}}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1$ $(\tilde{A}_i^U), H_2(\tilde{A}_i^U)),\ (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)))$, where $\tilde{A}_i^U$ and $\tilde{A}_i^L$ are type-1 fuzzy sets, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L$ and a_{i4}^L are the references points of the $\tilde{\tilde{A}}_i$, $H_j(\tilde{A}_i^U)$ denotes the membership value of the element $a_{j(j+1)}^U$ in the upper trepezodial membership function $(\tilde{A}_i^U)$ and $H_j(\tilde{A}_i^L)$ denotes the membership value of the element $a_{j(j+1)}^L$ in the lower trepezodial membership function $(\tilde{A}_i^L)$, $H_j(\tilde{A}_i^U) \in [0,1]$, $H_j(\tilde{A}_i^L) \in [0,1]$, and $j = 1, 2$.

Suppose that $\tilde{\tilde{A}}_1$ and $\tilde{\tilde{A}}_2$ are two trapezodial IT2Fs: $\tilde{\tilde{A}}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^U, a_{12}^U$ $, a_{13}^U, a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)),\ (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)))$ and $\tilde{\tilde{A}}_2 = (\tilde{A}_2^U,$ $\tilde{A}_2^L) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U)),\ (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(\tilde{A}_2^L), H_2(\tilde{A}_2^L)))$, then arithmetic operations are given in following definitions [10, 18].

2.4. Definition. [18] The addition operation is defined as follows:

$$\begin{aligned} \tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \oplus (\tilde{A}_1^U, \tilde{A}_1^L) \\ &= ((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \\ &\quad \min(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)), min(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U))), \\ &\quad (a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; \\ &\quad min(H_1(\tilde{A}_1^L); H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L); H_2(\tilde{A}_2^L)))) \end{aligned} \tag{2.4}$$

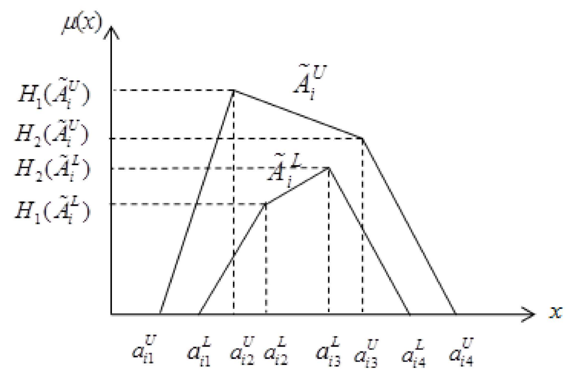
2.5. Definition. [18] The multiplication operation is defined as follows:

$$\begin{aligned} \tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \otimes (\tilde{A}_2^U, \tilde{A}_2^L) \\ &= ((a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U; \\ &\quad \min(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)), min(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U))), \\ &\quad (a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L \\ &\quad min(H_1(\tilde{A}_1^L); H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L); H_2(\tilde{A}_2^L)))) \end{aligned} \tag{2.5}$$

2.6. Definition. [10] The multiplication by a non-negative real number k is defined as follows:

$$k \times \tilde{\tilde{A}}_1 = ((k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), \\ (k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)). \quad (2.6)$$

Fig.2.1. $\tilde{A}_i^U$ **and** $\tilde{A}_i^L$ **of the IT2F** $\tilde{\tilde{A}}_i$ **[18]**



3. The Proposed Method

In this section, IT2FAHP and IT2FTOPSIS methods are given and then, the proposed integrated method of IT2FAHP and IT2FTOPSIS are presented.

3.1. IT2FAHP Method. AHP method developed by Saaty [23], is a popular approach for MCDM. The method is based on pair-wise comparison of criteria and alternatives. In many pratical cases, the decision makers might be unable to assign crisp values for the evaluation of criteria and alternatives. Therefore, various fuzzy AHP methods based on T1Fs were presented in the literature. In recent years, fuzzy AHP methods based on IT2Fs have been proposed to overcome high uncertainties in decision making process.

Kahraman *et al.* [16] presented an interval type-2 fuzzy AHP method into the literature for the first time. Then, Abdullah and Najib [1] proposed an AHP characterized by IT2Fs for linguistic variables to enhance judgment in the fuzzy decision-making environment. In this study, we consider IT2FAHP method proposed by Kahraman *et al.* [16]. The algorithm of this method is as follows:

Step 1: Construct the fuzzy pairwise comparison matrices for all the criteria. The linguistic terms and corresponding IT2Fs used in IT2FAHP are given in Table 3.1.

$$\tilde{\tilde{A}} = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} 1 & \tilde{\tilde{a}}_{12} & \dots & \tilde{\tilde{a}}_{1n} \\ 1/\tilde{\tilde{a}}_{12} & 1 & \dots & \tilde{\tilde{a}}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 1/\tilde{\tilde{a}}_{1n} & 1/\tilde{\tilde{a}}_{2n} & \dots & 1 \end{bmatrix} \end{array} \quad (3.1)$$

Step 2: Check the consistency of the fuzzy pairwise comparison matrices by using the DTriT or DTraT approach proposed by Kahraman *et al.* [16].

Step 3: Calculate the geometric mean of each row as follows:

$$\tilde{\tilde{r}}_i = \left[\tilde{\tilde{a}}_{i1} \otimes ... \otimes \tilde{\tilde{a}}_{in}\right]^{1/n}, \qquad i = 1, 2, ..., n \tag{3.2}$$

Step 4: Calculate the fuzzy weights of each criterion by

$$\tilde{\tilde{w}}_j = \tilde{\tilde{r}} \otimes \left[\tilde{\tilde{r}}_1 \oplus ... \oplus \tilde{\tilde{r}}_n\right]^{-1}, \qquad j = 1, 2, ..., n \tag{3.3}$$

Table 3.1.Linguistic terms and corresponding IT2Fs for evaluation of the criteria [16]

Linguistic Terms	**Interval Type-2 Fuzzy Sets**
Exactly Equal (EE)	((1,1,1,1;1,1)(1,1,1,1;1,1))
Slightly Strong (SS)	((1,2,4,5;1,1)(1.2,2.2,3.8,4.8;0.8,0.8))
Fairly Strong (FS)	((3,4,6,7;1,1)(3.2,4.2,5.8,5.8;0.8,0.8))
Very Strong (VS)	((5,6,8,9;1,1)(5.2,6.2,7.8,8.8;0.8,0.8))
Absolutely Strong (AS)	((7,8,9,9;1,1)(7.2,8.2,8.9,9;0.8,0.8))

3.2. IT2FTOPSIS Method. TOPSIS method developed by Hwang and Yoon [15], is one of the well known MCDM methods. This method is based on selection of alternative which have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. Since the TOPSIS method is not suitable to represent uncertainties, fuzzy TOPSIS method based on T1Fs were proposed by Chen [6]. In the last decade, fuzzy TOPSIS methods based on IT2Fs have been proposed by different researchers in order to better represent uncertainties. The algorithm of IT2FTOPSIS method proposed by Chen and Lee [7] is as follows:

Step 1: Construct the decision matrix $\tilde{\tilde{D}}$ by using linguistic terms and corresponding IT2Fs given in Table 3.2.

$$\tilde{\tilde{D}} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & ... & C_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{x}}_{11} & \tilde{\tilde{x}}_{12} & ... & \tilde{\tilde{x}}_{1n} \\ \tilde{\tilde{x}}_{21} & \tilde{\tilde{x}}_{22} & ... & \tilde{\tilde{x}}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{x}}_{m1} & \tilde{\tilde{x}}_{m2} & ... & \tilde{\tilde{x}}_{mn} \end{bmatrix} \end{array} \tag{3.4}$$

Construct the weighting vector $\tilde{\tilde{W}}$ by using linguistic terms and corresponding IT2Fs given in Table 3.1.

$$\begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & ... & C_n \end{array} \\ \tilde{\tilde{W}} = & \left[\begin{array}{cccc} \tilde{\tilde{w}}_1 & \tilde{\tilde{w}}_2 & ... & \tilde{\tilde{w}}_n \end{array} \right] \end{array} \tag{3.5}$$

where: $\tilde{\tilde{x}}_{ij} = \left(\tilde{\tilde{x}}_{ij}{}^1 \oplus \tilde{\tilde{x}}_{ij}{}^2 \oplus ... \oplus \tilde{\tilde{x}}_{ij}{}^k\right)/k$, $\tilde{\tilde{w}}_j = \left(\tilde{\tilde{w}}_j{}^1 \oplus \tilde{\tilde{w}}_j{}^2 \oplus ... \oplus \tilde{\tilde{w}}_j{}^k\right)/k$, $i = 1, 2, ..., m$, $j = 1, 2, ..., n$ and k denotes the number of DMs.

Step 2: Construct the weighted decision matrix $\tilde{\tilde{D}}_w = [\tilde{\tilde{v}}_{ij}]_{m \times n}$ by using $\tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_j \otimes \tilde{\tilde{x}}_{ij}$.

Step 3: Construct the ranking weighted decision matrix $\bar{D}_w^* = (Rank(\tilde{\tilde{v}}_{ij}))_{m\times n}$ calculating the ranking value Rank($\tilde{\tilde{v}}_{ij}$) of the IT2Fs proposed Lee and Chen [18].

Step 4: Determine the positive ideal solution $A^+ = \left(v_1^+, v_2^+, ..., v_n^+\right)$ and the negative ideal solution $A^- = \left(v_1^-, v_2^-, ..., v_n^-\right)$ as follows:

$$(3.6) \qquad v_j^+ = \begin{cases} \max\limits_{1\le i\le m} \{Rank(\tilde{\tilde{v}}_{ij})\} & , c_j \in C_1 \\ \min\limits_{1\le i\le m} \{Rank(\tilde{\tilde{v}}_{ij})\} & , c_j \in C_2 \end{cases}$$

$$(3.7) \qquad v_j^- = \begin{cases} \min\limits_{1\le i\le m} \{Rank(\tilde{\tilde{v}}_{ij})\} & , c_j \in C_1 \\ \max\limits_{1\le i\le m} \{Rank(\tilde{\tilde{v}}_{ij})\} & , c_j \in C_2 \end{cases}$$

where C_1 denotes benefit criteria, C_2 denotes cost criteria.

Step 5: Calculate the distance between each alternative and the positive and the negative ideal solutions as follows:

$$(3.8) \qquad d_i^+ = \sqrt{\sum_{i=1}^{m} (Rank(\tilde{\tilde{v}}_{ij}) - v_i^+)^2}$$

$$(3.9) \qquad d_i^- = \sqrt{\sum_{i=1}^{m} (Rank(\tilde{\tilde{v}}_{ij}) - v_i^-)^2}$$

Step 6: Calculate the relative degree of closeness CC_i by:

$$(3.10) \qquad CC_i = \frac{d_i^-}{d_i^+ + d_i^-}$$

Step 7: Rank the values of CC_i, $i = 1, 2, ..., m$ in a descending order.

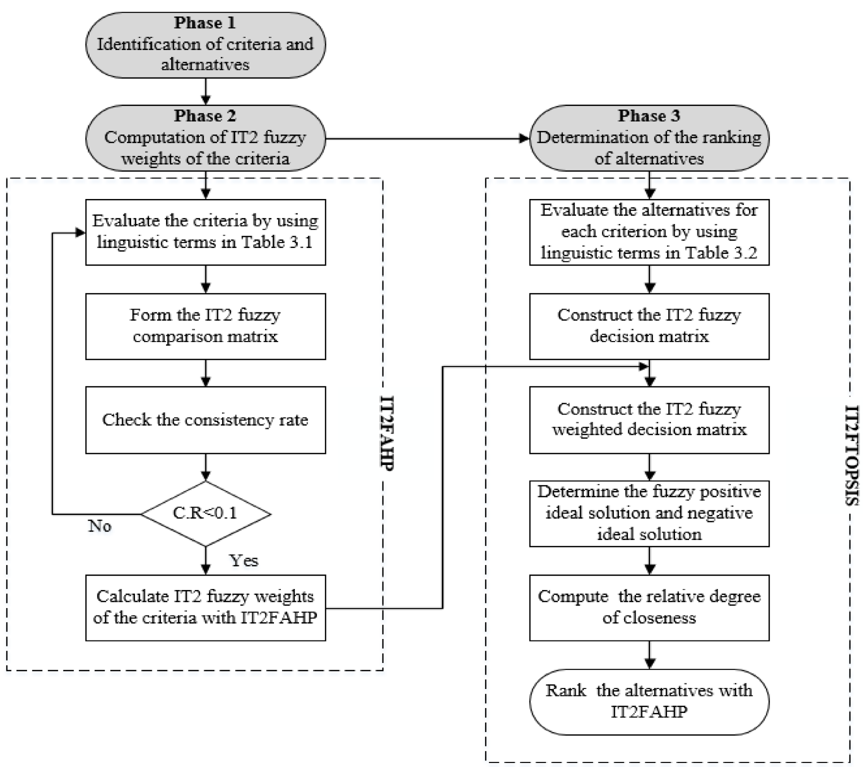
Table 3.2. Linguistic terms and corresponding IT2Fs for the evaluation of altenatives [1]

Linguistic Terms	Interval Type-2 Fuzzy Sets
Very Poor (VP)	((0,0.1,0.1,0.1;1;1) (0,0.1,0.1,0.05;0.9;0.9))
Poor (P)	((0.2,0.3,0.3,0.4;1;1) (0.25,0.3,0.3,0.35;0.9;0.9))
Medium (M)	((0.4,0.5,0.5,0.6;1;1) (0.45,0.5,0.5,0.55;0.9;0.9))
Good (G)	((0.6,0.7,0.7,0.8;1;1) (0.65,0.7,0.7,0.75;0.9;0.9))
Very Good(VG)	((0.8,0.9,0.9,1;1;1) (0.85,0.9,0.9,0.95;0.9;0.9))

3.3. The Integrated MCDM Method based on IT2Fs. Fuzzy AHP is an effective multicriteria technique for solving decision making problems including various conflicting criteria. Also, the fuzzy AHP comprise a useful mechanism for checking the consistency of the decision maker's evaluations, thus reducing the bias in the decision making process [23]. However, as the number of criteria/alternatives increases, numerous pair-wise comparisons are required in the fuzzy AHP method. In such cases, fuzzy TOPSIS in which no pair-wise comparison could be used for ranking of the alternatives to reduce computation time and provide ease of calculation. Hence, we integrated fuzzy AHP and fuzzy TOPSIS methods based on interval type-2 fuzzy sets. In the proposed integrated method, we used interval type-2 fuzzy sets on account of the fact that interval type-2 fuzzy sets are more flexible than type-1 fuzzy sets to overcome uncertainties and the fuzziness of the real world problems [7]. So, integrated MCDM method based on IT2Fs helps us providing more reliable, adaptable and sensitive results.

The proposed integrated method consists of three phases: (i) identification of criteria and alternatives, (ii) computation of IT2 fuzzy weights of the criteria via interval type-2 fuzzy AHP, (iii) determination of the ranking of alternatives with interval type-2 fuzzy TOPSIS. The framework of the proposed integrated method is given in Figure 3.1.

Fig.3.1. Framework of the proposed integrated method



4. Evaluation of Life Quality by Integrated Method based on IT2AHP and IT2TOPSIS

To show applicability of the proposed integrated method, we performed on a case study of evaluation of life quality for 28 EU countries and other six countries: Iceland, Macedonia, Montenegro, Serbia, Turkey and Kosovo, in terms of 30 criteria. The data set is taken from "Europan Quality of Life Surveys (EQLS) 2012" carried out by the Eurofound [13]. Table 4.1 shows criteria and sub-criteria for the EQLS 2012. There are 6 main criteria and related 30 sub-criteria including 18 benefit criteria and 12 cost criteria. Benefit criteria are written in bold font.

Table 4.1. Criteria and Sub-criteria in the EQLS 2012

Main Criteria	**Sub-criteria**
Employment and Work-Life Balance	C1. Coming home from work too tired to do household jobs
	C2. Having difficult to fulfill family responsibilities because of time spent on job
	C3. Having difficult to concentrate at work because of family responsibilities
	C4. The possibility of losing your job in the next 6 months
	C5. If you lost your job, the possibility of finding a job of similar salary
Family and Social Life	**C6. Face-to-face contact with children living outside the household**
	C7. Face-to-face contact with mother and father living outside the household
	C8. Face-to-face contact with other relatives living outside the household
	C9. Face-to-face contact with friends and neighbours
	C10. Phone, internet or postal contact with children living outside the household.
	C11. Phone, internet or postal contact with mother and father living outside the household.
	C12. Phone, internet or postal contact with other relatives living outside the household.
	C13. Phone, internet or postal contact with friends and neighbours
Health	**C14. Health status**
	C15. Feeling particularly tense
	C16. Feeling lonely
	C17. Feeling downhearted and depressed
Social Exclusion and Community Involvement	**C18. Attendance at religious services apart from weddings, funerals, christenings**
	C19. Usage of internet other than for work
	C20. Taking part in sports or physical exercise
	C21. Feeling left out of society
	C22. Feeling that life is too complicated
	C23. Feeling that some people look down on me because of my job situation or income
	C24. Feeling close to people in the area where I live
Standart of Living and Deprivation	**C25. Financial situation of household compared to most people in the country**
	C26. Struggling to meet household expense
Subjective Well-Being	**C27. Be optimistic about the future**
	C28. Feeling that what I do in life is worthwhile
	C29. Feeling free to decide how to live my life
	C30. Having time to do things I really enjoy

IT2FAHP method is used for determination of fuzzy weights of the criteria. At first, a DM evaluates the criteria by using the linguistic terms given in Table 3.1 and fuzzy pairwise comparison matrix is formed. Then, the consistency of the comparison matrix is checked. Fuzzy weights of the criteria are calculated by IT2FAHP and the results are presented in Table 4.2.

Table 4.2. Fuzzy Weights of Criteria

Criteria	Fuzzy Weights of Criteria
C1	((0.008,0.011,0.021,0.035;1,1)(0.008,0.012,0.020,0.031;0.8,0.8))
C2	((0.013,0.024,0.054,0.084;1,1)(0.015,0.026,0.049,0.077;0.8,0.8))
C3	((0.012,0.019,0.039,0.061;1,1)(0.013,0.021,0.036,0.055,0.8,0.8))
C4	((0.032,0.052,0.108,0.161;1,1)(0.036,0.056,0.100,0.148,0.8;0.8))
C5	((0.032,0.049,0.098,0.144;1,1)(0.035,0.053,0.092,0.133;0.8,0.8))
C6	((0.023,0.036,0.075,0.114;1,1)(0.025,0.039,0.070,0.105,0.8,0.8))
C7	((0.013,0.022,0.049,0.082;1,1)(0.015,0.024,0.045,0.073,0.8,0.8))
C8	((0.007,0.010,0.022,0.040;1,1)(0.007,0.011,0.020,0.035;0.8,0.8))
C9	((0.007,0.010,0.022,0.040;1,1)(0.007,0.011,0.020,0.035;0.8,0.8))
C10	((0.018,0.029,0.061,0.093;1,1)(0.020,0.031,0.056,0.085;0.8,0.8))
C11	((0.011,0.018,0.043,0.073;1,1)(0.012,0.020,0.039,0.065;0.8,0.8))
C12	((0.007,0.009,0.020,0.036;1,1)(0.007,0.010,0.018,0.032;0.8,0.8))
C13	((0.007,0.009,0.020,0.036;1,1)(0.007,0.010,0.018,0.032;0.8,0.8))
C14	((0.023,0.035,0.079,0.141;1,1)(0.025,0.037,0.071,0.123;0.8,0.8))
C15	((0.017,0.026,0.051,0.075;1,1)(0.018,0.028,0.048,0.069;0.8,0.8))
C16	((0.017,0.026,0.051,0.075;1,1)(0.018,0.028,0.048,0.069;0.8,0.8))
C17	((0.017,0.026,0.051,0.075;1,1)(0.018,0.028,0.048,0.069;0.8,0.8))
C18	((0.007,0.009,0.019,0.031;1,1)(0.007,0.010,0.017,0.028;0.8,0.8))
C19	((0.002,0.003,0.006,0.009;1,1)(0.002,0.003,0.005,0.008;0.8,0.8))
C20	((0.006,0.008,0.019,0.036;1,1)(0.006,0.009,0.017,0.031;0.8,0.8))
C21	((0.040,0.062,0.122,0.176;1,1)(0.044,0.066,0.114,0.164;0.8,0.8))
C22	((0.009,0.013,0.025,0.039;1,1)(0.010,0.014,0.023,0.035;0.8,0.8))
C23	((0.032,0.053,0.115,0.177;1,1)(0.035,0.057,0.107,0.162;0.8,0.8))
C24	((0.007,0.010,0.019,0.032;1,1)(0.008,0.011,0.018,0.028;0.8,0.8))
C25	((0.020,0.034,0.073,0.108;1,1)(0.023,0.038,0.068,0.100;0.8,0.8))
C26	((0.020,0.035,0.074,0.110;1,1)(0.023,0.038,0.069,0.101;0.8,0.8))
C27	((0.008,0.012,0.025,0.043;1,1)(0.009,0.013,0.023,0.038;0.8,0.8))
C28	((0.010,0.015,0.032,0.053;1,1)(0.011,0.016,0.029,0.047;0.8,0.8))
C29	((0.008,0.013,0.031,0.059;1,1)(0.009,0.014,0.028,0.051;0.8,0.8))
C30	((0.009,0.012,0.023,0.037;1,1)(0.010,0.013,0.022,0.033;0.8,0.8))

The ranking of life quality for the countries is constituted by IT2FTOPSIS. DMs use the linguistic terms in Table 3.2 to evaluate the countries with respect to each criteria and then decision matrix is constructed. The ranking results of life quality for the countries is determined by using IT2FTOPSIS method as shown in Table 4.3.

The three countries with the best quality of life are Iceland, Denmark and Sweden, wheras the three countries with the worst quality of life are Bulgaria, Latvia and Greece by the proposed method. Turkey is ranked as 26th among the considered 34 countries.

To show performance of the proposed method, the ranking results of life quality for the countries are also obtained by the methods of TOPSIS [21], FTOPSIS [6], IT2FTOPSIS [7], Integrated AHP and TOPSIS [3] and Integrated FAHP and FTOPSIS [24] as presented in Table 4.3. The ranking results of the proposed method are compared with them. The Spearman correlation coefficient is calculated to make this comparison as given in Table 4.4. According to the Table 4.4, it can be said that the ranking results of the

Table 4.3. The ranking of the countries in terms of life quality

Country Name	Proposed method	TOPSIS	FTOPSIS	IT2F TOPSIS	Integrated AHP& TOPSIS	Integrated FAHP& FTOPSIS
Austria	4	7	9	7	5	7
Belgium	11	18	20	18	14	16
Bulgaria	32	33	34	34	29	33
Croatia	22	17	16	15	22	18
Cyprus	31	24	18	21	24	20
Czech R.	29	30	33	30	30	32
Denmark	2	3	2	2	2	2
Estonia	28	28	32	29	31	31
Finland	5	4	4	4	10	4
France	10	22	29	19	16	28
Germany	7	8	12	11	3	10
Greece	34	32	26	32	21	25
Hungary	20	29	28	28	23	27
Iceland	1	1	1	1	1	1
Ireland	14	2	6	5	4	6
Italy	12	15	11	14	9	12
Kosova	30	26	25	24	26	26
Latvia	33	31	31	31	34	34
Lithuania	25	21	23	22	33	24
Luxemburg	8	11	13	8	12	13
Macedonia	21	19	17	16	19	17
Malta	16	10	10	13	13	11
Montenegro	13	13	7	9	17	8
Netherlands	6	5	3	6	6	3
Poland	24	23	22	25	28	23
Portugual	19	20	24	26	15	21
Romania	23	25	27	27	25	29
Serbia	27	27	21	23	27	22
Slovakia	18	16	14	17	20	15
Slovenia	9	9	8	10	11	9
Spain	17	14	15	20	7	14
Sweden	3	6	5	3	8	5
Turkey	26	34	30	33	32	30
United K.	15	12	19	12	18	19

proposed method are consistent with the other methods since all correlation coefficients are statistically significant.

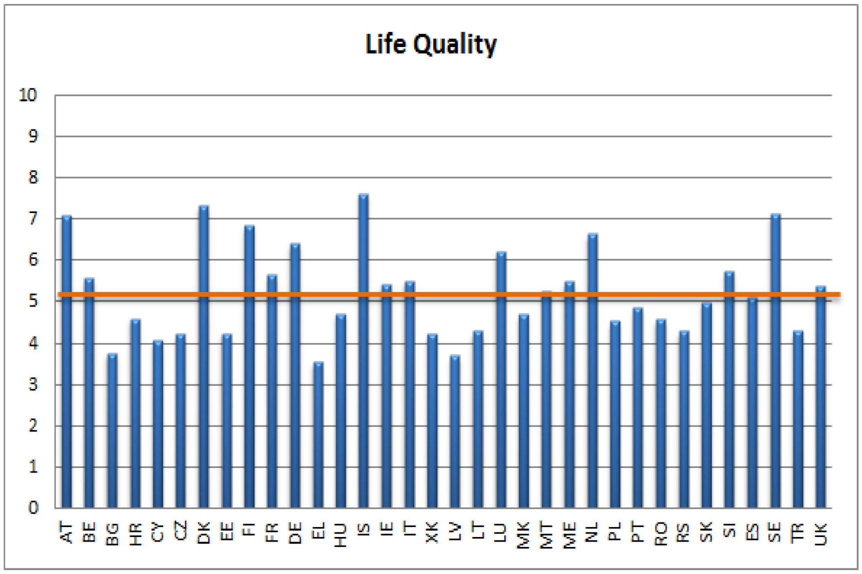
Table 4.4. Correlation between the proposed method and different methods

	TOPSIS	FTOPSIS	IT2F TOPSIS	Integrated AHP& TOPSIS	Integrated FAHP& FTOPSIS
Proposed method	0.736*	0.892*	0.814*	0.891*	0.854*

* Correlation is significant at the 0.01 level (2-tailed)

The average life quality is calculated as 5.23 on the scale between 0 and 10 by proposed method. In terms of life quality, countries with above the average are Austria, Belgium, Denmark, Finland, France, Germany, Iceland, Ireland, Italy, Luxemburg, Montenegro, Netherlands, Slovenia, Sweden, and United Kingdom (Fig. 4.1).

Fig.4.1. Life Quality Scores for the countries



5. Conclusions

MCDM methods are used to solve real world problems in the presence of multiple and usually conflicting criteria by considering different preferences/ judgements of decision makers. There have been many MCDM methods such as AHP, TOPSIS, VIKOR, COPRAS, ARAS, etc. developed by various researchers in the literature. Integration of two or more MCDM methods could give the better and more effective results than single MCDM method . Hence, we proposed an integrated method of AHP and TOPSIS based on interval type-2 fuzzy sets in this study. The proposed integrated method was applied a case study on evaluation of life quality for 34 countries in terms of 30 criteria

In the evaluation process, the proposed method doesn't require pair-wise comparison of alternatives with respect to the each criteria. Therefore, the computation time of the proposed method is less than traditional AHP and TOPSIS methods under crisp/ fuzzy environment. Due to this reason, it can be said that the proposed method is a useful way to handle multi-criteria decision making problems including too many alternatives. For further researches, it is possible to use different fuzzy MCDM methods under type-2 fuzzy sets to determine the weights of criteria and ranking of the alternatives.

Acknowledgment

This study is part of Master (M.Sc.) Thesis entitled "Multi Criteria Decision Making based on Type-2 Fuzzy Sets" prepared by Aynur SAHİN, submitted to Selcuk University, Graduate School of Natural and Applied Sciences, Department of Statistics, Konya, Turkey, 2016. This study is supported by Faculty Development Program (FDP), Project No: 2015-OYP-073.

References

[1] Abdullah, L. and Najib, L. *A new type-2 fuzzy set of linguistic variables for the fuzzy analytic hierarchy process.* Expert Systems with Applications **41** (7), 3297-3305, 2014.

[2] Ashtiani, B., et al. *Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets.* Applied Soft Computing **9** (2), 457-461, 2009.

[3] Bhutia, P.W. and Phipon, R. *Application of AHP and TOPSIS method for supplier selection problem.* IOSR Journal of Engineering **2** (10), 43-50, 2012.

[4] Buckley, J.J. *Fuzzy hierarchical analysis.* Fuzzy sets and systems **17** (3), 233-247, 1985.

[5] Chang, D.-Y. *Applications of the extent analysis method on fuzzy AHP.* European journal of operational research **95** (3), 649-655, 1996.

[6] Chen, C.-T. *Extensions of the TOPSIS for group decision-making under fuzzy environment.* Fuzzy sets and systems **114** (1), 1-9, 2000.

[7] Chen, S.-M. and Lee, L.-W. *Fuzzy multiple attributes group decision-making based on the interval type-2 TOPSIS method.* Expert systems with applications **37** (4), 2790-2798, 2010.

[8] Chen, T.-Y. *A PROMETHEE-based outranking method for multiple criteria decision analysis with interval type-2 fuzzy sets.* Soft Computing **18** (5), 923-940, 2014.

[9] Chen, T.-Y. *An interval type-2 fuzzy technique for order preference by similarity to ideal solutions using a likelihood-based comparison approach for multiple criteria decision analysis.* Computers & Industrial Engineering **85**, 57-72, 2015.

[10] Chen, T.-Y. *An interval type-2 fuzzy PROMETHEE method using a likelihood-based outranking comparison approach.* Information Fusion **25**, 105-120, 2015.

[11] Chen, T.-Y., Chang, C.-H., and Lu, J.-f.R. *The extended QUALIFLEX method for multiple criteria decision analysis based on interval type-2 fuzzy sets and applications to medical decision making.* European Journal of Operational Research **226** (3), 615-625, 2013.

[12] Chen, Y.-H., Wang, T.-C., and Wu, C.-Y. *Strategic decisions using the fuzzy PROMETHEE for IS outsourcing.* Expert Systems with Applications **38** (10), 13216-13222, 2011.

[13] Eurofound. [cited 12.01.2016]; Available from: http://www.eurofound.europa.eu/surveys/european-quality-of-life-surveys-eqls / european-quality-of-life-survey-2012, 2016.

[14] Goumas, M. and Lygerou, V. *An extension of the PROMETHEE method for decision making in fuzzy environment: Ranking of alternative energy exploitation projects.* European Journal of Operational Research **123** (3), 606-613, 2000.

[15] Hwang, C.-L. and Yoon, K. *Multiple criteria decision making.* Lecture Notes in Economics and Mathematical Systems, 1981.

[16] Kahraman, C., et al. *Fuzzy analytic hierarchy process with interval type-2 fuzzy sets.* Knowledge-Based Systems **59**, 48-57, 2014.

[17] Kilic, M. and Kaya, I. *Investment project evaluation by a decision making methodology based on type-2 fuzzy sets.* Applied Soft Computing **27**, 399-410, 2015.

[18] Lee, L.-W. and Chen, S.-M. *Fuzzy multiple attributes group decision-making based on the extension of TOPSIS method and interval type-2 fuzzy sets.* in *Machine Learning and Cybernetics, 2008 International Conference on.* IEEE. 2008.

[19] Mendel, J.M., John, R., and Liu, F. *Interval type-2 fuzzy logic systems made simple.* Fuzzy Systems, IEEE Transactions on **14** (6), 808-821, 2006.

[20] Nasab, F.G. and Malkhalifeh, M.R. *Extension of TOPICS for group DM based on the type two fuzzy positive and negative ideal solutions.* International Journal of Industrial Mathematics **2** (3), 199-213, 2010.

[21] Opricovic, S. and Tzeng, G.-H. *Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS.* European journal of operational research **156** (2), 445-455, 2004.

[22] Park, J.H., Cho, H.J., and Kwun, Y.C. *Extension of the VIKOR method for group decision making with interval-valued intuitionistic fuzzy information.* Fuzzy Optimization and Decision Making **10** (3), 233-253, 2011.

[23] Saaty, T.L. *The analytic hierarchy process: planning, priority setting, resources allocation.* New York: McGraw, 1980.

[24] Sun, C.-C. *A performance evaluation model by integrating fuzzy AHP and fuzzy TOPSIS methods.* Expert systems with applications **37** (12), 7745-7754, 2010.

[25] Triantaphyllou, E. and Lin, C.-T. *Development and evaluation of five fuzzy multiattribute decision-making methods.* international Journal of Approximate reasoning **14** (4), 281-310, 1996.

[26] Van Laarhoven, P. and Pedrycz, W. *A fuzzy extension of Saaty's priority theory.* Fuzzy sets and Systems **11** (1), 199-227, 1983.

[27] Wan, S.-P., Wang, Q.-Y., and Dong, J.-Y. *The extended VIKOR method for multi-attribute group decision making with triangular intuitionistic fuzzy numbers.* Knowledge-Based Systems **52**, 65-77, 2013.

[28] Wang, Y.M. and Elhag, T.M. *Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment.* Expert systems with applications **31** (2), 309-319, 2006.

[29] Yazdani, M., Alidoosti, A., and Zavadskas, E.K. *Risk analysis of critical infrastructures using fuzzy COPRAS.* Ekonomska istrazivanja **24** (4), 27, 2011.

[30] Zadeh, L.A. *Fuzzy sets.* Information and control **8** (3), 338-353, 1965.

[31] Zadeh, L.A. *The concept of a linguistic variable and its application to approximate reasoning—I.* Information sciences **8** (3), 199-249, 1975.

[32] Zavadskas, E.K. and Antucheviciene, J. *Multiple criteria evaluation of rural building's regeneration alternatives.* Building and Environment **42** (1), 436-451, 2007.