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Ideal Convergence in 2-Metric Spaces

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Abstract

In this paper, we firstly introduce the notions of $\mathscr{I}$-convergence and $\mathscr{I}^*$-convergence and also, we investigate some inclusion relations between $\mathscr{I}$-convergence and $\mathscr{I}^*$-convergence in 2-metric space. Then, we introduce the notions of $\mathscr{I}$-Cauchy sequence and $\mathscr{I}^*$-Cauchy sequence and also, we investigate some inclusion relations between $\mathscr{I}$-Cauchy sequence and $\mathscr{I}^*$-Cauchy sequence in 2-metric space.

Keywords: *Ideal convergence, 2-metric spaces, Ideal Cauchy Sequence*
2010 Mathematics Subject Classification: *40A05, 40A35, 47H10*

1. Introduction

Statistical convergence of number sequences was given by Fast [4]. Schoenberg [27] obtained some basic properties of statistical convergence. The idea of $\mathscr{I}$-convergence was introduced by Kostyrko et al. [17] as a generalization of statistical convergence which is based on the structure of the ideal $\mathscr{I}$ of subset of the set of natural numbers. Nabiev et al. [21] introduced the notions of $\mathscr{I}$-Cauchy sequence and $\mathscr{I}^*$-Cauchy sequence.
The concept of 2-metric by generalizing the concept of metric introduced by Gähler [8, 9]. They also, studied the various properties of 2-metric spaces. A 2-metric space is not topologically equivalent to an ordinary metric. For example, every metric space is first countable, but 2-metric spaces may not be first countable [18]. In this case, there is no simple relationship between the results obtained in metric spaces and the results obtained in 2-metric spaces. In a metric space a convergent sequence is a Cauchy sequence but in a 2-metric space may not be a Cauchy sequence, but if the 2-metric d is continuous on X, then each convergent sequence becomes a Cauchy sequence [22]. Although a metric is continuous on X, the 2-metric may not be continuous. Nuray [24] introduced the concepts of Cesàro convergence, statistical convergence, statistically Cauchy sequence, lacunary convergence and lacunary statistical convergence in 2-metric space.
Now, we recall the basic definitions and concepts (see [1, 5, 3, 2, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33]).
Let $K \subseteq \mathbb{N}$, K_n denotes the set $\{k \in K : k \leq n\}$, and $|K_n|$ denotes the cardinality of K_n. The natural density of K is given by

$$\delta(K) = \lim_{n\to\infty} \frac{1}{n}|K_n|,$$

if it exists.
Let (X,d) be a metric space. A sequence (x_n) is statistically convergent to x in (X,d) if for every $\varepsilon > 0$ $\delta(K) = 0$ i.e.,

$$\lim_{n\to\infty} \frac{1}{n}|\{k \leq n : d(x_k,x) \geq \varepsilon\}| = 0.$$

For this case, we write $st-\lim x_n = x$.
A family of sets $\mathscr{I} \subseteq 2^{\mathbb{N}}$ is called an ideal if and only if
(i) $\emptyset \in \mathscr{I}$, **(ii)** For each $A,B \in \mathscr{I}$ we have $A\cup B \in \mathscr{I}$, **(iii)** For each $A \in \mathscr{I}$ and each $B \subseteq A$ we have $B \in \mathscr{I}$.
An ideal is called nontrivial if $\mathbb{N} \notin \mathscr{I}$ and nontrivial ideal is called admissible if $\{n\} \in \mathscr{I}$ for each $n \in \mathbb{N}$. Then after, we let $\mathscr{I} \subseteq 2^{\mathbb{N}}$ be an admissible ideal.
A family of sets $\mathscr{F} \subseteq 2^{\mathbb{N}}$ is called a filter if and only if
(i) $\emptyset \notin \mathscr{F}$, **(ii)** For each $A,B \in \mathscr{F}$ we have $A\cap B \in \mathscr{F}$, **(iii)** For each $A \in \mathscr{F}$ and each $B \supseteq A$ we have $B \in \mathscr{F}$.
If $\mathscr{I}$ is a nontrivial ideal in X, $X \neq \emptyset$, then the class

$$\mathscr{F}(\mathscr{I}) = \{M \subset X : (\exists A \in \mathscr{I})(M = X\backslash A)\}$$

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is a filter on X, called the filter associated with $\mathscr{I}$.
A sequence (x_n) in X is $\mathscr{I}$-convergent to $L \in X$ if for every $\varepsilon > 0$

$$A(\varepsilon) = \{n \in \mathbb{N} : |x_n - L| \geq \varepsilon\} \in \mathscr{I}.$$

Denote by $\mathscr{I}_\delta$ the class of all $A \subset \mathbb{N}$ with $\delta(A) = 0$. Then, $\mathscr{I}_\delta$ is non-trivial admissible ideal and $\mathscr{I}_\delta$-convergence coincides with the statistical convergence.
A function f is said to be ideal continuous at x_0 if, given a sequence (x_n), $\mathscr{I} - \lim x_n = x_0$ implies that $\mathscr{I} - \lim f(x_n) = f(x_0)$.
Let $X \neq \emptyset$. $d : X^3 \to \mathbb{R}$ is said to be a 2-metric on X if
(M1) given distinct elements $a, b \in X$, there exists an element $c \in X$ such that $d(a,b,c) \neq 0$,
(M2) $d(a,b,c) = 0$ when at least two of a,b,c are equal,
(M3) $d(a,b,c) = d(a,c,b) = d(b,c,a)$ for all $a,b,c \in X$, and
(M4) $d(a,b,c) \leq d(a,b,z) + d(a,z,c) + d(z,b,c)$ for all $a,b,c,z \in X$.
When d is a 2-metric on X, then the ordered pair (X,d) is called a 2-metric space.
Very typical example of 2-metric $d(a,b,c)$ is the area of the triangle spanned by a,b,c. After that, throughout the paper, let $X = (X,d)$ be a 2-metric space.
(X,d) is said to be bounded if $sup\{d(a,b,c) : a,b,c \in X\} < \infty$.
A sequence (x_n) in X is said to be a Cauchy sequence if for all $a \in X$,

$$\lim_{n,m\to\infty} d(x_n, x_m, a) = 0.$$

A sequence (x_n) in X is said to be convergent to an element $x \in X$ if for all $a \in X$, $\lim_{n\to\infty} d(x_n, x, a) = 0$.
A complete 2-metric space is one in which every Cauchy sequence in X converges to an element of X.

Example 1.1. *Take $X = [0,1]$. Define $d : X^3 \to \mathbb{R}$ as*

$$d(a,b,c) = min\{|a-b|, |b-a|, |c-a|\}$$

where $a,b,c \in X$. Now, (X,d) is a 2-metric space.

A sequence (x_n) in X is said to be statistically convergent to an element $x \in X$ if for all $a \in X$ and for every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : d(x_k, x, a) \geq \varepsilon\}| = 0$$

or equivalently

$$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : d(x_k, x, a) < \varepsilon\}| = 1.$$

In this case, we write $st - \lim_{n\to\infty} d(x_n, x, a) = 0$.
A sequence (x_n) in X is said to be a statistically Cauchy sequence if for all $a \in X$ and for every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n} |\{k, \ell \leq n : d(x_k, x_\ell, a) \geq \varepsilon\}| = 0$$

or equivalently

$$\lim_{n\to\infty} \frac{1}{n} |\{k, \ell \leq n : d(x_k, x_\ell, a) < \varepsilon\}| = 1.$$

In this case, we write

$$st - \lim_{n,m\to\infty} d(x_n, x_m, a) = 0$$

Lemma 1.2. *[21] Let $\{Q_i\}_1^\infty$ be a countable a collection of subsets of $\mathbb{N}$ such that $Q_i \in \mathscr{F}(\mathscr{I})$ for each i, where $\mathscr{F}(\mathscr{I})$ is a filter associate with an admissible ideal $\mathscr{I}$ with property (AP). Then there exists a set $Q \subset \mathbb{N}$ such that $Q \in \mathscr{F}(\mathscr{I})$ and the set $Q \backslash Q_i$ is finite for all i.*

2. Main Results

In this section, we firstly introduced the notions of $\mathscr{I}$-convergence and $\mathscr{I}^*$-convergence in 2-metric space.

Definition 2.1. *A sequence (x_n) in X is $\mathscr{I}$-convergent to $L \in X$ if for all $a \in X$ and every $\varepsilon > 0$*

$$\{n \in \mathbb{N} : d(x_n, L, a) \geq \varepsilon\} \in \mathscr{I}$$

In this case, we write

$$\mathscr{I} - \lim_{n\to\infty} d(x_n, L, a) = 0.$$

Theorem 2.2. *If a sequence (x_n) in X is convergent to L, then (x_n) is $\mathscr{I}$-convergent to L.*

Proof. Let the sequence (x_n) is convergent to L in X. Then, for every $\varepsilon > 0$ there is a $n_0 = n_0(\varepsilon) \in \mathbb{N}$ such that for all $a \in X$ and $n > n_0$ we have

$$d(x_n, L, a) < \varepsilon.$$

Therefore, for every $\varepsilon > 0$ and all $a \in X$ we have

$$A_\varepsilon = \{n \in \mathbb{N} : d(x_n, L, a) \geq \varepsilon\} \subseteq \{1, 2, 3, ..., n_0\}.$$

Since $\mathscr{I}$ is an admissible ideal, it is clear that $A_\varepsilon \in \mathscr{I}$ and so, (x_n) is $\mathscr{I}$-convergent to L in X. □

Definition 2.3. *A sequence (x_n) in X is $\mathscr{I}^*$-convergent to $L \in X$ if there exists a set $M = \{m_1 < m_2 < \cdots < m_k < \cdots\} \in \mathscr{F}(\mathscr{I})$ (that is, $\mathbb{N} \setminus M \in \mathscr{I}$) such that for all $a \in X$*

$$\lim_{k\to\infty} d(x_{m_k}, L, a) = 0.$$

In this case, we write

$$\mathscr{I}^* - \lim_{n\to\infty} d(x_n, L, a) = 0.$$

Theorem 2.4. *If a sequence (x_n) in X is $\mathscr{I}^*$-convergent to L, then it is $\mathscr{I}$-convergent to L.*

Proof. Since the sequence (x_n) is $\mathscr{I}^*$-convergent to L in X, there exists a set $M = \{m_1 < m_2 < \cdots < m_k < \cdots\} \in \mathscr{F}(\mathscr{I})$ (that is, $H = \mathbb{N} \setminus M \in \mathscr{I}$) such that for all $a \in X$

$$\lim_{k\to\infty} d(x_{m_k}, L, a) = 0.$$

Then, for every $\varepsilon > 0$ there is a $k_0 = k_0(\varepsilon) \in \mathbb{N}$ such that for all $a \in X$ and $k > k_0$ we have

$$d(x_{m_k}, L, a) < \varepsilon.$$

Therefore, for every $\varepsilon > 0$ and all $a \in X$ we have

$$A_\varepsilon = \{n \in \mathbb{N} : d(x_n, L, a) \geq \varepsilon\} \subseteq H \cup \{m_1 < m_2 < \cdots < m_{k_0}\}.$$

Since $\mathscr{I}$ is an admissible ideal, it is clear that

$$H \cup \{m_1 < m_2 < \cdots < m_{k_0}\} \in \mathscr{I}$$

and so $A_\varepsilon \in \mathscr{I}$. Hence, (x_n) is $\mathscr{I}$-convergent to L in X. □

Theorem 2.5. *If the ideal $\mathscr{I}$ has property (AP) and a sequence (x_n) in X is $\mathscr{I}$-convergent to L, then (x_n) is $\mathscr{I}^*$-convergent to L.*

Proof. Suppose that the ideal $\mathscr{I}$ has property (AP) and the sequence (x_n) is $\mathscr{I}$-convergent to L in X. Then, for every $\varepsilon > 0$ and all $a \in X$

$$U_\varepsilon = \{n \in \mathbb{N} : d(x_n, L, a) \geq \varepsilon\} \in \mathscr{I}.$$

Now, put

$$U_1 = \{n \in \mathbb{N} : d(x_n, L, a) \geq 1\} \text{ and } U_n = \left\{n \in \mathbb{N} : \frac{1}{n} \leq d(x_n, L, a) < \frac{1}{n-1}\right\}$$

for $n \in \mathbb{N}$ and $n \geq 2$. It is clear that, for $r \neq s$, $U_r \bigcap U_s = \emptyset$. By property (AP) there exists a sequence of sets $\{V_n\}_{n\in\mathbb{N}}$ such that $U_s \Delta V_s$ are finite sets for $s \in \mathbb{N}$ and

$$V = \bigcup_{s=1}^{\infty} V_s \in \mathscr{I}.$$

We should prove that

$$\lim_{\substack{n\to\infty \\ (n\in M)}} d(x_n, L, a) = 0 \tag{2.1}$$

for $M = \mathbb{N} \setminus V$. Select $i \in \mathbb{N}$ such that $\frac{1}{i+1} < \delta$. Then

$$\{n \in \mathbb{N} : d(x_n, L, a) \geq \delta\} \subset \bigcup_{s=1}^{i+1} U_s.$$

Because of $U_s \Delta V_s$ are finite sets $(s = 1, 2, \cdots)$, there exists $n_0 \in \mathbb{N}$ such that

$$\left(\bigcup_{s=1}^{i+1} V_s\right) \cap \{n \in \mathbb{N} : n > n_0\} = \left(\bigcup_{s=1}^{i+1} U_s\right) \cap \{n \in \mathbb{N} : n > n_0\}. \tag{2.2}$$

If $n > n_0$ and $n \notin V$, then $n \notin \bigcup_{s=1}^{i+1} V_s$ and, so by (2.2) $n \notin \bigcup_{s=1}^{i+1} U_s$. But then,

$$d(x_n, L, a) < \frac{1}{n+1} < \delta$$

and hence we get (2.1). □

Now, we introduced the notions of $\mathscr{I}$-Cauchy sequence and $\mathscr{I}^*$-Cauchy sequence in 2-metric space.

Definition 2.6. *A sequence (x_n) in X is $\mathscr{I}$-Cauchy sequence if for all $a \in X$ and every $\varepsilon > 0$ there exists $N = N(\varepsilon)$ such that*

$$\{n \in \mathbb{N} : d(x_n, x_N, a) \geq \varepsilon\} \in \mathscr{I}.$$

Example 2.7. *Let $X=\left\{0,1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\ldots\right\}$ and take the ideal $\mathscr{I}$ as the $\mathscr{I}_\delta$ ideal. Define $d:X^3\to[0,\infty)$ by*

$$d(x,y,z)=\begin{cases}1, & \text{if } x\neq y\neq z \text{ and } \left\{\frac{1}{n},\frac{1}{n+1}\right\}\subset\{x,y,z\} \text{ for } n\in\mathbb{N}\\ 0, & \text{otherwise}\end{cases}$$

Then, (X,d) is a complete 2-metric space. Let define the sequence (x_n) by

$$x_n=\begin{cases}n, & n \text{ is square integer}\\ \frac{1}{n}, & \text{otherwise}\end{cases}$$

The sequence (x_n) is $\mathscr{I}_\delta$-convergent to 0 but is not a $\mathscr{I}_\delta$-Cauchy sequence.

Theorem 2.8. *Let (X,d) be 2-metric space and d be statistically continuous on X. If the sequence (x_n) is $\mathscr{I}$-convergent then (x_n) is $\mathscr{I}$-Cauchy sequence.*

Proof. Let $\mathscr{I}-\lim x_n=x$. Then by ideal continuity of d, we have

$$\mathscr{I}-\lim_{n,m\to\infty} d(x_n,x_m,x)=\mathscr{I}-\lim_{n\to\infty} d(x_n,x,x). \tag{2.3}$$

Let $N=N(\varepsilon)=m$, for every $\varepsilon>0$. By the property (iv) of 2-metric, for all $a\in X$ we can write

$$d(x_n,x_N,a)\leq d(x_n,x,a)+d(x,x_N,a)+d(x_n,x_N,x). \tag{2.4}$$

From inequality (2.4) and equation (2.3), for all $a\in X$ we get that

$$\mathscr{I}-\lim_{n\to\infty} d(x_n,x_N,a)=0$$

that is, (x_n) is $\mathscr{I}$-Cauchy sequence. □

Definition 2.9. *A sequence (x_n) in X is $\mathscr{I}^*$-Cauchy sequence if there exists a set $M=\{m_1<m_2<\cdots<m_k<\cdots\}\in\mathscr{F}(\mathscr{I})$ (that is, $\mathbb{N}\setminus M\in\mathscr{I}$) such that the subsequence $x_M=x_{m_k}$ is an ordinary Cauchy sequence in X, that is, for all $a\in X$*

$$\lim_{k,p\to\infty} d(x_{m_k},x_{m_p},a)=0.$$

Theorem 2.10. *If a sequence (x_n) in X is $\mathscr{I}^*$-Cauchy sequence, then (x_n) is $\mathscr{I}$-Cauchy sequence.*

Proof. Let the sequence (x_n) in X is $\mathscr{I}^*$-Cauchy sequence. Then by definition, there exists a set $M=\{m_1<m_2<\cdots<m_k<\cdots\}\in\mathscr{F}(\mathscr{I})$ (that is, $\mathbb{N}\setminus M\in\mathscr{I}$) such that for every $\varepsilon>0$ and all $a\in X$

$$d(x_{m_k},x_{m_p},a)<\varepsilon,$$

for all $k,p>k_0=k_0(\varepsilon)\in\mathbb{N}$. Let $N=N(\varepsilon)=m_{k_0+1}$. Then, for every $\varepsilon>0$ and all $a\in X$

$$d(x_{m_k},x_N,a)<\varepsilon,$$

for all $k>k_0$. Then, let $\mathbb{N}\setminus M=H\in\mathscr{I}$ and for every $\varepsilon>0$ and all $a\in X$

$$A_\varepsilon=\{n\in\mathbb{N}:d(x_{m_k},x_N,a)\geq\varepsilon\}\subseteq H\cup\{m_1<m_2<\cdots<m_{k_0}\}.$$

Since $\mathscr{I}$ is an admissible ideal, it is clear that

$$H\cup\{m_1<m_2<\cdots<m_{k_0}\}\in\mathscr{I}$$

and so $A_\varepsilon\in\mathscr{I}$. Hence, (x_n) is $\mathscr{I}$-Cauchy sequence in X. □

Theorem 2.11. *If $\mathscr{I}$ is an admissible ideal with property (AP) then the concepts $\mathscr{I}$-Cauchy sequence and $\mathscr{I}^*$-Cauchy sequence coincide in 2-metric space X.*

Proof. Let the sequence (x_n) in X is $\mathscr{I}^*$-Cauchy sequence. Then by Theorem 2.10, without property (AP), (x_n) is $\mathscr{I}$-Cauchy sequence in X. Now, let the sequence (x_n) in X is $\mathscr{I}$-Cauchy sequence. Then by definition, for every $\varepsilon>0$ there exists $N=N(\varepsilon)$ such that for all $a\in X$ we have

$$\{n\in\mathbb{N}:d(x_n,x_N,a)\geq\varepsilon\}\in\mathscr{I}.$$

For all $a\in X$ and $i=1,2,\cdots$ let

$$P_i=\left\{n\in\mathbb{N}:d(x_n,x_{m_i},a)<\frac{1}{i}\right\},$$

where $m_i=N\left(\frac{1}{i}\right)$. For $i=1,2,\cdots$ it is clearly that $P_i\in\mathscr{F}(\mathscr{I})$. With the property (AP) and Lemma 1.2, there exists a set $Q\in\mathscr{F}(\mathscr{I})$ and $Q\setminus Q_i$ is finite for all $i\in\mathbb{N}$. For proof, let $\varepsilon>0$ and $i\in\mathbb{N}$ such that $i>\dfrac{2}{\varepsilon}$. If $m,n\in Q$ then, $Q\setminus Q_i$ is a finite set, so there exists $u=u(i)$ such that $m\in Q$ and $n\in Q$ for all $m,n>u(i)$. Thus, for all $a\in X$ and all $m,n>u(i)$, we have

$$d(x_m,x_{m_i},a)<\frac{1}{i} \text{ and } d(x_n,x_{m_i},a)<\frac{1}{i}$$

and so,

$$\begin{aligned} d(x_m,x_n,a) &< d(x_m,x_{m_i},a)+d(x_n,x_{m_i},a)\\ &< \frac{1}{i}+\frac{1}{i}=\frac{2}{i}<\varepsilon. \end{aligned}$$

Hence, for all $a\in X$ we have

$$\lim_{\substack{m,n\to\infty\\ (m,n\in Q)}} d(x_m,x_n,a)=0$$

and so (x_n) is $\mathscr{I}^*$-Cauchy sequence in X. □

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