http://www.newtheory.org **ISSN: 2149-1402**

Journal of **New Theory**

Received: 29.04.2018 *Year*:**2018**, *Number*: **24**, *Pages*: **35-43**
Published: 18.08.2018 *Original Article*

Simple Forms of Nano Open Sets in an Ideal Nano Topological Spaces

Ilangovan Rajasekaran[1,*] <sekarmelakkal@gmail.com>
Ochanan Nethaji[2] <jionetha@yahoo.com>

[1]Department of Mathematics, Tirunelveli Dakshina Mara Nadar Sangam College, T. Kallikulam-627 113, Tirunelveli District, Tamil Nadu, India.
[2]School of Mathematics, Madurai Kamaraj University, Madurai, Tamil Nadu, India.

Abstaract — In this paper, we introduce and study the new concepts called α-nI-open sets, semi-nI-open sets, pre-nI-open sets, b-nI-open sets and β-nI-open sets, which are simple forms of nano open sets in an ideal nano topological spaces. Also we characterize the relations between them and the related properties. And the independence of n-open sets and nI-open sets is established.

Keywords — *n-open set, semi-nI-open set, α-nI-open set, pre-nI-open set.*

1 Introduction

An ideal I [10] on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

1. $A \in I$ and $B \subset A$ imply $B \in I$ and
2. $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a topological space (X, τ) with an ideal I on X. If $\wp(X)$ is the family of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [2]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I, \tau)$ [9] is a Kuratowski closure operator which generates a topology $\tau^\star(I, \tau)$ called the $\star$-topology finer than τ. The topological space together with an ideal on X is called an ideal topological space or an ideal space denoted by (X, τ, I). We will simply write $A^\star$ for $A^\star(I, \tau)$ and $\tau^\star$ for $\tau^\star(I, \tau)$.

Some new notions in the concept of ideal nano topological spaces were introduced by Parimala et al. [4, 5].

**Corresponding Author.*

In this paper, we introduce and study the new concepts called α-nI-open sets, semi-nI-open sets, pre-nI-open sets, b-nI-open sets and β-nI-open sets, which are simple forms of nano open sets in an ideal nano topological spaces. Also we characterize the relations between them and the related properties. And the independence of n-open sets and nI-open sets is established.

2 Preliminaries

Definition 2.1. *[7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Definition 2.2. *[3] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$,*

2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*

3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by n-$int(A)$ and n-$cl(A)$, respectively.

Definition 2.3. *A subset A of a space $(U, \mathcal{N})$ is called*

1. *nano α-open [3] if $A \subseteq n\text{-}int(n\text{-}cl(n\text{-}int(A)))$.*

2. *nano semi-open [3] if $A \subseteq n\text{-}cl(n\text{-}int(A))$.*

3. *nano pre-open [3] if* $A \subseteq n\text{-}int(n\text{-}cl(A))$.

4. *nano b-open [6] if* $A \subseteq n\text{-}int(n\text{-}cl(A)) \cup n\text{-}cl(n\text{-}int(A))$.

5. *nano* β*-open [8] if* $A \subseteq n\text{-}cl(n\text{-}int(n\text{-}cl(A)))$.

The complements of the above mentioned sets are called their respective closed sets.

A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [4] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [4] the family of nano open sets containing x.

In future an ideal nano topological spaces $(U, \mathcal{N}, I)$ is referred as a space.

Definition 2.4. *[4] Let* $(U, \mathcal{N}, I)$ *be a space with an ideal* I *on* U*. Let* $(.)^\star_n$ *be a set operator from* $\wp(U)$ *to* $\wp(U)$ *(*$\wp(U)$ *is the set of all subsets of* U*). For a subset* $A \subseteq U$, $A^\star_n(I, \mathcal{N}) = \{x \in U : G_n \cap A \notin I$, *for every* $G_n \in G_n(x)\}$ *is called the nano local function (briefly, n-local function) of* A *with respect to* I *and* $\mathcal{N}$*. We will simply write* $A^\star_n$ *for* $A^\star_n(I, \mathcal{N})$.

Theorem 2.5. *[4] Let* $(U, \mathcal{N}, I)$ *be a space and* A *and* B *be subsets of* U*. Then*

1. $A \subseteq B \Rightarrow A^\star_n \subseteq B^\star_n$,

2. $A^\star_n = n\text{-}cl(A^\star_n) \subseteq n\text{-}cl(A)$ *(*$A^\star_n$ *is a n-closed subset of* $n\text{-}cl(A)$*),*

3. $(A^\star_n)^\star_n \subseteq A^\star_n$,

4. $(A \cup B)^\star_n = A^\star_n \cup B^\star_n$,

5. $V \in \mathcal{N} \Rightarrow V \cap A^\star_n = V \cap (V \cap A)^\star_n \subseteq (V \cap A)^\star_n$,

6. $J \in I \Rightarrow (A \cup J)^\star_n = A^\star_n = (A - J)^\star_n$.

Theorem 2.6. *[4] Let* $(U, \mathcal{N}, I)$ *be a space with an ideal* I *and* $A \subseteq A^\star_n$*, then* $A^\star_n = n\text{-}cl(A^\star_n) = n\text{-}cl(A)$.

Definition 2.7. *[4] Let* $(U, \mathcal{N}, I)$ *be a space. The set operator* $n\text{-}cl^\star$ *called a nano* $\star$*-closure is defined by* $n\text{-}cl^\star(A) = A \cup A^\star_n$ *for* $A \subseteq X$.

It can be easily observed that $n\text{-}cl^\star(A) \subseteq n\text{-}cl(A)$.

Theorem 2.8. *[5] In a space* $(U, \mathcal{N}, I)$*, if* A *and* B *are subsets of* U*, then the following results are true for the set operator* $n\text{-}cl^\star$.

1. $A \subseteq n\text{-}cl^\star(A)$,

2. $n\text{-}cl^\star(\phi) = \phi$ *and* $n\text{-}cl^\star(U) = U$,

3. *If* $A \subset B$*, then* $n\text{-}cl^\star(A) \subseteq n\text{-}cl^\star(B)$,

4. $n\text{-}cl^\star(A) \cup n\text{-}cl^\star(B) = n\text{-}cl^\star(A \cup B)$,

5. $n\text{-}cl^\star(n\text{-}cl^\star(A)) = n\text{-}cl^\star(A)$.

Definition 2.9. *[5]*

A subset A *of a space* $(U, \mathcal{N}, I)$ *is said to be nano-I-open (briefly,* nI*-open) if* $A \subseteq n\text{-}int(A^\star_n)$.

3 Simple Forms of n-open Sets in $(U, \mathcal{N}, I)$

Definition 3.1. *A subset A of space $(U, \mathcal{N}, I)$ is said to be*

1. *nano α-I-open (briefly α-nI-open) if $A \subset n\text{-}int(n\text{-}cl^\star(n\text{-}int(A)))$,*
2. *nano semi-I-open (briefly semi-nI-open) if $A \subset n\text{-}cl^\star(n\text{-}int(A))$,*
3. *nano pre-I-open (briefly pre-nI-open) if $A \subset n\text{-}int(n\text{-}cl^\star(A))$,*
4. *nano b-I-open (briefly b-nI-open) if $A \subset n\text{-}int(n\text{-}cl^\star(A)) \cup n\text{-}cl^\star(n\text{-}int(A))$,*
5. *nano β-I-open (briefly β-nI-open) if $A \subset n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A)))$.*

The complements of the above mentioned sets are called their respective closed sets.

Theorem 3.2. *In a space $(U, \mathcal{N}, I)$, for a subset A, the following relations hold.*

1. A is n-open $\Rightarrow$ A is α-nI-open.
2. A is α-nI-open $\Rightarrow$ A is semi-nI-open.
3. A is α-nI-open $\Rightarrow$ A is pre-nI-open.
4. A is semi-nI-open $\Rightarrow$ A is b-nI-open.
5. A is pre-nI-open $\Rightarrow$ A is b-nI-open.
6. A is b-nI-open $\Rightarrow$ A is β-nI-open.

Proof.

1. A is n-open $\Rightarrow A = n\text{-}int(A)$. But $A \subseteq n\text{-}cl^\star(A) = n\text{-}cl^\star(n\text{-}int(A)) \subseteq n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A)))$ which proves that A is α-nI-open.
2. A is α-nI-open $\Rightarrow A \subseteq n\text{-}int(n\text{-}cl^\star(n\text{-}int(A))) \subseteq n\text{-}cl^\star(n\text{-}int(A))$ which proves that A is semi-nI-open.
3. A is α-nI-open $\Rightarrow A \subseteq n\text{-}int(n\text{-}cl^\star(n\text{-}int(A))) \subseteq n\text{-}int(n\text{-}cl^\star(A))$ which proves that A is pre-nI-open.
4. A is semi-nI-open $\Rightarrow A \subseteq n\text{-}cl^\star(n\text{-}int(A)) \subseteq n\text{-}cl^\star(n\text{-}int(A)) \cup n\text{-}int(n\text{-}cl^\star(A))$ which proves that A is b-nI-open.
5. A is pre-nI-open $\Rightarrow A \subseteq n\text{-}int(n\text{-}cl^\star(A)) \subseteq n\text{-}int(n\text{-}cl^\star(A)) \cup n\text{-}cl^\star(n\text{-}int(A))$ which proves that A is b-nI-open.
6. A is b-nI-open $\Rightarrow A \subseteq n\text{-}int(n\text{-}cl^\star(A)) \cup n\text{-}cl^\star(n\text{-}int(A)) \subseteq n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A))) \cup n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A))) = n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A)))$ which proves that A is β-nI-open.

Remark 3.3. *These relations are shown in the diagram.*

$$\begin{tikzcd} n\text{-}\boldsymbol{open} \arrow[r] & \alpha\text{-}nI\text{-}\boldsymbol{open} \arrow[r] \arrow[d] & \boldsymbol{pre}\text{-}nI\text{-}\boldsymbol{open} \arrow[d] \\ & \boldsymbol{semi}\text{-}nI\text{-}\boldsymbol{open} \arrow[r] & \boldsymbol{b}\text{-}nI\text{-}\boldsymbol{open} \arrow[d] \\ & & \beta\text{-}nI\text{-}\boldsymbol{open} \end{tikzcd}$$

The converses of each statement in Theorem 3.2 are not true as shown in the following Example.

Example 3.4. *1. Let $U = \{e_1, e_2, e_3, e_4, e_5\}$ with $U/R = \{\{e_1\}, \{e_2, e_3\}, \{e_4, e_5\}\}$ and $X = \{e_1, e_2\}$. Then $\mathcal{N} = \{\phi, U, \{e_1\}, \{e_2, e_3\}, \{e_1, e_2, e_3\}\}$. Let the ideal be $I = \{\phi, \{e_2\}\}$.*

(a) Then $A = \{e_1, e_2, e_3, e_4\}$ is α-nI-open but not n-open.
n-int$(A) = \{e_1, e_2, e_3\}$ and $\{e_1, e_2, e_3\}^\star_n = \{e_1, e_2, e_3, e_4, e_5\} = U$. Therefore n-cl*(n-int$(A)) = U$ and n-int(n-cl*(n-int$(A))) = U \supseteq A$. Thus A is α-nI-open. But A is not n-open.

(b) $B = \{e_2, e_3, e_4\}$ is semi-nI-open but not α-nI-open.

(c) $F = \{e_3, e_4\}$ is β-nI-open but not b-nI-open.

2. Let $U = \{e_1, e_2, e_3, e_4\}$ with $U/R = \{\{e_1\}, \{e_3\}, \{e_2, e_4\}\}$ and $X = \{e_1, e_2\}$. Then $\mathcal{N} = \{\phi, U, \{e_1\}, \{e_2, e_4\}, \{e_1, e_2, e_4\}\}$. Let the ideal be $I = \{\phi, \{e_1\}\}$.

(a) $C = \{e_2\}$ is pre-nI-open but not α-nI-open.

(b) $D = \{e_1, e_4\}$ is b-nI-open but not semi-nI-open.

(c) $E = \{e_2, e_3, e_4\}$ is b-nI-open but not pre-nI-open.

Remark 3.5. *In a space the family of n-open sets and the family of nI-open sets are independent.*

Example 3.6. *In Example 3.4(2), $A = \{e_2\}$ is nI-open but not n-open and $B = \{e_1, e_2, e_4\}$ is n-open but not nI-open.*

Theorem 3.7. *A subset A of a space $(U, \mathcal{N}, I)$ is α-nI-open $\iff$ A is semi-nI-open and pre-nI-open.*

Proof. $\Rightarrow$ Part follows from (2) and (3) of Theorem 3.2.

$\Leftarrow$ If A is semi-nI-open and pre-nI-open then $A \subseteq n\text{-}int(n\text{-}cl^\star(A))$ and $A \subseteq n\text{-}cl^\star(n\text{-}int(A))$.

Thus $A \subseteq n\text{-}int(n\text{-}cl^\star(A)) \subseteq n\text{-}int(n\text{-}cl^\star(n\text{-}cl^\star(n\text{-}int(A))) = n\text{-}int(n\text{-}cl^\star(n\text{-}int(A)))$ which proves that A is α-nI-open.

Remark 3.8. *In a space $(U, \mathcal{N}, I)$, the family of semi-nI-open sets and the family of pre-nI-open sets are independent of each other as shown in the following Example.*

Example 3.9. *Let $U = \{p, q, r, s\}$ with $U/R = \{\{p\}, \{s\}, \{q, r\}\}$ and $X = \{p, r\}$. Then $\mathcal{N} = \{\phi, U, \{p\}, \{q, r\}, \{p, q, r\}\}$. Let the ideal be $I = \{\phi, \{r\}\}$. Then the subset*

1. *$\{p, s\}$ is semi-nI-open but not pre-nI-open.*

2. *$\{q\}$ is pre-nI-open but not semi-nI-open.*

Theorem 3.10. *If a subset A of a space $(U, \mathcal{N}, I)$ is both $n\star$-closed and β-nI-open, then A is semi-nI-open.*

Proof. Since A is β-nI-open, $A \subset n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A))) = n\text{-}cl^\star(n\text{-}int(A))$, A being $n\star$-closed. Therefore A is semi-nI-open.

Theorem 3.11. *A subset A of a space $(U, \mathcal{N}, I)$ is semi-nI-open if and only if $n\text{-}cl^\star(A) = n\text{-}cl^\star(n\text{-}int(A))$.*

Proof. Let A be semi-nI-open. Then $A \subset n\text{-}cl^\star(n\text{-}int(A))$ and $n\text{-}cl^\star(A) \subset n\text{-}cl^\star(n\text{-}int(A))$. But $n\text{-}cl^\star(n\text{-}int(A)) \subset n\text{-}cl^\star(A)$. Thus $n\text{-}cl^\star(A) = n\text{-}cl^\star(n\text{-}int(A))$.

Conversely, let the condition hold. We have $A \subset n\text{-}cl^\star(A) = n\text{-}cl^\star(n\text{-}int(A))$, by assumption. Thus $A \subset n\text{-}cl^\star(n\text{-}int(A))$ and hence A is semi-nI-open.

Proposition 3.12. *In (U, $\mathcal{N}$, I) if A is a b-nI-open set such that $n\text{-}cl^\star(A) = \phi$, then A is semi-nI-open.*

Theorem 3.13. *A subset A of a space $(U, \mathcal{N}, I)$ is semi-nI-open if and only if there exists a n-open set G such that $G \subset A \subset n\text{-}cl^\star(G)$.*

Proof. Let A be semi-nI-open. Then $A \subset n\text{-}cl^\star(n\text{-}int(A))$. Take $n\text{-}int(A) = G$. Then $G \subset A \subset n\text{-}cl^\star(G)$, where G is n-open.

Conversely, let $G \subset A \subset n\text{-}cl^\star(G)$ for some n-open set G. Since $G \subset A$, $G \subset n\text{-}int(A)$ and $A \subset n\text{-}cl^\star(G) \subset n\text{-}cl^\star(n\text{-}int(A))$ which implies A is semi-nI-open.

Theorem 3.14. *If A is a semi-nI-open set in a space $(U, \mathcal{N}, I)$ and $A \subset B \subset n\text{-}cl^\star(A)$, then B is semi-nI-open.*

Proof. By assumption $B \subset n\text{-}cl^\star(A) \subset n\text{-}cl^\star(n\text{-}cl^\star(n\text{-}int(A)))$ (for A is semi-nI-open) $= n\text{-}cl^\star(n\text{-}int(A)) \subset n\text{-}cl^\star(n\text{-}int(B))$ by assumption. This implies B is semi-nI-open.

Theorem 3.15. *In a space $(U, \mathcal{N}, I)$, for a subset A, the following results hold.*

1. A is nI-open $\Rightarrow$ A is pre-nI-open.
2. A is nI-open $\Rightarrow$ A is β-nI-open.
3. A is nI-open $\Rightarrow$ A is b-nI-open.

Proof.

1. A is nI-open $\Rightarrow A \subseteq n\text{-}int(A_n^\star) \subseteq n\text{-}int(n\text{-}cl^\star(A))$ which proves that A is pre-nI-open.
2. A is nI-open $\Rightarrow A \subseteq n\text{-}int(A_n^\star) \subseteq n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A)))$ which proves that A is β-nI-open.
3. A is nI-open $\Rightarrow A \subseteq n\text{-}int(A_n^\star) \subseteq n\text{-}int(n\text{-}cl^\star(A)) \subseteq n\text{-}int(n\text{-}cl^\star(A)) \cup n\text{-}cl^\star(n\text{-}int(A))$ which proves that A is b-nI-open.

Remark 3.16. *The converses of (1), (2) and (3) in Theorem 3.15 are not true as shown in the following Example.*

Example 3.17. *In Example 3.4 (2),*

1. $A = \{e_1\}$ *is pre-nI-open but not nI-open and b-nI-open but not nI-open.*

2. *$A = \{e_3, e_4\}$ is β-nI-open but not nI-open.*

Remark 3.18. *1. In a space $(U, \mathcal{N}, I)$, the family of nI-open sets and the family of α-nI-open sets are independent of each other.*

2. In a space $(U, \mathcal{N}, I)$, the family of nI-open sets and the family of semi-nI-open sets are independent of each other.

Example 3.19. *In Example 3.4(2),*

1. $A = \{e_2\}$ is nI-open but not α-nI-open.
2. $B = \{e_1\}$ is α-nI-open but not nI-open.
   Examples (1) and (2) verify (1) of Remark 3.18.
3. $C = \{e_2\}$ is nI-open but not semi-nI-open.
4. $D = \{e_1\}$ is semi-nI-open but not nI-open.
   Examples (3) and (4) verify (2) of Remark 3.18.

Proposition 3.20. *For a subset of A a space $(U, \mathcal{N}, I)$, the following properties hold:*

1. *A is α-nI-open $\Rightarrow$ A is nano α-open.*
2. *A is pre-nI-open $\Rightarrow$ A is nano pre-open.*
3. *A is b-nI-open $\Rightarrow$ A is nano b-open.*
4. *A is β-nI-open $\Rightarrow$ A is nano β-open.*

Proof. 1. Let A be a α-nI-open set. Then $A \subset n\text{-}int(n\text{-}cl^\star(n\text{-}int(A))) \subset n\text{-}int(n\text{-}cl(n\text{-}int(A)))$. This shows that A is nano α-open.

2. Let A be a pre-nI-open set. Then $A \subset n\text{-}int(n\text{-}cl^\star(A)) \subset n\text{-}int(n\text{-}cl(A))$. This shows that A is nano pre-open.

3. Let A be a b-nI-open set. Then $H \subset n\text{-}int(n\text{-}cl^\star(A)) \cup n\text{-}cl^\star(n\text{-}int(A)) \subset n\text{-}int(n\text{-}cl(A)) \cup n\text{-}cl(n\text{-}int(A))$. This shows that A is nano b-open.

4. Let A be a β-nI-open set. Then $A \subset n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A))) \subset n\text{-}cl(n\text{-}int(n\text{-}cl(A)))$. This shows that A is nano β-open.

Remark 3.21. *The converses of Proposition 3.20 are not true in general as shown in the following Example.*

Example 3.22. *1. Let $U = \{e_1, e_2, e_3, e_4, e_5\}$ with $U/R = \{\{e_1\}, \{e_2, e_3\}, \{e_4, e_5\}\}$, $X = \{e_1, e_2\}$ and $I = \wp(U)$. Then in the space $(U, \mathcal{N}, I)$, $\mathcal{N} = \{\phi, U, \{e_1\}, \{e_2, e_3\}, \{e_1, e_2, e_3\}\}$. $A = \{e_1, e_2, e_3, e_4\}$ is nano α-open but not α-nI-open since $n\text{-}cl^\star(A) = A$.*

2. Let $U = \{k_1, k_2, k_3\}$ with $U/R = \{\{k_1\}, \{k_2, k_3\}\}$ and $X = \{k_1, k_2\}$. Then $\mathcal{N} = \{\phi, U, \{k_1\}, \{k_2, k_3\}\}$. Let the ideal be $I = \{\phi, k_2\}$. Then in $(U, \mathcal{N}, I)$, $B = \{k_1, k_2\}$ is nano pre-open but pre-nI-open.

3. *In Example 3.4(2),*

   (a) $C = \{e_1, e_3\}$ *is nano b-open but not b-nI-open.*

   (b) $D = \{e_1, e_3\}$ *is nano β-open but not β-nI-open.*

Lemma 3.23. *Let $(U, \mathcal{N}, I)$ be a space and A a subset of U. If H is n-open in $(U, \mathcal{N}, I)$, then $H \cap n\text{-}cl^\star(A) \subseteq n\text{-}cl^\star(H \cap A)$.*

Proof. $H \cap\ n\text{-}cl^\star(A) = H \cap (A^\star_n \cup A) = (H \cap A^\star_n) \cup (H \cap A) \subseteq (H \cap A)^\star_n \cup (H \cap A)$ by (5) of Theorem 2.5. Thus $H \cap n\text{-}cl^\star(A) \subseteq (H \cap A)^\star_n \cup (H \cap A) = n\text{-}cl^\star(H \cap A)$.

Proposition 3.24. *The intersection of a pre-nI-open set and n-open set is pre-nI-open.*

Proof. Let A be pre-nI-open and G be n-open. Then $A \subset n\text{-}int(n\text{-}cl^\star(A))$ and $G \cap A \subset n\text{-}int(G) \cap n\text{-}int(n\text{-}cl^\star(A)) = n\text{-}int(G \cap n\text{-}cl^\star(A)) \subset n\text{-}int(n\text{-}cl^\star(G \cap A))$ by Lemma 3.23. This shows that $G \cap A$ is pre-nI-open.

Proposition 3.25. *The intersection of a semi-nI-open set and n-open set is semi-nI-open.*

Proof. Let A be semi-nI-open and G be n-open in U. Then $A \subset n\text{-}cl^\star(n\text{-}int(A))$ and $n\text{-}int(G) = G$. $G \cap A \subset G \cap n\text{-}cl^\star(n\text{-}int(A)) \subseteq n\text{-}cl^\star(G \cap n\text{-}int(A)) = n\text{-}cl^\star(n\text{-}int(G) \cap n\text{-}int(A)) = n\text{-}cl^\star(n\text{-}int(G \cap A))$ by Lemma 3.23. Hence A is semi-nI-open.

Proposition 3.26. *The intersection of a α-nI-open set and n-open set is α-nI-open.*

Proof. Let G be a n-open and A be an α-nI-open in a space $(U, \mathcal{N}, I)$. Then A is both pre-nI-open and semi-nI-open by (2) and (3) of Theorem 3.2. $A \cap G$ is both pre-nI-open and semi-nI-open by Proposition 3.24 and 3.25. Hence by Theorem 3.7, $A \cap G$ is α-nI-open.

Proposition 3.27. *The intersection of a b-nI-open set and n-open set is b-nI-open.*

Proof. Let A be b-nI-open and G be n-open. Then $A \subset n\text{-}int(n\text{-}cl^\star(A)) \cup n\text{-}cl^\star(n\text{-}int(A))$ and $G \cap A \subset G \cap [n\text{-}int(n\text{-}cl^\star(A)) \cup n\text{-}cl^\star(n\text{-}int(A))] = [G \cap n\text{-}int(n\text{-}cl^\star(A))] \cup [G \cap n\text{-}cl^\star(n\text{-}int(A))] = [n\text{-}int(G) \cap n\text{-}int(n\text{-}cl^\star(A))] \cup [G \cap n\text{-}cl^\star(n\text{-}int(A))] \subset [n\text{-}int(G \cap n\text{-}cl^\star(A))] \cup [n\text{-}cl^\star(G \cap n\text{-}int(A))]$ by Lemma 3.23. Thus $G \cap A \subset [n\text{-}int(n\text{-}cl^\star(G \cap A))] \cup [n\text{-}cl^\star(n\text{-}int(G \cap A))]$. This shows that $G \cap A$ is b-nI-open.

Proposition 3.28. *The intersection of a β-nI-open set and n-open set is β-nI-open.*

Proof. Let A be β-nI-open and G be n-open. Then $A \subset n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A)))$ and $G \cap A \subset G \cap\ n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A))) \subset n\text{-}cl^\star(G \cap\ n\text{-}int(n\text{-}cl^\star(A))) \subset n\text{-}cl^\star(n\text{-}int(G) \cap n\text{-}int(n\text{-}cl^\star(A))) = n\text{-}cl^\star(n\text{-}int(G \cap n\text{-}cl^\star(A))) \subset n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(G \cap A)))$ by Lemma 3.23. This shows that $G \cap A$ is β-nI-open.

Remark 3.29. *The intersection of two semi-nI-open (resp. pre-nI-open, b-nI-open, β-nI-open) sets need not be semi-nI-open (resp. pre-nI-open, b-nI-open, β-nI-open) as shown in the following Example.*

Example 3.30. 1. *In Example 3.9, $H = \{p, s\}$ and $K = \{q, r, s\}$ are semi-nI-open. But $H \cap K = \{s\}$ is not semi-nI-open.*

2. *In Example 3.4(2),*

   (a) $H = \{e_1, e_2, e_3\}$ *and* $K = \{e_1, e_3, e_4\}$ *are pre-nI-open. But* $H \cap K = \{e_1, e_3\}$ *is not pre-nI-open.*

   (b) $H = \{e_1, e_2, e_3\}$ *and* $K = \{e_2, e_3, e_4\}$ *are b-nI-open. But* $H \cap K = \{e_2, e_3\}$ *is not b-nI-open.*

   (c) $H = \{e_2, e_3\}$ *and* $K = \{e_3, e_4\}$ *are* β*-nI-open. But* $H \cap K = \{e_3\}$ *is not* β*-nI-open.*

References

[1] D. Jankovi'c and T. R. Hamlett, *Compatible extensions of ideals*, Boll. Un. Mat. Ital., 7 (6-B) (1992), 453-465.

[2] K. Kuratowski, *Topology*, Vol I. Academic Press (New York) 1966.

[3] M. L. Thivagar and C. Richard, *On nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention, 1 (1) (2013), 31-37.

[4] M. Parimala, T. Noiri and S. Jafari, *New types of nano topological spaces via nano ideals* (to appear).

[5] M. Parimala and S. Jafari, *On some new notions in nano ideal topological spaces*, International Balkan Journal of Mathematics (IBJM), 1(3)(2018), 85-92.

[6] M. Parimala, C. Indirani and S. Jafari, *On nano b-open sets in nano topological spaces*, Jordan Journal of Mathematics and Statistics 9 (3) (2016), 173-184.

[7] Z. Pawlak, *Rough sets*, International journal of computer and Information Sciences, 11 (5) (1982), 341-356.

[8] A. Revathy and G. Ilango, *On nano β-open sets*, Int. Jr. of Engineering, Contemporary Mathematics and Sciences, 1 (2) (2015), 1-6.

[9] R. Vaidyanathaswamy, *The localization theory in set topology*, Proc. Indian Acad. Sci., 20 (1945), 51-61.

[10] R. Vaidyanathaswamy, *Set topology*, Chelsea Publishing Company, New York, 1946.