Hacettepe Journal of Mathematics and Statistics
Volume 47 (4) (2018), 921 – 936

Hierarchies in communities of Borsa Istanbul Stock Exchange

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Abstract

Nowadays, increase of the analyzing stock markets as complex systems lead graph theory to play key role. For instance detecting graph communities is an important task in the analysis of stocks, and minimum spanning trees let us to get important information for the topology of the market. In this paper, we introduce a method to build a connected graph representation of Borsa Istanbul based on the spectrum. We, then, detect graph communities and internal hierarchies by using the minimum spanning trees. The results suggest that the approach is demonstrably effective for Borsa Istanbul sessionally data returns.

Keywords: Financial networks, Graph communities, Hierarchy structures, Spectral graph theory, Minimum spanning trees

Mathematics Subject Classification (2010): AMS 91B26, 91B80, 62P20, 05C12, 90C35, 05C90

Received : 01.02.2016 *Accepted* : 31.08.2016 *Doi* : 10.15672/HJMS.201614520777

1. Introduction

Investigation of financial markets as complex systems is becoming increasingly accepted and recently majored in the statistical analysis of stock interaction networks. This kind of approach was first directed by Mantegna in [17] using the daily logarithmic price return correlation between of each stocks to obtain hierarchical networks. Analyzing this kind of networks let us to get the topological properties of a market and its core information. By the help of an appropriate metric that is based on the correlation distance, a connected graph in which vertices represent stocks can be build and the generated minimum spanning trees would yield the hierarchies.

Since companies interact with each other by cooperation and competition, financial markets can be characterized as evolving complex systems [2]. In [3], authors briefly introduced that empirical trees obtained from surrogated data simulated by using simple

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market models has features of a complex network that cannot be reproduce by a random market model and by the widespread one-factor model. By using the certain geometric measures, generalization of motif scores and clustering coefficient to weighted networks, and its application to complex networks such as financial markets and metabolic networks are considered in [25]. Lately, the minimum spanning tree techniques and theory of complex networks are used to study dynamics of financial networks [5, 22, 24]. In [23], authors showed that the length of minimum spanning trees shrinks during a stock market crisis and reconfiguration takes place strongly. Most of these studies show that stocks are tending to group in clusters and motive us to study graph communities in financial markets. Graph communities can be seen as vertex clusters which probably share common properties and/or play similar roles within the graph [11]. The internal hierarchical tree characterization of graph communities can be used to analyze each agent of the network that shares important features and to understand deeply evolution of the financial markets. Clusters of companies are identified by means of minimum spanning tree. However, this kind of clustering may bring us the loss of information of hierarchies. To overcome this problem, we study cluster by communities and obtain hierarchies by studying the minimum spanning trees of each community.

In this paper we study 93 companies that continuously operating in Borsa Istanbul 100 Index ($XU100$) and the exchange rate of USD to TRY from the period January 2013 to January 2015. There are 100 companies operating in the Index ($XU100$), since our analysis depend on the dimensional equality of the time series, we choose 93 of them which have trading operations during the chosen time interval. To represent this network as a connected graph, we consider each stock's daily sessional logarithmic returns which are the ends of midday and day prices and their Pearson correlations. In section 2, we first present some basics of graph theory and the spectrum of a graph. The main idea we present in this manuscript depends on the multiplicity of the 0 element in the spectrum of the graph. Then, in section 3, the method is presented. This method can be thought as in three steps. First we build a non-weighted undirected graph representation by studying the control parameter which in between 0 and 1. The optimal parameter is the largest one where the graph becomes with more than one component, i. e., multiplicity of the 0 eigenvalue of the graph is more than 1. Afterwards the obtained non-weighted graph, we determine the communities as the vertex sets by using the high modularity method [1], then obtain hierarchical organization of each stocks by studying the minimal spanning trees [17]. The main results of the method presented in Section 4 and also a comparative analysis respect to Planar Maximal Filter Graphs are presented. Finally in Section 5the discussion to the results and the topology of Borsa Istanbul ($BIST$) is given.

2. Preliminaries

An undirected graph G is the tuples (V, E), where V is the set of vertices (or nodes) and E is the set of edges. Each elements of E is an unordered pair of vertices for an undirected graph G. Strictly speaking, we are considering simple graphs in which all edges go between distinct vertices and in which there can be at most one undirected edge between a given pair of vertices. For any vertices $v_i, v_j \in V$ the graph G is called connected if there is a path , i.e. a sequence of edges, whose end points are v_i and v_j. A simple undirected graph in which every pair of distinct vertices is connected by a unique edge is called complete graph. Given an undirected graph $G = (V, E)$, a vertex cover is a set S subset of V that is incident to every elements of E. The smallest possible vertex cover for a given graph G is called minimum vertex cover.

In many real world applications, each edge of G has an associated non-negative numerical value, called a weight. Such a weighted graph can be represented by a triple (V, E, w) where $w : E \to \mathbb{R}^+$ is a function mapping edges to a numerical value.

An adjacency matrix A_G of a graph G is defined by

$$A_G(i,j) = \begin{cases} 1, & \text{if } (v_i, v_j) \in E \\ 0, & \text{otherwise.} \end{cases}$$

Note that the matrix A_G is symmetric, thus has an orthonormal basis of eigenvectors and the number of vertices many eigenvalues, counted with multiplicity [29].

A tree is a graph with no circuits, that is a connected graph that does not involve any sequence of vertices $(v_1, v_2, \ldots, v_k)$ such that $v_i = v_j$, $\exists i, j \in \{1, \ldots, k\}$. A spanning tree of a network is a subgraph that connects all the vertices. Among all the spanning trees of a weighted and connected graph, the one and possibly more with the least total weight is called a minimum spanning tree (MST) [13]. It can be easily concluded that for an unweighted graph all spanning trees are at the minimum cost. There are several ways to determine a minimum spanning tree, we refer readers [13] for the history and the solution of the problem.

Degree of vertex in an undirected graph G is the number of edges incident to the vertex, and let us denote it with d_v. By the introducing the degree of a vertex we can define the discrete analogue of a Laplacian operator for a graph which will lead us to the spectral graph theory. Given an undirected graph $G = (V, E)$, the Laplacian Matrix of G is $|V| \times |V|$ matrix whose entries are

$$L_G(i,j) = \begin{cases} d_{v_i}, & \text{if } i = j \\ -1, & \text{if } A_G(i,j) = 1 \\ 0, & \text{otherwise.} \end{cases}$$

The Laplacian Matrix L_G can also defined as $L_G = D_G - A_G$, where D_G is diagonal matrix with $D_G = [d_{v_i}]_{n \times n}$. It can be also concluded that Graph Laplacian does not depend on an ordering of the vertices of G. Let us now denote the spectrum of L_G by $S_G = \{\lambda_1, \ldots, \lambda_n\}$ for the graph with $|V| = n$. The Laplacian is positive-semidefinite, i.e. all of its eigenvalues have $\lambda_i \geq 0$ with the least one 0 [12].

2.1. Theorem. ***(Number of connected components and the spectrum of*** L_G***)*** *Let G be an undirected graph with nonnegative weights. Then the multiplicity k of the eigenvalue 0 of L_G equals the number of connected components $A_1, \ldots, A_k$ in the graph. The eigenspace of eigenvalue 0 is spanned by the indicator vectors $\mathbf{1}_{A_1}, \ldots, \mathbf{1}_{A_k}$ of those components.*

Proof. See [29]. □

3. Data and The Methodology

In this study, the undirected and unweighted graph based on Pearson Correlation Distance of Turkish companies, issued and traded on Borsa Istanbul between 2013 and 2014, was algorithmically built.

Borsa Istanbul (BIST), formerly called as Istanbul Stock Exchange, started operations at the beginning of 1986 and has memberships in various international federations and associations such as the World Federation of Exchanges, Federation of Euro-Asian Stock Exchanges, Federation of European Securities Exchanges, and International Capital Market Association [31]. The general trading is regulated in [32]. Trading hours for the Stocks are held by two sessions on business days, and one session in some official holidays.

3.1. Data. The data used in this paper consists of daily data from the period January 2013 to January 2015. 93 companies operating in Borsa Istanbul 100 Index (XU100) and the exchange rate of USD to TRY to validate the method are used throughout the rest of the paper. The daily price limit is set as ±20% of the base price which is found by rounding the previous daily settlement price to nearest price tick. If the price limits found by this method is not a valid price tick, for upper limit it is rounded up, while the lower limit is rounded down to the nearest price tick. Sessionally return is calculated as the logarithmic return in the value of index compared to previous session's closing value as follows:

$$Cl_i = \log P_i(t) - \log P_i(t-1),$$

where $P_i(t)$ is the closure price of the stock i at the daily session t.

Plot of the logarithmic return data that is presented by temperature mapping is given in Figure 1.

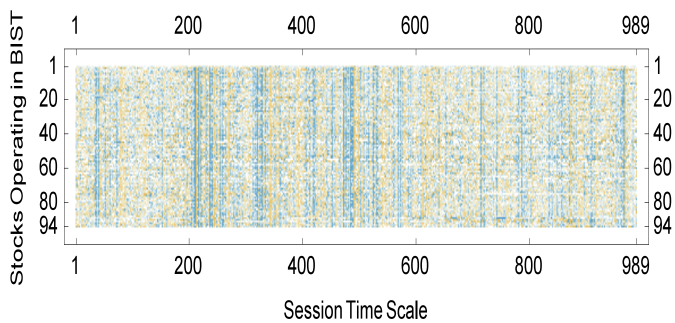

Figure 1. Sessionally data from the period January 2013 to January 2015. The vertical axis represents the stocks operating in BIST and the horizontal axis is for the time scale of operating sessions. The logarithmic return for each stock is represented in the matrix plot.

3.2. Methodology. Graph communities are cluster of vertices that is densely connected internally and can be used to analyze the data and links in the network [7, 16]. Community detection in graphs aims to identify these clusters, and their hierarchies, by using the topology of graph. The most common methods to detect communities can be summarized as Minimum-cut method [19, 20], Hierarchical clustering [15, 27], Girvan-Newman algorithm [21], High modularity [1], and Clique based methods [10, 9, 26].

Our method first aims to determine the network topology of stocks by studying their correlations. Rather than the weighted graph representation of the network, we first build a non-weighted graph to catch optimized many links between the stocks. This internally connectedness lead us to detect communities more precisely. For this purpose, we first consider the Pearson correlation of each stock as

$$\rho_{ij} = \frac{< Cl_i Cl_j > - < Cl_i >< Cl_j >}{\sqrt{(< Cl_i^2 > - < Cl_i >^2)(< Cl_j^2 > - < Cl_j >^2)}}$$

where $< .. >$ is a temporal average performed on all the trading days of the investigated time period which ranges from January 2, 2013 to December 30, 2015, $1 \leq i, j \leq n$ are the numerical labels of stocks, and $1 \leq t \leq m$. Then to determine edges, we introduce a distance function respect to correlation coefficients as $CorrDist := \sqrt{2(1-\rho_{ij})}/2$. Since $-1 \leq \rho_{ij} \leq 1$, $0 \leq CorrDist \leq 1$ for all Cl_i.

Our algorithm initially starts with the n-complete graph, i.e. a graph with only one 0 eigenvalue. Afterwards, we determine the edges by a control parameter which is the element of the fraction of $[0, 1]$ interval as the correlation distance of two stocks is lesser than the control parameter. The way that we choose the control parameter let us to catch highly correlated stocks; i.e., stocks with lesser correlation distance. Since as the control parameter is increasing from 0 to 1 the number of edges decreases, there exists such a control parameter that graph becomes with more than one component. The general outline of the algorithm is given in Table 1.

Table 1. Algorithm

Input:	D: $m \times n$ type data matrix
	h: fraction size
Initial:	G: n-complete graph with the A_G
	$t \leftarrow 0$
	while Number of 0 eigenvalue of $L_G = 1$ **do**
	$t \leftarrow t + 1$; $CP \leftarrow t/h$
	for $i = 1$ **to** $n - 1$
	for $j = i + 1$ **to** n
	if $CorrDist(Cl_i, Cl_j) \leq CP$
	then $A_G(i, j) \leftarrow 1$ **and** $A_G(j, i) \leftarrow 1$
	end if
	end for
	end for
	$G \leftarrow$ Graph with the A_G
	Compute the Eigenvalues of L_G
	end while
Output:	G with a tuned topology

The computational complexity of the algorithm is $O(hm^2n^6)$ in worst case, see [12] for the eigenvalue complexity and [30] for the correlation distance complexity.

Following the edge optimized graph, we determine the graph communities as the vertex clusters by using the methods mentioned above. For each cluster, it is possible to build weighted graph representation of each community and analyze internal minimum spanning trees that represent hierarchical structures. To construct hierarchical structures of stock markets, we refer readers [4, 17, 18].

4. Results

In order to investigate the communities of the Borsa Istanbul Stock Exchange, we first apply our algorithm to the data set. For the fraction size 100, the algorithm determines the control parameter as 0.65. In Figure 2, the number of connected components respect to the control parameter is shown for the fraction sizes 10,50,100,500,1000, and 5000.

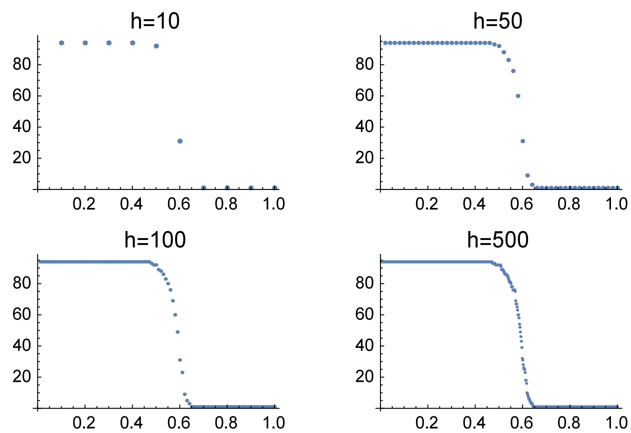

Figure 2. Horizontal axis represents the control parameter while the vertical axis represents number of connected components of the graph

Afterwards the obtained graph, the communities can be determined as follows by using the high modularity method:

Community 1:

USDTR	ADEL	AKBNK	ANACM	ARCLK
ASELS	BAGFS	DOAS	ECILC	EGEEN
FROTO	GSRAY	GLYHO	GSDHO	GUBRF
KARSN	KARTN	KOZAL	KOZAA	MGROS
NTTUR	PRKME	PETKM	SODA	TKNSA
TEKST	TOASO	TUPRS	TRCAS	VKGYO
YKBNK				

Community 2:

AFYON	AKSA	AKSEN	ALARK	ALBRK
ALKIM	AYGAZ	BJKAS	BRSAN	CLEBI
ECZYT	ENKAI	EREGL	HALKB	IHLAS
IPEKE	KCHOL	MNDRS	SAFGY	SAHOL
SASA	GARAN	TRGYO	TMSN	ULKER
VESTL				

Community 3:

AEFES	ALGYO	BIMAS	BIZIM	BRISA
CIMSA	DOHOL	ERBOS	FENER	GOLTS
GOZDE	HURGZ	ISGYO	KRDMD	SKBNK
TSKB	TAVHL	TKFEN	TRKCM	THYAO
TTKOM	TTRAK	ISCTR	ZOREN	

Community 4:

ASUZU	KONYA	OTKAR	SISE	TCELL
VAKBN	VESBE	YAZIC		

Community 5:

CCOLA	GOODY	METRO	NETAS	SNGYO

We visualize communities and their relations in Figure 3. The corresponding stock to each symbol in communities can be found in [31].

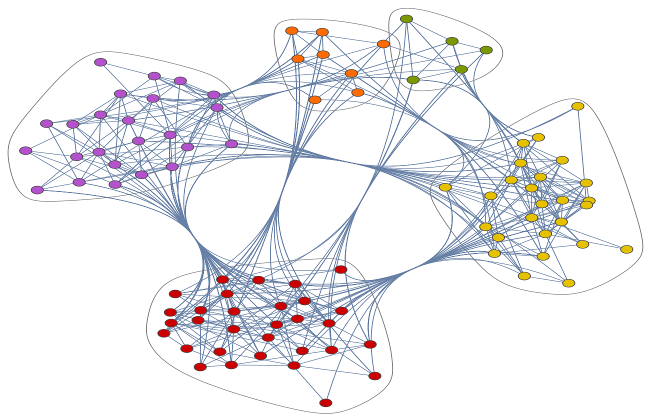

Figure 3. Communities of the graph. The red, yellow, purple, orange, and green nodes represent Community 1, Community 2, Community 3, Community 4, and Community 5; respectively.

Table 2 shows the sector of each operating stocks.

Table 2. Sectors of each considered stock

Financials
AKBNK, SKBNK, SNGYO, TSKB, TEKST, TRGYO, VKGYO, ALGYO, ISGYO, GARAN, ALBRK, GLYHO, ISCTR, YKBNK, SAHOL, GOZDE, HALKB, VAKBN, ECZYT, SAFGY, EKGYO, SAHOL, GSDHO
Industrials
ASELS, TAVHL, TKFEN, TTRAK, CLEBI
Consumer Discretionary
ASUZU, TKNSA, TOASO, YAZIC, AKSA, ARCLK, GSRAY, KARSN, THYAO, BRISA, DOAS, FENER, MNDRS, METRO, VESBE, ADEL , BJKAS, NTTUR, GOODY, OTKAR, TMSN, EGEEN, FROTO, IHLAS
Energy
AYGAZ, TUPRS, IPEKE , KCHOL
Technology
NETAS, VESTL
Materials
SASA, AFYON, ANACM, BAGFS, CIMSA, KONYA, KOZAA, ERBOS, KRDMD, PRKME, SISE, ALKIM , TRKCM, GUBRF, KOZAL, BRSAN, KARTN, PETKM, GOLTS, EREGL
Communications
TTKOM, TCELL, DOHOL, HURGZ
Consumer Staples
AEFES, CCOLA, BIZIM, ECILC, BIMAS, MGROS, SODA, ULKER
Utilities
AKSEN, ALARK, TRCAS, ZOREN, ENKAI

Now, to construct hierarchies in each community, we first consider the related distance matrix where vertices are the stocks in each community respect to the correlation distance

CorrDist. Then, we obtain weighted minimum spanning trees in each community by using Kruskal Algorithm. The resulted trees are given in Figure 4–8. In order to demonstrate the stocks' sectors, we used the coloring rule for Financials, Industrials, Consumer Discretionary, Energy, Technology, Materials, Communications, Consumer Staples, and Utilities as Blue, Purple, Red, Brown, Pink, Green, Claret Red, Orange, and Cyan, respectively. For the color images, we refer the reader to the web version of this article.

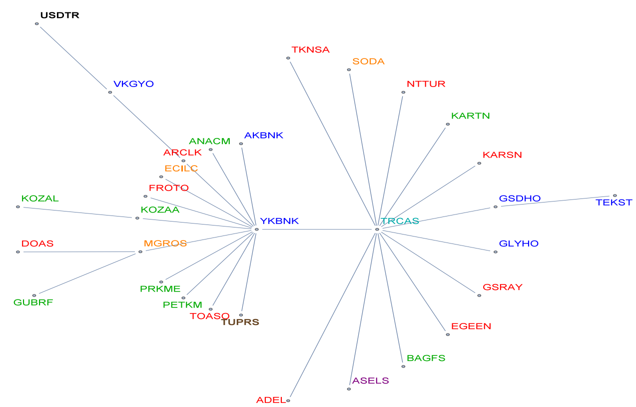

Figure 4. Hierarchy of the Community 1. In this hierarchy, stocks in Financial and Utilities sectors have the highest vertex degrees as the junction of the MST. The stocks adjacent to junction points are mostly in Finance, Metarials, and Consumer Discretionary sectors. Just one each stocks from the Industrial and Energy sectors occur in this hierarchy.

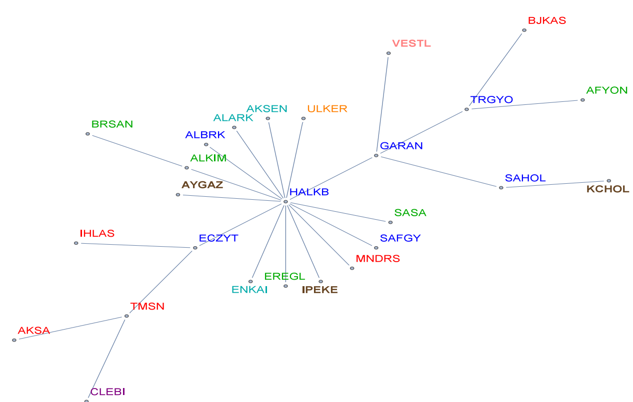

Figure 5. Hierarchy of the Community 2. The junction point with the highest degree is in the Financial sector. The rest of the stocks from Energy sector occur in this hierarchy as peripherals. Also stocks from the Utilities sector are in this hierarchy densely.

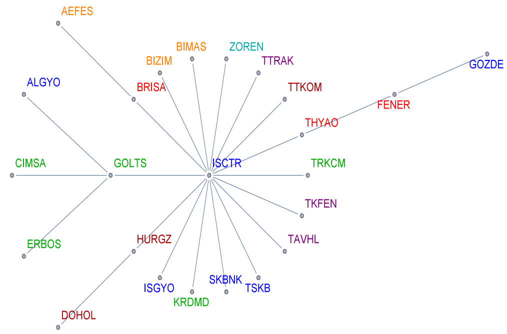

Figure 6. Hierarchy of the Community 3. In this hierarchy, a stock in Financial sector has has the highest vertex degrees as the junction of the MST. The other Financial stocks appear as peripherals. Stocks from the Communication sector are in this hierarchy densely.

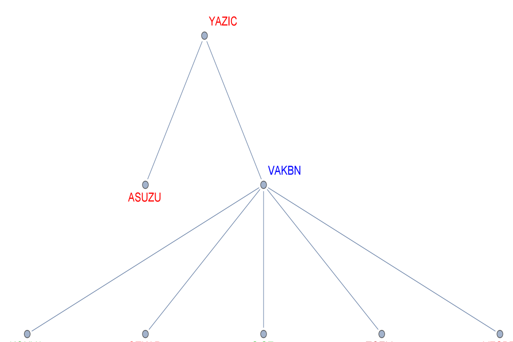

Figure 7. Hierarchy of the Community 4. A stock in Financial sector has the highest vertex degree and the peripherals are mostly Consumer Discretionary stocks.

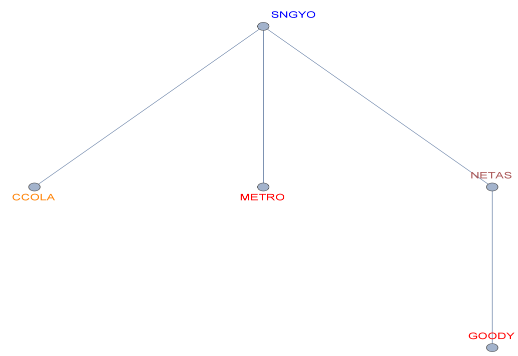

Figure 8. Hierarchy of the Community 5. In this hierarchy, a stock in Financial sector has the highest vertex degrees as the junction of the MST.

4.1. Comparative Analysis. Planar graphs have the same hierarchical structure of MST but they contain a larger amount of edges, loops and cliques. The idea of the construction of planar graphs is based on connecting the most correlated agents iteratively while constraining the resulting network to be embedded on a surface with genus g. In [28], authors briefly studied the special case for $g = 0$; i.e., the graph embedded on a sphere and called it as Planar Maximal Filter Graph (PMFG). PMFG are the topological triangulation of the sphere, hence they are only allowed to have three or four cliques [8].

An analysis on all of the 4-cliques in the PMFG reveals a high degree of homogeneity with respect to the stocks in each community of BIST. In Tables 3-7, we present all 4-cliques inside each community with the mean correlation distance $< CorrDist >$ among stocks and the mean of disparity measure $< y >$ where

$$y(i) = \sum_{j \neq i, j \in clique} \left(\frac{CorrDist(i,j)}{s_i} \right)^2$$

over the clique, where i is a generic element of the clique and

$$s_i = \sum_{j \neq i, j \in clique} CorrDist(i,j).$$

The disparity measure we present here is the direct analogy of the measure given in [28].

The level of correlation of the 4-cliques does not significantly vary amongst the communities. The largest mean correlation distance is in a clique of the Community 1 with 0.562571, whereas the smallest mean correlation distance is in a clique of the Community 4 with 0.440146. For 4-cliques, the value of the disparity measure is expected to be close to 1/3 [28]. Tables 3-7 show that most of the cliques have a disparity measure very close to 1/3. Hence, the pair correlations between stocks belonging to the cliques have higher homogeneity for each communities.

Table 3. 4-Cliques belonging to the Community 1

Stock 1	Stock 2	Stock 3	Stock 4	$< CorrDist >$	$< y >$
USDTR	ARCLK	BAGFS	GUBRF	0.56084	0.334559
USDTR	TEKST	TUPRS	VKGYO	0.561018	0.336105
USDTR	SODA	TEKST	VKGYO	0.562411	0.33542
USDTR	PRKME	PETKM	VKGYO	0.556716	0.33958
USDTR	NTTUR	SODA	VKGYO	0.561256	0.334755
USDTR	MGROS	TRCAS	VKGYO	0.544714	0.347718
USDTR	KARTN	PRKME	VKGYO	0.552557	0.340344
USDTR	KARSN	KOZAL	VKGYO	0.562546	0.334903
USDTR	GUBRF	KOZAL	VKGYO	0.557238	0.333821
USDTR	GSDHO	TUPRS	VKGYO	0.559784	0.33643
USDTR	GLYHO	KOZAA	VKGYO	0.559397	0.335824
USDTR	GLYHO	KARSN	VKGYO	0.559483	0.335843
USDTR	GSRAY	TOASO	VKGYO	0.560975	0.336893
USDTR	GSRAY	TKNSA	VKGYO	0.557004	0.33707
USDTR	FROTO	PETKM	VKGYO	0.558749	0.338899
USDTR	EGEEN	KOZAA	VKGYO	0.561081	0.335357
USDTR	EGEEN	FROTO	VKGYO	0.562571	0.336061
USDTR	ECILC	TOASO	VKGYO	0.559343	0.337999
USDTR	DOAS	GSDHO	VKGYO	0.559662	0.33674
USDTR	DOAS	ECILC	VKGYO	0.555571	0.339171
USDTR	BAGFS	NTTUR	VKGYO	0.551877	0.333554
USDTR	BAGFS	GUBRF	VKGYO	0.546378	0.333431
USDTR	ASELS	VKGYO	YKBNK	0.551052	0.343698
USDTR	AKBNK	ASELS	VKGYO	0.551889	0.342653
USDTR	ANACM	MGROS	VKGYO	0.549612	0.344103
USDTR	ANACM	KARTN	VKGYO	0.554096	0.340951
USDTR	ADEL	TKNSA	VKGYO	0.554364	0.339116
USDTR	ADEL	AKBNK	VKGYO	0.556661	0.340245

Table 4. 4-Cliques belonging to the Community 2

Stock 1	Stock 2	Stock 3	Stock 4	$< CorrDist >$	$< y >$
BJKAS	BRSAN	IHLAS	ULKER	0.541016	0.333386
AKSA	BJKAS	BRSAN	SAFGY	0.533513	0.333862
BRSAN	CLEBI	SASA	TRGYO	0.522965	0.335505
BRSAN	CLEBI	MNDRS	VESTL	0.534995	0.333749
BRSAN	CLEBI	MNDRS	SAFGY	0.536956	0.333666
BRSAN	CLEBI	EREGL	VESTL	0.53155	0.334408
BRSAN	CLEBI	EREGL	SASA	0.526299	0.335077
AYGAZ	BRSAN	CLEBI	IHLAS	0.539532	0.333416
AFYON	BRSAN	CLEBI	SAHOL	0.5226	0.334938
AFYON	AYGAZ	BRSAN	CLEBI	0.528309	0.334314
BJKAS	CLEBI	KCHOL	TMSN	0.519746	0.336212
BJKAS	CLEBI	IPEKE	SAFGY	0.537736	0.333436
BJKAS	CLEBI	ENKAI	IHLAS	0.541551	0.333408
BJKAS	CLEBI	ECZYT	GARAN	0.507557	0.34052
BJKAS	CLEBI	ECZYT	KCHOL	0.519198	0.336691
BJKAS	BRSAN	CLEBI	SAFGY	0.545495	0.333352
BJKAS	BRSAN	CLEBI	IHLAS	0.544061	0.333349
ALKIM	BJKAS	CLEBI	ENKAI	0.534149	0.333813
ALARK	ALKIM	BJKAS	CLEBI	0.525663	0.334946
ALBRK	BJKAS	CLEBI	TMSN	0.522824	0.335556
ALBRK	BJKAS	CLEBI	IPEKE	0.530757	0.334222
AKSEN	BJKAS	CLEBI	HALKB	0.503814	0.342573
AKSEN	ALARK	BJKAS	CLEBI	0.515865	0.337742

Table 5. 4-Cliques belonging to the Community 3

Stock 1	Stock 2	Stock 3	Stock 4	$< CorrDist >$	$< y >$
AEFES	BIMAS	THYAO	ZOREN	0.502937	0.33412
BIMAS	DOHOL	SKBNK	TTRAK	0.513207	0.333633
BIMAS	DOHOL	ISGYO	TTRAK	0.49895	0.334383
BIMAS	DOHOL	FENER	SKBNK	0.525654	0.33352
AEFES	BIMAS	FENER	GOLTS	0.506934	0.334118
AEFES	BIMAS	DOHOL	FENER	0.525571	0.333415
BIMAS	DOHOL	ERBOS	TAVHL	0.507615	0.333733
BIMAS	GOZDE	TKFEN	TRKCM	0.487189	0.33777
BIMAS	DOHOL	GOZDE	TAVHL	0.526903	0.333562
BIMAS	GOZDE	TSKB	ZOREN	0.51248	0.333992
BIMAS	GOZDE	KRDMD	TTKOM	0.503623	0.334692
BIMAS	GOZDE	HURGZ	TTKOM	0.520676	0.333602
BIMAS	GOZDE	HURGZ	TAVHL	0.523696	0.333494
BIMAS	CIMSA	GOZDE	TSKB	0.5051	0.334377
BIMAS	BRISA	GOZDE	TRKCM	0.495557	0.33631
BIMAS	BIZIM	CIMSA	GOZDE	0.505719	0.334604
BIMAS	BIZIM	BRISA	GOZDE	0.502829	0.335137
ALGYO	BIMAS	GOZDE	ISCTR	0.493407	0.336081
ALGYO	BIMAS	GOZDE	KRDMD	0.500806	0.335324
AEFES	BIMAS	GOZDE	ZOREN	0.525665	0.333471
AEFES	BIMAS	DOHOL	GOZDE	0.53155	0.333469

Table 6. 4-Cliques belonging to the Community 4

Stock 1	Stock 2	Stock 3	Stock 4	$< CorrDist >$	$< y >$
ASUZU	SISE	VAKBN	VESBE	0.440146	0.338944
SISE	TCELL	VESBE	YAZIC	0.454943	0.335381
OTKAR	TCELL	VESBE	YAZIC	0.461373	0.334814
ASUZU	SISE	TCELL	VESBE	0.464141	0.335094
ASUZU	KONYA	TCELL	VESBE	0.459768	0.334875

Table 7. 4-Cliques belonging to the Community 5

Stock 1	Stock 2	Stock 3	Stock 4	$< CorrDist >$	$< y >$
CCOLA	GOODY	METRO	SNGYO	0.463271	0.335471
CCOLA	GOODY	METRO	NETAS	0.464905	0.335804

Another interesting result appears from the construction of the communities such that each PMFG have only 4-cliques. This yields a very strong connection amongst the communities. In Table 8, intracommunity connection strength is given for the number of stocks n_s and the number of 4-cliques c_4.

Table 8. Intracommunity connection strength

Community	n_s	c_4	$c_4/(n_s-3)$
1	31	28	1
2	26	23	1
3	24	21	1
4	8	5	1
5	5	2	1

5. Conclusion

A certain connection criterion for stock market networks is first studied in [6], and determined as 0.7 in [14] for the analysing the stability of the network. However, this connection criterion is not permissive for our method since it yields only one community with densely connected nodes. In our study, for the fraction size 100, we determine the connection criterion which we called the control parameter as 0.65. It can also easily be seen in Figure 2 that the control parameter tends to 0.6 as the fraction size increases.

The exchange rate of USD to TRY appears in Community 1 adjacent to a strong Financial stock $VKGYO$ with the symbol $USDTR$. The multiplicity of the 0 eigenvalue of the connected graph becomes 2 when the control parameter is 0.64, then $BIMAS$ and $CLEBI$ becomes the isolated vertices. For the control parameter 0.63, $NTTUR$ and $BJKAS$ are also become isolated vertices, then for the lesser control parameters the number of the isolated vertices set exponentially grows and starts to form an internal cluster. It can be concluded that stocks that is becoming isolated for lesser control parameters are the peripheral ones in the respected community.

One of the effective methods to analyze hierarchies is the finding vertex covers of the representing minimum spanning trees. Vertex cover sets of the Communities 1–5 can be obtained as

$$\{\text{GSDHO}, \text{KOZAL}, \text{MGROS}, \text{TRCAS}, \text{VKGYO}, \text{YKBNK}\}$$
$$\{\text{ALKIM}, \text{ECZYT}, \text{HALKB}, \text{KCHOL}, \text{GARAN}, \text{TRGYO}, \text{TMSN}\}$$
$$\{\text{AEFES}, \text{DOHOL}, \text{FENER}, \text{GOLTS}, \text{ISCTR}\}$$
$$\{\text{VAKBN}, \text{YAZIC}\}$$
$$\{\text{NETAS}, \text{SNGYO}\}$$

respectively.

From the hierarchies, it can be concluded that stocks operating in Financial sectors play key role for Borsa Istanbul, i.e. junction points with the highest vertex degrees in MST of each community. Amongst the Financial sector stocks, especially companies in Banking industry occur as junction points. Banking industry has the highest weight in BIST as %36.76 [33], therefore our result is also consistent with the empirical data. The other significant sectors are Materials and Utilities in the topologies of the hierarchies. Stocks operating in these sectors which are the junctions are also adjacent to financial sector stocks. The stocks operating in Consumer Discretionary, Consumer Staples, Communication, and Industry sectors are occur as the adjacent points to the junctions. They are mostly adjacent to Financial sector stocks, then Materials sector stocks, as it is expected for the topology of Borsa Istanbul Stock Exchange.

6. Acknowledgement

The author would like to extend sincere thanks to Borsa Istanbul for providing data, and Ö. Akgüller for valuable comments.

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