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# On Grill $S_p$-Open Set in Grill Topological Spaces

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**Abstract -** In this paper, we introduce a new type of grill set namely; G$s_p$-open sets, which is analogous to the G-semiopen sets in a grill topological space ($X$, $\tau$, G). Further, we define G$s_p$-continuous and G$s_p$-open functions by using a G$s_p$-open set and we investigate some of their important properties.

***Keywords*** - *G$s_p$-open set, G$s_p$$O(X)$, G$s_p$-continuous function, G$s_p$-open function.*

## 1. Introduction and Preliminaries

Choquet [2] introduced the concept of grill on a topological space and the idea of grills has shown to be a essential tool for studying some topological concepts. A collection G of nonempty subsets of a topological space $(X, \tau)$ is called a grill on $X$ if (i) $A \in$ G and $A \subseteq B$ implies that $B \in$ G, and (ii) $A$, $B \subseteq X$ and $A \cup B \in$ G implies that $A \in$ G or $B \in$ G. A triple ($X$, $\tau$, G) is called a grill topological space.

Roy and Mukherjee [17] defined a unique topology by a grill and they studied topological concepts. For any point $x$ of a topological space $(X, \tau)$, $\tau(x)$ denotes the collection of all open neighborhoods of $x$. A mapping $\varphi : P(X) \to P(X)$ is defined as $\varphi(A) = \{x \in X : A \cap U \in$ G for all $U \in \tau(x)\}$ for each $A \in P(X)$. A mapping $\psi : P(X) \to P(X)$ is defined as $\psi(A) = A \cup \varphi(A)$ for all $A \in P(X)$. The map $\psi$ satisfies Kuratowski closure axioms:

(i) $\psi(\emptyset) = \emptyset$,
(ii) if $A \subseteq B$, then $\psi(A) \subseteq \psi(B)$,
(iii) if $A \subseteq X$, then $\psi(\psi(A)) = \psi(A)$, and

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(iv) if $A$, $B \subseteq X$, then $\psi(A \cup B) = \psi(A) \cup \psi(B)$.

Corresponding to a grill G on a topological space $(X, \tau)$, there exists a unique topology $\tau_G$ (say) on $X$ given by $\tau_G = \{U \subseteq X : \psi(X - U) = X - U\}$, where for any $A \subseteq X$, $\psi(A) = A \cup \varphi(A) = \tau_G$-cl$(A)$ and $\tau \subseteq \tau_G$.

The concept of decompositions of continuity on a grill topological space and some classes of sets were defined with respect to grill (see [3, 7, 10] for details). A subset $A$ in $X$ is said to be

(i) $\varphi$-open if $A \subseteq \text{int}(\varphi(A))$,
(ii) G-$\alpha$.open if $A \subseteq \text{int}(\psi(\text{int}(A)))$,
(iii) G-preopen if $A \subseteq \text{int}(\psi(A))$,
(iv) G-semiopen if $A \subseteq \psi(\text{int}(A))$,
(v) G-$\beta$.open if $A \subseteq \text{cl}(\text{int}(\psi(A)))$.

The family of all G-$\alpha$.open (resp. G-preopen, G-semiopen, G-$\beta$.open) sets in a grill topological space $(X,\tau,G)$ is denoted by $G\alpha O(X)$ (rep. $GPO(X)$, $GSO(X)$, $G\beta O(X)$). A function $f: (X,\tau,G) \rightarrow (Y, \sigma)$ is said to be G-semicontinuous if $f^{-1}(V) \in GSO(X)$ for each $V \in \sigma$.

Mashhour et al. [14] introduced a class of preopen sets and he defined pre interior and pre closure in a topological space. A subset $A$ in $X$ is said to be preopen if $A \subseteq \text{int}(\text{cl}(A))$ and $PO(X)$ denotes the family of preopen sets. For any subset $A$ of $X$, (i) pint$(A) = \cup\{U : U \in PO(X)$ and $U \subseteq A\}$; (ii) pcl$(A) = \cap\{F : X - F \in PO(X)$ and $A \subseteq F\}$.

In this paper, we define a G$s_p$-open set in a grill topological space $(X, \tau, G)$ and we study some of its basic properties. Moreover, we define G$s_p$-continuous, G$s_p$-open, G$s_p$-closed and G$s_p^*$-continuous functions on a grill topological space $(X, \tau, G)$ and we discuss some of their essential properties.

**Proposition 1.1.** [17] Let $(X, \tau, G)$ be a grill topological space. Then for all $A$, $B \subseteq X$:
(i) $A \subseteq B$ implies that $\varphi(A) \subseteq \varphi(B)$;
(ii) $\varphi(A \cup B) = \varphi(A) \cup \varphi(B)$;
(iii) $\varphi(\varphi(A)) \subseteq \varphi(A) = \text{cl}(\varphi(A)) \subseteq \text{cl}(A)$.

## 2. G$s_p$-Open Sets

**Definition 2.1.** Let $(X,\tau,G)$ be a grill topological space and let $A$ be a subset $A$ of $X$. Then $A$ is said to be G$s_p$-open if and only if there exist a $U \in PO(X)$ such that $U \subseteq A \subseteq \psi(U)$. A set $A$ of $X$ is G$s_p$-closed if its complement $X - A$ is G$s_p$-open. The family of all G$s_p$-open (resp. G$s_p$-closed) sets is denoted by G$s_pO(X)$ (resp. G$s_pC(X)$).

**Example 2.1.** Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a, b\}, \{c, d\}, \{a,b,c\}, \{a,b,d\}\}$ and $G = \{\{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, X\}$. Then G$s_pO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$.

**Theorem 2.1.** Let $(X,\tau,\mathrm{G})$ be a grill topological space and let $A \subseteq X$. Then $A \in \mathrm{G}s_pO(X)$ if and only if $A \subseteq \psi(\mathrm{pint}(A))$.

Proof. If $A \in \mathrm{G}s_pO(X)$, then there exist a $U \in PO(X)$ such that $U \subseteq A \subseteq \psi(U)$. But $U \subseteq A$ implies that $U \subseteq \mathrm{pint}(A)$. Hence $\psi(U) \subseteq \psi(\mathrm{pint}(A))$. Therefore $A \subseteq \psi(\mathrm{pint}(A))$. Conversely, let $A \subseteq \psi(\mathrm{pint}(A))$. To prove that $A \in \mathrm{G}s_pO(X)$, take $U = \mathrm{pint}(A)$, then $U \subseteq A \subseteq \psi(U)$. Hence $A \in \mathrm{G}s_pO(X)$.

**Corollary 2.1.** If $A \subseteq X$, then $A \in \mathrm{G}s_pO(X)$ if and only if $\psi(A) = \psi(\mathrm{pint}(A))$.

Proof. Let A $\in \mathrm{G}s_pO(X)$. Then as $\psi$ is monotonic and idempotent, $\psi(A) \subseteq \psi(\psi(\mathrm{pint}(A))) = \psi(\mathrm{pint}(A)) \subseteq \psi(A)$ implies that $\psi(A) = \psi(\mathrm{pint}(A))$. The converse is obvious.

**Theorem 2.2.** Let $(X,\tau,\mathrm{G})$ be a grill topological space. If A $\in \mathrm{G}s_pO(X)$ and $B \subseteq X$ such that $A \subseteq B \subseteq \psi(\mathrm{pint}(A))$, then $B \in \mathrm{G}s_pO(X)$.

Proof. Given $A \in \mathrm{G}s_pO(X)$. Then by Theorem 2.1, $A \subseteq \psi(\mathrm{pint}(A))$. But $A \subseteq B$ implies that $\mathrm{pint}(A) \subseteq \mathrm{pint}(B)$ and hence by Theorem 2.4[17], $\psi(\mathrm{pint}(A)) \subseteq \psi(\mathrm{pint}(B))$. Therefore $B \subseteq \psi(\mathrm{pint}(A)) \subseteq \psi(\mathrm{pint}(B))$. Hence $B \in \mathrm{G}s_pO(X)$.

**Corollary 2.2.** If A $\in \mathrm{G}s_pO(X)$ and $B \subseteq X$ such that $A \subseteq B \subseteq \psi(A)$, then $B \in \mathrm{G}s_\alpha O(X)$.

Proof. Follows from the Theorem 2.2 and Corollary 2.1.

**Proposition 2.1.** If $U \in PO(X)$, then $U \in \mathrm{G}s_pO(X)$.

Proof. Let $U \in PO(X)$, it implies that $U = \mathrm{pint}(U) \subseteq \psi(\mathrm{pint}(U))$. Hence $U \in \mathrm{G}s_pO(X)$.

Note that the converse of the above proposition need not be true. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathrm{G} = \{\{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. Then $PO(X) = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and $\mathrm{G}s_pO(X) = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. Here $\{b, d\}$ and $\{a, b, d\}$ are $\mathrm{G}s_p$-open sets but not preopen.

**Theorem 2.3.** Let $(X,\tau,\mathrm{G})$ be a grill topological space. If $A \in \mathrm{G}SO(X)$, then $A \in \mathrm{G}s_pO(X)$.

Proof. Given $A \in \mathrm{G}SO(X)$. Then $A \subseteq \psi(\mathrm{int}(A))$. Since $\mathrm{int}(A) \subseteq \mathrm{pint}(A)$, we have that $\psi(\mathrm{int}(A)) \subseteq \psi(\mathrm{pint}(A))$ (by Theorem 2.4[17]). Hence $A \subseteq \psi(\mathrm{pint}(A))$ and thus $A \in \mathrm{G}s_pO(X)$.

Note that the converse of the above theorem need not be true. By Example 2.1, we have that $\mathrm{G}SO(X) = \{\emptyset, X, \{a, b\}, \{c, d\}, \{a,b,c\}, \{a,b,d\}\}$. Therefore $\{a\}$, $\{b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{a,c,d\}$ and $\{b,c,d\}$ are $\mathrm{G}s_p$-open sets but not G-semiopen.

**Proposition 2.2.** If $PO(X) = \tau$, then $\mathrm{G}s_pO(X) = \mathrm{G}SO(X)$.

Proof. By Theorem 2.3, $\mathrm{G}SO(X) \subseteq \mathrm{G}s_pO(X)$. Let $A \in \mathrm{G}s_pO(X)$. Then by Theorem 2.1, $A \subseteq \psi(\mathrm{pint}(A))$. Since $PO(X) = \tau$, we have that $\mathrm{pint}(A) = \mathrm{int}(A)$ implies that $A \subseteq \psi(\mathrm{pint}(A)) = \psi(\mathrm{int}(A))$ and hence $A \in \mathrm{G}SO(X)$. Thus $\mathrm{G}s_pO(X) \subseteq \mathrm{G}SO(X)$.

**Theorem 2.4.** Let $(X,\tau,\mathrm{G})$ be a grill topological space.
(i) If $A_i \in \mathrm{G}s_pO(X)$ for each $i \in J$, then $\cup_{i\in J}A_i \in \mathrm{G}s_pO(X)$;
(ii) If $A \in \mathrm{G}s_pO(X)$ and $U \in PO(X)$, then $A \cap U \in \mathrm{G}s_pO(X)$.

Proof. (i) Since $A_i \in \mathrm{G}s_pO(X)$, we have that $A_i \subseteq \psi(\mathrm{pint}(A_i))$ for each $i \in J$. Thus, we obtain $A_i \subseteq \psi(\mathrm{pint}(A_i)) \subseteq \psi(\mathrm{pint}(\cup_{i\in J}A_i))$ and hence $\cup_{i\in J}A_i \subseteq \psi(\mathrm{pint}(\cup_{i\in J}A_i))$. This shows that $\cup_{i\in J}A_i \in \mathrm{G}s_pO(X)$.

(ii) Let $A \in \mathrm{G}s_pO(X)$ and $U \in PO(X)$. Then $A \subseteq \psi(\mathrm{pint}(A))$ and $\mathrm{pint}(U) = U$. Now, $A \cap U \subseteq \psi(\mathrm{pint}(A)) \cap U = (\mathrm{pint}(A) \cup \varphi(\mathrm{pint}(A))) \cap U = (\mathrm{pint}(A) \cap U) \cup (\varphi(\mathrm{pint}(A)) \cap U) \subseteq \mathrm{pint}(A \cap U) \cup \varphi(\mathrm{pint}(A) \cap U)$ (by Theorem 2.10[17]) $= \mathrm{pint}(A \cap U) \cup \varphi(\mathrm{pint}(A \cap U)) = \psi(\mathrm{pint}(A \cap U))$. Therefore $A \cap U \in \mathrm{G}s_pO(X)$.

**Remark 2.1.** The following example shows that if $A, B \in \mathrm{G}s_pO(X)$, then $A \cap B \notin \mathrm{G}s_pO(X)$.

From Example 2.1, take $A = \{b, c\}$ and $B = \{c, d\}$, then $A, B \in \mathrm{G}s_pO(X)$ but $A \cap B = \{c\} \notin \mathrm{G}s_pO(X)$.

**Theorem 2.5.** Let $(X,\tau,\mathrm{G})$ be a grill topological space and $A \subseteq X$. If $A \in \mathrm{G}s_pC(X)$, then $\mathrm{pint}(\psi(A)) \subseteq A$.

Proof. Suppose $A \in \mathrm{G}s_pC(X)$. Then $X - A \in \mathrm{G}s_pO(X)$ and hence $X - A \subseteq \psi(\mathrm{pint}(X - A)) \subseteq \mathrm{pcl}(\mathrm{pint}(X - A)) = X - \mathrm{pint}(\mathrm{pcl}(A)) \subseteq X - \mathrm{pint}(\psi(A))$, implies that $\mathrm{pint}(\psi(A)) \subseteq A$.

**Theorem 2.6.** Let $(X,\tau,\mathrm{G})$ be a grill topological space and $A \subseteq X$ such that $X - \mathrm{pint}(\psi(A)) = \psi(\mathrm{pint}(X - A))$. Then $A \in \mathrm{G}s_pC(X)$ if and only if $\mathrm{pint}(\psi(A)) \subseteq A$.

Proof. Necessary part is proved by Theorem 2.5. Conversely, suppose that $\mathrm{pint}(\psi(A)) \subseteq A$. Then $X - A \subseteq X - \mathrm{pint}(\psi(A)) = \psi(\mathrm{pint}(X - A))$, implies that $X - A \in \mathrm{G}s_pO(X)$. Hence $A \in \mathrm{G}s_pC(X)$.

**Definition 2.2.** Let $(X,\tau,\mathrm{G})$ be a grill topological space and $A \subseteq X$. Then
(i) $\mathrm{G}s_p$-interior of $A$ is defined as union of all $\mathrm{G}s_p$-open sets contained in $A$.
Thus $\mathrm{G}s_p\mathrm{int}(A) = \cup\{U : U \in \mathrm{G}s_pO(X) \text{ and } U \subseteq A\}$;
(ii) $\mathrm{G}s_p$-closure of $A$ is defined as intersection of all $\mathrm{G}s_p$-closed sets containing $A$.
Thus $\mathrm{G}s_p\mathrm{cl}(A) = \cap\{F : X - F \in \mathrm{G}s_pO(X) \text{ and } A \subseteq F\}$.

**Theorem 2.7.** Let $(X,\tau,\text{G})$ be a grill topological space and $A \subseteq X$. Then
(i) $\text{G}s_p\text{int}(A)$ is a $\text{G}s_p$-open set contained in $A$;
(ii) $\text{G}s_p\text{cl}(A)$ is a $\text{G}s_p$-closed set containing $A$;
(iii) $A$ is $\text{G}s_p$-closed if and only if $\text{G}s_p\text{cl}(A) = A$;
(iv) $A$ is $\text{G}s_p$-open if and only if $\text{G}s_p\text{int}(A) = A$;
(v) $\text{G}s_p\text{int}(A) = X - \text{G}s_p\text{cl}(X - A)$;
(vi) $\text{G}s_p\text{cl}(A) = X - \text{G}s_p\text{int}(X - A)$.

Proof. Follows form the Definition 2.15 and Theorem 2.4(i).

**Theorem 2.8.** Let $(X,\tau,\text{G})$ be a grill topological space and $A$, $B \subseteq X$. Then the following are hold: (i) If $A \subseteq B$, then $\text{G}s_p\text{int}(A) \subseteq \text{G}s_p\text{int}(B)$;
(ii) $\text{G}s_p\text{int}(A \cup B) \supseteq \text{G}s_p\text{int}(A) \cup \text{G}s_p\text{int}(B)$;
(iii) $\text{G}s_p\text{int}(A \cap B) = \text{G}s_p\text{int}(A) \cap \text{G}s_p\text{int}(B)$.

Proof. Follows from the Theorem 2.8.

**Definition 2.3.** A function $f$: $(X,\tau,\text{G}) \to (Y, \sigma)$ is said to be $\text{G}s_p$-continuous if $f^{-1}(V) \in \text{G}s_pO(X)$ for each $V \in PO(Y)$.

**Example 2.2.** Let $X = \{a, b, c, d\}$, $Y = \{1, 2, 3, 4\}$, $\tau = \{\emptyset, X, \{a,b\}, \{c,d\}\}$, $\sigma = \{\emptyset, Y, \{1, 2\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}\}$and $\text{G} = \{\{a,b, c\}, X\}$. Then $\text{G}s_pO(X) = P(X)$ and $PO(Y) = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$. Define $f$: $(X, \tau,\text{G}) \to (Y, \sigma)$ by $f(a) = 2$, $f(b) = 1$, $f(c) = 4$ and $f(d) = 3$. Then inverse image of every preopen sets in $Y$ is $\text{G}s_p$-open in $X$. Hence $f$ is $\text{G}s_p$-continuous.

**Remark 2.2.** The concepts of G-semicontinuous and $\text{G}s_p$-continuous are independent.

(i) From Example 2.2, we have that $\text{G}SO(X) = \{\emptyset, X, \{a,b\}, \{c,d\}\}$ and the function $f$ is $\text{G}s_p$-continuous. Also $f^{-1}(\{1, 2, 3\}) = \{a, b, d\}$ is not G-semiopen in $X$ for the open set $\{1, 2, 3\}$ of $Y$. Hence $f$ is not G-semicontinuous.

(ii) Let $X = \{a, b, c, d\}$, $Y = \{1, 2, 3, 4\}$, $\tau = \{\emptyset, X, \{a\}, \{a,b\}, \{c,d\}, \{a,c ,d\}, \{b,c,d\}\}$, $\sigma = \{\emptyset, Y, \{1, 2\}, \{3, 4\}\}$ and $\text{G} = \{\{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. Then $\text{G}SO(X) = \tau$, $\text{G}s_pO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $PO(Y) = P(Y)$. Define $f$: $(X, \tau,\text{G}) \to (Y, \sigma)$ by $f(a) = 4$, $f(b) = 3$, $f(c) = 2$ and $f(d) = 1$. Then the function $f$ is G-semicontinuous. Also the inverse image $f^{-1}(\{3\}) = \{b\}$ is not $\text{G}s_p$-open in $X$ for the preopen set $\{3\}$ of $Y$. Hence $f$ is not $\text{G}s_p$-continuous.

From (i) and (ii), we got the concepts of G-semicontinuous and $\text{G}s_p$-continuous are independent.

**Theorem 2.9.** For a function $f$: $(X,\tau,\text{G}) \rightarrow (Y, \sigma)$, the following are equivalent:
(i) $f$ is G$s_p$-continuous;
(ii) For each F $\in PC(Y)$, $f^{-1}(F) \in \text{G}s_pC(X)$;
(iii) For each $x \in X$ and each $V \in PO(Y)$ containing $f(x)$, there exists a $U \in \text{G}s_pO(X)$ containing $x$ such that $f(U) \subseteq V$.

Proof. (i) $\Leftrightarrow$ (ii): It is obvious.

(i) $\Rightarrow$ (iii): Let $V \in PO(Y)$ and $f(x) \in V(x \in X)$. Then by (i), $f^{-1}(V) \in \text{G}s_pO(X)$ containing $x$. Taking $f^{-1}(V) = U$, we have that $x \in U$ and $f(U) \subseteq V$.

(iii) $\Rightarrow$ (i): Let $V \in PO(Y)$ and $x \in f^{-1}(V)$. Then $f(x) \in V \in PO(Y)$ and hence by (iii), there exists a $U \in \text{G}s_pO(X)$ containing $x$ such that $f(U) \subseteq V$. Thus, we obtain $x \in U \subseteq \psi$ (pint($U$)) $\subseteq \psi$(pint($f^{-1}(V)$)). This shows that $f^{-1}(V) \subseteq \psi$(pint($f^{-1}(V)$)). Hence $f$ is G$s_p$-continuous.

**Theorem 2.10.** A function $f$: $(X,\tau,\text{G}) \rightarrow (Y, \sigma)$ is G$s_p$-continuous if and only if the graph function $g : X \rightarrow X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, is G$s_p$-continuous.

Proof. Suppose that $f$ is G$s_p$-continuous. Let $x \in X$ and $W \in PO(X \times Y)$ containing $g(x)$. Then there exist a $U \in PO(X)$ and $V \in PO(Y)$ such that $g(x) = (x, f(x)) \in U \times V \subseteq W$**.** Since $f$ is G$s_p$-continuous, there exists a $G \in \text{G}s_pO(X)$ containing $x$ such that $f(G) \subseteq V$. By Theorem 2.4(b), $G \cap U \in \text{G}s_pO(X)$ and $g(G \cap U) \subseteq U \times V \subseteq W$. This shows that $g$ is G$s_p$-continuous. Conversely, suppose that $g$ is G$s_p$-continuous. Let $x \in X$ and $V \in \alpha(Y)$ containing $f(x)$. Then $X \times V \in PO(X \times Y)$ and by G$s_p$-continuity of $g$, there exists a $U \in \text{G}s_pO(X)$ containing $x$ such that $g(U) \subseteq X \times V$. Thus we have that $f(U) \subseteq V$ and hence $f$ is G-$s_p$.continuous.

**Definition 2.3.** Let $(X, \tau)$ be a topological space and $(Y,\sigma,\text{G})$ a grill topological space. A function $f$: $(X, \tau) \rightarrow (Y,\sigma,\text{G})$ is said to be G$s_p$-open (resp. G$s_p$-closed ) if for each $U \in PO(X)$ (resp. for each $U \in PC(X)$), $f(U)$ is G$s_p$-open (resp. G$s_p$-closed) in $(Y, \sigma,\text{G})$.

**Theorem 2.11.** A function $f$: $(X, \tau) \rightarrow (Y,\sigma,\text{G})$ is G$s_p$-open if and only if for each $x \in X$ and each pre-neighbourhood $U$ of $x$, there exists a $V \in \text{G}s_pO(Y)$ such that $f(x) \in V \subseteq f(U)$.

Proof. Suppose that $f$ is a G-$s_p$.open function and let $x \in X$. Also let $U$ be any pre-neighbourhood of $x$. Then there exists $G \in PO(X)$ such that $x \in G \subseteq U$. Since $f$ is G$s_p$-open, $f(G) = V$ (say) $\in \text{G}s_pO(Y)$ and $f$ $(x) \in V \subseteq f(U)$. Conversely, suppose that $U \in PO(X)$. Then for each $x \in U$, there exists a $V_x \in \text{G}s_pO(X)$ such that $f(x) \in V_x \subseteq f(U)$. Thus $f(U) = \cup\{V_x : x \in U\}$ and hence by Theorem 2.4(a), $f(U) \in \text{G}s_pO(Y)$. This shows that $f$ is G$s_p$-open.

**Theorem 2.12.** Let $f$: $(X, \tau) \rightarrow (Y,\sigma,\text{G})$ be a G-$s_p$.open function. If $V \subseteq Y$ and $F \in PC(X)$ containing $f^{-1}(V)$, then there exists a $H \in \text{G}s_pO(Y)$ containing $V$ such that $f^{-1}(H) \subseteq F$.

Proof. Suppose that $f$ is G-$s_p$.open. Let $V \subseteq Y$ and $F \in PC(X)$ containing $f^{-1}(V)$. Then $X - F \in PO(X)$ and by G$s_p$-openness of $f$, $f(X - F) \in$ G$s_pO(X)$. Thus H = $Y - f(X - F) \in$ G$s_pC(Y)$ consequently $f^{-1}(V) \subseteq F$ implies that $V \subseteq H$. Further, we obtain that $f^{-1}(H) \subseteq F$.

**Theorem 2.13.** For any bijection $f$: $(X, \tau) \rightarrow (Y,\sigma,$G), the following are equivalent:
(i) $f^{-1}$: $(Y,\sigma,$G) $\rightarrow (X, \tau)$ is G$s_p$-continuous;
(ii) $f$ is G$s_p$-open;
(iii) $f$ is G$s_p$-closed.

Proof. It is obvious.

**Definition 2.4.** Let $(X,\tau,$G) be a grill topological space. A subset $A$ of $X$ is said to be a G$s_p^*$-set if $A = U \cap V$, where $U \in PO(X)$ and $\psi$(pint($V$)) = pint($V$).

**Theorem 2.14.** Let $(X,\tau,$G) be a grill topological space and let $A \subseteq X$. Then $A \in PO(X)$ if and only if $A \in$ G$s_pO(X)$ and $A$ is G$s_p^*$-set in $(X,\tau,$G).

Proof. Let $A \in PO(X)$. Then $A \in$ G$s_pO(X)$, implies that $A \subseteq \psi$(pint($A$)). Also $A$ can be expressed as $A = A \cap X$, where $A \in PO(X)$ and $\psi$(pint($X$)) = pint($X$). Thus $A$ is a G$s_p^*$-set. Conversely, Let $A \in$ G$s_pO(X)$ and $A$ be a G$s_p^*$-set. Thus $A \subseteq \psi$(pint($A$)) = $\psi$(pint($U \cap V$)), where $U \in PO(X)$ and $\psi$(pint($V$)) = pint($V$). Now $A \subseteq U \cap A \subseteq U \cap \psi$(pint($U \cap V$)) = $U \cap \psi(U \cap$ pint($V$)) $\subseteq U \cap \psi(U) \cap \psi$(pint($V$)) = $U \cap$ pint($V$) = pint($A$). Hence $A \in PO(X)$.

**Definition 2.5.** A function $f$: $(X,\tau,$G) $\rightarrow (Y, \sigma)$ is G$s_p^*$-continuous if for each $V \in PO(Y)$, $f^{-1}(V)$ is a G$s_p^*$-set in $(X, \tau,$G).

**Theorem 2.15.** Let $(X,\tau,$G) be a grill topological space. Then for a function $f$: $(X,\tau,$G) $\rightarrow (Y, \sigma)$, the following are equivalent:
(i) $f$ is precontinuous;
(ii) $f$ is G$s_p$-continuous and G$s_p^*$-continuous.

Proof. Straightforward.

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