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INTUITIONISTIC FUZZY DOT HYPER KU- (SUB ALGEBRA) IDEALS ON HYPER KU- ALGEBRAS

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ABSTRACT .It is known that, the concept of hyper KU-algebras is a generalization of KU-algebras. In this paper , the intuitionistic fuzzy dot theory of the (subalgebra- s-weak – strong) hyper KU-ideals in hyper KU-algebras are applied and the relations among them are obtained.

1. INTRODUCTION

Prabpayak and Leerawat [10,11] introduced a new algebraic structure which is called KU-algebras. They studied ideals and congruences in KU-algebras. Also, they introduced the concept of homomorphism of KU-algebra and investigated some related properties. Moreover, they derived some straightforward consequences of the relations between quotient KU-algebras and isomorphism. Mostafa et al. [8,12]introduced the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals .The hyper structure theory (called also multi-algebras) is introduced in 1934 by F. Marty [7] at the 8th congress of Scandinvian Mathematiciens. Around the 40's, several authors worked on hyper groups, especially in France and in the United States, but also in Italy, Russia and Japan. Hyper structures have many applications to several sectors of both pure and applied sciences. Since then numerous

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mathematical papers [2,3,5,9] have been written investigating the algebraic properties of the hyper BCK / BCI- KUalgebras . Jun and Xin [4] considered the fuzzification of the notion of a (weak, strong, reflexive) hyper BCK-ideal, and investigated the relations among them. In [9], Mostafa et al. applied the hyper structures to KU- algebras and introduced the concept of a hyper KU-algebra which is a generalization of a KU-algebra, and investigated some related properties. They also introduced the notion of a hyper KU-ideal, a weak hyper KU-ideal and gave relations between hyper KU-ideals and weak hyper KU-ideals. Borzooei and Jun[ ] introduced intuitionistic fuzzification of (strong, weak, s-weak) hyper BCK -ideals , and investigated related properties.

In this paper the concepts intuitionistic fuzzy dot theory of the (s-weak – strong) hyper KU-ideals in hyper KU-algebras are applied and the relations among them are obtained.

2. PRELIMINARIES

Let H be a nonempty set and $P^*(H) = P(H) \setminus \{\phi\}$ the family of the nonempty subsets of H . A multi valued operation (said also hyper operation) "$\circ$" on H is a function, which associates with every pair $(x, y) \in H \times H = H^2$ a non empty subset of H denoted $x \circ y$ .An algebraic hyper structure or simply a hyper structure is a non empty set H endowed with one or more hyper operations.

We shall use the $x \circ y$ instead of $x \circ \{y\}$, $\{x\} \circ y$ or $\{x\} \circ \{y\}$.

DEFINITION2.1[9]. Let H be a nonempty set and "$\circ$" a hyper operation on H , such that $\circ : H \times H \to P^*(H)$. Then H is called a hyper KU-algebra if it contains a constant "0" and satisfies the following axioms: for all $x, y, z \in H$

(HKU_1) $[(y \circ z) \circ (x \circ z)] << x \circ y$

(HKU_2) $x \circ 0 = \{0\}$

(HKU_3) $0 \circ x = \{x\}$

(HKU_4) $if\ x << y,\ y << x\ implies\ x = y$.

where x << y is defined by $0 \in y \circ x$ and for every $A, B \subseteq H$, $A << B$ is defined by $\forall a \in A, \exists b \in B \;\; such\ that\ \ a << b$. In such case, we call “<<” the hyper order in H .

Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup_{a \in A, b \in B} a \circ b \;\; of\ \ H$.

EXAMPLE 2.2.[9] . Let $H = \{0,1,2,3\}$ be a set. Define hyper operation $\circ$ on H as follows:

$\circ$	0	1	2	3
0	{0}	{1}	{2}	{3}
1	{0}	{0}	{1}	{3}
2	{0}	{0}	{0}	{0,3}
3	{0}	{0}	{1}	{0,3}

Then $(H, \circ, 0)$ is a hyper KU-algebra.

PROPOSITION 2.3.[9]. Let H be a hyper KU-algebra. Then for all $x, y, z \in H$, the following statements hold:

(P_1) A $\subseteq$ B implies $A << B$, for all nonempty subsets A, B of H .

(P_2) $0 \circ 0 = \{0\}$.

(P_3) $0 << x$.

(P_4) $z << z$.

(P_5) $x \circ z << z$

(P_6) $A \circ 0 = \{0\}$.

(P_7) $0 \circ A = A$.

(P_8) $(0 \circ 0) \circ x = \{x\}$ and $(x \circ (0 \circ x)) = \{0\}$.

(P_9) $x \circ x = \{x\} \Leftrightarrow x = 0$

Lemma 2.4.[9]. In hyper KU-algebra $(H, \circ, 0)$, the following hold:

$$x << y \quad \text{imply} \quad y \circ z << x \circ z \text{ for all } x, y, z \in H .$$

LEMMA 2.5.[9]. In hyper KU-algebra $(H,\circ,0)$, we have

$$z \circ (y \circ x) = y \circ (z \circ x) \text{ for all } x, y, z \in H .$$

LEMMA 2.6.[9].For all $x, y, z \in H$, the following statements hold:

(i) $x \circ y << z \Leftrightarrow z \circ y << x$,

(ii) $0 << A \Rightarrow 0 \in A$,

(iii) $y \in (0 \circ x) \Rightarrow y << x$.

DEfiNITION 2.8.[3] Let H be a hyper K-algebra. An element $a \in H$ is called to be a left(resp. right) scalar if $|a \circ x| = 1$(resp. $|x \circ a| = 1$) for all $x \in H$.

THEOREM 2,9. Let H be a hyper KU-algebra. Then the set $S(H) = \{x \in H \mid x \circ x = \{0\}\}$ is a KU-algebra.

THEOREM 2.10.[3] Let H be a hyper KU-algebra such that for all $x, y \in H$, $(y \circ x) \circ x << \{y\}$. Then, H is a KU -algebra.

DEFINITION 2.11.[9]. Let A be a non-empty subset of a hyper KU-algebra H . Then A is said to be a hyper ideal of H if

(HI_1) $0 \in A$,

(HI_2) $y \circ x << A$ and $y \in A$ imply $x \in A$ for all $x, y \in X$.

DEFINITION 2.12. [9].Let I be a non-empty subset of a hyper KU-algebra H and $0 \in I$. Then, (1) I is called a weak hyper ideal of H if $y \circ x \subseteq I$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.

(2) I is called a strong hyper ideal of H if $(y \circ x) \cap I \neq \phi$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.

DEFINITION 2.13. [9].For a hyper KU-algebras H , a non-empty subset $I \subseteq H$, containing 0 are :1- A weak hyper KU-ideal of H if $a \circ (b \circ c) \subseteq I$ and $b \in I$ imply $a \circ c \in I$.

2- A hyper KU-ideal of H if $a \circ (b \circ c) << I$ and $b \in I$ imply $a \circ c \in I$.

3- A strong hyper KU-ideal of H if $(\forall x, y \in H)((a \circ (b \circ c) \bigcap I \neq \phi)$ and $b \in I$ imply $a \circ c \in I$.

EXAMPLE 2.14.[9]. Let $H = \{0, a, b, c\}$ be a set with the following Cayley table

$\circ$	0	a	b	c
0	{0}	{a}	{b}	{c}
a	{0}	{0,a}	{0,b}	{b,c}
b	{0}	{0,b}	{0 }	{a}
c	{0}	{0,b}	{0 }	{0,a}

Then H is a hyper KU-algebra. Take $I = \{0, b\}$, then I is a weak hyper ideal, however, not a weak hyper KU-ideal of H as
$b \circ (b \circ c) \subseteq I,\ b \in I,\ but\ b \circ c = a \notin I$.

3. Intuitionistic fuzzy dot hyper KU –ideals

An Intuitionistic fuzzy set (briefly I F S see[1]) A in a nonempty set H is an object having the form $A = \{(x, \mu_A(x), \lambda(x)) \mid x \in H\}$, where the function μ_A: $H \to [0,1]$ and
λ_A: $H \to [0,1]$ denote the degree of membership and degree of non membership, respectively and $0 \le \mu_A(x) + \lambda_A(x) \le 1$ for all $x \in H$. An intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in H\}$ in H can be identified to an order pair (μ_A, λ_B) in $I^H \times I^H$. We shall use the symbol $A = (\mu_A, \lambda_A)$.
The compliment of μ, is the fuzzy set on H given by $\mu^C(x) := 1 - \mu(x)$ for all $x \in H$.

For every intuitionistic fuzzy sets A on H we define

$A^C = \{(x, \lambda_A(x), \mu\ (x)) \mid x \in X\}$.

Now some fuzzy logic concepts are reviewed .A fuzzy set μ in a set H is a function $\mu : H \to [0,1]$. A fuzzy set μ in a set H is said to satisfy the inf (resp. sup) property if for any subset T of H there exists $x_0 \in T$ such that $\mu(x_0) = \inf_{x \in T} \mu(x)$ (resp. $\mu(x_0) = \sup_{x \in T} \mu(x)$). For a fuzzy set μ in X and a $\in$ [0, 1] the set $\mu^t := \{x \in H \mid \mu(x) \geq t\}$

(resp $\mu_t := \{x \in H \mid \mu(x) \leq t\}$) is called an upper (resp. lower) a level set of μ.

If $A = \{\langle x, \mu_A(x), \lambda_A(x), x \in H \rangle\}$ and $B = \{\langle x, \mu_B(x), \lambda_B(x), x \in H \rangle\}$ are any two IFS of a set H , then

(1) If $A \subseteq B$,then $\mu_A(x) \leq \mu_B(x)$, $\lambda_A(x) \geq \lambda_B(x)$.

(2) If $A = B$,then $\mu_A(x) = \mu_B(x)$, $\lambda_A(x) = \lambda_B(x)$.

(3) $A \cap B = \{\langle x, (\mu_A \cap \mu_B)(x), (\lambda_A \cup \lambda_B)(x) \rangle \mid x \in H\}$, where

$(\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\}, (\lambda_A \cup \lambda_B)(x) = \max\{\lambda_A(x), \lambda_B(x)\}$;

(4) $A \cup B = \{\langle x, (\mu_A \cup \mu_B)(x), (\lambda_A \cap \lambda_B)(x) \rangle \mid x \in H\}$, where;

$(\mu_A \cup \mu_B)(x) = \max\{\mu_A(x), \mu_B(x)\}, (\lambda_A \cap \lambda_B)(x) = \min\{\lambda_A(x), \lambda_B(x)\}$;

LEMMA. If a, b, c, d $\in$ [0, 1], then

- –$\min\{a, b\} \geq a.b$
- –$\max\{a, b\} \leq \min\{1, a+b\}$
- –$\min\{a.b, c.d\} \geq \min\{a, c\}.\min\{b, d\}$.

DEfiNITION 3.1. An intuitionistic set $A = (\mu_A, \lambda_A)$ in H is said to be an intuitionistic fuzzy dot hyper KU-subalgebra of H if it satisfies the inequalities:

$\inf_{z \in x \circ y} \mu(z) \geq \mu(x) \bullet \mu(y)$ and

$\sup_{z \in x \circ y} \lambda_A(z) \leq \max\{\lambda(x), \lambda(y)\} \leq \min\{1, \lambda(x) + \lambda(y)\}$, $\forall x, y \in H$

EXAMPLE 3.2. Let $H=\{0,a,b\}$ be a set. Define hyper operation $\circ$ on H as follows:

$\circ$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0,a\}$	$\{a,b\}$
b	$\{0\}$	$\{0,a\}$	$\{0,a,b\}$

We know that $(H,\circ,0)$ is a hyper KU-algebra

Define an intuitionistic set $A=(\mu_A,\lambda_A)$ in H by μ(0) = 0.7,μ(a)=0.5 , μ(b)=0.3, $\lambda(0)=0.2$, $\lambda(a)=0.4$, $\lambda(b)=0.7$. It is easily verified that $A=(\mu_A,\lambda_A)$ is intuitionistic fuzzy dot hyper sub-algebra of H.

PROPOSITION 3.3. Let $A=(\mu_A,\lambda_A)$ be a intuitionistic fuzzy dot hyper KU-sub-algebra of H. Then $\mu(0)\geq[\mu(x)]^2$, $\lambda_A(0)\leq\sup\limits_{z\in x\circ x}\lambda_A(z)\leq\min\{1,2\lambda(x)\}$ for all x $\in H$.

PROOF. Using Proposition 2.3 (P_9), we see that 0 ∈ x ∘ x for all x $\in H$. Hence

$$\mu(0)\geq\inf_{0\in x\circ x}\mu(0)\geq\mu(x)\bullet\mu(x)=[\mu(x)]^2 \text{ for all } \forall x\in H.$$

$$\sup_{z\in x\circ x}\lambda_A(z)\leq\max\{\lambda(x),\lambda(x)\}\leq\min\{1,\lambda(x)+\lambda(x)\}$$

$$\lambda_A(0)\leq\sup_{z\in x\circ x}\lambda_A(z)\leq\max\{\lambda(x),\lambda(x)\}\leq\min\{1,\lambda(x)+\lambda(x)\}$$

$$\lambda_A(0)\leq\sup_{z\in x\circ x}\lambda_A(z)\leq\max\{\lambda(x),\lambda(x)\}\leq\min\{1,2\lambda(x)\}$$

LEMMA 3.4. If $A=(\mu_A,\lambda_A)$ is a intuitionistic fuzzy dot KU-sub-algebra of H, then so is $A^C=(\mu_A,\mu^C{}_A)$.

PROOF. $\inf\limits_{z\in y\circ x}\mu(z)\geq\min\{\mu(x),\mu(y)\}\geq\mu(x)\bullet\mu(y)$ and so

$$\min\{1-\mu^C(x),1-\mu^C(y)\}\leq\inf_{z\in y\circ x}(1-\mu^C(z))=1-\sup_{z\in y\circ x}\mu^C(z).$$ Hence

$$\min\{1-\mu^C(x),1-\mu^C(y)\} \le 1\text{-}\sup_{z\in y\circ x}\mu^C(z)$$, which implies

$\sup_{z\in y\circ x}\mu^C(z) \le 1\text{-}\min\{1-\mu^C(x),1-\mu^C(y)\}$. Therefore,

$\sup_{z\in y\circ x}\mu_A{}^C(z) \le \max\{\mu^C(x),\mu^C(y)\} \le \min\{1,\mu^C(x)+\mu^C(y)\} =, \forall x,y\in H$.

THEOREM 3.5 . Every intuitionistic fuzzy hyper KU-sub-algebra of H is an intuitionistic fuzzy dot hyper KU-sub-algebra of H .

PROOF. Let $A=(\mu_A,\lambda_A)$ be an intuitionistic fuzzy hyper KU-sub-algebra of H , therefore we have

$\inf_{z\in x\circ y}\mu(z) \ge \min\{\mu(x),\mu(y)\}$ and $\sup_{z\in x\circ y}\lambda_A(z) \le \max\{\lambda(x),\lambda(y)\}$ $\forall x,y\in H$, but

$\min\{\mu(x),\mu(y)\} \ge \mu(x)\bullet\mu(y)$ and

$\max\{\lambda(x),\lambda(y)\} \le \min\{1,\lambda(x)+\lambda(y)\}$ $\forall x,y\in H$.

Hence $\inf_{z\in x\circ y}\mu(z) \ge \min\{\mu(x),\mu(y)\} \ge \mu(x)\bullet\mu(y)$ and

$\sup_{z\in x\circ y}\lambda_A(z) \le \max\{\lambda(x),\lambda(y)\} \le \min\{1,\lambda(x)+\lambda(y)\}$ $\forall x,y\in H$. This proves the theorem

THEOREM 3.6 If $A=(\mu_A,\lambda_A)$ and $B=(\mu_B,\lambda_B)$ are two intuitionistic fuzzy dot hyper KU-sub-algebras of H , Then $A\cap B$ is also an intuitionistic fuzzy dot hyper KU-sub-algebra of H .

Proof. . Let $x,y\in\mu_A$ *and* μ_B, since μ_A *and* μ_B are fuzzy dot hyper KU subalgebras of H , Then we have:

$$(\mu_A\cap\mu_B)(y\circ x) \ge \min\{\inf_{z\in y\circ x}\mu_A(z), \inf_{z\in y\circ x}\mu_B(z)\} \ge \min\{\mu_A(x)\bullet\mu_A(y), \mu_B(x)\bullet\mu_B(y)\} \ge$$
$$\{\min\{\mu_A(x),\mu_B(x)\}\}\bullet\{\min\{\mu_A(y),\mu_B(y)\}\} = ((\mu_A\cap\mu_B)(x))\bullet((\mu_A\cap\mu_B)(y))$$

On other hand

$(\lambda_A \cup \lambda_B)(y \circ x) = \{\sup_{z \in y \circ x} \lambda_A(z), \sup_{z \in y \circ x} \lambda_B(z)\} \leq \max\{\sup_{z \in y \circ x} \lambda_A(z), \sup_{z \in y \circ x} \lambda_A(z)\} \leq$

$\max\{\min\{1, \lambda_A(x) + \lambda_A(y)\}, \min\{1, \lambda_B(x) + \lambda_B(y)\}\} \leq \min\{1, \min[1, \lambda_A(x) + \lambda_B(x)] + \min[1, \lambda_B(y) + \lambda_B(y$

$\min\{1, \max[\lambda_A(x), \lambda_B(x)] + \max[\lambda_B(y), \lambda_B(y)]\} = \min\{1, ((\lambda_A \cup \lambda_B)(x)) + ((\lambda_A \cup \lambda_B)(y))\}$

Hence $A \cap B$ is also an intuitionistic fuzzy dot hyper KU-sub-algebra of H .

COROLLARY 3.7. Let $\{A_i\}_{i \in \Lambda} = \{(\mu_i, \lambda_i) \mid i \in \Lambda\}$ be a family of intuitionistic fuzzy dot hyper KU–sub algebras of H . Then $\bigcap_{i \in \Lambda} (\mu_i, \lambda_i)$ is also a intuitionistic fuzzy dot hyper KU – sub algebras of H .

DEfiNITION3.8. An intuitionistic set $A = \langle \mu_A(x), \lambda_A(x) \rangle$ in H is called a intuitionistic fuzzy dot hyper ideal of H , if it satisfies the following conditions:

$K_1 :\ x << y$ *implies* $\mu(x) \geq \mu(y)$ and $\lambda_A(x) \leq \lambda_A(y)$

$K_2 : \mu(x) \geq \inf_{u \in (y \circ z)} \mu(u) \bullet \mu(y)$,

$$\lambda_A(x) \leq \max\left\{\sup_{a \in (y \circ x)} \lambda(a), \lambda(y)\right\} \leq \min\{1, \sup_{a \in (y \circ x)} \lambda(a) + \lambda(y)\}$$

DEfiNITION3.9. For a hyper KU-algebra H , a intuitionistic $A = \langle \mu_A(x), \lambda_A(x) \rangle$" in H is called:

(I) intuitionistic fuzzy dot hyper KU-ideal of H ,if

$K_1 :\ x << y$ *implies* $\mu(x) \geq \mu(y)$, $\lambda_A(x) \leq \lambda_A(y)$,

$\mu_A(x \circ z) \geq \inf_{u \in (x \circ (y \circ z))} \mu_A(u) \bullet \mu_A(y)$, and

$$\lambda_A(x \circ z) \leq \max\left\{\sup_{a \in x \circ (y \circ z)} \lambda(a), \lambda(y)\right\} \geq \min\{1, \sup_{a \in x \circ (y \circ z)} \lambda(a) + \lambda(y)\}.$$

(II) intuitionistic fuzzy dot weak hyper KU-ideal of H" if, for any x; y; z $\in$ H

$\mu_A(0) \geq \mu_A(x \circ z) \geq \inf_{u \in (x \circ (y \circ z))} \mu_A(u) \bullet \mu_A(y)$

$$\lambda_A(0) \le \lambda_A(x \circ z) \le \max\left\{ \sup_{u \in (x \circ (y \circ z)} \lambda_A(u), \lambda_A(y) \right\} \le \min\{1, \sup_{u \in (x \circ (y \circ z)} \lambda_A(u) + \lambda_A(y)\}$$

(III) intuitionistic fuzzy dot strong hyper KU-ideal of H " if, for any x; y; z $\in$ H

$$\inf_{u \in x \circ (y \circ z)} \mu_A(u) \ge \mu_A(z) \ge \sup_{u \in x \circ (y \circ z)} \mu_A(u) \bullet \mu_A(y),$$

$$\sup_{u \in x \circ (y \circ z)} \lambda_A(u) \le \lambda_A(z) \le \max\left\{ \inf_{u \in x \circ (y \circ z)} \lambda_A(u), \lambda_A(y) \right\} \le \min\{1, \inf_{u \in x \circ (y \circ z)} \lambda_A(u) + \lambda_A(y)\}$$

DEFINITION 3.10. A intuitionistic set $A = \langle \mu_A(x), \lambda_A(x) \rangle$ in H is called a fuzzy dot s-weak hyper KU-ideal of H if

(i) $\mu_A(0) \ge \mu_A(x)$, $\lambda_A(0) \le \lambda_a(x)$ $\forall x \in H$,

(ii) for every $x, y, z \in H$ there exists $a \in x \circ (y \circ z)$ such that

$\mu_A(x \circ z) \ge \mu_A(a) \bullet \mu_A(y)$.

(iii) $\lambda_A(x \circ z) \le \max\left\{ \sup_{a \in x \circ (y \circ z)} \lambda_A(a), \lambda_A(y) \right\} \le \min\{1, \sup_{a \in x \circ (y \circ z)} \lambda_A(a) + \lambda_A(y)\}$.

EXAMPLE 3.11. Let $H = \{0, a, b\}$ be a set with a binary operation $\circ$ on H as follows

$\circ$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0\}$	$\{b\}$
b	$\{0\}$	$\{a\}$	$\{0, b\}$

Define an intuitionistic set

$A = (\mu_A, \lambda_A)$ in H by μ(0) = 0.9,μ(a)=0.6 , μ(b)=0.3, $\lambda(0) = 0.09$, $\lambda(a) = 0.16$, $\lambda(b) = 0.23$. It is easily verified that $A = (\mu_A, \lambda_A)$ is intuitionistic fuzzy dot strong hyper KU-ideal of H .

THEOREM 3.12. Any intuitionistic fuzzy dot (weak, strong) hyper KU-ideal is a intuitionistic fuzzy dot (weak, strong) hyper ideal.

PROOF. Let $A=\langle \mu_A(x),\lambda_A(x)\rangle$ be intuitionistic a fuzzy dot strong hyper KU-ideal of H, we get for any x; y; z $\in$ H , $\mu_A(x\circ z)\geq \sup\limits_{u\in x\circ(y\circ z)} \mu_A(u)\bullet\mu_A(y)$ Put x = 0 , we get $\mu_A(0\circ z)\geq \sup\limits_{u\in 0\circ(y\circ z)} \mu_A(u)\bullet\mu_A(y)$, which gives,

$\mu_A(z)\geq \sup\limits_{u\in(y\circ z)} \mu_A(u)\bullet\mu_A(y)$, and

$\lambda_A(x\circ z)\leq \max\left\{ \inf\limits_{u\in x\circ(y\circ z)} \lambda_A(u),\lambda_A(y) \right\}\leq \min\{1, \inf\limits_{u\in x\circ(y\circ z)} \lambda_A(u)+\lambda_A(y)\}$.Take x = 0 , we get

$\lambda_A(0\circ z)\leq \max\left\{ \inf\limits_{u\in 0\circ(y\circ z)} \lambda_A(u) , \lambda_A(y) \right\}\leq \min\{1, \inf\limits_{u\in 0\circ(y\circ z)} \lambda_A(u) +\lambda_A(y)\} =$

$\min\{1, \inf\limits_{u\in(y\circ z)} \lambda_A(u) +\lambda_A(y)\}$, which gives

$\lambda_A(z)\leq \max\left\{ \inf\limits_{u\in(y\circ z)} \lambda_A(u) , \lambda_A(y) \right\}\leq \min\{1, \inf\limits_{u\in(y\circ z)} \lambda_A(u) + \lambda_A(y)\}$.

Similarly ,we can prove for intuitionistic fuzzy dot weak is a intuitionistic fuzzy dot weak hyper ideal. Ending the proof.

PROPOSITION 3.13. Let $A=\langle \mu_A(x),\lambda_A(x)\rangle$ be a intuitionistic fuzzy dot weak hyper KU-ideal of H. If $A=\langle \mu_A(x),\lambda_A(x)\rangle$ satisfies the inf-sup property, then $A=\langle \mu_A(x),\lambda_A(x)\rangle$ is a intuitionistic fuzzy dot s-weak hyper KU -ideal of H.

PROOF. Since $A=\langle \mu_A(x),\lambda_A(x)\rangle$ satisfies the inf property, there exists $a_0\in x\circ(y\circ z)$,such that $\mu_A(a_0)=\inf\limits_{a_0\in x\circ(y\circ z)} \mu_A(a_0)$ .It follows that $\mu_A(x\circ z)\geq \inf\limits_{a\in x\circ(y\circ z)} \mu_A(a)\bullet\mu_A(y)$. Similarly ,since $A=\langle \mu_A(x),\lambda_A(x)\rangle$ satisfies the sup property, there exists $b_0\in x\circ(y\circ z)$,such that $\lambda_A(b_0)=\sup\limits_{b_0\in x\circ(y\circ z)} \lambda_A(a_0)$ It follows that

$$\lambda_A(x\circ z)\leq \max\left\{ \sup\limits_{b\in x\circ(y\circ z)} \lambda_A(b) , \lambda_A(y) \right\}\leq \min\{1, \sup\limits_{b\in x\circ(y\circ z)} \lambda_A(b)+ \lambda_A(y)\}.$$

Ending the proof.

Note that, in a finite hyper KU-algebra, every intuitionistic fuzzy dot set satisfies (inf -sup) property. Hence the concept of intuitionistic fuzzy dot weak hyper KU - ideals and intuitionistic fuzzy dot s-weak hyper KU-ideals coincide in a finite hyper KU -algebra.

PROPOSITION 3.14. Let $A=\langle \mu_A(x), \lambda_A(x)\rangle$ be a intuitionistic fuzzy dot strong hyper KU-ideal of H and let x; y; z $\in$ H. Then

(i) $\mu_A(0) \geq \mu_A(x)$, $\lambda_A(0) \leq \lambda_A(x)$

(ii) x << y implies $\mu_A(x) \geq \mu_A(y)$.

(iii) $\mu_A(x \circ z) \geq \sup\limits_{a \in x \circ (y \circ z)} \mu_A(a) \bullet \mu_A(y), \forall a \in x \circ (y \circ z)$

(v) x << y implies $\lambda_A(x) \leq \lambda_A(y)$

(iv)

$\lambda_A(x \circ z) \leq \max\left\{ \inf\limits_{b \in x \circ (y \circ z)} \lambda_A(b) \text{ , } \lambda_A(y) \right\} \leq \min\{1, \inf\limits_{b \in x \circ (y \circ z)} \lambda_A(b) + \lambda_A(y)\}$.

PROOF. (i) Since $0 \in x \circ x \ \forall x \in H$,we have $\mu(0) \geq \inf\limits_{a \in x \circ x} \mu(a) \geq \mu(x)$,

$\lambda(0) \leq \sup\limits_{b \in x \circ x} \lambda(b) \leq \lambda(x)$,which proves (i).

(ii) Let x; y $\in$ H be such that x << y. Then $0 \in y \circ x \ \forall x, y \in H$ and so

$\inf\limits_{b \in (y \circ x)} \mu_A(b) \leq \mu_A(0)$, it follows from (i) that ,

$\mu_A(x) \geq \min\left\{ \sup\limits_{a \in (0 \circ x)} \mu_A(a), \mu_A(y) \right\} \geq \min\{\mu_A(0), \mu_A(y)\} = \mu_A(y)$,

$\lambda_A(x) \leq \max\left\{ \inf\limits_{b \in (0 \circ x)} \lambda_A(b), \lambda_A(y) \right\} \leq \max\{\lambda_A(0), \lambda_A(y)\} = \lambda_A(y)$,and

(iii) $\mu_A(x \circ z) \geq \sup\limits_{a \in x \circ (y \circ z)} \mu_A(a) \bullet \mu_A(y), \forall a \in x \circ (y \circ z)$,

$\lambda(x \circ z) \leq \max\left\{ \inf\limits_{b \in x \circ (y \circ z)} \lambda(b), \lambda(y) \right\} \leq \max\{\lambda(b), \lambda(y)\} \leq \min\{1, \lambda(b) + \lambda(y)\}, \forall b \in x \circ (y \circ z)$

we conclude that (iii), (v), (iv) are true. Ending the proof.

COROLLARY 3.15. Every intuitionistic fuzzy strong hyper KU-ideal is both a intuitionistic fuzzy dot s-weak hyper KU-ideal (and hence a fuzzy dot weak hyper ideal) and a intuitionistic fuzzy dot hyper KU -ideal.
PROOF. Straight forward.

PROPOSITION 3.16. Let $A=\langle \mu_A(x), \lambda_A(x)\rangle$ be a intuitionistic fuzzy dot hyper KU -ideal of H and let $x, y, z \in H$. Then,(i) $\mu_A(0) \geq \mu_A(x)$, $\lambda_A(0) \leq \lambda_A(x)$
(ii) if $A=\langle \mu_A(x), \lambda_A(x)\rangle$ satisfies the inf -sup property, then

$\mu_A(x \circ z) \geq \mu_A(a) \bullet \mu_A(y), \forall a \in x \circ (y \circ z)$,

$\lambda_A(x \circ z) \leq \max\left\{ \sup\limits_{b \in x \circ (y \circ z)} \lambda_A(b) \ , \ \lambda_A(y) \right\} \leq \{1, \sup\limits_{b \in x \circ (y \circ z)} \lambda_A(b) + \lambda_A(y)\}$

PROOF. (i) Since 0 << x for each $x \in H$; we have $\mu_A(0) \geq \mu_A(x)$, $\lambda_A(0) \leq \lambda_A(x)$ by (Definition 3.9(I)) and hence (i) holds.
(ii) Since $A=\langle \mu_A(x), \lambda_A(x)\rangle$ satisfies the inf property, there is $a_0 \in x \circ (y \circ z)$,such that $\mu(a_0) = \inf\limits_{a \in x \circ (y \circ z)} \mu_A(a)$ .Hence $\mu_A(x \circ z) \geq \mu_A(a_0) \bullet \mu_A(y)$, Since $A=\langle \mu_A(x), \lambda_A(x)\rangle$ satisfies the sup- property, there is $b_0 \in x \circ (y \circ z)$,such that $\lambda(b_0) = \sup\limits_{b \in x \circ (y \circ z)} \lambda_A(b)$.

Hence $\lambda_A(x \circ z) \leq \max\left\{ \sup\limits_{b \in x \circ (y \circ z)} \lambda_A(b) \ , \ \lambda_A(y) \right\} \leq \min\{1, \lambda_A(b_0) + \lambda_A(y)\}$,

which implies that (ii) is true. The proof is complete.

COROLLARY 3.17
(i) Every intuitionistic fuzzy hyper KU -ideal of H is a intuitionistic fuzzy dot weak hyper ideal of H.
(ii) If $A=\langle \mu_A(x), \lambda_A(x)\rangle$ is an intuitionistic fuzzy dot hyper KU -ideal of H satisfying inf-sup property, then $A=\langle \mu_A(x), \lambda_A(x)\rangle$ is an intuitionistic fuzzy dot s-weak Hyper KU -ideal of H.

PROOF. Straightforward.

THEOREM 3.18 . If $A=\langle \mu_A(x), \lambda_A(x)\rangle$ is an intuitionistic fuzzy dot strong hyper KU-ideal of H , then the set $\mu_{t,s}=\{x\in H, \mu_A(x)\geq t, \lambda_A(x)\leq s\}$ is a strong hyper KU-ideal of H ,when $\mu_{t,s}\neq \Phi,\ for\ t,s\in[0,1]$.

PROOF. Let $A=\langle \mu_A(x), \lambda_A(x)\rangle$ be an intuitionistic fuzzy dot strong hyper KU-ideal of H and $\mu_{t,s}\neq \Phi,\ for\ t,s\in[0,1]$ .Then there $a\in \mu_{t,s}$ and so $\mu_A(a)\geq t, \lambda_A(a)\leq s$.

By Proposition 3.14 (i), $\mu_A(0)\geq \mu_A(a)\geq t, \lambda(0)\geq \lambda(a)\leq s$ and so $0\in \mu_{t,s}$. Let $x,y,z\in H$ such that $x\circ(y\circ z)\cap \mu_{t,s}\neq \Phi\ and\ y\in \mu_{t,s}$, Then there exist $a_0\in x\circ(y\circ z)\cap \mu_{t,s}\ and\ hence\ \mu_A(a_0)\geq t, \lambda_A(a_0)\leq s$ .By Definition 3.9 (III), we have

$$\mu_A(x\circ z)\geq\left\{\sup_{a\in x\circ(y\circ z)}\mu_A(a)\bullet\mu_A(y)\right\}\geq\{\mu_A(a)\bullet\mu_A(y)\}\geq\{t\bullet t\}=t^2\leq t \text{ ,and}$$

$$\lambda_A(x\circ z)\leq S\left\{\inf_{a\in x\circ(y\circ z)}\lambda_A(a)\ ,\ \lambda_A(y)\right\}=S\{\ \lambda_A(a_0)\ ,\ \lambda_A(y)\ \}=S\{\ s\ ,\ s)\ \}=s$$

.

$$\lambda_A(x\circ z)\leq \max\left\{\inf_{b\in x\circ(y\circ z)}\lambda_A(b)\ ,\ \lambda_A(y)\right\}\leq \max\{\ \lambda_A(b_0)\ ,\ \lambda_A(y)\ \}=s\leq\{1,s+s\}=\{1,2s\}$$

So $(x\circ z)\in \mu_{t,s}$. It follows that $\mu_{t,s}$ is a strong hyper KU -ideal of H .

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CONFLICTS OF INTEREST

State any potential conflicts of interest here or “The author declare no conflict of interest”.

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