

Conformable Differential Geometry of Some Special Curves in the Equi-Affine Space

Anıl Altınkaya¹ 

Article Info

Received: 30 Aug 2025

Accepted: 30 Sep 2025

Published: 30 Sep 2025

Research Article

Abstract— This study investigates the conformable differential geometry of some special curves defined in the equi-affine space. The conformable derivative, a generalization of fractional calculus, is a flexible operator controlled by a parameter α that enables the modeling of nonlocal behavior in functions. This paper aims to offer a new perspective by combining the modern concept of derivative with equi-affine differential geometry. First, this paper introduces the conformable equi-affine arc length parameter and the corresponding conformable Frenet frame in the equi-affine space. The main focus is on characterizing special classes of curves, such as helices, slant helices, and rectifying curves, within the conformable equi-affine frame. These results expand the geometric applications of conformable calculus and provide a broader theoretical framework for curve analysis in equi-affine geometry. Finally, the accuracy of the results is observed with an example, and the curvatures are plotted as a function of α with the MATLAB R2022b.

Keywords — Conformable derivative, non-Newtonian calculus, curve theory, equi-affine space

Mathematics Subject Classification (2020) 26A33, 53A15

1. Introduction

Equi-affine (or equi-volume affine) space is a more general and complex branch of geometry derived by combining Euclidean and affine geometry. This concept, while preserving the structure of affine space, abandons rigid Euclidean measurements such as length and angle, focusing instead on transformations that preserve only the measurement of volume or area. Its fundamental philosophy is to examine how a shape can be transformed while preserving parallel lines (affine), while also investigating how these transformations keep the oriented volumes (areas) of objects constant. Thanks to this volume-preserving property, equi-affine geometry provides a powerful and elegant framework for many areas of mathematics and theoretical physics, from analyzing surface curvature to visual rendering, and even some models in relativistic physics. In recent years, many researchers [1–5] have conducted studies in the equi-affine space (or plane). Aydın et al. [1] introduced the Frenet formulas and equi-affine curvature and torsion in the equi-affine plane using fractional derivatives. In [3], researchers considered the concepts of general helix and slant helix in the equi-affine space. In [4], Öğrenmiş provided some characterizations for the arc length parameter, Frenet formulas, and curvatures for curves in the equi-affine space using fractional derivatives.

¹anilaltinkaya@gazi.edu.tr (Corresponding Author)

¹Department of Mathematics, Faculty of Science, Gazi University, Ankara, Türkiye

Curve theory, one of the primary subjects of study in differential geometry, is concerned with identifying certain types of curves that describe a curve's local and global behavior. The general helix, slant helix, and rectifying curves are notable for their historical and mathematical significance. General helices are curves whose tangent vector forms a constant angle with a fixed direction (perpendicular axis), which is a natural generalization of helical motion in space. The notion of the slant helix broadens the definition of the general helix by considering curves whose binormal vector forms a constant angle with a fixed direction, providing a fresh viewpoint on classical structure. Rectifying curves, as their name suggests, are straightenable curves. They exhibit important properties despite their principal normal vectors lying in a plane passing through the origin, and therefore not exhibiting zero torsion.

Fractional calculus is a field of mathematics that extends the principles of integer derivatives and integrals to any real or complex order. The origins of this field date back 300 years to the correspondence between Leibniz and L'Hôpital. Traditionally, the Riemann-Liouville, Caputo, and Grunwald-Letnikov definitions of fractional derivatives have been popular, with applications spanning from differential equations and signal processing. However, various issues with these traditional techniques (such as nonlocality and solitary kernel) have limited their practical application. Khalil et al. [6] introduced the conformable derivative as a fresh and elegant solution to these difficulties. This operator, which is famous for being local in its fundamental limit formulation, meeting classical differentiation criteria (e.g., the product and quotient rules), and providing computational simplicity, has quickly acquired prominence, notably in engineering and physics applications. This new operator quickly attracted great interest and inspired numerous subsequent studies [7–11]. In [7], researchers provided the Frenet operators for a conformable curve. Has et al. [9] obtained results for some special conformable curves in Euclidean 3-space.

Section 2 of this study presents some basic notions and properties. Section 3 introduces the concepts of conformable general helix and slant helix in the equi-affine space. Then, it proposes the conditions necessary for a curve to be a conformable general helix and slant helix. Furthermore, this section establishes the conditions necessary for a curve to be conformably rectifying in the equi-affine space $\mathbb{R}^3$. Afterward, it provides a general helix example to support the results. The final section discusses the need for further research.

2. Preliminaries

Consider the affine space $\mathbb{R}^3$. Then,

$$\det(x, y, z) = x_1(y_2z_3 - y_3z_2) - x_2(y_1z_3 - y_3z_1) + x_3(y_1z_2 - y_2z_1)$$

where $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3), z = (z_1, z_2, z_3) \in \mathbb{R}^3$. Consider the special linear group

$$SL(3, \mathbb{R}) = \{A \in GL(3, \mathbb{R}) \mid \det(A) = 1\}$$

i.e., the subgroup of $GL(3, \mathbb{R})$ consisting of all 3×3 real matrices with determinant one [12].

Let $\gamma : I \rightarrow \mathbb{R}^3$ be a regular curve in $\mathbb{R}^3$. For equi-affine space curves, the non-degenerate condition $\det(\gamma'(t), \gamma''(t), \gamma'''(t)) \neq 0$, for all $t \in I$, where $\gamma'(t) = \frac{d\gamma}{dt}$, $\gamma''(t) = \frac{d^2\gamma}{dt^2}$, and $\gamma'''(t) = \frac{d^3\gamma}{dt^3}$. The equi-affine arc length of γ is as follows [12]:

$$s(t) = \int_{t_0}^t \det(\gamma'(u), \gamma''(u), \gamma'''(u)) \, du$$

If γ is a regular curve with the parameter s in the equi-affine space, then the following condition is satisfied [12]:

$$\det(\gamma'(s), \gamma''(s), \gamma'''(s)) = 1$$

Definition 2.1. [3] In the equi-affine geometry, a non-degenerate curve in $\mathbb{R}^3$ is classified as a general helix when a constant direction, defined by a nonzero vector $\vec{u}$ (the axis), is always contained within the instantaneous plane determined by the curve's equi-affine tangent t and binormal b vectors.

Theorem 2.2. [3] A curve γ is an equi-affine general helix in $\mathbb{R}^3$ if and only if its equi-affine curvatures satisfy the following condition:

$$\tau(s) = \int_a^s \kappa(u) du$$

where s is equi-affine arc length parameter.

Definition 2.3. [3] An equi-affine slant helix is a non-degenerate parameterized curve in $\mathbb{R}^3$ for which a constant nonzero vector $\vec{u}$ (the axis) exists such that $\vec{u} = \xi t + \lambda n + \mu b$ where t , n , and b are the equi-affine Frenet frame vectors and the scalar function $\mu(s)$ is non zero.

Theorem 2.4. [3] A parameterized curve in $\mathbb{R}^3$ is said to be an equi-affine slant helix if and only if its equi-affine curvatures satisfy the following condition:

$$s \tau(s) = \int_a^s u \kappa(u) du$$

where s denotes the equi-affine arc-length parameter.

Definition 2.5. [6] Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a function, $t > 0$, and $0 < \alpha < 1$. Then, the conformable fractional derivative of order α of f at t is defined as follows:

$$D_\alpha f(t) = \lim_{\varepsilon \rightarrow 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}$$

provided that the limit exists. If this limit exists, then f is said to be α -differentiable at t .

Theorem 2.6. [6] Suppose that f and g are α -differentiable at $t > 0$ and $0 < \alpha < 1$. Then, the following properties are satisfied:

$$i. D_\alpha(af + bg) = aD_\alpha(f) + bD_\alpha(g), \text{ for all } a, b \in \mathbb{R}$$

$$ii. D_\alpha(t^p) = pt^{p-\alpha}, \text{ for all } p \in \mathbb{R}$$

$$iii. D_\alpha(c) = 0, \text{ for all constant functions } f(t) = c$$

$$iv. D_\alpha(fg) = fD_\alpha(g) + gD_\alpha(f)$$

$$v. D_\alpha\left(\frac{f}{g}\right) = \frac{gD_\alpha(f) - fD_\alpha(g)}{g^2}, \text{ where } g \neq 0$$

$$vi. \text{ If } f \text{ is } \alpha\text{-differentiable, then } D_\alpha(f(t)) = t^{1-\alpha} \frac{df(t)}{dt}$$

Definition 2.7. [6] Let $\alpha > 0$ and $f : [0, \infty) \rightarrow \mathbb{R}$ be an α -differentiable function. Then,

$$I_0^\alpha f(s) = I_1^\alpha f(s^{\alpha-1} f) = \int_a^s \frac{f(x)}{x^{\alpha-1}} dx$$

is referred to as the conformable integral.

Theorem 2.8. [6] Suppose that $f : [a, \infty) \rightarrow \mathbb{R}$ is α -differentiable function. Then, for all $s > 0$,

$$D_\alpha I_0^\alpha f(s) = f(s)$$

Proposition 2.9. [4] Assume that γ is a regular conformable curve in the equi-affine space $\mathbb{R}^3$. Then, the arc-length function is as follows:

$$s(t) = \int_{t_0}^t u^{1-\alpha} \det(\gamma'(u), \gamma''(u), \gamma'''(u)) du$$

Definition 2.10. [4] Let $\gamma = \gamma(s)$ be a regular conformable curve in the equi-affine space, $t = D_\alpha(\gamma)s^{\alpha-1}$ be unit tangent vector, $n = D_\alpha(t)s^{\alpha-1}$ be unit normal vector, $b = D_\alpha(n)s^{\alpha-1}$ be unit binormal vector, and the $\det(D_\alpha(\gamma)s^{\alpha-1}, D_\alpha(t)s^{\alpha-1}, D_\alpha(n)s^{\alpha-1}) = 1$. Then, the set $\{t(s), n(s), b(s)\}$ is the conformable equi-affine Frenet frame of the curve γ . Thus, the Frenet formulas for the curve γ are given by

$$\begin{pmatrix} D_\alpha(t(s)) \\ D_\alpha(n(s)) \\ D_\alpha(b(s)) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\kappa_\alpha(s) & -\tau_\alpha(s) & 0 \end{pmatrix} \begin{pmatrix} t(s) \\ n(s) \\ b(s) \end{pmatrix} \quad (2.1)$$

where $\kappa_\alpha(s) = -\det(D_\alpha(t)(s), D_\alpha(n)(s), D_\alpha(b)(s))$ and $\tau_\alpha(s) = \det(D_\alpha(\gamma)(s), D_\alpha(n)(s), D_\alpha(b)(s))$ are curvature and torsion of the curve γ , respectively.

Definition 2.11. [9] A unit-speed conformable curve $\gamma : I \subset \mathbb{R} \rightarrow E^3$ in Euclidean 3-space is called a conformable rectifying curve whenever its position vector lies entirely within the rectifying plane associated with the conformable frame.

3. Some Special Conformable Curves in the Equi-Affine Space

In this section, we define conformable general helices and slant helices in the equi-affine space. Moreover, we provide the relationship between the curvatures of the curve in the case of being an equi-affine general helix, a slant helix, or a rectifying curve.

Definition 3.1. A conformable general helix in the equi-affine space is defined as a non-degenerate curve $\alpha(s)$ in $\mathbb{R}^3$, parameterized by equi-affine arc length, for which a constant vector $\vec{u} \neq \vec{0}$ (the axis) exists that is always a linear combination of the equi-affine tangent t and binormal b vectors of the Frenet frame.

Theorem 3.2. A conformable curve γ in $\mathbb{R}^3$ is an equi-affine general helix if and only if its conformable equi-affine curvatures satisfy the following condition:

$$\tau_\alpha(s) = \int_a^s \frac{\kappa_\alpha(t)}{t^{1-\alpha}} dt$$

where s denotes the conformable equi-affine arc-length parameter.

PROOF. Let γ be a conformable general helix in the equi-affine space $\mathbb{R}^3$. Then, there exists a vector $u \in \mathbb{R}^3$ such that

$$\vec{u} = \xi(s)t(s) + \mu(s)b(s) \quad (3.1)$$

Differentiating both side of (3.1),

$$\begin{aligned} D_\alpha(\vec{u}) &= D_\alpha(\xi(s)t(s) + \mu(s)b(s)) \\ &= D_\alpha(\xi(s))t(s) + \xi(s)D_\alpha(t(s)) + D_\alpha(\mu(s))b(s) + \mu(s)D_\alpha(b(s)) \\ &= 0 \end{aligned}$$

Using (2.1), the last equation can be rewritten in the form

$$(D_\alpha(\xi(s)) - \kappa_\alpha\mu(s))t(s) + (\xi(s) - \tau_\alpha\mu(s))n(s) + D_\alpha(\mu(s))b(s) = 0$$

Thus,

$$\begin{aligned} D_\alpha(\xi(s)) - \kappa_\alpha \mu(s) &= 0 \\ \xi(s) - \tau_\alpha \mu(s) &= 0 \end{aligned} \quad (3.2)$$

and

$$D_\alpha(\mu(s)) = 0 \quad (3.3)$$

Hence, $\mu(s)$ is constant. Since $\mu(s) = 0$ leads to the contradiction $\vec{u} = \vec{0}$, $\mu(s) \neq 0$. Thereby, $\mu(s) = 1$ can be taken by scaling the axis appropriately. From (3.2) and (3.3), $D_\alpha(\xi(s)) = \kappa_\alpha(s)$ and $\xi(s) = \tau_\alpha(s)$. Therefore,

$$\tau_\alpha(s) = \int_a^s \frac{\kappa_\alpha(t)}{t^{1-\alpha}} dt$$

Conversely, let γ be a conformable curve in the equi-affine space $\mathbb{R}^3$. Suppose that

$$\tau_\alpha(s) = \int_a^s \frac{\kappa_\alpha(t)}{t^{1-\alpha}} dt$$

Then,

$$D_\alpha(\tau_\alpha(s)t(s) + b(s)) = 0 \quad (3.4)$$

From (3.4), γ is a conformable general helix in the equi-affine space $\mathbb{R}^3$. $\square$

Definition 3.3. A conformable equi-affine slant helix is a non-degenerate curve $\alpha(s)$ in $\mathbb{R}^3$, parameterized by equi-affine arc length, for which a constant vector $\vec{u} \neq \vec{0}$ (the axis) exists that can be expressed in its Frenet frame as follows:

$$\vec{u} = \xi(s)t(s) + \lambda n(s) + \mu(s)b(s)$$

where λ is a nonzero constant, and t , n , and b form the equi-affine Frenet frame.

Theorem 3.4. A non-degenerate curve in $\mathbb{R}^3$ is said to be a conformable equi-affine slant helix if and only if its conformable equi-affine curvatures satisfy

$$s^\alpha \tau(s) = \int_a^s t^{2\alpha-1} \kappa_\alpha(t) dt$$

where s denotes the equi-affine arc-length parameter.

PROOF. Let γ be a conformable equi-affine slant helix in $\mathbb{R}^3$. Then,

$$\vec{u} = \xi(s)t(s) + \lambda n(s) + \mu(s)b(s) \quad (3.5)$$

where $\xi(s)$ and $\mu(s)$ are smooth functions and $\lambda \neq 0$. Differentiating both sides of (3.5),

$$\begin{aligned} D_\alpha(\vec{u}) &= D_\alpha(\xi(s)t(s) + \lambda n(s) + \mu(s)b(s)) \\ &= D_\alpha(\xi(s))t(s) + \xi(s)D_\alpha(t(s)) + \lambda D_\alpha(n(s)) + D_\alpha(\mu(s))b(s) + \mu(s)D_\alpha(b(s)) \end{aligned}$$

By (2.1),

$$(D_\alpha(\xi(s)) - \kappa_\alpha \mu(s))t(s) + (\xi(s) - \mu(s)\tau_\alpha)n(s) + (\lambda + D_\alpha(\mu(s)))b(s) = 0$$

From the above equations, $D_\alpha(\xi(s)) - \kappa_\alpha \mu(s) = 0$, $\xi(s) - \mu(s)\tau_\alpha = 0$, and

$$\lambda + D_\alpha(\mu(s)) = 0 \quad (3.6)$$

If $\mu(s) = 0$, then this is the contradiction because $\vec{u} = 0$. If we take the conformable integral of (3.6),

then $\mu(s) = -\frac{\lambda}{\alpha}s^\alpha + c$, where c is a constant. From the above equations,

$$s^\alpha \tau_\alpha(s) = - \int_a^s t^{2\alpha-1} \kappa_\alpha(t) dt$$

Conversely, let γ be a conformable curve in the equi-affine space. Suppose that

$$s^\alpha \tau_\alpha(s) = - \int_a^s t^{2\alpha-1} \kappa_\alpha(t) dt$$

Then,

$$D_\alpha \left(\frac{s^\alpha}{\alpha} \tau_\alpha(s) t(s) + n(s) + \frac{s^\alpha}{\alpha} b(s) \right) = 0$$

Therefore, γ is a conformable equi-affine slant helix. $\square$

Theorem 3.5. Let γ be a conformable equi-affine rectifying curve in $\mathbb{R}^3$. Then, the following is satisfied:

$$\tau_\alpha(s) = \int_a^s \frac{\kappa_\alpha(t)}{t^{1-\alpha}} dt + \frac{c_1 s^\alpha}{\alpha} + c_2 \quad (3.7)$$

where s is the conformable equi-affine arc length parameter of the curve γ . Conversely, if (3.7) is satisfied, then γ is a conformable equi-affine rectifying curve.

PROOF. Suppose that γ is a conformable rectifying curve. Then,

$$\gamma(s) = \xi(s)t(s) + \mu(s)b(s) \quad (3.8)$$

If we take the conformable derivative of both sides of (3.8), then

$$\begin{aligned} D_\alpha(\gamma(s)) &= D_\alpha(\xi(s)t(s) + \mu(s)b(s)) \\ &= D_\alpha(\xi(s))t(s) + \xi(s)D_\alpha(t(s)) + D_\alpha(\mu(s))b(s) + \mu(s)D_\alpha(b(s)) \\ &= t(s) \end{aligned}$$

By (2.1),

$$(D_\alpha(\xi(s)) - \kappa_\alpha \mu(s)) t(s) + (\xi(s) - \tau_\alpha \mu(s)) n(s) + D_\alpha(\mu(s)) b(s) = t(s) \quad (3.9)$$

From (3.9), $D_\alpha(\xi(s)) - \kappa_\alpha \mu(s) = 1$, $\xi(s) - \tau_\alpha \mu(s) = 0$, and $D_\alpha(\mu(s)) = 0$. It follows from the last equations that $\mu(s)$ is constant and

$$D_\alpha(\tau_\alpha) = \kappa_\alpha + \frac{1}{\mu(s)} \quad (3.10)$$

If we represent $\frac{1}{\mu(s)} = c_1$ and we take the conformable integral both sides of (3.10), then

$$\tau_\alpha(s) = \int_a^s \frac{\kappa_\alpha(t)}{t^{1-\alpha}} dt + \frac{c_1 s^\alpha}{\alpha} + c_2 \quad (3.11)$$

where c_1 and c_2 are constant. Conversely, suppose that γ satisfies (3.11), then

$$D_\alpha \left(\gamma(s) - \frac{\tau_\alpha}{c_1} t(s) - \frac{1}{c_1} b(s) \right) = 0$$

where $c_1 \neq 0$. Therefore, γ is a conformable equi-affine rectifying curve. $\square$

Example 3.6. Let $\gamma(s) = \frac{1}{\alpha^2} \left(\frac{2}{3-\sqrt{3}} (s^\alpha)^{\frac{3-\sqrt{3}}{2}}, \frac{2}{3+\sqrt{3}} (s^\alpha)^{\frac{3+\sqrt{3}}{2}}, \frac{(s^\alpha)^3}{3\sqrt{3}} \right)$ be a conformable general helix in the equi-affine space $\mathbb{R}^3$. Then, the conformable equi-affine Frenet vectors are as

follows:

$$t(s) = \left(\alpha^{-\frac{1}{2}} s^{\frac{\alpha(1-\sqrt{3})}{2}}, \alpha^{-\frac{1}{2}} s^{\frac{\alpha(1+\sqrt{3})}{2}}, \frac{2\sqrt{3}}{9\alpha^2} s^{2\alpha} \right)$$

$$n(s) = \left(\alpha^{\frac{1}{2}} s^{\frac{\alpha(1-\sqrt{3})}{2}}, \alpha^{\frac{1}{2}} s^{\frac{\alpha(-1+\sqrt{3})}{2}}, \frac{4\sqrt{3}}{9\alpha} s^{\alpha} \right)$$

and

$$b(s) = \left(\alpha^{\frac{3}{2}} s^{\frac{-\alpha(3+\sqrt{3})}{2}}, \alpha^{\frac{3}{2}} s^{\frac{\alpha(3-\sqrt{3})}{2}}, \frac{4\sqrt{3}}{9} \right)$$

where $\det(t(s), n(s), b(s)) = 1$. Thus, $\kappa_{\alpha}(s) = \frac{\alpha^3}{s^{3\alpha}}$ and $\tau_{\alpha}(s) = -\frac{\alpha^2}{2s^{2\alpha}}$. Therefore,

$$\vec{u} = \tau_{\alpha}(s)t(s) + b(s) = \left(0, 0, \frac{1}{\sqrt{3}} \right)$$

The graphs of conformable equi-affine curvature κ_{α} and torsion τ_{α} for different values of α are demonstrated in Figures 1 and 2.

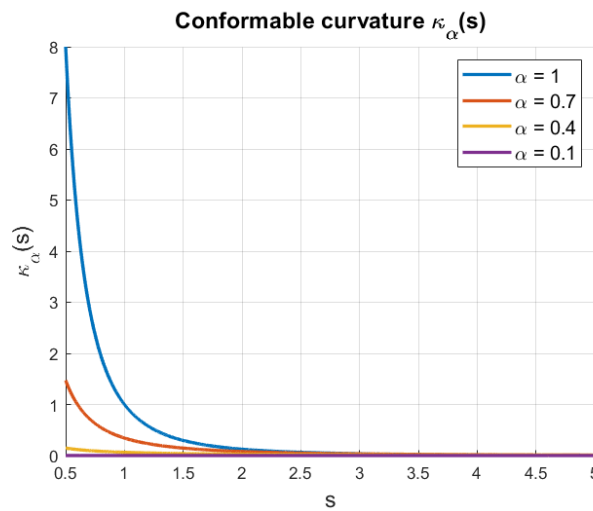


Figure 1. Conformable equi-affine curvatures κ_{α} for $\alpha = 0.1$, $\alpha = 0.4$, $\alpha = 0.7$, and $\alpha = 1$

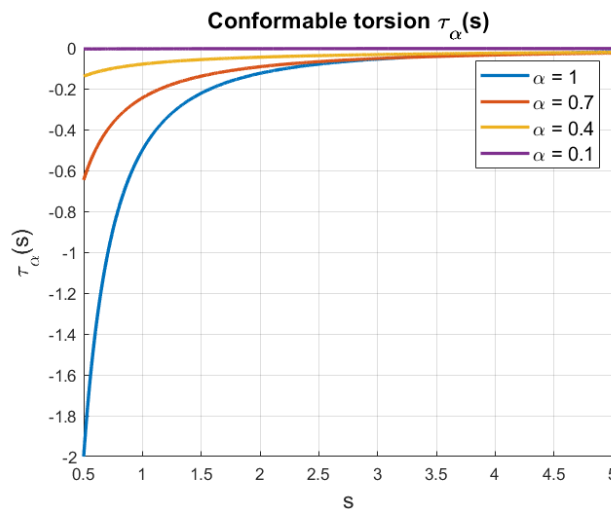


Figure 2. Conformable equi-affine torsions τ_{α} for $\alpha = 0.1$, $\alpha = 0.4$, $\alpha = 0.7$, and $\alpha = 1$

4. Conclusion

This study investigates the conformable differential geometry of special curves defined in the equi-affine space. By combining the flexible nature of conformable derivatives with the invariance properties of equi-affine geometry, we reveal the geometric structures that emerge from the intersection of these two theories. The main focus of this study is on special curve families such as helical curves, slant helices, and rectifying curves. The results of the study clearly demonstrate how the conformable derivative parameter α plays a critical role in the fundamental differential geometric properties of these curves (curvature, torsion, etc.). In future work, researchers can focus on extending this approach to surface theory, investigating conformably equi-affine minimal surfaces, or carefully and systematically comparing similar geometries under different definitions of fractional derivatives.

Author Contributions

The author read and approved the final version of the paper.

Conflicts of Interest

The author declares no conflict of interest.

Ethical Review and Approval

No approval from the Board of Ethics is required.

References

- [1] M. E. Aydın, A. Mihai, A. Yokuş, *Applications of fractional calculus in equiaffine geometry: Plane curves with fractional order*, Mathematical Methods in the Applied Sciences 44 (2021) 13659–13669.
- [2] M. Takahashi, *Equi-affine plane curves with singular points*, Journal of Geometry 113 (1) (2022) 16.
- [3] M. Ögrenmiş, A. O. Ögrenmiş, *Lancret-type theorems in equiaffine space*, Journal of Geometry 115 (2) (2024) 25.
- [4] M. Ögrenmiş, *Fractional curvatures of equiaffine curves in three-dimensional affine space*, Journal of New Theory (46) (2024) 11–22.
- [5] M. Ögrenmiş, A. O. Ögrenmiş, *Curves in multiplicative equiaffine space*, Mathematics 13 (2025) 1107.
- [6] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, *A new definition of fractional derivative*, Journal of Computational and Applied Mathematics 264 (2014) 65–70.
- [7] U. Gözütok, H. A. Çoban, Y. Sağiroğlu, *Frenet frame with respect to conformable derivative*, Filomat 33 (6) (2019) 1541–1550.
- [8] B. Yılmaz, *A new type electromagnetic curves in optical fiber and rotation of the polarization plane using fractional calculus*, Optik 247 (2021) 168026.
- [9] A. Has, B. Yılmaz, A. Akkurt, H. Yıldırım, *Conformable special curves in Euclidean 3-space*, Filomat 36 (14) (2022) 4687–4698.

- [10] A. Has, B. Yilmaz, *Measurement and calculation on conformable surfaces*, Mediterranean Journal of Mathematics 20 (5) (2023) 274.
- [11] E. Karaca, A. Altinkaya, *Minkowski geometry of special conformable curves*, International Journal of Geometric Methods in Modern Physics 22 (05) (2025) 2450318.
- [12] D. Davis, *Generic affine differential geometry of curves in $\mathbb{R}^n$* , Proceedings of the Royal Society of Edinburgh 136A (2006) 1195–1205.