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Probing the Photon Mass through the Electromagnetic Memory Effect

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ABSTRACT

Electromagnetic memory effect is the residual velocity (a kick) acquired by free charges following the passage of electromagnetic waves. We compute the electromagnetic memory effect in de Broglie-Proca theory for particle scattering in four dimensional Minkowski spacetime as a function of the photon mass, and show that it is suppressed by Yukawa decay compared to usual Maxwell electrodynamics. Therefore, we suggest that its experimental detection would place bounds on, or potentially exclude, the existence of non-zero photon mass.

Keywords: Electromagnetic Memory, de Broglie-Proca Equation, Massive Photon

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Introduction

The observation of gravitational waves is one of the most promising achievements of Einstein's gravity [1]. The observation offers a new view of point in gravitational physics and cosmology; however, gravitational wave memory, an important prediction associated with the low-frequency sector of gravitational waves, remains unobserved. Essentially, the gravitational memory effect, a residual change, manifests as a permanent displacement in the relative positions of free particles following the passage of a gravitational wave pulse. Gravitational memory arises in two forms ; ordinary (linear regime), discovered by Zel'dovich ve A. G. Polnarev [2] and null (non linear regime) found by Christodoulou [3]. In calculating ordinary memory, Weyl tensor and geodesic deviation equation are taken into account. On the other hand, in the case of non-linear memory, the focus is on studying the change at the null infinity when a null stress energy-tensor arrives at there. Following the detection of gravitational waves, many works have been devoted to study of the gravitational memory effect [4-14], motivated by the expectation of measuring it. Detection of memory effect would serve as an observational test of alternative gravity theories, potentially imposing constraints or ruling them out. For example, in [10], memory effect was studied within the framework of Fierz-Pauli massive gravity, shown that its detection could provide evidence regarding whether the graviton is massive or massless. Although it is widely believed that the LIGO/Virgo wave observations already contain the necessary data to measure the memory effect, distinguishing the corresponding signal remains extremely challenging due to the small magnitude of the effect [15]. Nevertheless, gravitational memory effect is expected to be observed in next generation detectors.

At this point, a natural question appears here: does an electromagnetic analog of gravitational wave memory exist? The answer is affirmative, the idea was first suggested in [16]. Furthermore, a recent study proposed an experimental scheme to measure electromagnetic memory [17]. Electromagnetic memory is the residual velocity (a kick) experienced by free charges after the passage of electromagnetic waves, which impart momentum to the particles. Whereas the gravitational wave memory is associated with residual relative displacement, electromagnetic wave memory refers to a residual velocity. Furthermore, electromagnetic wave memory is expected to be more easily detectable than gravitational wave memory [18].

According to Maxwell electrodynamics, the photon has vanishing rest mass and is described a massless spin-1 particle. However, numerous alternative theories and studies have proposed the possibility of a small non-zero finite photon mass [19-22] (such as de Broglie-Proca theory [23,24]), which remains one of the unsolved problems in modern physics. Additionally, Boulware and Deser proposed that Aharonov Bohm effect might provide an experimental finite limit on the photon mass [25]. On the other hand, according to Particle Data Group [26], the mass of the photon is bounded (upper) by $m_\gamma < 10^{-18}$ eV. In this study, we compute the electromagnetic memory for particle scattering in de Broglie-Proca theory which describes massive spin-1 particle and compare this with the result of Maxwell theory. We express the memory as a function of photon mass, and propose that a potential observation of memory would impose on, or possibly exclude, the existence of photon mass. Furthermore, we discussed the result for some numerical values.

Electromagnetic Wave Memory for Particle Scattering in de Broglie-Proca Theory

The Lagrangian for the massive spin-1 field is described by de-Broglie-Proca's Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{ab}\,F^{ab} - \frac{1}{2}\,{m_\gamma}^2 A_a A^a, \tag{1}$$

where A_a defines the vector potential, $F_{ab} = \partial_a A_b - \partial_b A_a$ is the field strength tensor, and m_γ is the mass of spin-1 particle. Note that, from a field theory perspective in four dimensions, while free Maxwell electrodynamics describes massless vector field with two degrees of freedom: namely, two transverse modes, de Broglie-Proca theory describes massive spin-1 vector field that propagates three degrees of freedom: namely two transverse modes and one longitudinal mode. In the Lorenz condition ($\partial_a A^a = 0$, which is a constraint that follows from the field equations, not a gauge choice), source coupled de Broglie-Proca wave equation yields

$$(\eta^{ab}\,\partial_a\partial_b - {m_\gamma}^2)A^a = -4\pi\, j^a, \tag{2}$$

in which $\eta_{ab} = diag(-1,+1,+1,+1)$ is the Minkowski metric and j^a is current density. It should be emphasized that, mass term breaks gauge invariance in de Broglie-Proca theory. The retarded Green's function for the Eq. (2) is given

$$G^a{}_{a'}(x,x') = \eta^a{}_{a'}G(x,x'), \tag{3}$$

In which $\eta^a{}_{a'}$ is the parallel propagator, $G(x,x')$ is the Green's function which can be obtained as

$$G(x,x') = \frac{e^{-m_\gamma r}}{4\pi r}\delta(t-t'-r), \tag{4}$$

which leads to deviation from Coulomb's inverse-square law. We perform the computation analogous to that in Maxwell theory [4,5]. We can solve the massive electromagnetic wave equation for a given current density via Green's function technique. For this purpose, let us consider the source to consist of free particles colliding at the spacetime point (t=0,x) , with other particles (possibly different ones) emerging from that same point. Then, we now introduce current density for ingoing and outgoing particles, respectively :

$$J^a_{out} = \sum_{i,out} q_i^{out}\frac{d\tau_i}{dt}\,u_i^a\,\delta\big(x-y_i(t)\big)\Theta(t), \tag{5}$$
$$J^a_{in} = \sum_{j,in} q_j^{in}\frac{d\tau_j}{dt}\,u_j^a\,\delta\left(x-y_j(t)\right)\Theta(-t),$$

wherein $q_{i,j}$ represent electromagnetic charges, $u^a_{i,j}$ correspond to the normalized four velocities, and $\tau_{i,j}$ denote proper times along the worldliness. By employing Eqs.(3), (4), and (5), the retarded solution for de Broglie-Proca theory can be obtained up to leading order in 1/r as

$$A^a = (\alpha^a(r)\Theta(U) + \beta^a(r)\Theta(-U))\frac{e^{-mr}}{r}, \tag{6}$$

where $U = t - r$ denotes retarded time and $\alpha^a(r)$ and $\beta^a(r)$ are defined as

$$\alpha^a(r) = \sum_{i,out}\frac{d\tau_i}{dt}\frac{q_i^{out}u_i^a}{1-\hat{\boldsymbol{r}}.\boldsymbol{v_i}}, \tag{7}$$
$$\beta^a(r) = \sum_{j,in}\frac{d\tau_j}{dt}\frac{q_j^{out}u_j^a}{1-\hat{\boldsymbol{r}}.\boldsymbol{v}_j}.$$

Notably, the retarded solution is derived by boosting the general solution of the field equation, in order to obtain the field of a particle created at O moving with the coordinate velocity v = dy/dt. On the other hand, the field strength tensor can be obtained as, to the leading order

$$F_{ab} = -\frac{2\,e^{-m_\gamma r}}{r}\,(K_{[a}\alpha_{b]} - K_{[a}\beta_{b]})\delta(U) \tag{8}$$
$$-\frac{2m_\gamma\,e^{-m_\gamma r}}{r}\left(r_{[a}\alpha_{b]}\Theta(U) + r_{[a}\beta_{b]}\Theta(-U)\right),$$

where we have used $K^a = -\partial^a U$ and $r^a = \partial^a r$. The force exerted on a charged test particle Q can be expressed as

$$f^a = QF^{ab}V_b. \tag{9}$$

Now, consider the case where the test particle is at rest ($V_b = t_b$); the force takes the following form

$$f^a = -\frac{e^{-m_\gamma r}}{r}\,Q\,\Delta^a\delta(U), \tag{10}$$

here we have defined

$$\Delta^a = \sum_{i,out} q_i^{in}\frac{d\tau_i}{dt}\frac{u_i^a}{1-\hat{\boldsymbol{r}}.\boldsymbol{v}_i} - \sum_{j,in} q_j^{out}\frac{d\tau_i}{dt}\frac{u_j^a}{1-\hat{\boldsymbol{r}}.\boldsymbol{v}_j}. \tag{11}$$

The momentum change experienced by the charged test particle can be computed as

$$\Delta P^a = \int_{-\infty}^{U} dU'\,f^a(U',x) = -\frac{e^{-m_\gamma r}}{r}\,Q\,\Delta^a\Theta(U). \tag{12}$$

Observe that in the zero photon mass limit, Eq. (12) reduces to the usual result for the Maxwell theory [4]. Furthermore, for the upper bounds on the photon mass $\mathrm{m}_\gamma < 10^{-18}\mathrm{eV}$ and $r \approx 1\mu pc \approx 10^{10}m$ (distance from the electromagnetic source to a detector), Yukawa potential term decays very weakly and gives meaningful contribution to the result. Hence, careful observations could constraint the photon mass or rule it out. On the other hand, in de Broglie-Proca theory, memory effect vanishes at large separations, as seen in the case of GRB 130427A (gamma ray burst) [27], which was at distance 1104 Mpc.

Conclusion

Research on the gravitational memory effect has recently gained more attention, due to the potential for

its detection by advanced detectors (such as eLISA). It is believed that current gravitational wave observations by LIGO/Virgo already contain signs of the memory effect, but it is difficult to detect because of the background noise. Here, we consider the electromagnetic analog of the gravitational memory effect, known as electromagnetic memory effect. We calculated electromagnetic memory effect within de Broglie-Proca theory for particle scattering in 3+1 dimensional Minkowski spacetime as dependent on photon mass, showing that it is suppressed by Yukawa decay compared to the standard Maxwell electrodynamics. We have shown that, in the large separations, electromagnetic memory takes the same form as in pure Maxwell theory. On the other hand, in the solar system distances or smaller, careful observations could give stringent upper limits on photon mass or even rule out its existence.

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Conflicts of interest

There are no conflicts of interest in this work.

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