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Soft Almost b-Continuous Mappings

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Abstaract − In the present paper, we introduce one class of soft mappings,namely soft almost b-continuous mappings and investigate several properties of these mappings.This notion is stronger than soft almost β-continuous mappings and is weaker than both soft almost pre-continuous mappings and soft almost semi-continuous mappings.The diagrams of implications among these soft classes of mappings and some known classes of mappings have been established.

Keywords − *Soft regular open set, Soft b-open set, Soft δ-open set, Soft almost continuous mappings, Soft b-continuous mappings.*

1 Introduction

In 1999, Molodtsov [14] introduced the concept of soft sets to deal with uncertainties while modelling the problems with incomplete information. In 2011 Shabir and Naz [15] initiated the study of soft topological spaces.Theoretical study of soft sets and soft topological spaces have been by some authors in [6, 8, 9, 10, 11, 14, 15, 23, 25, 26]. Soft regular-open sets[5], soft semi-open sets[12],soft preopen sets [2], soft α-open sets [4],soft β-open sets [3], soft b-open sets [1] play an important part in the researches of generalizations of continuity in soft topological spaces. The aim of this paper is to introduce one class of soft mappings, namely soft almost b-continuous mappings by utilizing the notions of soft b-open sets due to [1]. We investigate several properties of this class. The class of soft almost b-continuous mappings is a generalization of soft almost pre-continuous mappings and soft almost semi-continuous mappings. At the same time, the class of soft almost β-continuous mappings is a generalization of soft almost b-continuous mappings.

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2 Preliminary

Let U is an initial universe set, E be a set of parameters, P(U) be the power set of U and A $\subseteq$ E.

Definition 2.1. [14] A pair (F, A) is called a soft set over U, where F is a mapping given by F: A $\rightarrow$ P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For all e $\in$ A, F(e) may be considered as the set of e-approximate elements of the soft set (F, A).

Let X and Y be an initial universe sets and E and K be the non empty sets of parameters, S(X, E) denotes the family of all soft sets over X and S(Y, K) denotes the family of all soft sets over Y.

Definition 2.2. [15] A subfamily τ of S(X , E) is called a soft topology on X if:

1. $\widetilde{\phi}$, $\widetilde{X}$ belong to τ.
2. The union of any number of soft sets in τ belongs to τ.
3. The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are called soft open sets in X and their complements called soft closed sets in X.

Definition 2.3. [25] The soft set (F, E)$\in$ S(X, E) is called a soft point if there exist x $\in$ X and e $\in$ E such that F(e) = {x} and F(e') = ϕ for each e' $\in$ E – {e}, and the soft point (F, E) is denoted by $(x_e)_E$.

Definition 2.4. [23, 4, 7, 5, 1, 3] A soft set (F,E) in a soft topological space (X,τ,E) is said to be :
(a) Soft regular open if (F,E)= Int(Cl(F,E)).
(b) Soft α-open if (F,E) $\subseteq$ Int(Cl(Int(F,E))).
(c) Soft semi-open if (F,E) $\subseteq$ Cl(Int(F,E)).
(d) Soft pre-open if (F,E) $\subseteq$ Int(Cl(F,E)).
(e) Soft b-open if (A, E) $\subset$ Int(Cl(A, E)) $\cup$ Cl(Int (A, E)).
(f) Soft β-open if (A, E) $\subset$ Cl(Int(Cl(A, E))).
The complement of soft α-open set (resp. soft semi-open set, soft pre-open,soft b-open, soft β-open) set is called Soft α-closed(resp. soft semi-closed ,soft pre-closed,soft b-closed, soft β-closed) set.

Definition 2.5. [17]A soft point $(x_e)_E$ in a soft topological space (X,τ,E) is called δ-cluster point of a soft set (A,E) of X if Int(Cl(V,E)) $\cap$ (A,E) $\neq \phi$ for each soft open set (V,E) containing $(x_e)_E$.The union of all δ-cluster points of (A, E) is called δ-closure of (A,E) and is denoted by δCl(A, E).

Remark 2.6. [4, 23, 1]
(a) Every soft regular open (resp. soft regular closed) set is soft open (resp. closed), every soft open (resp. soft closed) set is soft α-open (resp. soft α-closed), every soft α-open (resp. soft α-closed) set is soft pre-open (resp. pre-closed) and soft semi-open (resp. semi-closed) but the converses may not be true.

(b) The concepts of soft semi-open (resp. soft semi-closed) and soft pre-open (resp.soft pre-closed)sets are independent to each other.

(c) Every soft pre-open (resp. pre-closed) and soft semi-open (resp. semi-closed) is soft b-open(resp. soft b-closed) set and every soft b-open(resp. soft b-closed) set is soft β-open((resp. soft β-closed) set but the converses may not be true.

Definition 2.7. [1] Let (F,E) be a soft set in a soft topological space (X ,τ,E).

(a) The soft b-closure of (F, E) is defined as the smallest soft b-closed set over which contains (F, E) and it is denoted by bCl(F,E).

(b) The soft b-interior of (F, E) is defined as the largest soft b-open set over which is contained in (F, E) and is denoted by bInt(F,E).

Definition 2.8. [23, 4, 7, 5, 1, 3] Let (X,τ,E) and (Y,υ,K) be a soft topological spaces. A soft mapping f_{pu} : (X,τ,E)$\rightarrow$ (Y,υ,K) is said to be soft continuous (resp. soft α-continuous,soft semi-continuous ,soft pre-continuous,soft b-continuous,soft β-continuous) mapping if f_{pu}^{-1}(G, K) is soft open(resp. soft α-open ,soft semi-open,soft pre-open,soft b-open,soft β-open) over X , for all soft open set(G,K) over Y.

Definition 2.9. [1] A soft mapping f_{pu} : (X,τ,E)$\rightarrow$ (Y,υ,K) is said to be soft b-irresolute if f_{pu}^{-1}(G, K) is soft b-open over X , for all soft b-open set (G, K) over Y.

Definition 2.10. [23, 4, 7, 5, 1, 3] Let (X,τ,E) and (Y,υ,K) be a soft topological spaces. A soft mapping f_{pu} : (X,τ,E)$\rightarrow$ (Y,υ,K) is said to be soft open(resp. soft α-open,soft semi-open,soft pre-open,soft b-open,soft β-open) mapping if f_{pu} (F, E) is soft open(resp. soft α-open ,soft semi-open,soft pre-open,soft b-open,soft β-open) over Y, for all soft open set (F, E) over X.

Remark 2.11. [4, 3, 1]

(a) Every soft continuous (resp. soft open) mapping is soft α-continuous(resp. soft α-open) mapping ,every soft α-continuous (resp. soft α-open) mapping is soft pre-continuous (resp. soft pre-open) and soft semi-continuous (resp. soft semi-open) mapping but the converse may not be true.

(b) The concepts of soft semi-continuous and soft pre-continuous (resp. soft semi-open and soft pre-open) mappings are independent.

(c) Every soft pre-continuous (resp. soft pre-open) and soft semi-continuous (resp. soft semi-open) mappings are soft b-continuous and every soft b-continuous mapping is soft β-continuous mapping but the converse may not be true.

Definition 2.12. [18, 19, 20, 21, 22] A soft mapping f_{pu} : (X ,τ,E) $\rightarrow$ (Y,ϑ,K) is said to be soft almost(resp. α-continuous,semi-continuous,pre-continuous,β-continuous) mapping if the inverse image of every soft regular open set over Y is soft open(soft α-open,soft semi-open,soft pre-open,soft β-open) over X.

In this paper ,we use the abbreviations of soft almost continuous mapping , soft almost α-continuous mapping, soft almost semi-continuous mapping, soft almost pre-continuous mapping, soft almost β-continuous mapping by s.a.c., s.a.α.c., s.a.s.c., s.a.p.c., s.a.β.c. respectively.

Remark 2.13. [18, 19, 20, 21, 22]

(a) Every soft continuous mapping is soft almost continuous but the converse may not be true.

(b) Every soft α-continuous mapping is soft almost α-continuous but the converse may not be true.

(c) Every soft almost continuous(resp. soft almost-open) mapping is soft almost α-continuous(resp. soft almost α-open) but the converse may not be true.

(d) Every soft almost α-continuous(resp. soft almost α-open) mapping is almost pre-continuous(resp. soft almost pre-open) and almost semi-continuous(resp. soft almost semi-open) but the converse may not be true.

(e) Every soft semi-continuous mapping (resp. soft semi-open) is soft almost semi-continuous(resp. soft almost semi-open) but the converse may not be true.

(f) Every soft pre-continuous(resp. soft pre-open) mapping is soft almost pre-continuous(resp. soft almost pre-open) but the converse may not be true.

(g) The concepts of soft almost semi-continuous and soft almost pre-continuous (resp. soft almost semi-open and soft almost pre-open) mappings are independent.

(h) Every soft β-continuous(resp. soft β-open) mapping is soft almost β-continuous(resp. soft almost β-open) but the converse may not be true.

(i) Every soft almost pre-continuous(resp. soft almost pre-open) and soft almost semi-continuous(resp. soft almost semi-open) mapping is soft almost β-continuous(resp. soft almost β-open) but the converse may not be true.

Definition 2.14. [20] A soft topological space (X ,τ,E) is said to be soft semiregular if for each soft open set (F,E) and each soft point $(x_e)_E \in$ (F,E), there exists a soft open set (G,E) such that $(x_e)_E \in$ (G,E) and (G,E) $\subset$ Int(Cl(G,E)) $\subset$ (F,E).

Definition 2.15. [16] Let f_{pu} : (X ,τ , E) $\rightarrow$ (Y , ϑ , K) be a soft mapping. Then a soft mapping $G_{g_p g_u}$: (X ,τ,E) $\rightarrow$ (X $\times$ Y, $\tau \times \vartheta$, E $\times$ K) is said to be soft graph mapping of f_{pu} where g_u and g_p are respectively defined by g_u(x) = (x , u(x)) for all x $\in$ X and g_p(e) = (e , p(e)) for all e $\in$ E.

3 Soft Almost b-Continuous Mappings

Definition 3.1. A soft mapping f_{pu} : (X ,τ,E) $\rightarrow$ (Y,ϑ,K) soft almost b-continuous (brie?y s.a.b.c.) for each soft point $(x_e)_E$ over X and each soft regular open set (V,K) over Y containing $f_{pu}((x_e)_E)$, there exists soft b-open set(U,E) over X containing $(x_e)_E$ such that f_{pu}(U,E) $\subset$ (V,K).

Theorem 3.2. Let f_{pu} : (X ,τ,E) $\rightarrow$ (Y,ϑ,K) be a soft mapping. Then the following conditions are equivalent:

(a) f_{pu} soft almost b-continuous.

(b) For each soft point $(x_e)_E$ over X and each soft open set (V,K) over Y containing $f_{pu}((x_e)_E)$, there exists soft b-open set (U,E) over X containing $(x_e)_E$ such that f_{pu}(U,E) $\subset$ Int(Cl(V,K)).

(c) f_{pu}^{-1}(V,K) be a soft b-open set over X, for every soft regular open set (V,K) over Y.

Proof: It is obvious.

Remark 3.3. Every soft b-continuous mapping is soft almost b-continuous but the converse may not be true.

Example 3.4. Let X = $\{x_1, x_2\}$, E = $\{e_1, e_2\}$ and Y = $\{y_1, y_2\}$, K = $\{k_1, k_2\}$. The soft sets (F,E), (G,K) are defined as follows :

F(e_1) = $\{x_2\}$, F(e_2) = $\{x_1\}$

G(k_1) = $\{y_1\}$, G(k_2) = $\{y_2\}$

Let $\tau = \{\phi$,(F,E), $\tilde{X}\}$, and $\upsilon = \{\phi$,(G,K), $\tilde{Y}\}$ are topologies on X and Y respectively. Then soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,υ,K) defined by u(x_1) = y_1 , u(x_2) = y_2 and p(e_1)= k_1, p(e_2) = k_2 is soft almost b-continuous but not soft b-continuous.

Remark 3.5. Every soft almost semi-continuous is soft almost b-continuous but the converse may not be true.

Example 3.6. Let X = $\{x_1, x_2\}$, E = $\{e_1, e_2\}$ and Y = $\{y_1, y_2\}$, K = $\{k_1, k_2\}$. The soft sets (F_1,E), (F_2,E), (G,K) are defined as follows :

$G_1(k_1) = \{y_1\}$, $G_1(k_2) = \{y_2\}$,

$G_2(k_1) = \{y_2\}$, $G_2(k_2) = \{y_1\}$,

Let $\tau = \{\phi, \tilde{X}\}$, and $\upsilon = \{\phi$, (G_1,K), (G_2,K), $\tilde{Y}\}$ are topologies on X and Y respectively. Then soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,υ,K) defined by u(x_1) = y_1, u(x_2)= y_2 and p(e_1)= k_1, p(e_2) = k_2 is soft almost b-continuous mapping but not soft almost semi-continuous.

Remark 3.7. Every soft almost pre-continuous is soft almost b-continuous but the converse may not be true.

Example 3.8. Let X = $\{x_1, x_2\}$, E = $\{e_1, e_2\}$ and Y = $\{y_1, y_2\}$, K = $\{k_1, k_2\}$. The soft sets (F_1,E), (F_2,E), (G,K) are defined as follows :

$F_1(e_1) = \phi$, $F_1(e_2) = \{x_1\}$,

$F_2(e_1) = \{x_1\}$, $F_2(e_2) = \phi$,

$F_3(e_1) = \{x_1\}$, $F_3(e_2) = \{x_1\}$,

$G_1(k_1) = \{y_1\}$, $G_1(k_2) = \{y_2\}$,

$G_2(k_1) = \{y_2\}$, $G_2(k_2) = \{y_1\}$.

Let $\tau = \{\phi$, (F_1,E) ,(F_2,E),(F_3,E), $\tilde{X}\}$, and $\upsilon = \{\phi$,(G_1,K),(G_2,K), $\tilde{Y}\}$ are topologies on X and Y respectively. Then soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,υ,K) defined by u(x_1) = y_1 , u(x_2)= y_2 and p(e_1)= k_1 , p(e_2) = k_2 is soft almost b-continuous mapping but not soft almost pre-continuous.

Remark 3.9. Every soft almost b-continuous mapping is soft almost β-continuous but the converse may not be true.

Example 3.10. Let X = $\{x_1 , x_2,x_3,x_4\}$, E = $\{e_1 , e_2\}$ and Y = $\{y_1 , y_2,y_3,y_4\}$, K = $\{k_1 , k_2\}$. The soft sets (F_1,E) ,(F_2,E),(F_3,E),(G_1,K),(G_2,K) and (G_3,K) are defined as follows :

$F_1(e_1) = \{x_3\}$, $F_1(e_2) = \phi$,

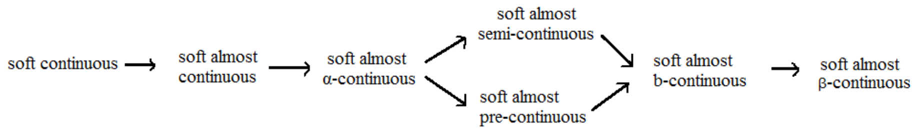
$F_2(e_1) = \{x_1,x_4\}$, $F_2(e_2) = \phi$,

$F_3(e_1) = \{x_1,x_3,x_4\}$, $F_3(e_2) = \phi$,

$G_1(k_1) = \{y_3\}$, $G_1(k_2) = \phi$,

$G_2(k_1) = \{y_1,y_4\}$, $G_2(k_2) = \phi$,
$G_3(k_1) = \{y_1,y_3,y_4\}$, $G_3(k_2) = \phi$,
Let
$\tau = \{\phi$, $(F_1$,E), $(F_2$,E),$(F_3$,E), $\tilde{X}$ } and $\upsilon = \{\phi$, $(G_1$,K), $(G_2$,K), $(G_3$,K), $\tilde{Y}$ } are topologies on X and Y respectively. Then soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,υ,K) defined by u(x_1) = u(x_2)= y_1, u(x_3) = y_3, u(x_4) = y_4 and p(e_1)= k_1, p(e_2) = k_2 is soft almost β-continuous mapping but not soft almost b-continuous.

Thus we reach at the following diagram of implications.



Theorem 3.11. Let f_{pu} : (X ,τ,E) $\rightarrow$ (Y,ϑ,K) be a soft mapping. Then the following conditions are equivalent:

(a) f_{pu} is soft almost b-continuous.

(b) f_{pu}^{-1} (G,K) is soft b-closed set in X for every soft regular closed set (G,K) over Y.

(c) f_{pu}^{-1} (A,K) $\subset$ bInt(f_{pu}^{-1}(Int(Cl(A,K)))) for every soft open set (A,K) over Y.

(d) bCl(f_{pu}^{-1}(Cl(Int(G,K)))) $\subset$ f_{pu}^{-1}(G,K) for every soft closed set (G,K) over Y.

(e) For each soft point $(x_e)_E$ over X and each soft regular open set (G,K) over Y containing f_{pu} $((x_e)_E)$,there exists a soft b-open set (F,E) over X such that $(x_e)_E \in$ (F,E) and (F,E) $\subset$ f_{pu}^{-1}(G,K).

(f) For each soft point $(x_e)_E$ over X and each soft regular open set (G,K) over Y containing $f_{pu}((x_e)_E)$,there exists a soft b-open set (F,E) over X such that $(x_e)_E \in$ (F,E) and f_{pu}(F,E) $\subset$ (G,K).

Proof: (a)$\Leftrightarrow$(b) Since f_{pu}^{-1} ($(G,K)^C$) = $(f_{pu}^{-1}(G,K))^C$ for every soft set (G,K) over Y.

(a) $\Rightarrow$(c) Since (A,K) is soft open set over Y, (A,K) $\subset$ Int(Cl(A,K)) and hence, f_{pu}^{-1} (A,K) $\subset$ f_{pu}^{-1}(Int(Cl(A,K))).Now Int (Cl(A,K)) is a soft regular open set over Y. By (a), f_{pu}^{-1} (Int(Cl(A,K))) is soft b-open set over X. Thus, f_{pu}^{-1} (A,K) $\subset$ f_{pu}^{-1} (Int(Cl(A,K))) = bInt(f_{pu}^{-1}(Int(Cl(A,K)))).

(c) $\Rightarrow$(a) Let (A,K) be a soft regular open set over Y, then we have f_{pu}^{-1} (A,K) $\subset$ bInt(f_{pu}^{-1}(Int(Cl(A,K)))) = bInt(f_{pu}^{-1}(A,K)).Thus, f_{pu}^{-1} (A,K) = bInt(f_{pu}^{-1}(A,K)) shows that f_{pu}^{-1} (A,K) is a soft b-open set over X.

(b) $\Rightarrow$(d) Since (G,K) is soft closed set over Y, Cl(Int(G,K)) $\subset$ (G,K) and f_{pu}^{-1}(Cl(Int (G,K))) $\subset$ f_{pu}^{-1} (G,K). Cl(Int(G,K)) is soft regular closed set over Y. Hence, f_{pu}^{-1}(Cl(Int(G,K) is soft b-closed set over X.Thus, bCl(f_{pu}^{-1}(Cl(Int(G,K)))) = f_{pu}^{-1}(Cl(Int(G,K))) $\subset$ f_{pu}^{-1} (G,K).

(d) $\Rightarrow$(b) Let (G,K) be a soft regular closed set over Y,then we have bCl(f_{pu}^{-1}(G,K)) = βCl(f_{pu}^{-1}(Cl(Int(G,K)))) $\subset$ f_{pu}^{-1} (G,K) .Thus, bCl(f_{pu}^{-1}(G,K)) $\subset$ f_{pu}^{-1} (G,K), shows that f_{pu}^{-1} (G,K) is soft b-closed set over X.

(a) $\Rightarrow$(e) Let $(x_e)_E$ be a soft point over X and (G,K) be a soft regular open set over Y such that $f_{pu}((x_e)_E) \in$ (G,K), Put (F,E) = f_{pu}^{-1}(G,K) .Then by (a), (F,E) is soft b-open set, $(x_e)_E \in$ (F,E) and (F,E) $\subset f_{pu}^{-1}$(G,K).

(e) $\Rightarrow$(f) Let $(x_e)_E$ be a soft point over X and (G,K) be a soft regular open set over Y such that $f_{pu}((x_e)_E)$. By (e) there exists a soft b-open set (F,E) such that $(x_e)_E \in$ (F,E), (F,E) $\subset f_{pu}^{-1}$(G,K). And so , we have $(x_e)_E \in$ (F,E), f_{pu}(F,E) $\subset$ $f_{pu}(f_{pu}^{-1}$(G,K)) $\subset$ (G,K).

(f) $\Rightarrow$(a) Let (G,K) be a soft regular open set over Y and $(x_e)_E$ be a soft point over X such that $(x_e)_E \in f_{pu}^{-1}$(G,K). Then $f_{pu}((x_e)_E) \in f_{pu}(\ f_{pu}^{-1}$(G,K)) $\subset$(G,K). By (f) ,there exists a soft b-open set (F,E) such that $(x_e)_E \in$ (F,E) and f_{pu}(F,E) $\subset$(G,K) .This shows that $(x_e)_E \in$ (F,E) $\subset f_{pu}^{-1}$(G,K). it follows that f_{pu}^{-1}(G,K) is soft b-open set and hence f_{pu}^{-1} is soft almost b-continuous.

Definition 3.12. Let (X ,τ,E) be soft topological space and (A,E) be a soft set over X is called soft δ-open if for each soft point $(x_e)_E \in$ (A,E),there exists a soft regular open set (F,E) such that $(x_e)_E \in$ (F,E) $\subset$ (A,E) and its complement is called soft δ-closed.

Definition 3.13. Let (X ,τ,E) be soft topological space and (A,E) be a soft set over X,

The intersection of all soft δ-closed sets containing a soft set (A,E) is called the δ-closure of (A,E) and is denoted by δCl(A,E).

Theorem 3.14. Let f_{pu} : (X ,τ,E) $\rightarrow$ (Y,ϑ,K) be a soft mapping. Then the following conditions are equivalent:

(a) f_{pu} is soft almost b-continuous.

(b) f_{pu}(bCl(A,E)) $\subset \delta$Cl(f_{pu}(A,E)), for every soft set (A,E) over X.

(c) bCl(f_{pu}^{-1}(B,K)) $\subset f_{pu}^{-1}(\delta$Cl(B,K)), for every soft set (B,K) over Y.

(d) f_{pu}^{-1}(F,K) is soft b-closed set over X, for every soft δ-closed set (F,K) over Y.

(e) f_{pu}^{-1}(V,K) is soft b-open set over X, for every soft δ-open set (V,K) over Y.

Proof: (a) $\rightarrow$ (b)Let (A,E) be a soft set over X.Since, δCl(f_{pu}(A,E)) is a soft δ-closed set over Y.By theorem 3.11,we have (A,E) $\subset f_{pu}^{-1}(\ \delta$Cl($f_{pu}$(A,E))) which is soft b-closed set over X.Hence,bCl(A,E) $\subset f_{pu}^{-1}(\ \delta$Cl($f_{pu}$(A,E))).Hence,we obtain f_{pu}(bCl(A,E)) $\subset \delta$Cl(f_{pu}(A,E)).

(b) $\rightarrow$ (c) Let (B,K) be a soft set over Y . We have f_{pu}(bCl(f_{pu}^{-1}(B,K))) $\subset$ δCl($f_{pu}(f_{pu}^{-1}$(B,K)) $\subset \delta$Cl(B,K) and hence, bCl(f_{pu}^{-1}(B,K)) $\subset f_{pu}^{-1}(\delta$Cl(B,K)).

(c) $\rightarrow$ (d) Let (F,K) be a soft δ-closed set over Y. We have bCl(f_{pu}^{-1}(F,K)) $\subset$ $f_{pu}^{-1}(\delta$Cl(F,K)) = f_{pu}^{-1}(F,K) and f_{pu}^{-1}(F,K) is soft b-closed over X.

(d) $\rightarrow$ (e) Let (V,K) be a soft δ-open set over Y. By (d), we have $f_{pu}^{-1}(V,K)^c$ = $(f_{pu}^{-1}(V,K))^c$, which is soft b-closed over X and so f_{pu}^{-1}(V,K) is soft b-open set in X.

(e)$\rightarrow$ (a) Let (V,K) be a soft regular open set over Y.Since (V,K) is soft δ-open set over Y, f_{pu}^{-1}(V,K) is soft b-open over X and hence by theorem 3.11,f_{pu} is soft almost b-continuous.

Theorem 3.15. Let f_{pu} : (X, τ, E) $\rightarrow$ (Y, ϑ, K) be a soft mapping and $G_{g_pg_u}$: (X ,τ,E) $\rightarrow$ (X $\times$ Y, $\tau \times \vartheta$, E $\times$ K) be the soft graph mapping of f_{pu}. Then g_{pu} is soft almost b-continuous mapping if and only if f_{pu} is soft almost b-continuous.

Proof: Necessity : Let $(x_e)_E \in$ be a soft point over X and (V,K) be a soft regular open set over Y containing $f_{pu}((x_e)_E)$. Then, we have $G_{g_pg_u} = ((x_e)_E, f_{pu}((x_e)_E)) \in$

(X $\times$ Y, τ $\times$ ϑ, E $\times$ K) which is soft regular open over (X $\times$ Y, τ $\times$ ϑ, E $\times$ K). Since $G_{g_pg_u}$ is soft almost b-continuous,there exists a soft b-open set (U,E) over X containing $(x_e)_E$ such that $G_{g_pg_u}$ (U,E) $\subset$ (X $\times$ Y, τ $\times$ ϑ , E $\times$ K). Therefore, we obtain f_{pu}(U,E) $\subset \tilde{Y}$ and hence, f_{pu} is soft almost b-continuous.

Sufficiency: Let $(x_e)_E$ be a soft point over X and (W, E $\times$ K) be a soft regular open set over (X $\times$ Y, τ $\times$ ϑ , E $\times$ K) containing $G_{g_pg_u}$ $((x_e)_E)$. There exist (U_1,E) be a soft regular open set Over X and (V,K) be a soft regular open set over Y such that $((x_e)_E$,$f_{pu}((x_e)_E))$ $\in$ (U_1,E) $\times$ (V,K) $\subset$ (W, E $\times$ K).Since f_{pu} is soft almost b-continuous ,there exists (U_2,E) be a soft b-open set Over X such that $(x_e)_E)$ $\in$ (U_2,E) and f_{pu}(U_2,E) $\subset$ (V,K). Put (U,E) = (U_1,E) $\cap$ (U_2,E).we obtain $(x_e)_E)$ $\in$ (U,E) which is soft b-open set over X and $G_{g_pg_u}$ (U,E) $\subset$ (U_1,E) $\times$ (V,K) $\subset$ (W, E $\times$ K).This shows that $G_{g_pg_u}$ is soft almost b-continuous.

Theorem 3.16. Let f_{pu} : (X ,τ,E) $\to$ (Y,ϑ,K) be a soft mapping from a soft topological space (X ,τ,E) to a soft semiregular space (Y,ϑ,K).Then f_{pu} is soft almost b-continuous if and only if f_{pu} is soft b-continuous.

Proof: Necessity: Let $(x_e)_E$ be a soft point over X and (F,K) be a soft open set over Y such that f_{pu}($(x_e)_E$) $\in$ (F,K) .Since (Y,ϑ,K) is soft semiregular there exists a soft open set (G,K) over Y such that $f_{pu}((x_e)_E)$ $\in$ (G,K) and (G,K) $\subset$ Int(Cl(G,K) $\subset$ (F,K). Since Int(Cl(G,K)) is soft regular open over Y and f_{pu} is soft almost b-continuous,by theorem 3.11 (f) there exists a soft b-open set (A,E) over X such that $(x_e)_E$ $\in$ (A,E) and f_{pu}(A,E) $\subset$ Int(Cl(G,K)).Thus, (A,E) is soft b-open set such that $(x_e)_E$ $\in$ (A,E) and f_{pu}(A,E) $\subset$ (F,K). Hence, f_{pu} is soft b-continuous.

Sufficiency : Obvious.

Lemma 3.17. If f_{pu} : (X ,τ,E) $\to$ (Y,ϑ,K) be a soft mapping and f_{pu} is a soft open and soft continuous mapping then f_{pu}^{-1}(G,K) is soft b-open over X for every (G,K) is soft b-open over Y.

Proof: Let (G,K) is soft b-open over Y. Then, (G,K) $\subseteq$ Int(Cl(Int(G,K))). Since f_{pu} is soft continuous we have,

f_{pu}^{-1}(G,K) $\subseteq$ f_{pu}^{-1}(Int(Cl(Int(G,K)))) $\subseteq$ Int(f_{pu}^{-1}(Cl(Int(G,K)))).

By the openness of f_{pu}, we have

f_{pu}^{-1}(Cl(Int(G,K))) $\subseteq$ Cl(f_{pu}^{-1}(Int(G,K))).

Again f_{pu} is soft continuous

f_{pu}^{-1}(Int(G,K)) $\subseteq$ Int(f_{pu}^{-1}(G,K)).

Thus,

f_{pu}^{-1}(G,K)$\subseteq$ Int(Cl(Int(f_{pu}^{-1}(G,K)))).

Consequently, f_{pu}^{-1}(G,K) is soft b-open over X.

Theorem 3.18. If soft mapping $f_{p_1u_1}$: (X ,τ,E) $\to$ (Y,ϑ,K) is soft open soft continuous and soft mapping $g_{p_2u_2}$:(Y,ϑ,K) $\to$ (Z,η,T) is soft almost b-continuous, then $g_{p_2u_2}$ o$f_{p_1u_1}$: (X ,τ,E) $\to$ (Z,η,T) is soft almost b-continuous.

Proof : Suppose (U,T) is a soft regular open set over Z. Then $g_{p_2u_2}^{-1}$(U,T) is a soft b-open set over Y because $g_{p_2u_2}$ is soft almost b-continuous. Since $f_{p_1u_1}$ being soft open and continuous.By lemma 3.17 ($f_{p_1u_1}^{-1}$ ($g_{p_2u_2}^{-1}$(U,T)) is soft b-open over X. Consequently, $g_{p_2u_2}$ o$f_{p_1u_1}$: (X ,τ,E) $\to$ (Z,η,T) is soft almost b-continuous.

Lemma 3.19. If (A,E) be a soft b-open set over X and (Y,E) is soft open in a soft topological space (X ,τ,E). Then (A,E) $\cap$ (Y,E) is soft b-open in (Y,E).

Proof: Obvious.

Theorem 3.20. Let f_{pu} : (X ,τ,E) $\rightarrow$ (Y,ϑ,K) be a soft almost b-continuous mapping and (A,E) is soft open set over X, Then f_{pu}/ (A,E) is soft almost b-continuous.

Proof : Let (G,K) be a soft regular open set in Y then f_{pu}^{-1} (G,K) is soft b-open in X. Since (A,E) is soft open in X, By lemma 3.19 (A,E) $\cap$ f_{pu}^{-1} (G,K) = $[f_{pu}/(A, E)]^{-1}$ (G,K) is soft b-open in (A,E). Therefore, f_{pu}/ (A,E) is soft almost b-continuous.

4 Soft Almost b-Open Mappings

Definition 4.1. A soft mapping f_{pu} : (X ,τ,E) $\rightarrow$ (Y,ϑ,K) is said to be soft almost b-open if for each soft regular open set (F,E) over X, f_{pu}(F,E) is soft b-open in Y.

Remark 4.2. Every soft b-open mapping is soft almost b-open but the converse may not be true.

Example 4.3. Let X = $\{x_1, x_2\}$, E = $\{e_1, e_2\}$ and Y = $\{y_1, y_2\}$, K = $\{k_1, k_2\}$. The soft sets (F,E), (G,K) are defined as follows :

F(e_1) = $\{x_1\}$, F(e_2) = $\{x_2\}$,

G(k_1) = $\{y_2\}$, G(k_2) = $\{y_1\}$.

Let τ = $\{\phi$,(F,E), $\tilde{X}$ $\}$, and υ = $\{\phi$,(G,K), $\tilde{Y}$ $\}$ are topologies on X and Y respectively. Then soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,υ,K) defined by u(x_1) = y_1 , u(x_2) = y_2 and p(e_1)= k_1, p(e_2) = k_2 is soft almost b-open but not soft b-open.

Remark 4.4. Every soft almost semi-open is soft almost b-open but the converse may not be true.

Example 4.5. Let X = $\{x_1, x_2\}$, E = $\{e_1, e_2\}$ and Y = $\{y_1, y_2\}$, K = $\{k_1, k_2\}$. The soft sets (F_1,E), (F_2,E) are defined as follows :

$F_1(e_1) = \{x_1\}$, $F_1(e_2) = \{x_2\}$,

$F_2(e_1) = \{x_2\}$, $F_2(e_2) = \{x_1\}$.

Let τ = $\{\phi$,(F_1,E) ,(F_2,E), $\tilde{X}$ $\}$, and υ = $\{\phi$, $\tilde{Y}$ $\}$ are topologies on X and Y respectively. Then soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,υ,K) defined by u(x_1) = y_1 , u(x_2)= y_2 and p(e_1)= k_1 , p(e_2) = k_2 is soft almost b-open mapping but not soft almost semi-open.

Remark 4.6. Every soft almost pre-open is soft almost b-open but the converse may not be true.

Example 4.7. Let X = $\{x_1, x_2\}$, E = $\{e_1, e_2\}$ and Y = $\{y_1, y_2\}$, K = $\{k_1, k_2\}$. The soft sets (F_1,E), (F_2,E), (G_1,K), (G_2,K) and (G_3,K) are defined as follows :

$F_1(e_1) = \{x_1\}$, $F_1(e_2) = \{x_2\}$,

$F_2(e_1) = \{x_2\}$, $F_2(e_2) = \{x_1\}$,

$F_3(e_1) = \phi$, $F_3(e_2) = \{y_1\}$,

$G_1(k_1) = \{y_1\}$, $G_1(k_2) = \phi$,

$G_2(k_1) = \{y_1\}$, $G_2(k_2) = \{y_1\}$.

Let τ = $\{\phi$, (F_1,E), (F_2,E), $\tilde{X}$ $\}$, and υ = $\{\phi$, (G_1,K),(G_2,K), (G_3,K),$\tilde{Y}$ $\}$ are topologies on X and Y respectively. Then soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,υ,K) defined by u(x_1) = y_1 , u(x_2)= y_2 and p(e_1)= k_1 , p(e_2) = k_2 is soft almost b-open but not soft almost pre-open.

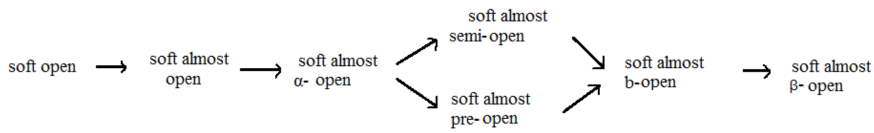
Remark 4.8. Every soft almost b-open mapping is soft almost β-open but the converse may not be true.

Example 4.9. Let X = $\{x_1,\ x_2,x_3,x_4\ \}$, E = $\{e_1\ ,\ e_2\ \}$ and Y = $\{y_1,\ y_2,y_3,y_4\}$, K = $\{k_1,\ k_2\ \}$. The soft sets (F_1,E),(F_2,E),(F_3,E),(G_1,K),(G_2,K) and (G_3,K) are defined as follows :

$F_1(e_1) = \{x_3\}$, $F_1(e_2) = \phi$,
$F_2(e_1) = \{x_1,x_4\}$, $F_2(e_2) = \phi$,
$F_3(e_1) = \{x_1,x_3,x_4\}$, $F_3(e_2) = \phi$,
$G_1(k_1) = \{y_3\}$, $G_1(k_2) = \phi$,
$G_2(k_1) = \{y_1,y_4\}$, $G_2(k_2) = \phi$,
$G_3(k_1) = \{y_1,y_3,y_4\}$, $G_3(k_2) = \phi$.

Let $\tau = \{\phi$, (F_1,E), (F_2,E), (F_3,E), $\tilde{X}$ $\}$, and $\upsilon = \{\phi$,(G_1,K),(G_2,K),(G_3,K), $\tilde{Y}$ $\}$ are topologies on X and Y respectively. Then soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,υ,K) defined by u(x_1) = u(x_2)= y_1 ,u(x_3) = y_3,u(x_4) = y_4 and p(e_1)= k_1 , p(e_2) = k_2 is soft almost β-open mapping but not soft almost b-open.

Thus we reach at the following diagram of implications.



Theorem 4.10. Let $f_{p_1u_1}$: (X ,τ,E) $\rightarrow$ (Y,ϑ,K) and $g_{p_2u_2}$:(Y,ϑ,K) $\rightarrow$ (Z,η,T) be two soft mappings, If $f_{p_1u_1}$ is soft almost open and $g_{p_2u_2}$ is soft b-open. Then the soft mapping $g_{p_2u_2}$ o$f_{p_1u_1}$: (X ,τ,E) $\rightarrow$ (Z,η,T) is soft almost b-open.

Proof : Let (F,E) be soft regular open in X. Then $f_{p_1u_1}$(F,E) is soft open over Y because $f_{p_1u_1}$ is soft almost open.Therefore, $g_{p_2u_2}$ ($f_{p_1u_1}$)(F,E) is soft b-open over Z. Because $g_{p_2u_2}$ is soft b-open. Since ($g_{p_2u_2}$ o$f_{p_1u_1}$)(F,E) = ($g_{p_2u_2}$($f_{p_1u_1}$ (F,E)), it follows that the soft mapping ($g_{p_2u_2}$ o$f_{p_1u_1}$) is soft almost b-open.

Theorem 4.11. Let $f_{p_1u_1}$: (X ,τ,E) $\rightarrow$ (Y,ϑ,K) and $g_{p_2u_2}$:(Y,ϑ,K) $\rightarrow$ (Z,η,T) be two soft mappings, such that $g_{p_2u_2}$ o$f_{p_1u_1}$: (X ,τ,E) $\rightarrow$ (Z,η,T) is soft almost b-open and $g_{p_2u_2}$ is soft b-irresolute and injective then $f_{p_1u_1}$ is soft almost b-open.

Proof : Suppose (F,E) is soft regular open set over X. Then $g_{p_2u_2}$ o$f_{p_1u_1}$(F,E) is soft b-open over Z because $g_{p_2u_2}$ o$f_{p_1u_1}$ is soft almost b-open. Since $g_{p_2u_2}$ is injective, we have ($g^{-1}_{p_2u_2}$ ($g_{p_2u_2}$ o$f_{p_1u_1}$)(F,E)) = $f_{p_1u_1}$ (F,E). Therefore $f_{p_1u_1}$ (F, E) is soft b-open over Y, because $g_{p_2u_2}$ is soft b-irresolute. This implies $f_{p_1u_1}$ is soft almost b-open.

Theorem 4.12. Let soft mapping f_{pu} : (X ,τ,E) $\rightarrow$ (Y,ϑ,K) be soft almost b-open mapping. If (G,K) is soft set over Y and (F,E) is soft regular closed set of X containing f^{-1}_{pu} (G,K) then there is a soft b-closed set (A,K) over Y containing (G,K) such that f^{-1}_{pu} (A,K) $\subset$ (F,E).

Proof: Let (A, K) = $(f_{pu}(F,E)^C)^C$. Since f^{-1}_{pu} (G,K) $\subset$ (F,E) we have $f_{pu}(F,E)^C$ $\subset$ (G,K). Since f_{pu} is soft almost b-open then (A,K) is soft b-closed set of Y and f^{-1}_{pu} (A,K) = (f^{-1}_{pu} $(f_{pu}(F,E)^C)^C \subset ((F,E)^C)^C$ = (F,E). Thus, f^{-1}_{pu} (A,K) $\subset$ (F,E).

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