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Reflective and reflexive beliefs of two pre-service secondary Mathematics teachers

Shashidhar Belbase[1]

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Abstract

The objective of this paper is to present two preservice secondary mathematics teachers' beliefs about teaching Geometric Transformations (GTs) using Geometer's Sketchpad (GSP). The study comprised of series of five task-based interviews with each of two participants, who were senior undergraduate preservice teachers, at a medium-sized public university in the Rocky Mountain Region of the United States. I used radical constructivist grounded theory (RCGT) as a theoretical frame to guide this study process. The results of study include reflective and reflexive beliefs about teaching GTs with GSP. These beliefs have been further explored as pre-reflective, in-reflective, post-reflective, pre-reflexive, in-reflexive, and post-reflexive beliefs of two preservice secondary mathematic teachers about teaching GTs with GSP. Pedagogical implications of these belief categories are widely discussed.

Keywords: Mathematics Teachers' Beliefs, Beliefs about Technology Integration, Radical Constructivist Grounded Theory (RCGT), Reflective Beliefs, Reflexive Beliefs, Geometric Transformation, Geometers Sketchpad, Interviews

Author 1
Assistant Professor
University College,
Zayed University, Dubai
United Arab Emirates
z9865@zu.ac.ae

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INTRODUCTION

Integration of technology in mathematics teaching and learning has been growing since the last few decades. Many research scholars (e.g., Ertmer, 2006) agrees that teachers' beliefs affect the way they use technological tools for teaching mathematics. A proper use of technological tools may play a positive role in mathematics education by making it interesting to the learners with a deeper sense of problem-solving, creative thinking, and reasoning (Ertmer, 2006; Ertmer et al. 2012). Use of technology in the classroom can support in making abstract mathematical concepts visual otherwise not possible to see with equations or formulas (Foley and Ojeda,2007). Teachers can provide students with tasks for collaborative or individual problem solving by using calculators, computers, and so forth. Hence, technology may play a significant role in helping students to learn mathematics in more way that is meaningful. Teachers can design their lessons in a creative way by providing students with constructive and flexible learning tools (Garry, 1997). In this context, many mathematics education researchers and scholars have pointed to the challenges of technology integration without the change of teachers' mindset of using technological tools (Chai, Wong and Teo, 2011; Leatham, 2002; Wachira, Keengwe and Onchwari, 2008). In some cases, these beliefs were inconsistent in classroom practices (Ertmer et al. 2012).

There are several studies on mathematics teachers' beliefs about technologies integration in teaching and learning mathematics (Lin, 2008). Some researchers identified mathematics teachers' technology related beliefs as no technology beliefs, pre-mastery beliefs, post-mastery beliefs, and exploratory beliefs (Misfeldt et al., 2016). Erens and Eichler (2015) identified four categories of teachers based on their beliefs about technology integration – initiators, explorers, reinforcers and symbiotic collaborators. Chen (2011) categorizes teacher beliefs about technology integration in mathematics teaching as instrumental (technology is just a neutral instrument to solve problem) and empowerment (technology enhances power of visualizing, representing, and complex problem solving). A positive belief about technology integrations in mathematics teaching and learning stems from teachers' knowledge of mathematics, mathematics pedagogy, content, and technology. Mishra and Koehler (2006) termed this combination as technological pedagogical and content knowledge (TPACK). Now, technology integration in mathematics education is considered as an integral part of classroom practices. With such a need, TPACK has been an important aspect of teacher education and training to enhance effective use of technological tools in the classrooms (Hunter, 2015). Cuevas (2010) suggested that use of technology in the mathematics classroom can take students "into the domain of non-routine tasks" (p. 374). Teachers require the ability to integrate technologies with paper and pencil so that students can have opportunity to learn concepts and procedures both by hands-on and technological constructions. In doing so, teachers may use technology in mathematics teaching as an exploring tool, connecting tool, and thinking tool (Cuevas, 2010). These skills play a significant role in developing positive beliefs about technology integrations among mathematic teachers for teaching of mathematics meaningfully (Misfeldt et al., 2016).

Most of the studies on mathematics teachers' beliefs about technology integrations have focused on the generic beliefs in relation to uses of technological tools in mathematics classes. In many studies on preservice or in-service mathematics, teachers' beliefs are categorized as the dichotomies of positive or negative beliefs and constructivist or instrumentalist beliefs. In some studies teachers beliefs were based on the function of technology in teaching and learning mathematics, for example, technology for algebraic thinking and reasoning, technology for meaning of mathematical relations, technology for interpretation, and contextualization of mathematical content and pedagogy (Polly, 2015). These studies drew conclusions from quantitative and descriptive qualitative analysis. In the qualitative study, the mathematics teachers' beliefs did not present the data with adequate details for the reader to contextualize their expressed beliefs. Also, a strong theoretical frame helps reader to understand the perspective and viewpoint within the data collection, analysis, and interpretation in the study. This study aims to present the requirement on preservice secondary mathematics teachers' beliefs about teaching geometric transformations (GTs) with Geometer's Sketchpad (GSP).

The research question for the study was - What beliefs do preservice secondary mathematics teachers hold about their future practices of teaching geometric transformations with Geometer's Sketchpad?

In the similar study previously conducted, findings were discussed about preservice secondary mathematics teachers' beliefs about teaching geometric transformations (GTs) with Geometer's Sketchpad (GSP) (Belbase, 2015). In this paper, two key categories – reflective and reflexive beliefs about teaching GTs with GSP are discussed. This paper introduces reflective and reflexive beliefs in general followed by a theoretical frame of radical constructivist grounded theory (RCGT).

Reflective and Reflexive Beliefs

Mathematics teaching is both a reflective and reflexive process (Lowery, 2003; Smitherman, 2006). This supports that beliefs about actions can be a reflective beliefs. This reflexive belief is associated with looking back at his or her practices, and making sense to reflect the "learning from experiences" (Wilson et al., 1987 as cited in Lowery, 2003, p. 23). Teachers' lived experiences can help in forming the personal theories through practices, towards shaping their beliefs in teaching mathematics (Shulman, 1987). Whereas, reflexive belief is associated with active conscious roles and actions of teaching mathematics through recursive and generating thoughtful dialogues (Smitherman, 2006). Reflexive belief is also not only associated with thinking of oneself in relation to practice and action, but it is also about belief toward one's own ability and confidence gained through such dialogue and experiences. Hence, reflexive belief is relational thinking within the community of practice (Jawarski, 2006).

Teachers' reflective beliefs about teaching and learning mathematics may influence their observation of how they make sense of their teaching and how they can learn better from their teaching process (Leikin and Zazkis, 2010). Their reflexive beliefs may affect their

ability and action to become a critical self-learner with self-awareness and consciousness to the subject matter, context, and relationship with students (Whittock, 1997). Reflective beliefs seem to be dependent on the external source such as a phenomenon, action, or object that has a reliable representation (Sperber, 1997). Whereas, reflexive beliefs do not have such external source and hence they are non-representational. Table 1 summarizes the major characteristics of reflective and reflexive beliefs about teaching mathematics with technology.

Table1. Reflective and Reflexive Beliefs

Belief Type	Belief Object	Characteristics
Reflective Belief	Action, phenomenon	Reflective belief is related to an action or phenomenon. This kind of belief is related to the external world in which one gains experiences. It is explicit.
Reflexive Belief	Self-other relation, own biography and identity	Reflexive belief is related to one's awareness and consciousness to the self-other relations. This belief is about the internal world. It is implicit and self-referential.

Reflective Beliefs

The reflective beliefs focus on one's awareness toward actions, objects, and phenomena. "Reflective beliefs are those beliefs we consciously hold; truths we explicitly endorse" (Barrett and Lanman, 2008, p. 111). The reflective beliefs may be rooted in reflective thinking and reflective practice. Reflective practice in teaching and learning has been widely discussed by researchers in various disciplines- for example, nursing (e.g. Duffy, 2009; Jarvis, 1992; Hargreaves, 2004), engineering (e.g., Adams et al., 2003; Bucciarelli, 1984), and sports management (e.g., Edwards, 1999). Other fields that apply reflective beliefs are medical education (e.g., Koepke 2009) and teacher education (e.g., Adler, 1993; Fletcher, 1997; Harford and Mac Ruairc, 2008). Reflective beliefs can also be associated with these reflective thinking and practices. Schön (1983) introduced reflection-in-practice, reflection-on-practice, or reflection-for-practice which generated a state of mind with confidence toward an action or practice. Thus, beliefs arise from reflective actions are the reflective beliefs.

Reflective beliefs are widely discussed in terms of reflective practice. In modern education, John Dewey (1933) was accredited for beginning with the reflective practice in education. He discussed human thought processes as reflective phenomena. He implied to "a state of perplexity, hesitation, doubt, and act of search or investigation directed toward bringing to light further facts associated with certain belief" (p. 9). Dewey's version of reflective practice has become helpful in developing some key aspects of reflective beliefs in terms of the course of action of the past, present, and the future. Therefore, one may have pre-action beliefs, during action beliefs, and post-action beliefs.

Moreover, theory of reflective beliefs is also associated with Schön's (1983) 'the reflective practitioner: how professionals think in action'. Schön's (1983) works outlined the reflection-in-action and reflection-on-action as a part of reflective beliefs which people developed in terms of professional growths. Furthermore, reflective beliefs is anticipatory (future-oriented), retrospective (past-oriented), and contemporary (current) (Loughran, 2002). Thompson and Thompson (2008) extended Schön's (1983) idea of reflection-on-action and reflection-in-action by adding another domain "reflection-for-action". They discussed various ways to promote critical reflective practices by focusing whether one is reflecting-on-action or reflecting-in-action or reflecting-for-action. One may acquire reflective beliefs in a "reflective mode" even through "partial understanding of their true conditions" (Engel, 2000, p. 24). Reflective beliefs are, therefore, associated truth-values ascribed to one's actions or any external phenomena. The degree of the truth-values might relate to strength of one's beliefs. These beliefs are not isolated from one's self-awareness, consciousness, and deeply rooted values. The relationship of self-awareness, consciousness, and values to one's actions and thoughts generates the next level of beliefs towards reflexive beliefs.

Reflexive Beliefs

Van der Hart et al. (2006) discussed reflexive beliefs in terms of how traumatized persons believe about themselves and others. These beliefs associated with "feelings, prejudice, suggestions, and restricted views of ourselves and others" (Van der Hart et al. 2006, p. 181). Such beliefs even lead the believer to "fixed cognitive schemas, that is, maladaptive core beliefs about self, others, and the world" (p. 181). Reflexive beliefs also relate to one's awareness of self and others, especially "beliefs about the referents of one's states" (Braude, 1995, p. 72). This in terms gives meaning of reflexive beliefs which are self-referential to person's awareness, consciousness, and values. Campbell (2010) theory of consciousness states, "…an entity is conscious if and only if it has reflexive beliefs" (p. 9). Hence, reflexive beliefs are about self and other relations and core beliefs in terms of one's consciousness. This kind of belief even relates to one's being amongst the others that reciprocates roles and responsibilities as a teacher and student (Smitherman, 2006).

Van der Hart et al. (2006) discussed different action tendencies as part of reflexive beliefs. The lower action tendencies serve short-term goals for living with basic reflexes, presymbolic and basic symbolic actions. The intermediate action tendencies include reflective and reflexive actions. The higher-level action tendencies includes of prolonged reflective, experimental, and progressive actions. These tendencies are associated with reflexive beliefs that may act at different levels of consciousness and awareness of the person. According to Jovchelovitch (1996), reflexive beliefs of teachers are associated with "who they are, how they understand themselves and others, where they locate themselves and others, and which are the cognitive and affective resources that are available to them in a given historical time" (p. 125). Braude (1995) relates reflexive beliefs to indexicality of one's own mental state. This indexicality refers to his or her epistemological state that has a relational property in terms of experiences and cognitive state of the mind (Braude, 1995).

Jansen (2008) introduced the idea of reflexive beliefs in terms of discourse in mathematics classrooms. She interrelated students' psychological factors to their participation and opportunities to participate at individual level and classroom as a community. Their reflexive beliefs in terms of 'who they are' influenced their engagement in the classroom discourse. Davis and Harré (1999, p. 37) proposed the idea of "reflexive positioning" in which an individual "positions oneself" based on his or her personal self-referential beliefs. Despite these examples, there is still a lack of sufficient literature in mathematics education contributing explicitly towards reflexive beliefs of in-service and preservice mathematics teachers.

THEORETICAL FRAME

I conceptualized five theoretical assumptions of RCGT from the literature of radical constructivism (e.g., von Glasersfeld, 1978 & 1995) and grounded theory (e.g., Charmaz, 2006; Glaser and Strauss, 1967; Strauss and Corbin, 1998) to guide the methodology of the study including data construction, description, analysis, and interpretation. These assumptions integrate essential features of grounded theory methodology in qualitative research within interpretive and radical constructivist epistemology. These assumptions focus on mutualism as a symbiosis between the researcher and participants, balancing voice of research participants and the researcher, notion of research as a cognitive and adaptive function, and praxis as criteria to examine theory constructed or hypothesized from the data. Hence, these assumptions synthesized from the literature of grounded theory and radical constructivism helped me to conceptualize the research input, process, and outcomes.

1. Symbiotic relation

I assumed that the participants and researchers have a symbiotic relationship while co-constructing knowledge from this research. This is supported by radical constructivists (e.g. Steffe and Thompson, 2000; von Glasersfeld, 1995) and grounded theorists (e.g. Charmaz, 2006; Corbin and Strauss, 2008) accept that the researchers and participants have a symbiotic relationship. The researchers and the participants share different epistemic roles during the research process. The researchers may construct data through teaching experiments (Steffe, 2002; Steffe and Thompson, 2000), clinical interviews, or task-based interviews (Goldin, 2000; Maher, 1998) in which the participants contribute to the study through their participation, construction of narratives, and reflections on their experiences. At the same time, they also learn something new from the research process. In this study, the researcher-participant relation was a mutualism. The participants went through a series of problem-based task situations. These situations provided them with new insights or experiences in terms of teaching GTs with GSP. At the same time, in this study, I processed immediate reflections, points of view, and artefacts relevant as sources of data for this study.

2. Researchers and participants voice

The qualitative research assumption carries participants' voice in different forms (e.g., vignette, protocols, narratives, life stories, to name a few). These voices reflect direct expressions of the participants in different genres (both verbal and nonverbal). The research carries the voice of the participants as the first person perspective (direct voice) and the researcher's voice as the second and the third person perspectives (indirect voice) in the form of reflexivity (Hertz, 1997). Here, reflexivity of the researcher is related to his or her sense, awareness, and consciousness to the issues through deep abstraction of meanings. Grounded theorists (e.g., Bergkamp, 2010; Warfield, 2013) used researcher and participants' voice. The constructivist grounded theory assumes that the voice of the researcher is reflexive (Charmaz, 2006; Warfield, 2013). The research process for this study was different from the classical grounded theory, which focused on participants' voice through narrative protocols and the researcher voice through reflective and reflexive interpretations. The researcher voice is decentred by keeping the participants' voice upfront (Pierre 2009) for the fact that their voice might not "speak on their own" (Mauther and Doucet, 2003, p. 418).

3. Research as a cognitive function

For the processes of data constructing, coding, categorizing, theoretical sampling, constant comparing, and theoretical memoing active cognitive processes are initiated. Both the participants and the researcher are active cognizing subjects. For the same, interview and observation were implied. While doing this, in-situ data construction also validated through observation and record by asking the relevant questions (Friedhoff et al. 2013). While coding and categorizing of data, the theoretical memo was maintained in order to keep track of the process, about thinking, and personal hunches (Charmaz, 2006). A theoretical sampling and constant comparing of concepts or categories with additional data was undertaken. All of these functions are related to active cognitive processes (Bailyn, 1977). In this study of preservice secondary mathematics teachers' beliefs about teaching GTs with GSP, both participants and I engaged in interactions during the task-based interviews. The interactive, reflective, and constructive moments in the interview sessions provided me a pathway to look at their beliefs in conjunction with knowledge and comprehension of teaching GTs using GSP. The participants and I discussed the ways to use GSP in teaching GTs. The series of interactions helped us in "organization of our experiential world" (von Glasersfeld, 1990, p. 19) in forming and shaping our knowledge and beliefs. Hence, the entire research process was an active cognitive function.

4. Research as an adaptive function

Any endeavour to construct a new knowledge is an adaptation to a new experience, context, and challenges. The construction of codes and categories in the grounded theory approach is not one-time activity, but a continuous process through a series of new codes, categories, and meanings until the final categories are saturated with data. The construction or invention of grounded categories is an adaptive process (Lichtenstein, 2000). This gives the

meaning of the construction of grounded categories that undergo the reorganization with changing data, context, and interpretation. In this study of preservice secondary mathematics teachers' beliefs about teaching GTs with GSP, data were constructed in different phases. In the first phase, initial data through series of task-based interviews with the two participants from which three major conceptual categories emerged regarding beliefs about ability, action, and attributes were taken into consideration. In the second phase, these three concepts were taken as a basis for further interviews. Two more interviews were conducted with two participants in relation to their beliefs about teaching GTs with GSP especially focusing on the key aspects of environment and object of teaching GTs with GSP. This offered the study to associate with qualitative, constructive, and adaptive grounded categories (Layder, 1998; Welsh, 2009).

5. Fit and *viability of theory* (Praxis)

The analysis and interpretation of data provided major categories that were examined with praxis criteria, in examining the categories constructed or invented from the study (Charmaz, 2006; Glaser and Strauss, 1967; Strauss and Corbin, 1998). It is observed that the quality of the categorical findings or theory of the research can be related to the praxis dimension. This offered the dimension of fit and dimension of viability. The dimension of fit relates to whether the theory constructed or invented from the data that seem suitable in the context. In relations to the question of "To what extent the theory constructed or invented from the data resonates with the research context?" the praxis of viability relates to whether the category grounded on data carries a possible explanation of the phenomenon, and can it be transferred to a similar but another phenomenon. This also relates to the question of "Can the categories constructed or invented from the data explain a similar phenomenon?" Praxis dimension focuses on the "transformative possibilities of the research process and product" (Rodwell 1998, p. 79). In this study, attempts were made to explore 'to what extent these constructs fit within their belief system and whether they provide a viable explanation of their beliefs'. Supports were also rendered from peer associated for debriefing, reviewing, and auditing the codes, categories, and concepts (Rodwell, 1998). The peer associate collaboration offered additional support towards reliability of the inquiry process. The peer associate collaboration was a critical "affective and intellectual dimensions of the inquiry…" (Rodwell, 1998, p. 194) which significantly contributed towards data interpretation and findings.

These five assumptions of RCGT guided the methodology of the study of Preservice Secondary Mathematics Teachers' Beliefs about Teaching Geometric Transformations Using Geometer's Sketchpad (Belbase, 2015) by clearly outlining the roles and responsibilities of the participants and I towards identifying our position in the research process.

METHODOLOGY

This section discusses the recruitment of participants, administration of interviews, writing theoretical memos, and analysis and interpretation, these are discussed in the separate sub-sections.

Participants

Two preservice secondary mathematics teachers were appointed from a pool of students taking second methods of teaching secondary mathematics course in the fall of 2013 at a University in the Rocky Mountain Region of the U.S. The selection of two participants depended on access, availability of their time for interviews and their interest to volunteer in this study. One of them was a male participant who had re-joined the college to get a teaching degree after many years of his private job. He did not have prior experience of using GSP. Another participant was a female undergraduate preservice secondary mathematics teacher aiming to be a mathematics teacher after graduation. She had a prior experience of using GSP in a geometry class she took.

Administration of ınterviews

The first two interviews took place at the end of fall of 2013 term when the participants were taking the methods of teaching mathematics course while rest of the three interviews were conducted in the spring of 2014 during their student teaching internship. The first interview episode was designed for teaching reflection with GSP. The second interview episode was designed for teaching translation with GSP. The third interview episode was designed for teaching rotation with GSP. The fourth interview episode was designed for teaching of the composite transformations with GSP. The last interview focused on confirming their beliefs expressed in earlier interviews. Each interview episode was designed with task situations, discussions, and reflections. Each interview episode took 37-86 minutes to complete. All the interview episodes were recorded using a digital video recorder. I conducted the first two interviews and recorded them for transcribing and analysing. For the rest of the three interviews, I was assisted by a peer associated within the educational mathematicians to conduct the interviews and observation while I took note of important points. I transcribed the interview data verbatim for analysis and interpretation.

Writing theoretical memos

I wrote a reflective memo after each interview to support the analysis and interpretation of the data. These notes included major points that were discussed during the interview sessions, observations and nonverbal expressions of the participants. These memos helped me in constructing themes and categories during the coding process. It helped me in constant comparison of the codes, categories, and themes. This process helped me in keeping track of main theoretical constructs, ideas, concepts, and categories during data generation, analysis, and interpretation. This process also enabled me to reflect on the key ideas, events,

and processes at each stage of data generation, analysis, and interpretation. The initial codes from the data were connected together to a broader concept or category leading to the construction of final categories of beliefs. The writing of memos also helped me in forming second and third-order interpretive accounts during the interpretation of the data.

Analysis and interpretation

The analyses and interpretations of the interview were carried out in two phases. The first phase consisted of a classificatory analysis and interpretation using the principles of grounded theory (Charmaz, 2005; Strauss and Corbin, 1998). This kind of analysis was based on grounded theory approach to find concepts and categories from the pieces of data. In the second phase, the data were analyzed and interpreted using a holistic approach (Hall, 2008). This analysis was based on constructivist approach to find meanings of data as a whole. The whole analysis and interpretive approach was guided by the five assumptions of RCGT. I transcribed the interview data verbatim for each interview episode, and the transcribed texts were used for the analysis and interpretation. The analyses of data were carried out by open coding, axial coding, and selective coding to construct codes, categories, and themes (Corbin and Strauss, 2008; Strauss and Corbin, 1998). Theoretical sensitivity was maintained by remaining open and reflective to the elements of theoretical importance in the data (Charmaz, 2006; Corbin and Strauss, 2008).

FINDINGS

After the completion of categorical analysis of data using the grounded theory approach of coding and categorizing, re-analyzed and re-interpreted the data from a holistic perspective (Hall, 2008) were conducted. From the holistic analysis and interpretation of the entire data of the participants' beliefs, new dimensions of beliefs emerged in terms of reflective and reflexive beliefs. The reflective beliefs are associated with objects, environment, and phenomena outside the 'self' of the research participants. The reflexive beliefs are associated with 'selves' of the two participants in relation to others (students, content, and context) and hence those beliefs are internal to them. Reflective beliefs are related to critical examination of actions, objects, and environment. Whereas, reflexive beliefs are related to critical examination of participants' self-awareness and consciousness to the self-other interface. These beliefs are more affective, attitudinal, and cognitive in nature. The directional and temporal dimensions of these beliefs indicated to the possibility of pre-, in-, and post- reflective and reflexive beliefs (Fig. 1). Each of these holistic dimensions of beliefs is discussed in the following sub-section.

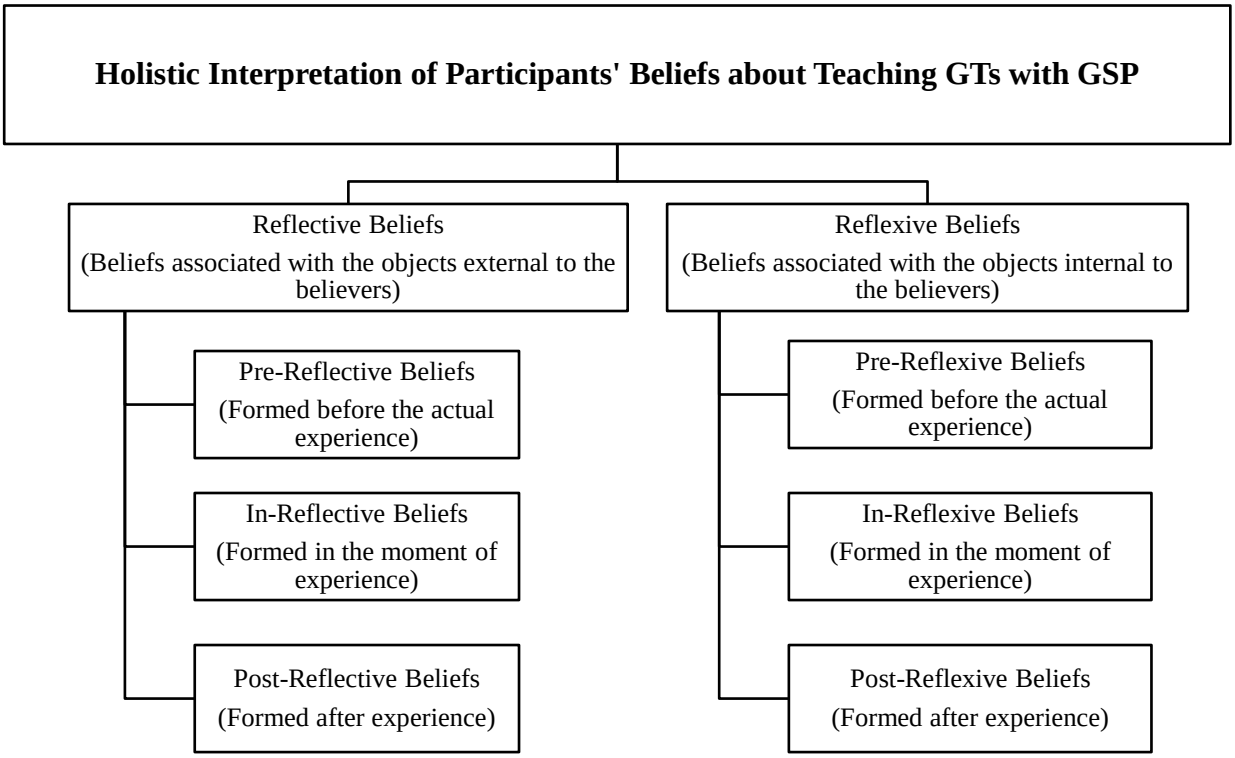

Figure 1. Holistic interpretation of beliefs in terms of reflective and reflexive beliefs

Reflective Beliefs

The observation of the entire interview transcript revealed that some beliefs that the participants formed were related to the belief objects outside their personal 'selves'. These beliefs were found to be associated with the phenomena external to the participants' mental states. This gives a meaning that the participants were found to have those beliefs about objects, persons, and environment external to them. The belief narratives in the forms of reflective beliefs were generated from the interview episodes by putting together bits and pieces of narratives to create a sensible belief-expression.

Cathy's reflective beliefs

The following belief narrative presents examples of Cathy's reflective beliefs extracted from the interview episodes. Cathy expresses her reflective beliefs in relation to the phenomenon of reflection transformation. These beliefs are also associated with GSP as a tool (object) to facilitate the study of GT processes. The narrative in protocol 1 is in the first person point of view where the narrator (speaker) is the research participant (Cathy), and it portrays the elements of her reflective beliefs.

Protocol-1

Reflection is about line, axis, and there has to be reflected across. You need to know here is a butterfly and how to reflect part of it. You need to know how to construct an image. Lines and points are important basic things to know. They (students) need to know the distance- yes. Um, congruency and similarity – yes. Ah, parallel, perpendicular, I don't know if you would need to know, but I think we need to know that shapes are similar. I would use it (GSP), but I wouldn't rule out the pencil and paper activity. I guess the title I would give it (GSP) as a discovery tool cause it's not doing the teaching, you are doing the learning. I want to know about their mistakes, first of all, cause their mistakes are good for learning. I think mistakes are fine, and they are the tools for learning.

The properties of GTs related to angles and side lengths can be explored with GSP. Under a rotation, angles stay preserved and side lengths are also preserved. Under a rotation, orientation is not preserved, but perimeters preserved, and areas preserved. I think that procedurally it (GSP) skips steps, but not really skipping steps; it is just quickening the steps. I think the individual activity would be the discussion. I think that in geometry it is incredibly important to discuss their ideas. Prompts would build their (students') confidence.

They (students) can explain to me like what they are doing, so like they decide they are gonna explore the area. I think that they need the ability to recognize their own and in groups, like simultaneously. They (students) could create like the Ferris wheel and then being able to kinda do animation of their own, which is meaningful to them. Yes, GSP helps in exploring properties of any transformation or even composite transformations.

GSP is a tool for the mathematics exploration. The only thing I don't want is that it does not tell you what you are doing really. It is just rotating, whatever that means, and it is just rotating. GSP is not so much procedural, I guess. It is more of exploring, conceptual, visual, and dynamic.

Jack's reflective beliefs

The following belief narrative is example of Jack's reflective beliefs in a cluster extracted from the interview episodes. His reflective beliefs are associated with the functions of GSP in explicating the GT processes. These beliefs are related to visualization of the GT processes within the dynamic environment of the GSP. The narrative in protocol 2 is in the first person point of view where the narrator (speaker) is the research participant (Jack), and it portrays the elements of his reflective beliefs.

Protocol-2

GSP has a lot of learning curve. They (students) have to know what they are trying to do. GSP is more visual. I think it's a great tool. I think we do the measurement of things and how it works. It (GSP) is like any computer program. I really like the simplicity with the sketchpad. They will be able to use the coordinates. Um, you know, think about how long I took just to

count those tiny squares. Here (in GSP), you can go to the coordinates. Then all of a sudden you can get to the algebra.

Properties, you could talk about how the shapes are congruent. The conjecture on rotation is gonna be pretty much the exploration or that kind of thing. I would choose angles, how they are related to each point and each object. Even you could talk about with the angles and sides and how they are the same. Like this here (with the rotation) that may not click right away, but if you can show them real life thing. Maybe even, you draw a satellite in the space and how it is orbiting. They can see that. So, that way it helps me in visualizing and explaining it.

It is always cool about this (plotting of areas). Kids when build this now they are also learning about linear functions. Kids are actually ready to do with functions and go into linear transformations. You can totally compare that and build into a kind of lesson that is built on itself and then you can talk about linear functions. It is really cool. I think GSP helps you enrich this type of environment. I like the visualization. How you can visualize angles, like you can put the angles in here so that they can see it right out there, and side lengths. I think that's cool. I think it (GSP) really can help build on it

GSP is a tool for conceptual understanding. It skips a lot of steps. It has short-cuts. GSP is both a tool for problem solving and mathematical exploration, but I say more mathematical exploration if I have to pick between the two. I think that GSP makes teaching GTs meaningful cause they can see real life applications. That's what kids want. They want to know when they leave the classroom they may use it. I think I can help students in making and proving conjectures. Just the hands-on stuff and show them what they are looking at and making animations. I think that GSP can help in designing different instructional approaches. You could design different lessons and do hands-on. You can approach different demonstrations or student based learning where they do it themselves.

Some of the key points from the participant's reflective belief narratives are – nature of GSP as a discovery tool, building students' confidence, exploring properties of GTs with GSP, transition from geometry to algebra, and making implicit processes explicit.

Cathy thinks that GSP is a discovery tool in relation to teaching GTs. She explains, "I guess the title I would give it (GSP) as a discovery tool cause it's not doing the teaching, you are doing the learning." She further accepts that use of GSP in teaching GTs facilitates the teacher in prompting that can build students' confidence. She accepts that the use of GSP helps in exploring different properties of GTs. In this sense, GSP is not much procedural for her because it is more of exploring, conceptualizing, visualizing, and animating tool.

For Jack, use of GSP helps students in making a transition from geometry to algebra. He expresses, "Here (in GSP), you can go to the coordinates. Then all of a sudden you can get to the algebra" and this clearly indicates his belief that the use of GSP in teaching GTs may facilitate teacher to make a smooth transition between geometry and algebra. While doing this, students can visualize and explain the geometry-algebra interface of the GTs.

I observed that these reflective belief narratives have temporal aspects associated with them. The time of actions or happening of something and forming beliefs based on them created three forms of reflective beliefs: pre-reflective, in-reflective, and post-reflective beliefs. Each of them has been discussed under a separate sub-heading.

Pre-reflective beliefs

The above belief narratives included the following statements identified as examples of pre-reflective beliefs. Cathy and Jack expressed these beliefs about the actions that have not yet happened, but they already formed these beliefs. They seem to form these belief states in their mind even without having the actual experiences of actions, which means these beliefs are anticipatory. Following are the samples of their pre-reflective beliefs about teaching GTs with the use of GSP.

> ***Cathy**: They will be able to move the vertices and see what's going on and what it (reflection) is doing.*
>
> ***Jack**: That (GSP) has a lot of learning curve. They (students) have to know what they are trying to do (with it).*

The first statement represents Cathy's belief about the dynamic nature of GSP that allows movement of vertices. Also, another relationship noticable is the temporal dimension. Cathy is indicating toward her students' ability to move the vertices and see what is going on with the object and the image under reflection as the result of the movement. This clearly indicates that her belief is anticipatory, and hence it is pre-reflective, which means such beliefs are her reflections of future actions about teaching GTs with the use of GSP.

Again, the second statement is a representation of Jack's belief about GSP and his students' knowledge about this tool. The first part about the learning curve seems to be non-temporal. However, the second part (the dominant one) is about his student's anticipated knowledge about what to do with GSP, which forms an anticipatory belief and hence it seems to be his pre-reflective belief. These beliefs seem to associate with pre-reflective being. Pre-reflective beliefs are non-representational (does not represent ontic world) and formed at the level of perception and future action (Romdenh-Romluc, 2007). "The notion of pre-reflective belief also account for the paradoxical situation where psychological breakdown has taken place, but has not been experienced" (Groarke, 2014, p. 35). Teachers form pre-reflective beliefs about their future actions with common sense beliefs. Such beliefs are formed with anticipatory and preactive reflections (Van Manen, 1991). These beliefs may influence the planned actions and anticipation of acting in a certain way to achieve a goal.

Hence, the characteristics of pre-reflective beliefs are – intuitive (but not experienced), common sense, anticipatory, and non-representational. Cathy and Jack's beliefs about the use of the GSP in their future teaching have these features and hence they are pre-reflective and pro-active beliefs.

In-reflective beliefs

The belief narratives included the statements identified as examples of in-reflective beliefs. Cathy and Jack expressed these beliefs about the actions that are on going and they formed these beliefs in the moment of experience. They seem to form these belief states in their mind at the time of actual experiences of actions, whichmeans these beliefs are participatory or beliefs within the moment of participation in the action. Following are the samples of their in-reflective beliefs about teaching GTs with the use of GSP.

***Cathy**: The coordinate (x, y) changes to (-x, y) under a reflection on Y- axis.*

***Jack**: GSP helps in the visualizing (of any GTs). I think that's the biggest thing, I could see that.*

In the moment of construction and discussion about the nature of transformation, Cathy observes the coordinates and expresses her belief based on the generalization of the coordinates that (x, y) changes to (-x, y) under a reflection in the Y-axis. Here, the first belief statement represents Cathy's belief about the nature of the change of coordinates of vertices of the object triangle into the image triangle. Likewise, the second belief statement represents Jack's belief about the nature of a GT in terms of visualization. Both of these belief statements are related to immediate action or operations about GTs with GSP as validating context. If belief has a validating context (through experience and observations) and can be represented in a form (a symbol) thus represents an in-reflective belief (Sperber, 1997). Such beliefs, according to Tremlin (2006), "probably require a demonstration before it is accepted" (p. 138 which offers in-reflective belief that has a context to validate it. A belief is represented and expressed by symbol can have a meaning that it can be codified and de-codified within a field of practice that generates a belief or is influenced by a belief. These beliefs act on moment of performing an action and also may modify with immediate confrontation of problems. Hence, such beliefs are head-on beliefs. Normally, one may not form a belief within the situation of confrontation or when reflecting on an action at the moment (Van Manen, 1991). There is proximity of belief object and the moment of forming the belief itself within that experience.

Hence, in-reflective beliefs, the characteristics contains of – contextual, immediate, and within the flow of an action and experience. The properties of in-reflective belief resonate with what Cathy and Jack expressed in the above expressions. Hence, some of the beliefs expressed by both Cathy and Jack (during the interviews) seem to be in-reflective beliefs.

Post-reflective beliefs

The belief narratives also included examples of post-reflective beliefs. Cathy and Jack expressed these beliefs about the actions that already happened, consequently, they formed beliefs as a consequence of their experience and reflection after the actions. They seem to form these belief states in their mind after having the actual experiences of actions, which

further reflected that these beliefs are experience oriented. Following are the samples of their post-reflective beliefs about teaching GTs with the use of GSP.

> ***Cathy****: I think it (construction with GSP) might be more visible, but I don't think it is more meaningful or more powerful.*
>
> ***Jack****: I think that the constructions of an object and image under a GT by using GSP are not a lot powerful, but it's definitely more visual. I don't know that it is meaningful. I think it is more powerful and more visual. You can hit meaning with a lot of different things. So, I don't know if GSP makes more on those, but it's definitely useful.*

When I presented some activities of geometric transformations using GSP to Cathy and Jack, they already had some experiences of working with GSP. Before the final interview, they already had gone through some activities with GSP, especially reflection, rotation, translation, and composite transformations during the earlier interviews 1, 2, 3 and 4, respectively. These beliefs expressed in the above examples were no more momentary (immediate effect of operations or actions). This reflected that they did not form these beliefs within the moment of actions. These beliefs are mental states after having some experiences with working on GTs with GSP (either in their classes or in the earlier interview sessions). Hence, these beliefs are post-experiential or retrospective beliefs.

Here, the first statement is a representation of Cathy's retrospective belief about the use of GSP and how it could make sense of teaching and learning of GTs. She expresses her thought that GSP provides visualization of GTs. However, she does not seem to believe that it is meaningful or powerful. In the same line, Jack also seems to believe that GSP is more a visual tool, but not necessarily meaningful and powerful. After having some experience with the tool, they appeared to form these beliefs. Cathy has a longer experience with GSP than Jack beside the interview sessions. Since these beliefs formed or retained as a result of their experience with GSP in the past, it further entails that they are historical or post-experiential. They may not be truly representational as the ones that at the moment (in-reflective). These beliefs may not be non-representational as the one that has not been experienced. Since they experienced using GSP (at least as learners and future teachers) and the psychological states actually perpetuated through their experience in the past. Hence, they present here a set of pseudo-representational or historical beliefs or post-historical or ex-post-facto beliefs. These beliefs have root actions in the past, but psychological or mental effect perpetuate until now and may transcend further to the future. Therefore, they can be considered as post-reflective beliefs.

Kwanvig (2013, p. 234) mentioned about post-reflective belief in relation to epistemic principles and theory of rationality, however, he did not explicitly discuss the term 'post-reflective belief'. Bouma (1997) mentioned about John B. Cobb Jr.'s contributions in theology stating that "…psychic wholeness leads us to match post-reflective belief systems with pre-reflective experience" (para 4, sub-title: Anthropology). It appears that philosophers have some sense of post-reflective beliefs, but none of them has spelled it out clearly. Post-

reflective beliefs are associated with recollective or retroactive reflections on experiences (Van Manen, 1991). There is a temporal and spatial distance between the object of belief and the time of forming or sustaining a belief about an action or a phenomenon (Van Manen, 1991). Hence, post-reflective beliefs have the characteristics - recollective, retroactive, distanced, and experience oriented. Cathy and Jack's beliefs about attributes of the GSP in terms of power, use, visibility of process, and meaning are post-reflective beliefs.

The reflective beliefs expressed by Cathy and Jack seem to have roots to their reflexive beliefs that are related to their consciousness of self, other, and the reciprocal relationships. I observed that there were many instances of beliefs in their narratives that focused on the participants' personal 'selves' in the forms of efficacy, awareness, and consciousness toward the phenomenon of teaching GTs with the use of GSP. These reflexive beliefs are discussed in the following sub-section.

Reflexive Beliefs

The observation of the entire interview transcript revealed that some beliefs the participant's formations about beliefs were related to the objects inside their personal 'selves'. These beliefs were found to be associated with the phenomena internal to the participants' mental states, which further means that the participants were found to have those beliefs about their actions, perceptions, and cognitions internal to them. These beliefs are about un-observable mental constructs related to self-awareness and self-consciousness. These beliefs cannot be represented with any external source object or a phenomenon. These are self-referential beliefs that the participants formed about themselves in terms of ability, cognition, affect, and attitude. The belief narratives in the forms of reflexive beliefs were generated from the interview episodes by putting together bits and pieces of narratives to create a sensible belief-expression.

Following sub-headings further illustrates the belief narratives of interpretation within such beliefs.

Cathy's reflexive beliefs

The belief statements in protocol 3 are a few examples of Cathy's reflexive beliefs extracted from the different interview episodes. The protocol shows that her reflexive beliefs are related to her consciousness and awareness to the self-other relation. The narrative in the protocol is in the first person point of view where the narrator (speaker) is the research participant (Cathy).

Protocol-3

I guess a bunch of students really liked the tutorial, but I didn't because I like to make my mistakes and learn from them. That's where I learned GSP. I think I learned it really well, and I still remember how to do most of the stuffs because I learned it so. In my own

classroom, I think I would still start with like folding something (e.g., a paper) before they come to this (GSP). If you can't see the computers, they (students) can go off-tasks a lot more. How I would go off-tasks is much different than most students would. I strongly believe in as a teacher I am just there to spark their interest into show you things that are interesting.

We can have them get a new do an activity completely, and they will open a new window, they will need to recreate a picture or something and send them to rotate. That way (with new construction) I can see they can rotate it that they know what rotation means, and they are not just following the procedure that they have just to follow. I guess, so having GSP on with it reaches more students, and I think more students would have a deeper understanding, but I can teach the concept in a different way.

You have to be able to distinguish between here is your personal work time to make your own cluster of thoughts, and here is your partner time or whatever you can share with others and make them grow into whatever direction. If we use it (GSP), it needs to be a tool and not the sole way of expressing concepts. That (GSP) is not gonna reach so many students like me who needs the entire semester to figure out, would be lost.

Once we know what rotation is I don't want them (students) to draw a rotation every single time. I want them to go quickly to it (GSP) and then explore with it. I have to get honestly more comfortable with the idea of teaching with it because I have to figure out the procedural side. If I have figured out the procedural side, I would feel more comfortable in teaching (with GSP). I think you (the teacher) and your class (students) develop the environment. So, I don't know if GSP would. I mean it would affect the environment in some way. That moment right there (finding the center of rotation), that made me really think about using GSP while teaching GTs.

Jack's reflexive beliefs

The following belief statements in the protocol 4 are a few examples of Jack's reflexive beliefs extracted from the interview episodes. The narrative in the protocol is in the first person point of view where the narrator (speaker) is the research participant (Jack).

Protocol-4

I think it (matrix of reflection) will be interesting to them. Some people are really interested in that, but some people don't. It just confuses. I like them, but I have a little more experience. It depends on where they are in matrices. They may have a problem with that. Um, if they are starting it (GTs) algebraically, they can derive this stuff probably. Probably they need to understand what does Y = X mean, and if you go from there, another problem is some students don't visualize things. I think GSP is gonna add to it (teaching and learning of GTs). It's really gonna help you out show what happens. Um, like you can move the line, and it moves the shapes.

I need to refresh with GSP. I have spent two classes on GSP, very limited actual uses of GSP. Yea, with practice I can do it. I have such a limited thing. In methods class, we used it a little

bit. Junior high does not have it. So, I haven't just messed up with it. I can just think about what's gonna interest them, what's gonna hold their attention. If you are just talking about rotation, you are just using formal terms, I think you are gonna lose their attention in ten minutes. I think you can gain their interest. You know maybe a kid is struggling with it (doing a rotation). All of a sudden the kid sees it in the computer what's happening. Maybe that just turns around and builds his huge confidence of what he or she is seeing.

I don't anticipate students explaining these (linear functions) themselves. It would be interesting to see if they can actually put it into words what they are seeing. It would be interesting to see if they can actually put it into words what they are doing. It would be interesting to hear their own language what they say. That would be a kind cool to hear. I think it's important to give somebody time to mess it up. As far as exploring, like I probably know two percent of what people do (with GSP). So, I wish I had more hands-on experience with it. In relation to use of GSP in the future teaching, I am half way there, but I am not getting there. The more I play with it (GSP), the more I like it.

I am both yes or no in thinking if I am ready to teach GTs with GSP, probably more toward no. I just need to get more comfortable with it. I have sort of internalized the use of GSP for teaching GTs. I need more practice. I think that GSP makes teaching and learning more interesting than without using it. I mean, especially kids love technology. They can see that. You can relate it to building a video game. You can have them do the real thing they get interested. Maybe I can use the jigsaw method when applying GSP in teaching of GTs, if I have the technology, like the access.

Some of the key points in the participants' reflexive beliefs are- awareness to the interest of self and others (students), a sense of understanding of understanding of others and own, and conciousness toward self-other relation. Further discussions of each points is presented briefly in relation to the excerpts of the above narratives.

Cathy's epxression, "I strongly believe in as a teacher I am just there to spark their interest into show you things that are interesting" shows her awareness to the interest of her students. At the same time she is exhibiting her awarness to self as a teacher, and also her role as a teacher. Jack expresses that use of GSP makes teaching and learning more intersting (for both teacher and students) and the teacher can use this tool have the students do the real thing they are interested in with it.

Both Cathy and Jack expressed their beliefs in relation to how the use of GSP in teaching may help in developing student understanding of the processes inherent with GTs. Cathy expresses "I guess, so having GSP on with it (GTs) reaches more students, and I think more students would have a deeper understanding." On the other hand Jack says, "Probably they need to understand what does Y = X mean, and if you go from there, another problem is some students don't visualize things." These narratives of the participants indicate to their understanding of students' understanding of GTs with GSP.

There several instances in the narratives in which both Cathy and Jack expressed their beliefs related to their own consciousness toward self-other relation. For example, Cathy

expresses, "I think you (the teacher) and your class (students) develop the environment. So, I don't know if GSP would." Here, she reveals her beliefs about self-other relation in the context of creating a classroom environment. She further reveals her belief that, "It (GSP) might develop teacher-student relation, but that's I care about, and they (students) don't care about that." She reflected her consciousness of her role being a teacher and further giving meaning to that relations. Jack expresses, "Everything a kid does is related to relationship (student-teacher relationship) cause they can tell you what he or she is doing." This reflects that Jack looks at the students' tasks from the viewpoint of self-other (teacher-student) relation. These views clearly show Cathy and Jack's pedagogical reflexivity and hence their reflexive beliefs.

I found that the reflexive beliefs discussed here have temporal aspects associated with them. The time of being aware of something and forming beliefs based on them created three forms of reflexive beliefs: pre-reflexive, in-reflexive, and post-reflexive beliefs. Each of them have been discussed under separate sub-sections.

Pre-reflexive beliefs

The belief narratives of Cathy and Jack have the elements of pre-reflexive beliefs. They expressed their personal beliefs toward self-awareness, consciousness, and dispositions before they experienced different phenomena associated with the GT processes. The following expressions are the examples of Cathy and Jack's pre-reflexive belief statements from their belief narrative.

> ***Cathy**: I would have a different inquiry path, like look at the sides and angles. Look at the sides and angles, and look at the area. Maybe having them (students) present to each other, come together, and have a discussion of them about what a reflection is (using GSP).*
>
> ***Jack**: I can just think about what's gonna interest them, what's gonna hold their attention. If you are just talking about rotation, you are just using formal terms, I think you are gonna lose their attention in ten minutes. I think you can gain their interest…one of the cool things about GSP is that's where video games come from. You can show them that's where it is coming from.*

In the first statement, Cathy seems to express her belief about her inquiry path in which she anticipates engaging her students in a collaborative way of learning reflection transformation using GSP. She seems to have a sense of connectedness to the students. This connectedness in the inquiry path is visionary or anticipatory. She has not yet experienced the connectedness, exploration, and learning of GTs by the students through this inquiry path. However, she seems to be aware of her roles, relations to students, and to the teaching. The first statement portrays an example of Cathy's pre-reflexive belief that is primordial and pre-historic. Here, it is pre-historic beliefs that formed before the happentence of phenomena (history).

In the next statement, Jack seems to express his beliefs about awareness toward his (own) thinking and toward students' ability and interest to learn GTs with GSP. He seems to be aware of his thinking about students' interest, attention, motivation, and connection. He appears to think that his students would be interested to see and use the dynamic features of GSP, possibly connected to the process of animation in the video games. Hence, his belief seems to be connected to his identity as a future teacher and awareness towards students' psychological or mental states, however, anticipatory. He seems to acquire this psychological/mental state without having actual experience in the classroom teaching of GTs with GSP. Hence, his belief statement in the above example appears to be his pre-reflexive belief that is primordial and pre-historic (before the happentance of phenomena).

Lizardo and Strand (2011) state, "….persons can form pre-theoretical, pre-conceptual beliefs about the world, and that they can reason and form expectations about the world at this pre-reflexive level" (p. 1). This indicated Cathy and Jack's pre-reflexive beliefs are pre-theoretical and pre-conceptual that they have not experienced the actual phenomena, but they already formed such beliefs. Eacott (2013) mentions 'pre-reflexive belief' in the context of leadership as a research object. There is not any description of the term except it brings a context of blurring "the boundaries between the epistemic and the empirical" (p. 227) sense of a person or agent in a social milleau. Zimmermann (2010, p. 185) states, "According to Bourdieu there is a kind of pre-reflexive belief, which is usually not questioned in the social practice." This offers the pre-reflexive beliefs are sometimes latent as a part of habitus. It is assumed that habitus has a power to maintain an order through the pre-reflexive beliefs that the new members in a field have to adapt with in order to gain dominant role in the field (Bourdiew 2001). Then here, pre-reflexive belief is a common sense belief as de-facto belief about one's roles, position, and power in the field in which none generally questions. Both Bourdiew and Zimmermann's views are related to social practice and field in terms of power relations and interactions among the social agents that maintains the social order.

Medeiros and Capela (2010, p. 43) mention about 'pre-reflexive belief' in the context of linking ethos to the construction of reality. They do not explicitly describe the term. However, it provides a glimpse of pre-reflexive beliefs within Bourdiew's eidos and ethos. Emmerich (2014) also mentions about 'pre-reflexive belief' in the context of grounding eidos as instrument of construction and also the object constructed within "ethos and habitus" (p. 14). These examples show that pre-reflexive beliefs are matters of personal awareness, consciousness, and a habitus (personal dispositions).

Hence, pre-reflexive beliefs have the characteristics of – conditioned state of acceptance of something without actual experience, pre-theoretic, pre-conceptual, grounded in iedos, and anticipatory about self and other relation, awareness, and future course of actions. Both Cathy and Jack's pre-reflexive beliefs about having a different inquiry path for teaching GTs with GSP, and gaining students' interest to what they want them to do in learning GTs with the use of GSP have these elements. However, there is a wide crevice in the literature in the area of pre-reflexive beliefs, and more research needs to be done in this area to understand the nature and functions of pre-reflexive beliefs of preservice mathematics teachers.

In-reflexive beliefs

The belief narratives of Cathy and Jack have the elements of in-reflexive beliefs. They expressed their personal beliefs toward self-awareness, consciousness, and dispositions at the moment they experienced different phenomena associated with the geometric transformations. The following excerpts from their narratives are the examples of in-reflexive beliefs.

> ***Cathy**: The moment right there (while finding the center of rotation), that made me really think about using GSP while teaching GTs. How that point of rotation relates to the ….(the object and the image). I mean I would have gone through about it, but I didn't go that far.*
>
> ***Jack**: I have been able to perceive different roles of GSP, like teaching and learning, using it as a conceptual versus procedural tool. I don't know if I can think even beside those. You can use it in the classroom to teach concepts. That's the biggest thing.*

In the first statement, Cathy seems to express her belief about her existence and awareness at the moment when she is struggling to find the center of rotation. She is making connection to her own conscious thinking about the problem to find the center point. In the second statement, Jack seems to be aware of his current beliefs about GSP as a conceptual versus procedural or teaching versus learning tool. His belief is not just about the role of GSP as a tool, but his ability, intention, and perception. This kind of belief seems to have a relation with the direct experience and awareness of the person through the experience, not just anticipatory. Cathy and Jack's belief statements in the above examples seem to show their in the moment awareness and disposition and hence, they portray examples of their in-reflexive beliefs. These beliefs also have some relevance to the literature.

Richard (2013, p. 104) mentions about in-reflexive beliefs in terms of ascribing such beliefs within a sentence that has 'such and such' in conjunction with 'self' or 'his or her own'. Nozick (1983) mentions in-reflexive belief in terms of "disposition to behave" (p. 81) and "self-reference" (p. 79). Braude (1995, p. 72) mentions in-reflexive belief in relation to self-indexicality that refers to "one's states", by this he seems to refer to the mental states of self-awareness. However, the literature lacks a clear explication of one's beliefs in terms of at the moment belief or in-reflexive belief.

Post-reflexive beliefs

The belief narratives of Cathy and Jack have the elements of post-reflexive beliefs. They expressed their personal beliefs toward self-awareness, consciousness, and dispositions after they experienced different phenomena associated with the GT processes by using GSP. The following excerpts from their narratives are the examples of post-reflexive beliefs.

> ***Cathy**: I guess a bunch of students really liked the tutorial, but I didn't because I like to make my mistakes and learn from them.*
>
> ***Jack**: Yea, with practice I can do it. I have such a limited thing.*

In the first statement, Cathy seems to make a connection of current belief to her past experience of learning GSP in the Foundation of Geometry class. She then connects the same experience to her learning of GSP and using it for teaching GTs. This can perhaps show her awareness of who she is. In the second statement, Jack seems to be aware of the situation that he did not have adequate practice on GSP and that limited his ability to use the tool for teaching. First, he anticipates his relationship to the content of GT and use of GSP. Then he seems to move further (mentally being more aware) of the consequence of his limited experience of using GSP. Hence, these examples show Cathy and Jack's post-reflexive beliefs with their awareness and anticipation after passing through situations of doing something with GSP. There is not explicit theory and literature to address the explicit meaning and context of post-reflexive beliefs about teaching mathematics with technology. It shows that literature of teacher beliefs has not reached that far to analyze post-reflexive beliefs although we hold such belief as a result of an experience in our everyday life and professional practice.

CONCLUSION AND IMPLICATIONS

The research question explored 'What beliefs do preservice secondary mathematics teachers hold about their future practices of teaching geometric transformations with Geometer's Sketchpad?' The research question further intended to characterize their holistic beliefs in terms of temporal dimensions and directions of beliefs about teaching GTs with GSP. The temporal dimensions are related to time of action or event and formation of beliefs. The findings and discussion related to beliefs of two preservice mathematics teachers in this study portrayed their nested beliefs in terms of reflective and reflexive beliefs as categories of directional beliefs. The further analysis of these beliefs revealed the nature of these beliefs within three temporal dimensions – pre-, in-, and post reflective and reflexive beliefs that could be associated with their anticipated practices of teaching GTs with GSP. These beliefs characterized the entire domains of beliefs in terms of belief objects as external or internal phenomena to the believers (the research participants). In this sense, these beliefs are directional.

The holistic analysis and interpretation of Cathy and Jack's beliefs revealed that some of their beliefs they consciously hold reflected a state of perplexity, hesitation, and doubt in a reflective mode. Cathy expresses her belief that a reflection is about a line, axis, and there has to be an object reflected across the line. She thinks that one needs to know where the object is and how to reflect part of it. To do this, he or she needs to know how to construct an image. She accepts that her students need to know the concepts of distance, congruency and similarity, parallel, and perpendicular. She appears to believe that she would use GSP, but she would not rule out the pencil and paper activity. The title she would give GSP is a discovery tool, because it is not doing the teaching, but the teacher does the teaching and the students do the learning. She reveals that she wants to know about their mistakes, this is because their mistakes can be used to enhance their learning.

Cathy further expresses her beliefs that under a rotation, angles stay preserved, and side lengths are also preserved. For her, under a rotation, orientation is not preserved, but perimeters and the areas are preserved. She seems confused in the case of orientation during rotation. She also thinks that procedurally it (GSP) skips steps, but not really skipping steps; it is just quickening the steps. She thinks that individual activity would be a discussion. She seems to believe that it is incredibly important to discuss their (students') ideas in geometry in general and GTs in particular. She considers prompts as an important tool for engaging students in creative thinking about properties of GTs. She accepts that students can explain to her what they are doing, and why they decides to explore the area or perimeter. She thinks that they need the ability to recognize their potential as an individual and in groups simultaneously. For her, GSP is a tool for mathematics exploration. She has concerns that 'it does not tell you what you are doing really with it'. She reveals her belief that GSP is not efficient as a procedural tool. It is more of a dynamic tool for exploring, conceptualizing, and visualizing.

Jack appears to believe that GSP requires a lot of learning curves. For him, the students have to know what they are trying to do. In the case of nature of GSP, he thinks that it is more visual, and he reflects that it's a great tool. With the tools in GSP, he thinks, students can do measurement of lengths, angles, areas, and perimeters and they can observe how it works. He considers that students should be able to use the coordinates for algebraic manipulation of GT processes. He accepts that students can talk about how the shapes are congruent and how they would choose angles, how they are related to each point and each object. He suggests that students can talk about the angles and sides and how they relates to each other. For him, it may not instantly perceived right away in the minds of the students, but if the teacher can show them real life applications, like asking them to draw a satellite in space and how it is orbiting within the dynamic environment of GSP, this can further emphasize towards the understanding which can be developed. He accepts that they can see that (the construction), and it helps them in visualizing and explaining it.

Jack seems to believe that when students are engaged in plotting of areas of object (e.g., a triangle) and image (e.g., an image of the object triangle under a transformation) they are also learning about linear functions. For him, students are actually ready to deal with functions and go into linear transformations. He thinks that GSP helps a teacher to enrich this type of environment, for example, constructions, visualizations, and verbalizations. He appears to believe that GSP really can help students build in these environments. Jack considers that GSP is a tool for conceptual understanding. For him, it skips a lot of steps because it has shortcuts. He purports that GSP is both a tool for problem solving and mathematical exploration, preferring the second use. He appears to believe that GSP makes teaching GTs meaningful because students can see real life applications, and for him that's what they want.

The analysis and interpretation of Cathy and Jack's beliefs revealed that some of their beliefs indicated their relational states with feelings, prejudice, suggestion, and restricted views of themselves and others that might have fixed their cognitive schemas with core

beliefs about self, others, and the world. These beliefs are related to their awareness of self and others in terms of action tendencies of varied levels. With these beliefs, Cathy and Jack revealed their identity as students and future teachers and positioned themselves in the time and space with both cognitive and affective resources.

Cathy appears to believe that she learned the use of GSP with her self-struggles and making mistakes. She accepts that she likes to make mistakes and learn from them, and that's where she learned GSP. She also exclaims that she learned it really well, and she can still remember how to do most of the works because she learned it so. In her own classroom, she thinks that she would still start with folding something (e.g., a paper) before her students come to GSP while teaching and learning GTs. She thinks that a teacher has to be careful about what students are doing on their computers while working on GTs with GSP. She seems to believe that if the teacher can't see the computers, the students can go off-task a lot more. She appears to believe that as a teacher, she is just there to spark their interest and show them the things that are interesting.

She would have her students recreate a picture and then rotate (or reflect or translate) it. This can be the reason with new construction (with new construction) that she can see if they are able to rotate it which reveals if they know what rotation means, and they are not just following the procedure. She believes that using GSP reaches more students, and she thinks that more students would have a deeper understanding, allowing her to teach the concept in a different way. She expresses that her students should be able to distinguish between personal work time where they develop personal understanding, and partner time where they share with others and develop communal knowledge. For her, if they use GSP for learning GTs, it needs to be a tool and not the sole way of expressing concepts.

She claims that once her students know what rotation is, she doesn't want them to draw a rotation every single time. She wants them to use the tools in GSP and then explore with it. She accepts that she has to get more comfortable with the idea of teaching with GSP because she has to figure out the procedural side. If she has figured out the procedural side, she would feel more comfortable in teaching (with GSP). She further thinks that the teacher and her students develop the classroom environment. She seems not to be sure what role does GSP play in it. She agrees that it would affect the environment in some way. She reveals that finding the center of rotation makes her really think about using GSP while teaching GTs. She seems to accept that she is not able to think of how to reverse and engineer the center of rotation from the object and image under a rotation. For this, she claims as a teacher, that she has to go through these processes before bringing it into the classroom.

During the task-based interviews, I engaged Jack in observing the process of reflection transformations from viewpoint of matrix operation. The algebraic manipulation of reflections on x and y-axes using coordinates of vertices of object and image polygons (e.g., triangles and quadrilaterals) lead to formulation of matrices for those reflections. Jack seems to believe that matrix of reflection could be interesting to his students. He accepts that some students might be really interested in that, but others would not be. For him, it just confuses them. He likes working with matrices of different transformations because he has a little more experience with it. He claims that it may depend on where they are in matrices. To him, if the students

don't know about matrices, then they may have a problem with matrices of different transformations. For him, if they are starting GTs algebraically, they can probably derive this. He accepts that they need to understand what the line Y = X means, and if he goes from there, another problem is some students don't visualize things like this. He appears to believe that GSP adds value to teaching and learning of GTs. For him, it's really going to help him show students what happens when they move the lines, and how that moves the shapes.

In relation to effective use of GSP for teaching GTs, he appears to believe that he does not have adequate knowledge and skill in GSP. He feels that he needs to refresh with GSP. He expresses that he has spent two classes on GSP with very limited actual uses of it for teaching/learning GTs. He seems to be optimistic that he can do it with a little more practice on the tool. He accepts that he has such a limited understanding about GSP. In methods class, he used it a little bit during a technology presentation, and during his practicum the school did not have it. He reveals his belief that he could have done more with GSP given more time to play with it. However, he still can think about what will interest his students, what will hold their attention, and what will motivate them. He claims that if he is just talking about rotation, he is just using formal terms and he is going to lose their attention in ten minutes. He thinks that if a kid is struggling with it (doing a rotation) and all of a sudden he or she sees what is happening on the computer, that builds his or her confidence.

He seems to believe that his students won't be able to explain the linear functions themselves. He claims that it would be interesting to see if they can actually put into words what they are seeing. For him, it would be interesting to see if they can actually put into words what they are doing. He accepts that it would be interesting to hear their language about what they have done or what is happening with a GT process. In addition, he thinks that it's important to give his students time to mess up with GTs by using GSP. He believes that he needs more hands-on experience with GSP. In relation to using GSP in future teaching, he feels that he is just half way there, but he is not getting there.

Jack appears to waiver on if he is ready to teach GTs with GSP. He thinks that he is probably not ready. He seems to believe that he just needs to get more comfortable with it. He feels that he has internalized the uses of GSP for teaching GTs, nonetheless he accepts that he needs more practice. He thinks that GSP makes teaching and learning more interesting than without using it. He seems to believe that use of GSP in teaching may provide more opportunities for both the teacher and students to explore the properties of the GTs.

The findings within category of reflective and reflexive beliefs have epistemological implications in the areas of forming and changing beliefs within temporal dimensions of pre-, in-, and post- reflective and reflexive beliefs. Pre-service teachers' beliefs about the relationship among tool, process, and outcome in temporal dimensions in terms of pre-reflective, in-reflective, and post-reflective beliefs can also influence their conscious/unconscious attitude toward the use of GSP in teaching GTs. Pre-service teachers' beliefs about their relationship with the tools, processes, and outcomes in temporal dimensions in terms of pre-reflexive, in-reflexive, and post-reflexive beliefs can also influence their awareness and anticipation of using GSP in teaching GTs.

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