Eastern Anatolian Journal of Science
Volume XI, Issue I-II, 2025, 26-31
Eastern Anatolian Journal of Science

Coefficient Bound Estimates and Fekete-Szegö Problem for the Certain Analytic and Univalent Function Classes Subordinated to the Improved q −Exponential Function

Nizami MUSTAFA [1] and Veysel NEZİR [2*]

[1] *Kafkas University, Faculty of Science and Letters, Department of Mathematics, Kars, Turkey,* nizamimustafa@gmail.com

[2] *Kafkas University, Faculty of Science and Letters, Department of Mathematics, Kars, Turkey,* veyselnezir@yahoo.com

Abstract

In this study, we introduce a new subclass of convex univalent functions in the open unit disk of the complex plane and subordinated to the improved q-exponential function. By using techniques from geometric function theory together with tools from q-calculus, we derive sharp coefficient bounds and solve the Fekete–Szegö problem for the proposed class. The obtained results extend and generalize several earlier findings for subclasses associated with classical exponential and special functions. Moreover, it is shown that the main results reduce to known estimates in the limiting case as $q \to 1^-$. Comparisons with recent related works demonstrate the effectiveness and generality of the introduced framework.

Keywords: Analytic function, convex function, univalent function, improved q −exponential function.

1. *Introduction and Preliminaries*

In this section, we recall some basic notions and definitions that will be useful throughout the paper.

Let $\mathbb{H}(\mathcal{U})$ denote the class of analytic functions in the open unit disk $\mathcal{U} = \{z \in \mathbb{C}: |z| < 1\}$.

Received:25.09.2025
Revised:20.12.2025
Accepted:25.12.2025
**Corresponding author:* Veysel NEZİR, *PhD*
Kafkas University, Faculty of Science and Letters, Department of Mathematics, Kars Turkey
E-mail: veyselnezir@yahoo.com
Cite this article as: N. Mustafa and V. Nezir, Coefficient Bound Estimates and Fekete-Szegö Problem for the Certain Analytic and Univalent Function Classes Subordinated to the Improved q-Exponential Function, *Eastern Anatolian Journal of Science, Vol. 11, Issue 1-2, 26-31, 2025.*

We denote by A the subclass of $\mathbb{H}(U)$ consisting of functions f of the form

$$f(z) = z + a_2z^2 + a_3z^3 + \cdots + a_nz^n + \cdots$$
$$= z + \sum_{n=2}^{\infty} a_nz^n. \qquad a_n \in \mathbb{C}, (1.1)$$

Clearly, every $f \in \mathcal{A}$ satisfies the conditions $f(0) = 0$ and $f'(0) = 1$. Functions satisfying these properties are referred to as *normalized* in the literature.

The well-known subclass of $\mathcal{A}$ consisting of univalent functions is denoted by $\mathcal{S}$. This class was first introduced by Koebe (1909) and has since become a central object of study in geometric function theory. In 1916, Bieberbach (1916) formulated the celebrated coefficient conjecture, which states that for each $n \geq 2$,

$$|a_n| \leq n \qquad (f \in \mathcal{S}).$$

This problem generated extensive research activity, and numerous contributions can be found in the literature (e.g., Alotaibi et al., 2020; Arif et al., 2019; Bano & Raza, 2020; Brannan & Kirwan, 1969; Cieśliński, 2011; Duren, 1983; Kumar & Arora, 2020; Mendiratta, Nagpal, & Ravichandran, 2015; Mustafa, Nezir, & Kankılıç, 2023a, 2023b; Mustafa & Demir, 2023a, 2023b; Sharma, Jain, & Ravichandran, 2016; Sokol, 2011; Sokol & Stankiewicz, 1996; Ullah et al., 2021).

The class of convex functions in $\mathcal{U}$ is defined analytically by

$$C = \left\{ f \in S: Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0,\ z \in \mathcal{U} \right\}$$

If $f, g \in \mathbb{H}(\mathcal{U})$, we say that f is *subordinate* to g, written $f \prec g$, if there exists a Schwarz function ω such that $f(z) = g(\omega(z))$.

In recent years, several subclasses of $\mathcal{S}$ have been investigated as particular choices related to C (see, e.g., Mustafa, Nezir, & Kankılıç, 2023c; Mustafa & Nezir, 2023; Mustafa & Demir, 2023a, 2023b).

Throughout this section and the sequel, we employ standard definitions from q -calculus.

The q -numbers and q -factorials are defined (see Cieśliński, 2011) by

$$[n]_q = \frac{1-q^n}{1-q} = 1 + q + \cdots + q^{n-1}$$
$$= \sum_{k=0}^{n-1} q^k, \qquad q \in (0,1),\ n \in \mathbb{N},$$

with $[0]_q! = 1$ and $[n]_q! = [n-1]_q!\,[n]_q$. It is immediate that $\lim_{q\to 1^-}[n]_q = n$ and $\lim_{q\to 1^-}[n]_q! = n!$.

In the classical approach to q-calculus, the q-exponential and improved q-exponential functions are defined (Cieśliński, 2011) as

$$e_q(z) = \sum_{n=0}^{\infty} \frac{z^n}{[n]_q!} = \prod_{n=0}^{\infty} \frac{1}{[1-(1-q)q^n z]},$$
$$0 < |q| < 1,\ |z| < \frac{1}{|1-q|},$$
$$E_q(z) = e_{1/q}(z) = \sum_{n=0}^{\infty} \frac{q^{\frac{n(n-1)}{2}}}{[n]_q!} z^n$$
$$= \prod_{n=0}^{\infty} [1 + (1-q)q^n z], \quad 0 < |q| < 1,\ z \in \mathbb{C}.$$

Clearly, $\lim_{q\to 1^-} e_q(z) = e^z$ and $\lim_{q\to 1^-} E_q(z) = e^z$.

Based on these notions, we now introduce some new subclasses of convex functions in $\mathcal{U}$.

Definition 1.1. For $\tau \in \mathcal{C}\backslash\{0\}$ and $q \in (0,1)$, a function $f \in \mathcal{S}$ is said to belong to the class $C(\tau, E_q(z))$ if

$$1 + \frac{1}{\tau}\left\{\frac{[zf'(z)]'}{f'(z)} - 1\right\} \prec E_q(z), \qquad z \in \mathcal{U}.$$

Definition 1.2. For $q \in (0,1)$, a function $f \in \mathcal{S}$ belongs to the class $C(E_q(z))$ if

$$\frac{[zf'(z)]'}{f'(z)} \prec E_q(z), \qquad z \in \mathcal{U}.$$

Definition 1.3. For $\tau \in \mathcal{C}\backslash\{0\}$, a function $f \in \mathcal{S}$ belongs to the class $C(\tau, e^z)$ if

$$1 + \frac{1}{\tau}\left\{\frac{[zf'(z)]'}{f'(z)} - 1\right\} \prec e^z, \qquad z \in \mathcal{U}.$$

Definition 1.4. For $\tau = 1$ in Definition 1.3, we obtain the subclass $C(e^z)$, consisting of all $f \in \mathcal{S}$ such that

$$\frac{[zf'(z)]'}{f'(z)} \prec e^z, \qquad z \in \mathcal{U}.$$

Let P denote the class of analytic functions in $\mathcal{U}$ with $p(0) = 1$ and $Re\{p(z)\} > 0$ $(z \in \mathcal{U})$. Each such function admits the series expansion

$$p(z) = 1 + p_1 z + p_2 z^2 + \cdots$$
$$= 1 + \textstyle\sum_{n=1}^{\infty} p_n z^n, \qquad z \in \mathcal{U}. \qquad (1.2)$$

The class P is known as the Carathéodory class of functions (Miller, 1975). We next recall two lemmas that will be used in the sequel.

Lemma 1.1 (Duren, 1983) If $p \in P$, then

$$|p_n| \le 2, n \in \mathbb{N}, |p_n - \nu p_k p_{n-k}| \le 2,$$
$$n > k,\ n,k \in \mathbb{N},\ \nu \in [0,1].$$

The inequalities are sharp for $p(z) = \frac{1+z}{1-z}$, $z \in \mathcal{U}$.

Lemma 1.2 (Duren, 1983) If $p \in P$, then

$$2p_2 = p_1^2 + (4 - p_1^2)x,$$

$$4p_3 = p_1^3 + 2(4 - p_1^2)p_1 x$$
$$-(4 - p_1^2)p_1 x^2 + 2(4 - p_1^2)(1 - |x|^2)y,$$

for some $x, y \in \mathcal{C}$ with $|x| \le 1$ and $|y| \le 1$.

In this paper, we derive coefficient bounds and solve the Fekete-Szegö problem for the class $C\left(\tau, E_q(z)\right)$.

2. *Main Results*

In this section, we establish coefficient estimates for functions in the class $C\left(\tau, E_q(z)\right)$ and solve Fekete-Szegö problem for this class.

Firstly, we give the following theorem on the coefficient estimates.

Theorem 2.1. Let $f \in C(\tau, E_q(z))$. Then the following inequalities hold:

$$|a_2| \le \frac{|\tau|}{2},$$

$$|a_3| \le \frac{|\tau|}{6}\begin{cases} 1, & \text{if } |\tau| \le 1 - \frac{q}{[2]_q!}, \\ |\tau| + \frac{q}{[2]_q!}, & \text{if } |\tau| \ge 1 - \frac{q}{[2]_q!}. \end{cases} \quad (2.1)$$

Proof. Let $f \in C(\tau, E_q(z))$. Then, by Definition 1.1, there exists a Schwarz function $\omega: \mathcal{U} \to \mathcal{U}$ such that

$$1 + \frac{1}{\tau}\left\{\frac{(zf'(z))'}{f'(z)} - 1\right\} = E_q^{\omega(z)}, \qquad z \in \mathcal{U}. \ (2.2)$$

Let $p \in P$ be defined as follows

$$p(z) = \frac{1+\omega(z)}{1-\omega(z)} = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots$$
$$= 1 + \sum_{n=1}^{\infty} p_n z^n, \quad z \in \mathcal{U}. \qquad (2.3)$$

It follows from that

$$\omega(z) = \frac{p(z)-1}{p(z)+1} = \frac{p_1}{2}z + \frac{1}{2}\left(p_2 - \frac{p_1^2}{2}\right)z^2$$
$$+ \frac{1}{2}\left(p_3 - p_1 p_2 - \frac{p_1^3}{4}\right)z^3 + \cdots, \quad z \in \mathcal{U}. \ (2.4)$$

Considering the series expansion of the improved q −exponential function E_q and expression (2.4), from (2.2) we obtain

$$2a_2z+(6a_3-4a_2^2)z^2+\cdots$$
$$=\tau\left\{\frac{p_1}{2}z+\frac{1}{2}\left[p_2-\frac{1}{2}\left(1-\frac{q}{[2]_q!}\right)p_1^2\right]z^2+\cdots\right\},z\in\mathcal{U}.$$
(2.5)

Comparing coefficients, we obtain

$$2a_2=\frac{\tau}{2}p_1,\qquad 3a_3-2a_2^2=\frac{\tau}{4}\left[p_2-\frac{1}{2}\left(1-\frac{q}{[2]_q!}\right)p_1^2\right].$$
(2.6)

From the first equality of (2.6), we can write $a_2=\frac{\tau}{4}p_1$. (2.7)

Applying Lemma 1.1 gives $|a_2|\le|\tau|/2$.

From the second relation in (2.6), we have

$a_3=\frac{1}{3}\left\{2a_2^2+\frac{\tau}{4}\left[p_2-\frac{1}{2}\left(1-\frac{q}{[2]_q!}\right)p_1^2\right]\right\}$. (2.8)

Using Lemma 1.2, $p_2=\frac{p_1^2}{2}+\frac{4-p_1^2}{2}x$ for some $x\in\mathbb{C}$ with $|x|\le1$. Substituting into (2.8) and using (2.7), we get

$$a_3=\frac{1}{3}\left[\frac{\tau^2}{8}p_1^2+\frac{\tau}{4}\left(\frac{q}{2\cdot[2]_q!}p_1^2+\frac{4-p_1^2}{2}x\right)\right].$$

Applying the triangle inequality yields

$$|a_3|\le\frac{|\tau|}{24}\left[\left(|\tau|+\frac{q}{[2]_q!}\right)t^2+(4-t^2)\xi\right],\xi\in[0,1],$$
(2.9)

where $\xi=|x|$, $t=|p_1|\le2$.

Maximizing the expression on the right hand side of the inequality (2.9) according to the parameter ξ, we obtain

$|a_3|\le\frac{|\tau|}{24}[c(\tau,q)t^2+4], t\in[0,2]$,

where $c(\tau,q)=|\tau|-1+\frac{q}{[2]_q!}$.

Then, maximizing the function

$$\varphi(t)=c(\tau,q)t^2+4,\qquad t\in[0,2]$$

It follows that $\varphi(t)\le4$ if $c(\tau,q)\le0$ and $\varphi(t)\le4\left(|\tau|+\frac{q}{[2]_q!}\right)$ if $c(\tau,q)\ge0$.

With this, the proof of the second inequality of (2.1) is provided.

Thus, the proof of the theorem is completed.

In the case $\tau=1$ and $q\to1^-$ from the Theorem 2.1, we have the following results, respectively.

Corollary 2.1. If $f\in C\left(E_q(z)\right)$, then

$|a_2|\le\frac{1}{2}$ and $|a_3|\le\frac{1}{6}\left(1+\frac{q}{[2]_q!}\right)$.

Corollary 2.2. If $f\in C(\tau,e^z)$, then

$|a_2|\le\frac{|\tau|}{2}$ and $|a_3|\le\frac{|\tau|}{6}\begin{cases}1 & if\ |\tau|\le\frac{1}{2},\\ \frac{1}{2}+|\tau| & if\ |\tau|\ge\frac{1}{2}.\end{cases}$

Also, taking $\tau=1$ in the Corollary 2.2 or in the case $q\to1^-$ from the Corollary 2.1, we obtain the following result.

Corollary 2.3. If $f\in C(e^z)$, then $|a_2|\le\frac{1}{2}$ and $|a_3|\le\frac{1}{4}$.

Now, we give the following theorem on the Fekete-Szegö problem for the class $C\left(\tau,E_q(z)\right)$.

Theorem 2.2. Let $f\in C\left(\tau,E_q(z)\right)$ and $\mu\in\mathbb{C}$, then

$$|a_3-\mu a_2^2|\le\frac{|\tau|}{6}\begin{cases}1 & if\ c(\tau,q,\mu)\le0,\\ \frac{|2-3\mu||\tau|[2]_q!+2q}{2\cdot[2]_q!} & if\ c(\tau,q,\mu)\ge0,\end{cases}$$
(2.10)

where $c(\tau,q,\mu)=\frac{[|2-3\mu||\tau|-2][2]_q!+2q}{[2]_q!}$.

Proof. Let $f\in C\left(\tau,E_q(z)\right)$ and $\mu\in\mathbb{C}$, then from the equalities (2.7) and (2.8), we can write

$$a_3-\mu a_2^2=\frac{(2-3\mu)\tau^2}{48}p_1^2+\frac{\tau}{12}\left\{p_2-\frac{1}{2}\left(1-\frac{q}{[2]_q!}\right)p_1^2\right\}.$$
(2.11)

Using Lemma 1.2, $p_2=\frac{p_1^2}{2}+\frac{4-p_1^2}{2}x$ for some $x\in\mathbb{C}$ with $|x|\le1$. Then, the expression $a_3-\mu a_2^2$ can be written as follows

$a_3-\mu a_2^2=\frac{\tau}{48}\left\{\begin{array}{l}\frac{[2]_q!(2-3\mu)\tau+2q}{[2]_q!}p_1^2\\ +2(4-p_1^2)x\end{array}\right\}$. (2.12)

Applying triangle inequality leads to

$|a_3-\mu a_2^2|\le\frac{|\tau|}{48}\left\{\begin{array}{l}\frac{[2]_q!|2-3\mu||\tau|+2q}{[2]_q!}t^2\\ +2(4-t^2)\xi\end{array}\right\},\xi\in[0,1]$, (2.13)

with $t=|p_1|$, $\xi=|x|$.

Maximizing the expression on the right hand side of the inequality (2.13) to the parameter ξ, we obtain

$|a_3-\mu a_2^2|\le\frac{|\tau|}{48}\{c(\tau,q,\mu)t^2+8\}$, $t\in[0,2]$,

Where $c(\tau,q,\mu)=\frac{[|2-3\mu||\tau|-2][2]_q!+2q}{[2]_q!}$.

Maximizing the function $\psi:[0,2]\to\mathbb{R}$ defined as follows

$\psi(t)=c(\tau,q,\mu)t^2+8,\ t\in[0,2]$,

we can easily see that $\psi(t)\le8$ if $c(\tau,q,\mu)\le0$ and $\psi(t)\le\frac{4\left(|2-3\mu||\tau|[2]_q!+2q\right)}{[2]_q!}$ if $c(\tau,q,\mu)\ge0$.

Thus, the proof of the theorem is completed.

In the case $\tau = 1$ and $q \to 1^-$ from the Theorem 2.2, we have the following results, respectively.

Corollary 2.4 If $f \in C\left(E_q(z)\right)$ and $\mu \in \mathbb{C}$, then

$$|a_3 - \mu a_2^2| \le \frac{1}{6}\begin{cases} 1 & if\ |2-3\mu| \le \frac{2([2]_q! - q)}{[2]_q!}, \\ \frac{|2-3\mu|[2]_q! + 2q}{2[2]_q!} & if\ |2-3\mu| \ge \frac{2([2]_q! - q)}{[2]_q!}. \end{cases}$$

Corollary 2.5 If $f \in C(\tau, e^z)$ and $\mu \in \mathbb{C}$, then

$$|a_3 - \mu a_2^2| \le \frac{|\tau|}{6}\begin{cases} 1 & if\ |2-3\mu||\tau| \le 2, \\ \frac{|2-3\mu||\tau|+1}{2} & if\ |2-3\mu||\tau| \ge 2. \end{cases}$$

Also, taking $\tau = 1$ in the Corollary 2.5 or in the case $q \to 1^-$from the Corollary 2.4, we obtain the following result.

Corollary 2.6 If $f \in C(e^z)$, then

$$|a_3 - \mu a_2^2| \le \frac{1}{6}\begin{cases} 1 & if\ |2-3\mu| \le 2, \\ \frac{|2-3\mu|+1}{2} & if\ |2-3\mu| \ge 2. \end{cases}$$

Also, taking $\mu = 0$ and $\mu = 1$in the Theorem 2.2, we obtain the following results, respectively.

Corollary 2.7 If $f \in C\left(\tau, E_q(z)\right)$, then

$$|a_3| \le \frac{|\tau|}{6}\begin{cases} 1 & if\ |\tau| \le 1 - \frac{q}{[2]_q!}, \\ |\tau| + \frac{q}{[2]_q!} & if\ |\tau| \ge 1 - \frac{q}{[2]_q!}. \end{cases}$$

Corollary 2.8. If $f \in C\left(\tau, E_q(z)\right)$, then

$$|a_3 - a_2^2| \le \frac{|\tau|}{6}\begin{cases} 1 & if\ |\tau| \le \frac{2([2]_q! - q)}{[2]_q!}, \\ \frac{|\tau|[2]_q! + 2q}{2[2]_q!} & if\ |\tau| \ge \frac{2([2]_q! - q)}{[2]_q!}. \end{cases}$$

Remark 2.1. We note that Corollary 2.7 confirms the second result of Theorem 2.1.

In the case $\tau = 1$ and $q \to 1^-$ from the Corollary 2.8, we obtain the following results, respectively.

Corollary 2.9. If $f \in C\left(E_q(z)\right)$, then $|a_3 - a_2^2| \le \frac{1}{6}$.

Corollary 2.10. If $f \in C(\tau, e^z)$, then

$$|a_3 - a_2^2| \le \frac{|\tau|}{6}\begin{cases} 1 & if\ |\tau| \le 1, \\ \frac{|\tau|+1}{2} & if\ |\tau| \ge 1. \end{cases}$$

Also, taking $\tau = 1$ in the Corollary 2.10 or in the case $q \to 1^-$from the Corollary 2.9, we obtain the following result.

Corollary 2.11 If $f \in C(e^z)$, then $|a_3 - a_2^2| \le \frac{1}{6}$.

3. Discussion

The results obtained in this study provide new insights into the coefficient behavior of analytic and univalent functions subordinated to the improved q-exponential function. In particular, the derived coefficient estimates and the solutions to the Fekete–Szegö problem extend and complement the classical results of Bieberbach (1916) and Koebe (1909), who laid the foundational work on coefficient problems and univalent functions.

Recent studies on coefficient bounds and Fekete–Szegö type inequalities within the framework of q-calculus have attracted considerable attention (Caglar, Orhan, & Srivastava, 2020; Alsoboh, Caglar, & Buyankara, 2022).

The use of the improved q-exponential function has been inspired by the generalizations proposed in q-calculus (Cieslinski, 2011), and the present results further enrich the framework introduced in earlier works on starlike and convex functions (Brannan & Kirwan, 1969; Duren, 1983). Recent contributions on subclasses associated with trigonometric and hyperbolic functions, such as those by Alotaibi et al. (2020), Bano and Raza (2020), and Kumar and Arora (2020), show a growing interest in extending geometric function theory toward more generalized structures. The findings of this paper align with these trends by incorporating the improved exponential function.

The results of the present study can be compared with several recent investigations on coefficient estimates and Fekete–Szegö problems for subclasses of analytic and univalent functions. In particular, Mustafa and Nezir (2024) obtained sharp coefficient bounds for subclasses associated with the sine hyperbolic function, while Caglar, Buyankara, and Karaman (2024) studied Fekete–Szegö inequalities for Sakaguchi-type bi-univalent functions defined by Gegenbauer polynomials. Compared with these studies, the current work employs the improved q-exponential function, which provides a more flexible framework and includes the classical exponential case as a limiting situation.

Furthermore, the bounds obtained here are consistent with the general methodology used in univalent function theory, particularly in relation to Carathéodory functions (Miller, 1975). The inequalities established in Lemmas 1.1 and 1.2 play a crucial role in proving the sharpness of the results, similar to approaches discussed by Sokol (2011) and Sharma et al. (2016).

It is worth noting the recent contributions of Mustafa, Nezir, and their collaborators, who studied subclasses associated with sine and cosine functions and addressed coefficient estimates as well as Fekete–Szegö problems (Mustafa, Nezir, & Kankılıç, 2023a, 2023b; Mustafa & Demir, 2023a, 2023b). The present work provides an additional perspective by considering the improved q-exponential framework, thereby generalizing these previously studied subclasses.

Overall, the results contribute to the broader program of unifying classical and modern approaches in geometric function theory by means of analytic techniques rooted in q-calculus and its improved functions. This framework opens several avenues for further research, particularly in extending the results to higher-order coefficients, to bi-univalent function classes, and to complex orders, as already suggested in the works of Mustafa, Nezir, and Demir (2023c, 2023d).

References

ALOTAIBI, A., ARIF, M., ALGHAMDI, M. A., HUSSAIN, S. (2020). Starlikeness associated with cosine hyperbolic function. Mathematics, 8, 1118.

ALSOBOH, A., ÇAĞLAR, M., BUYANKARA, M. (2022). Fekete–Szegö inequality for a subclass of bi-univalent functions linked to q-ultraspherical polynomials. Contemporary Mathematics, 5(2), 2366–2380.

ARIF, M., AHMAD, K., LIU, J.-L., SOKOL, J. (2019). A new class of analytic functions associated with Salagean operator. J. Funct. Spaces, Article ID 5157394.

BANO, K., RAZA, M. (2020). Starlike functions associated with cosine function. Bull. Iran. Math. Soc., 47, 1513–1532.

BIEBERBACH, L. (1916). Über die Koeffizienten derjenigen Potenzreihen welche eine schlichte Abbildung des Einheitskreises vermitteln. Sitzungsberichte Preuss. Akad. Wiss., 940–955.

BRANNAN, D. A., KIRWAN, W. E. (1969). On some classes of bounded univalent functions. J. Lond. Math. Soc., 2, 431–443.

CIESLINSKI, J. L. (2011). Improved exponential and trigonometric functions. Applied Mathematics Letters, 24(12), 2110–2114.

ÇAĞLAR, M., BUYANKARA, M., KARAMAN, A. (2024). Fekete–Szegö inequalities for Sakaguchi type bi-univalent functions defined by Gegenbauer polynomials. Turkish Journal of Science, 9(2), 126–136.

ÇAĞLAR, M., ORHAN, H., SRIVASTAVA, H. M. (2020). Coefficient bounds for q-starlike functions associated with q-Bernoulli numbers. Journal of Applied Analysis and Computation, 15(4), 2354–2364.

DUREN, P. L. (1983). Univalent Functions. Grundlehren der Mathematischen Wissenschaften, Vol. 259. Springer-Verlag, New York.

KOEBE, P. (1909). Über die Uniformisierung der algebraischen Kurven, durch automorphe Funktionen mit imaginärer Substitutionsgruppe. Nachr. Akad. Wiss. Göttingen Math.-Phys., 68–76.

KUMAR, S. S., ARORA, K. (2020). Starlike functions associated with a petal shaped domain. arXiv preprint arXiv:2010.10072.

MENDIRATTA, R., NAGPAL, S., RAVICHRANDRAN, V. (2015). On a subclass of strongly starlike functions associated with exponential function. Bull. Malays. Math. Soc., 38, 365–386.

MILLER, S. S. (1975). Differential inequalities and Carathéodory functions. Bull. Amer. Math. Soc., 81, 79–81.

MUSTAFA, N., NEZIR, V., KANKILIÇ, A. (2023a). Coefficient estimates for certain subclass of analytic and univalent functions associated with sine hyperbolic function. 13th Int. İstanbul Scientific Research Congress, April 29–30, 234–241, İstanbul, Turkey.

MUSTAFA, N., NEZIR, V., KANKILIÇ, A. (2023b). The Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine hyperbolic function. 13th Int. İstanbul Scientific Research Congress, April 29–30, 242–249, İstanbul, Turkey.

MUSTAFA, N., NEZIR, V. (2023c). Coefficient estimates and Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine hyperbolic function. 13th Int. İstanbul Scientific Research Congress, May 1–2, 475–481, İstanbul, Turkey.

MUSTAFA, N., NEZIR, V. (2024). Coefficient estimates and Fekete–Szegö problem for certain subclass of analytic and univalent functions associated with sine hyperbolic function of complex order. Turkish Journal of Science, 9(1), 45–53.

MUSTAFA, N., DEMIR, H. A. (2023d). Coefficient estimates for certain subclass of analytic and univalent functions associated with sine and cosine functions. 4th Int. Black Sea Congress on Modern Scientific Research, June 6–8, 2555–2563, Rize, Turkey.

MUSTAFA, N., DEMIR, H. A. (2023e). Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine and cosine functions. 4th Int. Black Sea Congress on Modern Scientific Research, June 6–8, 2564–2572, Rize, Turkey.

MUSTAFA, N., NEZIR, V., KANKILIÇ, A. (2023f). Coefficient estimates for certain subclass of analytic and univalent functions associated with sine hyperbolic function with complex order. Journal of Scientific and Engineering Research, 10(6), 18–25.

MUSTAFA, N., NEZIR, V., KANKILIÇ, A. (2023g). The Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine hyperbolic function with complex order. Eastern Anatolian Journal of Science, 9(1), 1–6.

MUSTAFA, N., DEMIR, H. A. (2023h). Coefficient estimates for certain subclass of analytic and univalent functions associated with sine and cosine functions with complex order. Journal of Scientific and Engineering Research, 10(6), 131–140.

SHARMA, K., JAIN, N. K., RAVICHRANDRAN, V. (2016). Starlike function associated with a cardioid. Afr. Math., 27, 923–939.

SOKOL, J. (2011). A certain class of starlike functions. Comput. Math. Appl., 62, 611–619.

SOKOL, J., STANKIEWICZ, J. (1996). Radius of convexity of some subclasses of strongly starlike functions. Zesz. Nauk. Politech. Rzesz. Math., 19, 101–105.

ULLAH, K., ZAINAB, S., ARIF, M., DARUS, M., SHUTAYI, M. (2021). Radius problems for starlike functions associated with the tan hyperbolic function. J. Funct. Spaces, Article ID 9967640.