

Fuzzy mathematical programming for assembly line balancing problem with uncertain parameters

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ABSTRACT

Assembly line balancing models aim at minimizing cycle time, and the number of stations becomes quite complex with precedence relations and various resource constraints. Various heuristic and fuzzy-based methods are used to ease the solution in such assembly line balancing models. This study proposes a two-stage model that simultaneously minimizes cycle time and the number of stations for a real-life case. The paper shows how to formulate a mixed integer non-linear programming model for an assembly line balancing problem. After the constraints on production size and task durations are fuzzy transformed, the model acquires new constraints. In the new model, the α coefficient takes values in the range $[0,1]$, bringing the fuzzy parameters closer to the lower or upper range. In cases where uncertain task duration and production size parameters are fuzzy, better results are achieved with the fuzzy model than the basic model. The fuzzy model provides to expand value range for parameters and evaluates different solution alternatives. In the study, the reflection of the value change in the parameters on the solution was examined with the experimental data set created in line with 11^2 different α coefficients. One-way ANOVA test was used to understand whether the fuzzy parameter values created a significant difference in the solution. The proposed model improves the solution quality through scenarios, provides a 7% saving for the total duration of the tasks at the stations within a cycle time and suggests a more suitable station layout for efficient production.

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1. Introduction

The assembly line balancing (ALB) problem serves the purpose of assigning tasks constituting a production process to workstations by considering their priority relationships and allowing the product to remain at each station for a maximum cycle time. For an efficient assembly line, total assembly times should be distributed equally among workstations and thus idle times should be minimized [1]. However, the indivisible structure of tasks and priority relationships make task-station assignment quite complex. Balance occurs by grouping tasks at stations along the line by considering their cycle time [2]. Materials pass through workstations, each responsible for performing certain tasks, and leave the line as finished products [3].

The diversity of elements in the problem changes the structure of the solution model. ALB problems were classified by Baybars [4] as simple assembly line balancing problem (SALBP) and generalized assembly line balancing problem (GALBP) [5]. According to the SALBP model, tasks cannot be divided, have definite durations, and it is possible to

perform these tasks at any station. This model performs assignments by considering the precedence relations and cycle time [5]. GALBPs work according to more constraints by diversifying the elements that may affect the assignment decision [3]. According to the classification made in line with the objective functions, the Type-1 model tries to minimize the number of stations based on the cycle time value, while the Type-2 model aims to minimize the cycle time according to a fixed number of stations. Similarly, the Type-E problem tries to minimize the cycle time and the number of stations simultaneously, while the Type-F model tries to make appropriate task assignments in line with the determined cycle time and the number of stations [6].

ALB problems can be defined as complex and difficult problems that can be considered NP-complete [7]. Many parameters and variables of the model may be uncertain. Models that consider the uncertain nature of the problem can be useful in determining the real efficiency of the line. Therefore, stochastic [8-11] and heuristic models [12-14] are used extensively in solving ALB problems. Fuzzy methods are also quite useful, especially in cases where past data is

insufficient, unreliable and some parameters are uncertain [2]. Fuzzy methods promise a preferable compromise solution even in limited information environments [15].

In this study, a Type-E ALB problem is considered in which the number of stations and the cycle time are simultaneously minimized. The following questions are sought to be answered within the scope of the study:

- How can the cycle time and the number of stations be optimized simultaneously?
- How can the mathematical model be transformed under uncertainty?
- How do the fuzzy parameter values affect the solution quality?
- What is the best station and task layout for the ALB problem under consideration?
- What is the impact of task duration and production size parameters on ALB efficiency?

The linear model is transformed into MINLP (Mix Integer Non-Linear Problem) format due to the multiplication of two variables in the objective function. In addition, since the task durations and production size are uncertain, a two-stage solution methodology is adopted in the study. In the first stage, the optimum result is reached over the average values, and in the second stage, the relevant parameters are fuzzified and the solutions presented by the model under different conditions are investigated. The proposed model differs in terms of fuzzy transformation methodology. In the second section of the study, the literature on ALB models and the preference of fuzzy methods in this problem is investigated. The proposed mathematical model and the fuzzy transformation process are presented in the third and fourth section. The fifth section includes the analysis and findings on the optimum solution and fuzzy solution alternatives to a real-life problem. The contributions and improvable aspects of the study are discussed in the conclusion section.

2. Literature Review

ALB models are solution tools for businesses that produce on a large scale or want to benefit from economies of scale [16]. Especially since demand is variable and personalized, flexibility and efficiency become much more important for large-scale production systems. Therefore, ALB models suitable for different conditions emerge and different classifications are made for ALB models [4, 16-19]. Different systematic classifications are seen in review studies.

ALB problems, which can be simply expressed as task distribution to stations based on a certain cycle time, can become quite complex when elements related to priority order and efficiency are considered. While techniques such as integer programming [20] and ranked positional weight technique [21] were used to solve the problem a long time ago,

more effective and practical models and techniques have emerged at this point. Various methods such as dynamic programming, branch-and-bound, heuristics and meta-heuristics are also used for ALB problems [18]. Bukchin and Raviv [1] propose a new constraint programming-based model for problems such as equipment selection and task assignment for SALBP-1, SALBP-2, U-shaped type 1 ALB problem. Topaloglu et al. [5] balanced the assembly line in line with alternative assembly plans with the rule-based constraint-programming model. Çil and Kizilay [22] developed a constraint-programming model that aims to minimize cycle time and total number of employees, ran the model with small and large data sets and achieved effective results. Pınarbaşı and Alakaş [6] considered compatible, incompatible and resource usage constraints as assignment constraints in the traditional type-1 assembly line balancing problem.

In cases where the optimum solution cannot be achieved due to the difficult structure of the problem, obtaining near-optimal solutions can be valuable. Therefore, metaheuristics are also quite useful for ALB problems with high difficulty levels. In this field, it is seen that metaheuristics such as simulated annealing [23], genetic algorithm [24-26], particle swarm optimization [10-11], ant colony optimization [27], and bee colony algorithm [14] are frequently used to solve ALB problems.

Production problems have elements that cannot be fully predicted due to machine and human factors. Generally, the input of the models, data and targets can be uncertain. Fuzzy logic stands out as a preferable solution methodology in cases where there is uncertainty. Fuzzy methods are quite successful in reflecting vagueness in data through fuzzy numbers and in determining the common fuzzy solution level that can satisfy uncertain targets [15].

There is variability in ALB task durations due to reasons such as malfunctions as well as employee or machine efficiency. In mathematical models established for such problems, process times can be considered stochastic or are considered within a set of interval values expressed as fuzzy [19]. Kara et al. [28] propose a model for straight and U-shaped lines with fuzzy workstation number and cycle time targets through binary fuzzy goal programming. Özcan and Toklu [29] proposed a fuzzy goal programming model in two-sided assembly lines that will first minimize the number of paired stations and then minimize the number of stations for a given cycle time as the second target. Lee et al. [30] targeted the minimization of cycle time, workstation number, workload variance and workstation idle times. Due to the conflicting objectives in the study, a fuzzy multi-objective linear programming model was used, and the solution was relaxed with a genetic algorithm. Zacharia and Nearchou [31] focused on maximizing the balance efficiency by trying to catch the appropriate number of workstations and cycle time in an assembly line with fuzzy

task durations. Xu and Xiao [7] tried to reduce the work overload times through fuzzy process times by considering the uncertainty in the process times.

Considering the task or process times as uncertain due to the performance of the employee and the machine increases the quality of the solution. Fuzzy methods are generally used to tolerate this uncertainty and reflect it in the solution. In ALB models, the production size target influences determining the

cycle time. Table 1 highlights the similar and different aspects of this study with similar recent studies. As seen in Table 1, the problems in most studies could not be solved with the existing mathematical model and a solution to the problem was sought with heuristic algorithms. In this study, the uncertainty in the parameters was reflected in the model with a fuzzy transformation and the model became operable under uncertainty.

Table 1. Articles with relevant characteristics to the research

Authors	Model Type	Objectives	Sources of Uncertainties	Methods
[38]	Mixed-Integer Linear Programming	Turnaround Time, Number of Operators	-	Mathematical Model
[39]	-	Takt Time, Comprehensive Balance Rate	Uncertain Assembly Time	Multi Objective Mathematical Model, Fuzzy Graphical Evaluation Review Technique, Adaptive Double-Layer Genetic Algorithm
[40]	Mixed-Integer Linear Programming	Cycle Time, Task Alteration	-	Multi Objective Mathematical Model, Enhanced Cooperative Co-Evolutionary Algorithm
[41]	Binary Linear Programming	Cycle Time	-	Binary Linear Mathematical Programming, Artificial Bee Colony
[42]	Mixed-Integer Linear Programming	Cycle Time	-	Mixed-Integer Linear Programming, Ant Colony Optimization
[43]	Mixed-Integer Linear Programming	Cycle Time	-	Mixed-Integer Linear Programming, Tabu Search
[44]	Mixed-Integer Linear Programming	Cycle Time	-	Mixed-Integer Linear Programming, Variable Neighborhood Search
[45]	Multi-Objective Linear Programming	Total Number of Utilized Workstations, Cycle Time, Total Cost of Additional Resources	-	Goal, Programming, Fuzzy Goal Programming
[30]	Multi-Objective Linear Programming	Cycle Time, The Number of Workstations, The Workload Variance, The Workstation Idle Time	Uncertain Objectives	Fuzzy Multi-Objective Linear Programming Model, Genetic Algorithm
[46]	Binary Linear Programming	Cycle Time	Fuzzy Processing Time	Bi-Level Fuzzy Mathematical Model,
[31]	-	Efficiency of the Line	Fuzzy Task Processing Times	Genetic Algorithm

Table 1. Continues

[47]	-	The Numbers of Stations, Fuzzy Line Efficiency, Fuzzy Idleness Percentage, Fuzzy Smoothness Index	Fuzzy Task Processing Times	Hybrid Multi-Objective Genetic Algorithm
[48]	-	Fuzzy Cycle Time and The Fuzzy Smoothness Index	Uncertain Task Processing Times	Genetic Algorithm
This study	Mixed-Integer Non-Linear Programming	Cycle Time, Total Number of Stations Opened	Fuzzy Duration of Task, Fuzzy Production Size	Mixed-Integer Non-Linear Programming,

As can be seen in Table 1, ALB models generally prioritize cycle time optimization. However, when the model focuses on optimizing multiple objectives, alternative solution methods are used to simplify the problem due to the difficulty of solving the model. While genetic algorithms, ant colony optimization, and tabu search algorithms are quite compatible with ALB models, they operate within a narrow solution range where the parameters are certain, and ultimately, these methods offer a heuristic solution. In such cases, a compromise solution can be found that satisfies conflicting objectives in an uncertain environment by fuzzing the variables, as in goal programming. However, even in this case, an additional heuristic method may be needed. This study specifically explores the best solution alternative within a wider solution range by defining a range of values for the parameters, not the variables, under uncertainty. By taking the parameters in a triangular fuzzy structure and within a range of values, a solution can be sought without the need for an additional heuristic method. The method used facilitates computational convenience. Considering the parameters as uncertain brings the problem closer to the real world. Furthermore, the fuzzy transformation methodology allows for a better solution than the solution obtained under the assumption of certainty.

3. Mathematical Model

ALB models traditionally have main objectives that they are expected to fulfill. The objective function and constraint set of the model are shaped in line with these objectives [3]. As different constraints such as source, station or distance enter the structure of the model, the model becomes more complex and, in this case, alternative solution algorithms are used for the model established in linear programming format [6]. There are three main objectives that we expect our model to serve [3, 32]:

- How many stations can complete tasks?
- What should be the cycle time?
- Which task should be assigned to which station?

The proposed model represents the lid line of bed base in an international furniture company. The job consists of six tasks. The current assembly line layout is shown in Fig 1. Task definitions, durations and priority relationships are given in Table 2.

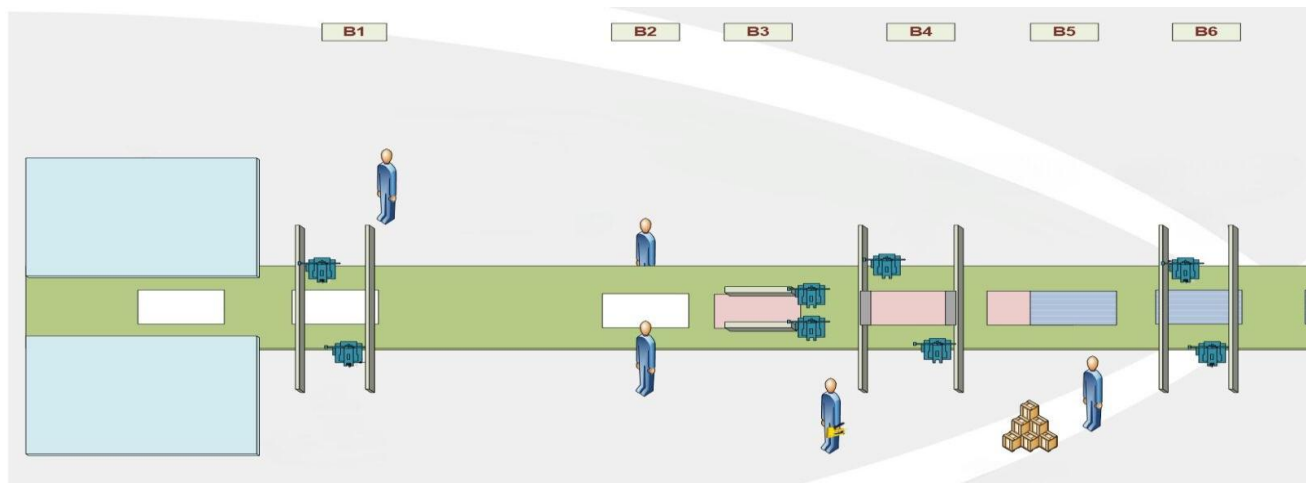


Figure 1. Current assembly line

Table 2. Characteristics of tasks

Tasks	Definition	Priority	Duration (sec)
B1	Driving screws into chipboard material	-	79
B2	Putting fabric and chipboard material on the production line	-	33
B3	Stapling long edges of chipboard material	B2	58
B4	Stapling the short edges of the chipboard material and completing the material's hand grip operations	B2	90
B5	Leaving the hydraulic mechanism on the chipboard material	B1	42
B6	Hydraulic mechanism assembly	B5	68

The workers in the assembly line constitute the workstation and more than one task can be performed at a workstation. The planned task cannot be divided into more than one workstation. There are a single shift and an 8-hour daily work routine within the company. The elements of the model and the equation sets related to the formulation are as follows:

Sets:

I : Set of tasks, $I = \{1,2, \dots, 6\}$

J : Set of stations, $J = \{1,2, \dots, 6\}$

i : Indices for the tasks

j : Indices for the stations expected to be opened on the assembly line

Parameters:

s_i : Duration of task i , see Table 2., $i \in I$

w_j : Priority weight of station j , $j \in J$

V : Shift duration, 28800 sec.

U : Production size in one shift, 300 units

M : Big number, 100

Variables:

K : Total number of stations opened

T : Cycle time, sec.

$$n_j = \begin{cases} 1, & \text{if station } j \text{ is opened,} \\ 0, & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1, & \text{if task } i \text{ is assigned to station } j, \forall i \in I; \forall j \in J \\ 0, & \text{otherwise} \end{cases}$$

Minimize $K * T$ (1)

s.t.

$$\sum_j x_{ij} = 1 \quad (2)$$

$$\sum_i s_i x_{ij} \leq T \quad (3)$$

$$\sum_j x_{1j} \leq \sum_j w_j x_{5j} \quad (4)$$

$$\sum_j x_{2j} \leq \sum_j w_j x_{3j} \quad (5)$$

$$\sum_j x_{2j} \leq \sum_j w_j x_{4j} \quad (6)$$

$$\sum_j x_{5j} \leq \sum_j w_j x_{6j} \quad (7)$$

$$\sum_i x_{ij} \leq Mn_j \quad (8)$$

$$n_{j+1} \leq n_j \quad (9)$$

$$K = \sum_j n_j \quad (10)$$

$$V = U * T \quad (11)$$

$$K, T \geq 0 \quad (12)$$

$$x_{ij}, n_j \in \{0,1\} \quad (13)$$

The main purpose of the model is to minimize the maximum time spent at stations within cycle time. For such a purpose, the total number of stations and the cycle time must be minimized simultaneously. When both variables are included in the model simultaneously as in Eqs. 1, the model becomes a non-linear structure. A task is assigned to only one station through Constraint (2). Constraint (3) ensures that the total workload of the stations remains below the cycle time. The equations between Constraint (4) and Constraint (7) regulate the priority relationships of the tasks. If it is appropriate to assign a task to a station through Constraint (8), the station is open. The stations must be opened in order and Constraint (9) determines this order. Constraint (10) reveals the total number of stations variable in the objective function. Constraint (11) helps to establish the balance between production size and cycle time. When this model solves the problem, an optimum result can be obtained. However, considering the uncertainty of some parameters in the problem, it is necessary to use heuristic algorithms or to transform and upgrade the model. Fuzzy transformation methodologies stand out as useful tools in tolerating uncertainty in the model.

4. Fuzzy transformation process

Fuzzy transformation methods play an important role in adapting mathematical models established with strict assumptions to uncertainties in real-life problems. Fuzzification and defuzzification processes can be easily integrated into different models regardless of the problem type. In the study, the fuzzification process developed by

Parra et al. [33] and Jiménez [34] and the defuzzification process introduced by Pishvae and Torabi [35] were used. This method pair, which has been shown to give good results in different models [36-37], is also quite useful for tolerating the uncertainty in some parameters of the ALB model. The fuzzy transformation approach is as follows [36-37]:

The fuzzy number \tilde{c} has triangular lower, middle and upper limits (c^p, c^m, c^o) . The membership function $\mu_{\tilde{c}}(x)$ of a triangular fuzzy number is defined as follows:

$$\mu_{\tilde{c}}(x) \begin{cases} f_c(x) = \frac{x-c^p}{c^m-c^p} & \text{if } c^p \leq x \leq c^m \\ 1 & \text{if } x = c^m \\ g_c(x) = \frac{c^o-x}{c^o-c^m} & \text{if } c^m \leq x \leq c^o \\ 0 & \text{if } x \leq c^p \text{ or } x \geq c^o \end{cases} \quad (14)$$

Through Eqs. 15, the fuzziness at the lower and upper limits of the fuzzy \tilde{c} number is modeled as the expected range (EI) and then the expected value (EV) in Eqs. 16 is reached.

$$EI(\tilde{c}) = [E_1^c, E_2^c] = \left[\int_0^1 f_c^{-1}(x)dx, \int_0^1 g_c^{-1}(x)dx \right] = \left[\frac{1}{2}(c^p + c^m), \frac{1}{2}(c^m + c^o) \right] \quad (15)$$

$$EV(\tilde{c}) = \frac{E_1^c + E_2^c}{2} = \frac{c^p + 2c^m + c^o}{4} \quad (16)$$

Fuzzy number pairs with $\tilde{a} > \tilde{b}$ relationship is needed and the membership function $\mu_M(\tilde{a}, \tilde{b})$ created for these numbers is given in Eqs. 17.

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & \text{if } E_2^a - E_1^b < 0 \\ \frac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & \text{if } 0 \in [E_1^a - E_2^b, E_2^a - E_1^b] \\ 1 & \text{if } E_2^a - E_1^b > 0 \end{cases} \quad (17)$$

The α parameter is needed to express the status of the relationship between the fuzzy number pairs \tilde{a} and \tilde{b} in the membership function and this relationship is defined as $\tilde{a} >_{\alpha} \tilde{b}$. The inequality is divided into two parts $\tilde{a} \geq_{\omega_2} \tilde{b}$ and $\tilde{a} \leq_{\omega_2} \tilde{b}$. The new membership function that emerges in line with the inequalities is shown in Eqs. 18:

$$\frac{\alpha}{2} \leq \mu_M(\tilde{a}, \tilde{b}) \leq 1 - \frac{\alpha}{2} \quad (18)$$

Eqs. 19 and Eqs. 20 are taken as basis for the defuzzification process.

$$\tilde{a}_i x \geq \tilde{b}_i \quad i = 1, \dots, l \quad (19)$$

$$\tilde{a}_i x = \tilde{b}_i \quad i = l + 1, \dots, m \quad (20)$$

Because of using the membership function equivalents of the variables, the transformation of Eqs. (21) and Eqs. (22) is realized.

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \geq \alpha \quad i = 1, \dots, l \quad (21)$$

$$\frac{\alpha}{2} \leq \frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \leq 1 - \frac{\alpha}{2} \quad i = l + 1, \dots, m \quad (22)$$

The arrangement of equations is as follows:

$$\begin{aligned} [(1 - \alpha)E_2^{a_i} + \alpha E_1^{a_i}]x &\geq \alpha E_2^{b_i} \\ + (1 - \alpha)E_1^{b_i} & \quad i = 1, \dots, l \end{aligned} \quad (23)$$

$$\begin{aligned} \left[\left(1 - \frac{\alpha}{2}\right)E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i} \right]x &\geq \frac{\alpha}{2}E_2^{b_i} \\ + \left(1 - \frac{\alpha}{2}\right)E_1^{b_i} & \quad i = l + 1, \dots, m \end{aligned}$$

$$\begin{aligned} \left[\frac{\alpha}{2}E_2^{a_i} + \left(1 - \frac{\alpha}{2}\right)E_1^{a_i} \right]x &\leq \left(1 - \frac{\alpha}{2}\right)E_2^{b_i} \\ + \frac{\alpha}{2}E_1^{b_i} & \quad i = l + 1, \dots, m \end{aligned} \quad (24)$$

In the last case, the defuzzification process to be used in the constraints is completed by using the number pairs a and b .

$$\begin{aligned} \left[(1 - \alpha) \frac{a_i^m + a_i^o}{2} + \alpha \frac{a_i^p + a_i^m}{2} \right]x &\geq \alpha \frac{b_i^m + b_i^o}{2} \\ + (1 - \alpha) \frac{b_i^p + b_i^m}{2} & \quad i = 1, \dots, l \end{aligned} \quad (25)$$

$$\begin{aligned} \left[\left(1 - \frac{\alpha}{2}\right) \frac{a_i^m + a_i^o}{2} + \left(\frac{\alpha}{2}\right) \frac{a_i^p + a_i^m}{2} \right]x &\geq \left(\frac{\alpha}{2}\right) \frac{b_i^m + b_i^o}{2} \\ + \left(1 - \frac{\alpha}{2}\right) \frac{b_i^p + b_i^m}{2} & \quad i = l + 1, \dots, m \end{aligned}$$

$$\begin{aligned} \left[\left(\frac{\alpha}{2}\right) \frac{a_i^m + a_i^o}{2} + \left(1 - \frac{\alpha}{2}\right) \frac{a_i^p + a_i^m}{2} \right]x &\geq \left(1 - \frac{\alpha}{2}\right) \frac{b_i^m + b_i^o}{2} \\ + \left(\frac{\alpha}{2}\right) \frac{b_i^p + b_i^m}{2} & \quad i = l + 1, \dots, m \end{aligned} \quad (26)$$

5. Analysis

5.1. Fuzzy parameters

In the ALB model, differences in demand, capacity and labor performance create variability in production size and task durations. Considering these parameters as sets of values within a range rather than a definite value has a direct impact on the results. Considering the two basic parameters in the problem under uncertainty brought the model closer to the real situation. The opinions of experts operating in the assembly

line field were considered in the parameter transformation. Therefore, the following parameters in the basic model are considered fuzzy and the relevant equations in the basic model are passed through a fuzzy process.

- \tilde{S}_i : Fuzzy duration of task i, sec.
- \tilde{U} : Fuzzy production size in one shift

After the relevant parameters were accepted as fuzzy, the fuzzy transformation system was shown in Eqs. 25 and Eqs. 26 was applied to Constraints (3) and Constraint (11) in the basic model. Accordingly, the constraint set in Eqs. 27 was used instead of Constraint (3), and the constraint sets in Eqs. 28 and Eqs. 29 were used instead of Constraint (11).

$$\sum_i \left[(1 - \alpha) \frac{s_i^o + s_i^m}{2} + (\alpha) \frac{s_i^p + s_i^m}{2} \right] x_{ij} \leq T \tag{27}$$

$$V \leq \left[\left(1 - \frac{\alpha}{2}\right) \frac{U^o + U^m}{2} + \left(\frac{\alpha}{2}\right) \frac{U^p + U^m}{2} \right] T \tag{28}$$

$$V \geq \left[\left(\frac{\alpha}{2}\right) \frac{U^o + U^m}{2} + \left(1 - \frac{\alpha}{2}\right) \frac{U^p + U^m}{2} \right] T \tag{29}$$

5.2. Findings

The model was adapted from a real-life problem, and it was seen that considering some parameters in the problem as fuzzy would increase the solution quality. After the constraints including production size and task durations pass through the fuzzy transformation, the model has new constraints. In the new model, the α coefficient takes values in the range of [0, 1] and brings the fuzzy parameters closer to the lower or upper value range [37]. This situation reflects the changes in production size and task durations to the solution. In other words, the sensitivity of the model can also be tested. In the

study, coefficients $\alpha = 0$, $\alpha = 0,5$ and $\alpha = 1$ were used for α and the results were compared with the basic model solution. In addition, due to the multiplication of two variables in the objective function, the linearity of the model is disrupted. MINLP operator of GAMS solver was used as solution tool. Table 3 shows the fuzzy parameter values.

Table 3. Fuzzy parameters values

i	\tilde{S}_i : Fuzzy duration of task i, sec.			\tilde{U} : Fuzzy production size in one shift		
	s_i^p	s_i^m	s_i^o	U^p	U^m	U^o
1	73	79	87	280	300	350
2	25	33	36			
3	53	58	68			
4	84	90	92			
5	40	42	50			
6	65	68	73			

Evaluating both models in terms of solutions provides the opportunity to address the problem in a broader perspective. The fuzzy model can achieve more realistic and better results. Table 4 includes the results obtained from the basic model and the fuzzy model using different α coefficients.

Fuzzy parameters cause the order and cycle time of the tasks to change. In all three scenarios, better results are obtained than the basic model. Small changes in the order create a difference in the total performance. Detecting this change is important for efficiency gains. The solution alternative is diversified by evaluating the production size and task durations independently. The results obtained by considering the α coefficient of the fuzzy parameters independently is given in Table 5.

Table 4. Comparison of solution alternatives

Model	K	T	Station distribution
Basic	5	96	1 → 2-5 → 6 → 4 → 3
$\alpha = 0$	5	91	1 → 2-5 → 4 → 3 → 6
$\alpha = 0,5$	5	91,06	1 → 2-5 → 3 → 4 → 6
$\alpha = 1$	5	93,65	1 → 2-5 → 6 → 4 → 3

Table 5. Fuzzy model results obtained with different α coefficients

$\alpha_{\tilde{S}_i}$	$\alpha_{\tilde{U}}$	K * T	Number of stations	$\alpha_{\tilde{S}_i}$	$\alpha_{\tilde{U}}$	K * T	Number of stations
0	0	455,00	5	0,5	1	468,29	5
0	0,5	455,33	5	1	0	443,07	5
0	1	468,29	5	1	0,5	455,33	5
0,5	0	445,00	5	1	1	468,29	5
0,5	0,5	455,33	5				

Independent fuzzy parameter α coefficients create changes in the total duration of tasks at stations within the cycle time. Minimization of the number of stations and cycle time has a positive effect on performance and efficiency. When all results are considered, although the total number of stations is the same, cycle times decrease. The model transformed with the fuzzy approach achieves better results than the basic model under all conditions. Taking the parameters as a range value set rather than a single value helps to apply the real situation to the current ALB layout more effectively. Accordingly, the best result is achieved for the total duration of tasks at stations within a cycle time when $\alpha_{\bar{s}_i} = 1$ and $\alpha_{\bar{t}} = 0$.

ANOVA analysis was performed to see whether the effect of fuzzy parameters on the total duration of tasks at stations within a cycle time, which is the objective function of the model, creates a significant difference. There are 11^2 data for the fuzzy production size and task duration parameters in the experimental set. Table 6 contains the sample data set for the total duration of tasks at stations within a cycle time obtained by using α coefficients in the value range of 0,1. In addition, total solver times of NLP and optimality gap for the experimental data set results are presented in Table 6. Conopt solver was used in the analysis of the data

The results of the one-way ANOVA test were performed to understand whether the fuzzy task duration and fuzzy production size parameters create a significant difference in the results of the model, as shown in Table 7.

Table 6. Experimental dataset example

Iteration	$\alpha_{\bar{s}_i}$	$\alpha_{\bar{t}}$	$K*T$	CPU (sec)	Gap (%)	Iteration	$\alpha_{\bar{s}_i}$	$\alpha_{\bar{t}}$	$K*T$	CPU (sec)	Gap (%)
1	0	0	455,00	0,21	1,97	:	:	:	:	:	:
2	0	0,1	455,00	0,16	1,57	113	1	0,2	447,90	0,16	12,43
3	0	0,2	455,00	0,17	1,17	114	1	0,3	450,35	0,28	14,32
4	0	0,3	455,00	0,32	0,77	115	1	0,4	452,83	0,15	14,87
5	0	0,4	455,00	0,18	0,28	116	1	0,5	455,34	0,15	0,00
6	0	0,5	455,34	0,19	0,00	117	1	0,6	457,87	0,29	0,00
7	0	0,6	457,87	0,19	0,00	118	1	0,7	460,43	0,20	0,00
8	0	0,7	460,43	0,16	0,00	119	1	0,8	463,02	0,20	0,00
9	0	0,8	463,02	0,17	0,00	120	1	0,9	465,64	0,33	0,00
10	0	0,9	465,64	0,32	0,00	121	1	1	468,29	0,26	0,00

Table 7 ANOVA results

	Source	SS	DF	MS	F	p
$\alpha_{\bar{s}_i}$	Between groups	136,25	10	13,62	0,24	0,991
	Within-groups	6223,77	110	56,58		
	Total	6360,02	120			
$\alpha_{\bar{t}}$	Between groups	5945,97	10	594,59	157,96	0,000
	Within-groups	414,05	110	3,76		
	Total	6360,02	120			

According to the ANOVA test, the variability in the task duration does not create a significant difference in the model results, whereas the production size creates a significant difference in the emergence of the ALB station number and in determining the cycle time. Table 8 presents the difference between the groups formed by the α coefficients in the 0,1

range for the production size. There are differences at every level between the fuzzy parameter groups of production size. In Table 8, the differences between $\alpha = 0$, $\alpha = 0,1$, $\alpha = 0,4$ ve $\alpha = 0,5$ and other groups are presented as examples. According to this situation, for example, there is a significant difference between $\alpha = 0,1 / \alpha = 1$ and due to the difference

between these two groups, a difference of 20,3 second occurs for the total duration of the tasks at the stations within a cycle time. The largest difference among the fuzzy α coefficient groups is seen between $\alpha = 0 / \alpha = 1$. There is a mean difference of 21,43 seconds between these two groups. Since the ALB task sequence can be in different stations according to each value, the mean difference does not follow any trend.

Figure 2 shows the change of the objective function in terms of fuzzy parameters from a different perspective. Here, the values of $\alpha_{\xi_i} \geq 0,5$ do not create a significant difference in terms of the objective function. Small values of α_{ξ_i} reveal variability for the objective function. However, the real difference in the results is created by the values of $\alpha_{\bar{y}}$.

The fuzzy approach can be applied to many problems with uncertainty. The proposed study shows that the fuzzy method is a useful tool when the variability in the production size and task duration is considered. Under the assumption that all parameters are certain, the optimum value reached by the model for the total duration of the tasks at the stations within a cycle time is 480 sec. However, much better results can be achieved with the transformed model by applying the fuzzy approach methodology. Especially, when the fuzzy parameters are $\alpha_{\xi_i} = 1$ and $\alpha_{\bar{y}} = 0$, a solution of 443,07 sec is reached. This situation provides a saving of approximately 7% of the total working time. This gain must be interpreted

from two perspectives. First, it should be noted that the solution methodology is two-stage. Before optimization, the assembly line consisted of six stations and had a cycle time of 90 seconds. There were five workers along the line. In the first stage, the base model reduced the number of stations to five in a real-world assembly line that consisted of six stations. Because the assembly line wasn't balanced efficiently before optimization, a worker was required to perform task B5, as seen in Figure 1. In the improved version, the base model combines tasks B2 and B5 at the same station, eliminating the need for a worker to perform task B5. This result offers a cost advantage for the company through resource savings. Then, in the study, the search for a better solution with a model that better reflects the real situation continued. The station distribution in the best solution obtained by using the fuzzy transformation approach is the same as the distribution pattern obtained in the base model. However, in the best solution, unlike the base model, the cycle time decreases to 88,615 seconds. This result demonstrates that even if the number of stations remains unchanged, existing stations can be used more efficiently. The fuzzy model eliminated the certainty constraints in the ALB problem, brought the problem closer to real life, and achieved better results even though the computational conditions became more difficult.

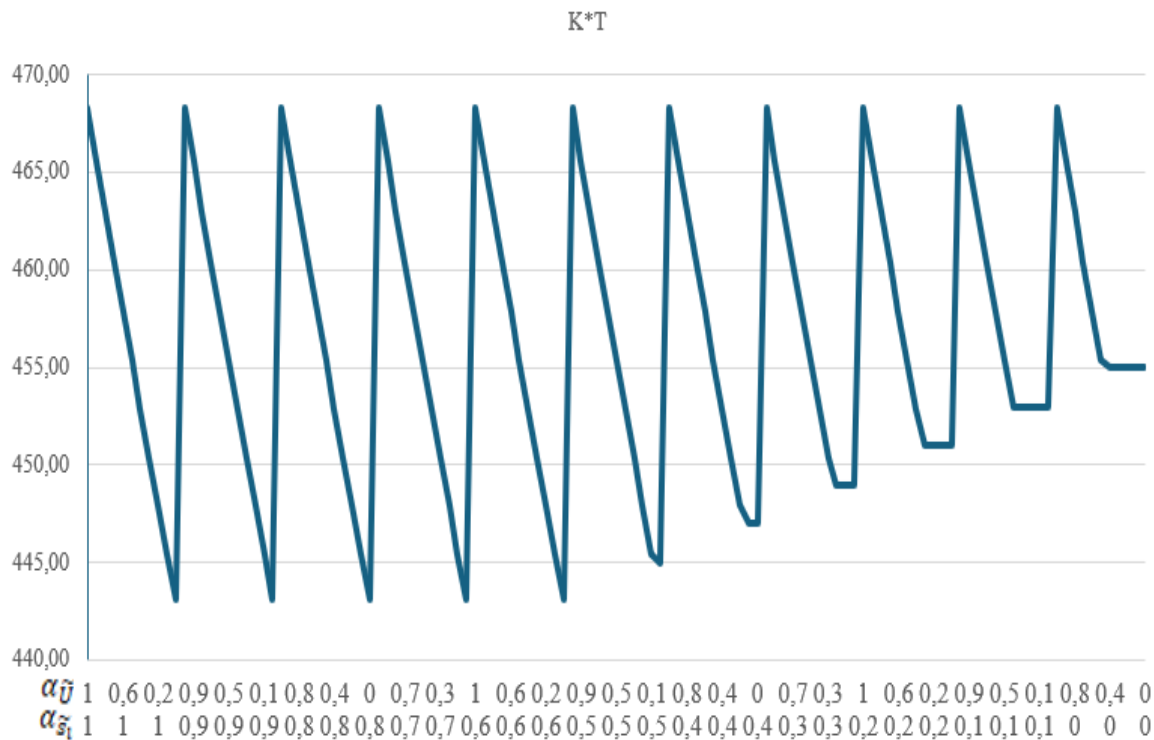


Figure 2. Change of objective function according to fuzzy parameters

Table 8. Fuzzy parameter groups $\alpha_{\bar{v}}$ that create differences

Groups	Mean	<i>p</i>	Groups	Mean	<i>p</i>
$\alpha = 0 / \alpha = 0,3$	-4,21*	0,007	$\alpha = 0,4 / \alpha = 0$	6,18*	0,000
$\alpha = 0 / \alpha = 0,4$	-6,18*	0,000	$\alpha = 0,4 / \alpha = 0,1$	5,05*	0,000
$\alpha = 0 / \alpha = 0,5$	-8,48*	0,000	$\alpha = 0,4 / \alpha = 0,2$	3,65*	0,046
$\alpha = 0 / \alpha = 0,6$	-11,01*	0,000	$\alpha = 0,4 / \alpha = 0,6$	-4,82*	0,001
$\alpha = 0 / \alpha = 0,7$	-13,57*	0,000	$\alpha = 0,4 / \alpha = 0,7$	-7,38*	0,000
$\alpha = 0 / \alpha = 0,8$	-16,16*	0,000	$\alpha = 0,4 / \alpha = 0,8$	-9,97*	0,000
$\alpha = 0 / \alpha = 0,9$	-18,78*	0,000	$\alpha = 0,4 / \alpha = 0,9$	-12,59*	0,000
$\alpha = 0 / \alpha = 1$	-21,43*	0,000	$\alpha = 0,4 / \alpha = 1$	-15,24*	0,000
$\alpha = 0,1 / \alpha = 0,4$	-5,05*	0,000	$\alpha = 0,5 / \alpha = 0$	8,48*	0,000
$\alpha = 0,1 / \alpha = 0,5$	-7,35*	0,000	$\alpha = 0,5 / \alpha = 0,1$	7,35*	0,000
$\alpha = 0,1 / \alpha = 0,6$	-9,88*	0,000	$\alpha = 0,5 / \alpha = 0,2$	5,94*	0,000
$\alpha = 0,1 / \alpha = 0,7$	-12,44*	0,000	$\alpha = 0,5 / \alpha = 0,3$	4,26*	0,000
$\alpha = 0,1 / \alpha = 0,8$	-15,03*	0,000	$\alpha = 0,5 / \alpha = 0,7$	-5,09*	0,000
$\alpha = 0,1 / \alpha = 0,9$	-17,65*	0,000	$\alpha = 0,5 / \alpha = 0,8$	-7,68*	0,000
$\alpha = 0,1 / \alpha = 1$	-20,30*	0,000	$\alpha = 0,5 / \alpha = 0,9$	-10,30*	0,000
			$\alpha = 0,5 / \alpha = 1$	-12,95*	0,000

6. Conclusion

ALB problems have direct effects on the efficiency and performance of manufacturing and service systems. Although production systems are moving towards more agile, flexible and automated resources, changes in the layout can overshadow the use of real capacities. In this study, a production line consisting of six tasks in a furniture factory was examined and the ALB model was established. In the model, which includes a two-stage solution hierarchy, first all parameters were assumed to be certain, and the optimum solution was reached. In the second stage, the fuzzy transformation methodology was used to tolerate uncertainty in the production size and task duration parameters, and better results were obtained with the new fuzzy model.

The type-E ALB model designed in this study has some differences from its previous counterparts. Since ALB problems are generally NP-hard [7], cooperation of different methods may be needed to ease the solution. These methods are generally heuristic methods and provide solutions close to the optimum. Models prepared in linear form can also facilitate reaching the solution. However, in this case, one of the variables, the number of stations or the cycle time, must be included in the objective function. For both variables to be included in the objective function, multi-objective models are needed. In this study, two variables are included together in the objective function and in this case the model is transformed into a non-linear form. However, the solution can be reached with the MINLP operator. In studies where ALB problems are modeled in a multi-objective manner, methods such as ϵ -constraint, goal programming and interactive fuzzy programming can be used [15]. The fuzzy methodology that

focuses on the transformation of uncertain parameters in the proposed study is an innovative approach in this field. In this way, the fuzzy method goes beyond just relieving the solution and offers the advantage of examining solution alternatives in a more realistic and broader perspective. The findings reveal the efficiency advantage of the method. In the study, a simulation with multiple iterations was performed with the model operated in line with different α coefficients. Such approaches are quite useful in testing the sensitivity of the solution. The proposed study, 11² different alternatives were examined and a search for a better solution was demonstrated. The method used has a useful decision support system for decision makers and implementers who want to establish an efficient ALB placement on the production line.

In fact, there are two separate problems and two separate data sets in the study. In the first stage, the variables are certain. In the second stage, due to some fuzzy defined parameters, the data set has changed, and uncertainty has emerged. In such cases, since the mathematical model constructed in the first stage is helpless, a different method is needed and these methods, which are generally heuristic, do not guarantee the optimum solution. Learning and applying such heuristic methods is quite time-consuming. This study shows that the current model can still be used with small modifications that can be made instead of such an intensive effort. Fuzzy transformation is much more practical, and the obtained gain shows the effective side of the transformation. The task duration and production size were determined as the basic parameters for the proposed ALB problem. The model includes station order and various assignment constraints. The model can become more complex by expanding the available data and including labor and various production resource

constraints in the model. The model can perform operator assignments as well as station assignments for tasks. The model established for a flat ALB line can be used in the U-type ALB model with some modifications. A broader examination and a more comprehensive data set will be needed for all these changes. As the positive effects of ALB models on efficiency and performance are examined from a broader perspective, interest in the issues in this area will increase.

Declarations

Use of AI tools

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Data availability statement

All data generated or analyzed during this study are included in this article.

Ethical approval

The authors state that this research adheres to the ethical standards. This research does not involve either human participants or animals.

Consent for publication

Not applicable

Conflicts of interest

The author declares that he has no conflict of interest.

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Author's contributions

Conceptualization, Methodology, Formal Analysis, Investigation, Writing – Original Draft, Writing – Review & Editing. The author has read and approved the final manuscript.

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