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Ortaokul Öğrencilerinin Sesi: Kesirlerdeki Kavramsal ve İşlemsel Bilgilerinin İncelenmesi

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Öz

Bu çalışma, altıncı ve yedinci sınıf öğrencilerinin kesirlere ilişkin kavramsal ve işlemsel bilgilerini incelemiştir. Katılımcılar, İstanbul'daki özel bir ortaokuldan amaçlı örnekleme yoluyla seçilen 30 öğrenciden (12–13 yaş aralığında) oluşmaktadır. Açıklayıcı sıralı karma yöntem deseninin kullanıldığı araştırmada, öncelikle öğrencilerin kavramsal ve işlemsel bilgi düzeylerini değerlendirmek amacıyla alan yazında geçerliliği kanıtlanmış bir kesir testi uygulanmış; ardından sınıf gözlemleri ve yarı yapılandırılmış görüşmeler gerçekleştirilmiştir. Bulgular, kavramsal ve işlemsel bilgi arasında istatistiksel olarak anlamlı bir korelasyon olduğunu ortaya koymuş; yüksek başarı gösteren öğrencilerin her iki bilgi türünü de bütünleştirerek kullandıkları, düşük başarı gösteren öğrencilerin ise ağırlıklı olarak işlemsel bilgiye dayandıkları belirlenmiştir. Bu çalışma, ortaokul öğrencilerinin matematiksel anlayış ve becerilerini geliştirmek için kavramsal ve işlemsel bilginin dengeli bir şekilde kazandırılmasının önemini vurgulamaktadır.

Anahtar kelimeler: *İşlemsel bilgi, kavramsal bilgi, kesirler, matematik anlayışı, ortaokul seviyesi.*

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Voice of Middle School Students: Illuminating Their Conceptual and Procedural Knowledge in Fractions

Abstract

This study explored sixth- and seventh-grade students' conceptual and procedural fractional knowledge. The participants comprised 30 purposely selected students (aged 12-13) from a private middle school in Istanbul. Employing an explanatory sequential mixed methods design; first, a well-established fractions test was utilized to assess the students' conceptual and procedural understanding; this was followed by classroom observations and semi-structured interviews. Findings revealed a statistically significant correlation between conceptual and procedural knowledge, and high-achieving students exhibited a combination of these knowledge domains, while low-achieving students relied predominantly on procedural knowledge. This study underscores the importance of fostering a balanced acquisition of both conceptual and procedural knowledge in fractions to enhance mathematical understanding and skills among middle school students.

Keywords: *Conceptual knowledge, fractions, procedural knowledge, math understanding, middle school.*

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Introduction

The middle school mathematics curriculum in Türkiye has been revamped to emphasize the relationships between mathematical concepts, the underlying meanings of processes, and the development of procedural skills (Turkish Ministry of National Education [MoNE], 2024). The curriculum places greater emphasis on building a solid foundation of conceptual mathematical knowledge, enabling students to derive mathematical meanings and abstract from concrete experiences and intuitions (MoNE, 2024). In mathematics education, effective instruction should address three core objectives: conceptual knowledge of mathematics, procedural knowledge, and the connections between conceptual and procedural knowledge (Hechter et al., 2022; Rittle-Johnson & Siegler, 1998; Van de Walle, 1990).

This paper aims to explore the reciprocal relationship between conceptual and procedural knowledge in mathematics education, specifically in the domain of fractions, by examining how sixth- and seventh-grade students understand and apply fraction concepts. The study investigates whether stronger procedural skills correspond to deeper conceptual understanding, using a combination of written assessments, interviews, classroom observations, and students' written work to provide a comprehensive view of how these two facets of knowledge interact. Previous studies have examined the development of students' conceptual understanding of fractions, a domain widely recognized as cognitively demanding and critical for later mathematical achievement (Tian & Siegler, 2017). In particular, studies focusing on the relationship between conceptual and procedural knowledge in the context of fractions in secondary education (Lenz & Wittmann, 2021; Sullivan et al., 2023) highlight its importance for students' mathematical learning. Specific attention to this relationship in fractions, using a new study group of middle school students and a mixed-methods approach, offers a unique perspective on the current trend in studies.

Conceptual Knowledge and Procedural Knowledge

Conceptual knowledge, often described as *knowing that,* involves the understanding of relationships between mentally connected entities (Byrnes, 1992). It emphasizes grasping underlying principles, structures, and the interconnections among ideas rather than isolated facts. Researchers (Hiebert & Lefevre, 1986; Fyfe et al., 2014; Rittle-Johnson et al., 2015) characterize conceptual knowledge as rich in relationships and entails absorbing both intuitive and formal information. This view is supported by more recent research highlighting that conceptual knowledge enables flexible problem solving, supports transfer across contexts, and deepens mathematical reasoning (Braithwaite et al., 2021; Clements & Sarama, 2020; Zohar & Ben-Ari, 2022). Kieren (1993) adds that conceptual knowledge is characterized by association, interweaving, and rich relationships. Unlike procedural knowledge, conceptual knowledge emphasizes analysis and connections rather than rote memorization. For instance, a conceptual knowledge question might require students to estimate an addition without performing calculations and explain their reasoning convincingly (Patterson et al., 2020; Tan et al., 2023). On the other hand, the Common Core State Standards emphasize developing strong conceptual knowledge of fractions. This means that students should be able to think flexibly and coherently about fractions—recognizing them as numbers, understanding them in different representations and real-world contexts, and using these understandings to make sense of and perform fraction arithmetic (Schwarzmeier & Obersteiner, 2025). Conceptual knowledge goes beyond procedures to include why operations work and how different representations relate to one another. For example, while iterating a unit fraction, 3/4 can be conceptualized as three iterations of the unit fraction 1/4. This perspective may support middle school students' understanding of fractions as measures and can be applied consistently to both proper and improper fractions (e.g., 7/4 as seven 1/4 units). When multiplying fractions, the concept can be interpreted as taking a fraction of another quantity. For example, 2/3 × 3/4 can be understood as "two-thirds of three-fourths," which can be visualized through area models, tape diagrams or iterated unit fraction reasoning. Importantly, fractions are not only central to students' conceptual mathematical development but also essential for real-world contexts, including applications in the physical, biological, and social sciences, as well as in a range of middle-income professions such as nursing, carpentry, and auto mechanics (Hoyles et al., 2001; Sformo, 2008).

On the other hand, procedural knowledge is referred to as knowing how to perform specific tasks. Byrnes (1992, p. 236) defines procedural knowledge as goal-directed action sequences, encompassing memory methods, mathematical procedures, and grammar rules. Hiebert and Lefevre (1986) and Rittle-Johnson et al. (2015) highlight that procedural knowledge involves using algorithms or rules to complete mathematical tasks. Procedural knowledge is typically linear and context-independent, focusing on following a defined set of actions to obtain the correct answer. A well-developed procedural knowledge refers to middle school students' ability to carry out basic arithmetic operations and algorithms involving fractions, such as addition, subtraction, multiplication, and division. This ability often exists without necessarily understanding the underlying concepts. This includes knowing rules (e.g., invert and multiply for division) and executing step-by-step procedures accurately (Schwarzmeier & Obersteiner, 2025). However, students who rely solely on procedural knowledge may apply rules heuristically without evaluating their appropriateness, or may fail to recognize when an algorithm is incorrectly applied. For instance, "1/2 + 1/3 = ⅖", indicating deficiencies in both procedural and conceptual knowledge. In contrast, procedural fluency combined with conceptual knowledge can support flexible and adaptive mathematical thinking, enabling students to perform complex fraction operations accurately and meaningfully.

Conceptual Knowledge, Procedural Knowledge, and Understanding Fractions

Byrnes and Wasik (1991) suggested that children tend to acquire conceptual understanding before gaining procedural proficiency in fractions. The study involved fourth and sixth-grade students who were asked to answer conceptual knowledge questions and solve various fraction-related problems. The results indicated that students in all grades exhibited a better understanding of conceptual aspects before mastering procedural knowledge. Furthermore, these findings support the "concepts-first" approach proposed by Rittle-Johnson et al. (2001). Conceptual knowledge facilitates the learning of procedural knowledge by enabling students to understand why procedures work and providing motivation to solve problems. Rittle-Johnson et al. (2001) argue that children can acquire both conceptual and procedural knowledge concurrently, utilizing both types of knowledge to enhance their understanding of fractions.

The important lines of work in fractions were also conducted by Izsak et al. (2019), Lewis and Perry (2017), Siegler et al. (2011, 2013), and Sullivan et al. (2023). Siegler et al. (2011) examined middle school students (ages 11-14) on their understanding of fraction magnitudes through number line estimation and magnitude comparison tasks, alongside analyzing their solution strategies in fraction arithmetic problems. The results linked stronger deep conceptual understanding of fractions enhanced broader mathematical skills. Izsak et al. (2019) examined mathematics teachers' understanding (grades 6–12) in a content course focused on multiplication and division concepts. In terms of fraction knowledge, they used a measurement-based approach (e.g., double number lines) to connect fraction operations to proportional reasoning (e.g., How many base units make one group?). They could generate valid methods such as reciprocal multiplication for division. Lewis and Perry (2017) explored the impact of lesson study on students' fraction understanding, showing that sustained professional development can lead to significant gains in both teacher content knowledge and student achievement in fractions. Their findings emphasized the value of collaborative, inquiry-based teacher learning in addressing persistent challenges in fraction instruction. Sullivan et al. (2023) experimented on students' common misconceptions in fraction knowledge. They showed us that students often used the gap reasoning approach (e.g., comparing 5/6 and 7/8 by noting both are "1 away from the whole"). Their key finding highlighted how a dominant part-whole conception of fractions hindered understanding of fraction magnitude and operations. The first study highlights students' understanding of fraction magnitude using number line concepts, the second focuses on students' misconceptions, the third demonstrates the effectiveness of professional development through lesson study, while the fourth shows how teacher training can overcome such gaps through structured, measurement-focused pedagogy.

In addition to these, Steffe and Olive (2010) have provided a comprehensive account of how children's fractional schemes develop, particularly focusing on the progression from part-whole understanding to more sophisticated measurement-based and iterative fraction schemes (Steffe & Olive, 2010). Their constructivist framework has been highly influential in framing how students build conceptual knowledge over time. Similarly, Hecht (1998) emphasized the distinct contributions of procedural and conceptual knowledge to fraction problem solving, showing that these two knowledge types are separable yet interdependent. His work also revealed the importance of working memory and reading comprehension in supporting students' fraction learning. Van Hoof et al. (2015) extended this by studying students' strategy use, noting that flexible switching between strategies (e.g., mental number line vs. part-whole reasoning) reflects deeper understanding. Their findings suggest that instructional design should not only teach procedures but also promote metacognitive awareness of when and why strategies work. These contributions further emphasize that fraction knowledge is not monolithic but multifaceted, involving magnitude understanding, strategic reasoning, representation fluency, and connections to proportionality. So, although already known about how students reason about fractions in controlled settings, there needs to be more integrated studies that trace students' conceptual and procedural understanding across assessment data, classroom behaviors, and student reflections.

The literature on conceptual and procedural knowledge in fractions revealed the need to address various aspects of students' understanding (Ahrendt et al., 2021; Hallett et al., 2010; Nahdi & Jatisunda, 2020; Sullivan et al., 2023). Some studies used experimental approaches to investigate conceptual and procedural knowledge, while others focused on pre-service teachers' theoretical and operational awareness of fractions. Several studies assessed students' levels of conceptual and procedural knowledge in fractions. Despite the rich body of work noted above, a gap remained in studies that integrated these perspectives to examine how both types of knowledge interact in real classroom settings with specific instructional tools. Additionally, Powell (2023) examined 35 second-grade students' conceptual understanding of fraction magnitude under an experimental design. The research team asked the students to utilize manipulatives such as Cuisenaire rods and to discuss fraction relations by grounding the study in a cultural-historical framework. Through physical manipulatives and mathematical conversation, the students grasped that a fraction represents a multiplicative comparison between two commensurable quantities of the same kind. This study found that the students were able to conceptualize the size of fractions of a quantity and develop symbolic expressions incorporating fractional comparisons based on their mental manipulations of non-numeric images.

Building on previous observations that some students struggle with developing both conceptual and procedural knowledge in fractions, this study aims to investigate how these two aspects of knowledge are related among sixth- and seventh-grade students in a private school. Specifically, the study examines whether students who demonstrate stronger procedural skills also show corresponding conceptual understanding. To achieve this, students' fraction knowledge is assessed through a written worksheet, complemented by qualitative data from interviews, observations, and students' written work, providing a comprehensive view of how conceptual and procedural knowledge interact in the learning process. Hence, the purpose of this study is to investigate students' conceptual and procedural understanding of fractions, examining how these two aspects of knowledge are interrelated, and to explore how students' reasoning is reflected in their verbal explanations, and written work.

Significance of the Study

This study is scientifically and academically significant in the field of mathematics education, particularly in understanding the relationship between conceptual and procedural knowledge in the context of fractions. The research contributes to the existing literature by providing empirical evidence of a positive correlation between these knowledge domains. The findings emphasize the importance of integrating conceptual and procedural knowledge to enhance students' mathematical understanding and proficiency.

This paper investigates the relationship between students' conceptual and procedural understanding of fractions in a middle school context through a mixed-methods approach. While studies have extensively explored this relationship within subjects like calculus (Hechter et al., 2022), algebra (Gerasimova et al., 2023) at higher education levels, and fractions in secondary school education (Lenz & Wittmann, 2021), as well as aspects of affective characteristics such as multifaceted math anxiety related to fractions (Rayner et al., 2009), certain gaps persist. Notably, most of these studies collected data from university students, particularly pre-service teachers (e.g., Dogan Coskun et al., 2023; Novikasari & Dede, 2023; Özpınar & Arslan, 2021).

This paper offers a contribution by shifting the focus by specifically examining the relationship between conceptual and procedural knowledge in fractions within a previously understudied group: middle school students. This shift in focus, coupled with a mixed-methods approach, presents a novel angle that enriches the existing body of research. By targeting this specific age group, it seeks to uncover findings that could significantly contribute to understanding how these foundational mathematical concepts interact and develop in younger learners. This approach stands as a departure from the current trend in studies, providing a fresh lens through which to analyze and comprehend the dynamics of conceptual and procedural knowledge acquisition in fractions.

The study's insights into the relationship between mathematics grades and both conceptual and procedural knowledge underscore the significance of these knowledge types for students' overall mathematical performance. The observed correlation indicates a potential link between conceptual knowledge and previous mathematics grades. This indicates that students with a solid conceptual foundation are more likely to excel in mathematics. Therefore, educators should prioritize the teaching of conceptual knowledge before introducing procedural skills. Byrnes and Wasik (1991) conducted two studies that recommended acquisition of conceptual understanding as a precursor to procedural competence with fractions, further supporting the notion that students should develop conceptual knowledge before delving into procedural aspects.

To support the integration of conceptual and procedural knowledge, instructional strategies may need to be carefully designed. Teachers should employ methods that encourage students to connect conceptual understanding with procedural application. This could involve providing real-world contexts, problem-solving activities, and opportunities for students to explain their reasoning and justify their solutions. By incorporating conceptual understanding into procedural practice, students can develop a deeper and more meaningful understanding of fractions.

Moreover, it is crucial to ensure that students receive appropriate support and scaffolding to bridge the gap between conceptual and procedural knowledge. Teachers should monitor students' progress closely, identify areas of difficulty, and provide targeted interventions to address misconceptions and strengthen conceptual understanding. Differentiated instruction can be implemented to cater to the diverse learning needs and abilities of students, allowing for individualized support as they navigate the complexities of fractions.

Methods

Research Design

This study employed a mixed-methods research design, namely, a quantitative approach with an explanatory sequential mixed-methods design, and a follow-up explanation procedure (Creswell & Plano-Clark, 2017) as presented in Table 1. The research was conducted in two stages, utilizing a conventional written assessment and using interviews, observations, and students' written work to explore whether stronger procedural skills correspond to deeper conceptual understanding.

Table 1. *Design process in the study.*

Research Que.	Research Paradigm	Information Source	Sampling Method	Data Collection Tool	Data Collection Time	Data Analysis Method
1	Quantitative	Middle school students (n = 30)	Convenient	Test of Fractions Knowledge	Pre-test before the mathematics weekly instructions	Descriptive analysis
2	Quantitative	Middle school students (n = 30)	Convenient	Test of Fractions Knowledge	8-months later	Inferential analysis
3	Qualitative	Middle school students (n = 8)	Purposeful	Interview protocol	Semi-structured interview	Narrative analysis
4	Qualitative	Documents	Purposeful	Observation protocol	Field notes	Document analysis
5	Mixed	Integration of the findings				Quantitative→ Qualitative (Creswell & Plano-Clark, 2017

Participants

The participants were students from a private school in Istanbul from families with relatively higher socio-economic backgrounds. The sample included sixth and seventh-grade middle school classes, comprising a total of 30 students (aged 12-13). They followed the national middle school mathematics curriculum, which incorporates constructivist pedagogical principles (MoNE, 2024). Its perspective lies within our values, basic competencies (e.g., communication in mother tongue and foreign language, mathematical and digital competencies, learning to learn, to take initiative and entrepreneurship, and cultural awareness. The authors had the opportunity to observe these students twice a week for eight months, allowing the researchers to develop a deeper familiarity with students' learning behaviors and reasoning patterns. To ensure confidentiality and unbiased data analysis, each student was assigned a code for testing and analysis purposes in a codebook. However, these codes were used solely for organizational purposes during analysis, and the order of the codes was based on the students' performance on the worksheet.

Data Collection Tools and Procedure

Written assessments (paper-and-pencil tests) are widely used and valuable method for assessing and evaluating students' achievements (Heyworth, 1999). In this study, the authors analyzed and evaluated the paper-pencil test completed by students to assess their conceptual and procedural knowledge. Each student individually completed the assessment (Test of Fractions Knowledge) that evaluated their knowledge of fractions, covering both procedural and conceptual aspects. The conceptual knowledge assessment consisted of 13 questions, while the procedural knowledge assessment included 8 questions. These worksheets were developed by a professor specializing in mathematics education at the Faculty of Education, following the objectives set by the MoNE (2024) and relevant literature (Chan et al., 2023; Pesen, 2007; Sowder et al., 1998; Van de Walle, 2007).

The paper-pencil tests were administered to all students at the Department of Mathematics Education. The classes were mixed in gender, and the students were familiar with the examination process. The authors provided instructions to the teacher, who then distributed the tests to the students. The students were given 45 minutes [i.e. The maximum duration permitted by the MoNE in middle schools] to complete the test.

The assessment of conceptual and procedural knowledge in fractions was conducted through a paper-pencil test administered individually to each student. The test was developed by adapting and compiling items from existing instruments used in previous studies by Alacacı (2010), Aksoy and Yazlik (2017), Kassim et al. (2017) and Rosli et al. (2024), and Örmeci (2012). This assessment tool has previously been validated for use with middle school students and has proven effective in measuring fraction knowledge in similar age groups (Örmeci, 2012).

The test items were structured to evaluate students' understanding across seven key areas related to fractions: (1) the meaning of fractions, (2) comparison of fractions, (3) reference to the whole, (4) addition and subtraction of fractions, (5) multiplication and division of fractions, (6) representation of fractions, and (7) rules of operations. These categories reflect common difficulties encountered in national assessments and are supported by prior literature (Alacaci, 2010; Chan et al., 2023; Charalambous & Pitta-Pantazi, 2007; Fisher & Dennis, 2023; Lamon, 2020).

After collecting the students' responses, a *Fraction Rubric* was used to systematically assess both students' conceptual understanding and their procedural skills in fraction-related tasks, ensuring a structured and consistent evaluation across their written responses. The conceptual part was prepared with items to score individually (0–2), resulting in a total possible score of 44 points, whereas the procedural part was prepared with restricted constructed-response items to score 0–1, yielding a total of 19 points. Its analytic scoring structure allowed for capturing nuanced differences in students' reasoning, particularly in restricted open-ended explanations that required evaluating the depth and accuracy of mathematical thinking. Using this rubric provided a reliable framework for quantifying qualitative variations in students' performance, thereby enabling meaningful statistical analysis of the relationship between conceptual and procedural knowledge (see Table 2 for more information).

Moreover, students were interviewed after completing the test. The interview questions were primarily designed to explore the reasoning behind students' responses to the fraction problems they completed on the test. Students were asked to explain their interpretations of each problem, the strategies they used to solve it, and the rationale for the steps they followed. The questions aimed to uncover both conceptual understanding (e.g., how they perceived fractions as parts of a whole) and procedural knowledge (e.g., the methods used to perform operations), as well as any connections between these two facets of their thinking. Additionally, students were encouraged to describe any visual representations, gestures, or analogies they used while solving the problems, providing richer insight into their cognitive processes.

Finally, one of the researchers participated in the classroom of the mathematics teacher. She observed instructional practices that emphasized conceptual foundations in mathematics, helping their students form mathematical meanings. The observations conducted in this study were guided by two primary objectives: (a) to document how students demonstrated conceptual understanding of fractions during problem-solving, and (b) to examine how this conceptual understanding translated into procedural actions. Observation content, therefore, focused on students' verbal reasoning, their use of visual or representational strategies (e.g., drawings, diagrams, gestures), and the extent to which they articulated underlying principles of fraction operations. In addition, the observations captured students' spontaneous problem-solving behaviors—such as drawing fraction bars or circles, using analogies, explaining steps to peers, or relying on memorized procedures. These observation notes were complemented by students' handwritten artefacts produced during the activities, allowing the researchers to trace connections between students' conceptual explanations and the procedural steps they executed. Together, the observations provided rich, context-embedded evidence supporting the analysis of students' conceptual and procedural knowledge in fractions.

Validity and Reliability

To ensure confidentiality and unbiased analysis, each of the 30 students was assigned a code for identification, with codes assigned for organizational purposes during analysis. To ensure the content-related validity of this rubric (Moskal & Leydens, 2019), the research team reviewed the rubric's content. Specifically, they scrutinized whether the questions incorporated complex sentence structures that might unintentionally measure students' reading comprehension skills rather than their fraction knowledge. Furthermore, to ensure construct validity, the research team aligned the learning outcomes with rubric items. As part of a fairness check (consequential validity), results were examined for gender-based discrepancies. Finally, a full professor specializing in mathematics education, along with the authors cross-checked the scoring decisions to assess interrater reliability. An agreement level of $\cong$ 82% between the authors and the professor (as an expert) suggests that the rubric items are conceptually coherent, indicating strong consistency (Miles et al., 2014). Any disagreements were duly addressed and resolved. Similarly, the interview and observation protocols were reviewed by the authors and the expert professor to ensure both content and construct validity, confirming that the questions and observational focus accurately captured students' conceptual and procedural understanding of fractions. Interrater reliability was also verified during the coding of students' explanations, gestures, and handwritten work, with an agreement level of $\cong$ 87%, with any discrepancies discussed and resolved to maintain consistent interpretation of the qualitative data.

Data Analysis

Fractions Test

The authors used a rubric to determine and evaluate the students' conceptual and procedural knowledge test results. Each question in the conceptual knowledge section was graded using a scale of 0, 1, or 2 points. A score of 0 indicated an incorrect answer or incorrect explanations, 1 indicated an incorrect answer with some reasonable explanations, and 2 indicated a correct answer with explanations. In total, the conceptual knowledge section consisted of 22 items. Some of these were open-ended and allowed for multiple sub-responses, contributing to a higher cumulative score. The maximum possible score for conceptual knowledge was therefore 44 points, calculated by summing up all the possible points across items (e.g., 22 items × 2 points for fully correct answers).

Each procedural knowledge question was graded on a scale of 0 to 1 point, where 0 indicated an incorrect answer and 1 indicated a correct answer. There were 19 distinct procedural tasks in total, some embedded within multi-part questions or scenarios, each scored individually, making the maximum possible score 19. Both scores were standardized to a 0–100 scale for comparison.

The conceptual and procedural knowledge levels were then examined for correlation using statistical software, SPSS 22 (George & Mallery, 2018; Ratner, 2009). Content validity was further supported through expert review. That is to say the expert assessed whether the items purporting to measure conceptual knowledge did in fact measure conceptual knowledge, and likewise for procedural knowledge (i.e., content validity of the rubric was ensured through expert review and alignment with curriculum standards).

Table 2. *Table of specifications for items' knowledge and cognitive process dimension.*

Rubric Section	Knowledge Dimension	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8	Item 9+	Total Point
1. Conceptual*	Understanding; Applying; Analyzing; Creating	✓	✓	✓	✓	✓	✓	✓	✓	✓	44
2. Procedural**	Applying & Analyzing	✓	✓	✓	✓	✓	✓	✓	✓	-	19

* Conceptual knowledge items included multi-part open-ended questions such as: analyzing shapes, comparing shaded areas to unit fractions, explaining reasoning using students' own words, and interpreting visual models. Several items consisted of sub-tasks, each scored individually (0–2), resulting in a total possible score of 44 points.

** Procedural knowledge items required solving restricted open-ended tasks involving the application of fraction operations (e.g., addition, subtraction, multiplication, division). These were discrete items or sub-items across different questions, with each scored 0–1, yielding a total of 19 points.

Notably, cognitive process dimensions were adapted from Revision of Bloom's Taxonomy (Anderson & Krathwohl, 2001), covering a range from understanding and applying to analyzing and creating. To sum up, in light of the research design and data collection procedures outlined above, this study aimed to address the following research questions: (1) What is the level of conceptual knowledge and procedural knowledge among sixth and seventh-grade students regarding fractions? (2) Is there a relationship between students' conceptual knowledge and procedural knowledge of fractions? To answer these research questions, a mixed-methods approach was employed, with an initial quantitative analysis of the test results to determine the students' conceptual and procedural knowledge scores. The correlation analysis using SPSS 22 allowed for an examination of the relationship between conceptual knowledge and procedural knowledge scores. By answering these research questions, this study seeks to contribute to more nuanced understanding of how these knowledge types relate.

Semi-structured Interviews, Observations and Student Artefacts

Following the initial open coding of interview transcripts, observation notes, and students' handwritten artefacts, codes were grouped into thematic categories. The number of students associated with each category was then estimated based on repeated patterns across the dataset ($n \approx 8$ interviews and corresponding observations). These frequencies were reported in approximate ranges to avoid overstating precision, consistent with qualitative research conventions that emphasize patterns rather than exact counts. This approach enhanced the interpretability of the findings and further detailed the results coming from the quantitative analysis.

Results

The results of this study provide insight into the relationship between conceptual knowledge and procedural knowledge in the context of fractions. Through the administration of conceptual and procedural knowledge tests to sixth- and seventh-grade students at a private school in Istanbul, Türkiye, the researchers aimed to examine the extent to which these two knowledge domains are correlated. The findings revealed a significant positive correlation between conceptual knowledge and procedural knowledge scores, indicating that students who demonstrated a strong conceptual understanding of fractions also exhibited proficiency in applying procedural skills. However, it is noteworthy that not all students displayed parallel performance across both knowledge domains, suggesting the presence of individual differences in their mathematical learning.

Additionally, the study investigated the correlation between mathematics grades and both conceptual knowledge and procedural knowledge, uncovering substantial associations that underscore the importance of these knowledge types for overall mathematical performance, respectively ($r = 0.902$, $p = 0.007$; $r = 0.622$, $p = 0.006$. In interpreting these results, it is important to consider the instructional context in which the data were collected. The participating school follows a curriculum aligned with national standards and integrates a mixture of traditional instruction and inquiry-based learning activities in mathematics. Fractions were taught through multiple representations—including area models, number lines, and word problems—and students engaged in structured practice followed by collaborative discussions designed to deepen conceptual understanding. The classroom instruction during the semester emphasized both the meaning of fractions (e.g., part-whole relationships, equivalence) and operations with fractions (e.g., algorithmic steps and justifications). As such, the observed correlations likely reflect not only students' individual aptitudes but also the influence of this dual emphasis in instructional materials and teaching methods.

Correlations

Regarding the items in the Fractions Rubric (refer to Table 2), there are twelve restricted open-ended items about fractions, nine of which are aimed to measure understanding (i.e. lower-level thinking) and three of which are creating (i.e. higher order thinking) cognitive process dimension (Krathwohl & Anderson, 2010). The correlation analysis revealed a significant positive correlation ($r = 0.416$, $p = 0.046$) between students' conceptual knowledge (CK) scores and procedural knowledge (PK) scores. This indicates a moderate positive linear relationship between the two types of knowledge. However, it should be noted that not all students who excelled in conceptual knowledge performed equally well in procedural knowledge, challenging the initial hypothesis.

Conceptual Knowledge and Procedural Knowledge Test

While there was a statistically significant relationship between procedural knowledge and conceptual knowledge in fractions, student-level variation was observed. Some students exhibited scores that deviated from the overall pattern. Several factors could contribute to these results, such as students losing focus or having better interpretive skills compared to operational skills. The finding that some students were more adept at procedural tasks further supports this observation.

Specifically, none of the students answered conceptual knowledge question 8 correctly, which required them to develop a problem using fraction operations. Conceptual Knowledge Question 8 asked students to construct a word problem corresponding to the given operation: 4/5 x 1/3= 4/5. This indicates that while students were able to solve the operation, they struggled to create a problem based on that operation. This highlights the need for students to interweave and establish rich relationships between conceptual knowledge components.

Figure 1 illustrates individual scores across conceptual and procedural domains. Notably, Student 23 demonstrated relatively high conceptual scores but lower procedural performance, illustrating a divergence in knowledge profiles.

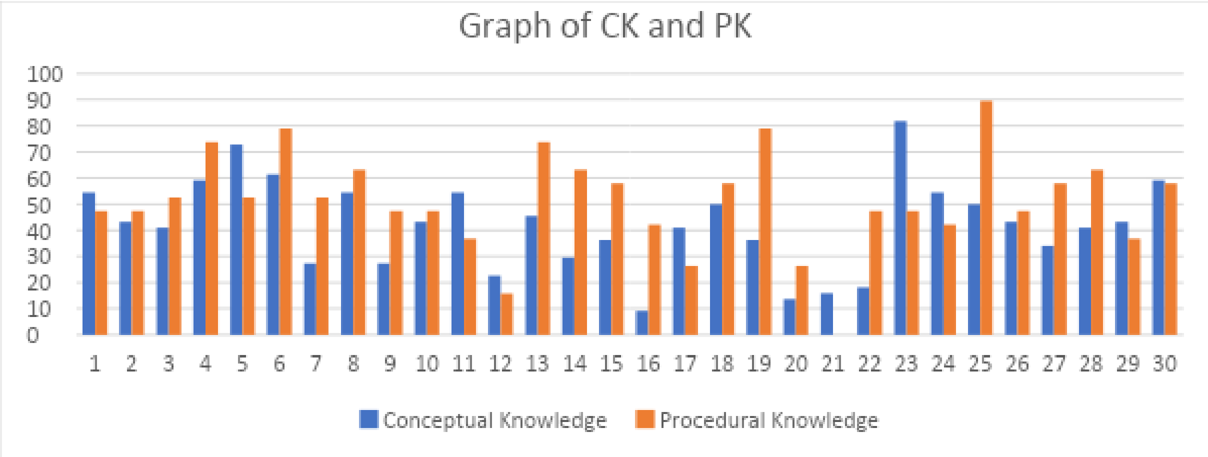

Figure 1. Student scores in each knowledge domain.

Among the target students, 18 demonstrated better performance in procedural knowledge, revealing an evident disparity between conceptual knowledge and procedural knowledge scores. In the conceptual knowledge test, the differences were not as pronounced as in the procedural knowledge test. Therefore, students generally exhibited higher scores in the procedural knowledge test compared to the conceptual knowledge test. Lastly, the scatter plot in Figure 2 indicates a weak positive relationship between conceptual and procedural knowledge in fractions.

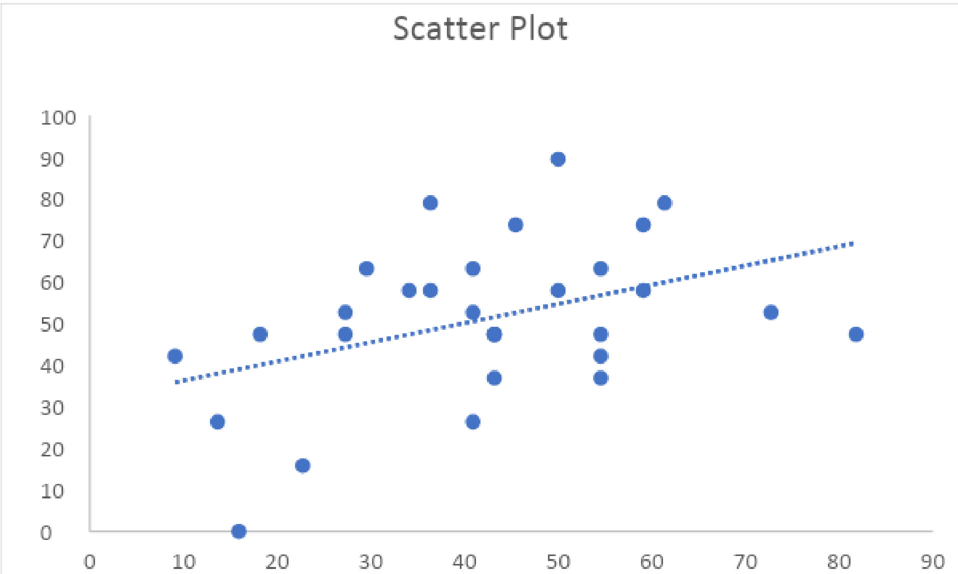

Figure 2. Scatter plot on a possible relationship between procedural and conceptual knowledge domains.

A strong positive correlation (r = 0.902, p = 0.007) was observed between students' mathematics grades (as indicated by their school report) and their conceptual knowledge (CK) scores. This suggests that students who achieved higher mathematics grades in the previous year also performed well in the conceptual knowledge test.

A moderate positive correlation ($r = 0.622$, $p = 0.006$) was found between students' mathematics grades (end-of-term scores) and their procedural knowledge (PK) scores. The correlation coefficient for procedural knowledge and mathematics grade was lower compared to the correlation between conceptual knowledge and mathematics grade. This suggests that conceptual knowledge may be more closely aligned with overall mathematics performance than procedural knowledge.

In summary, the results indicate a statistically significant correlation between conceptual and procedural knowledge. However, some students with higher conceptual scores did not show comparable procedural performance. The correlation between conceptual knowledge and mathematics grade was stronger than the correlation between procedural knowledge and mathematics grade.

The correlations between mathematics grades and both conceptual knowledge and procedural knowledge further emphasize the importance of these knowledge domains in students' overall mathematical performance. Students who demonstrated strong conceptual knowledge scores also achieved higher mathematics grades, indicating a stronger association between conceptual understanding and mathematics grades. Similarly, students with higher procedural knowledge scores tended to have higher mathematics grades, although the correlation coefficient was lower. This suggests that while procedural knowledge is important, it may be comparatively easier for students to retain and apply conceptual knowledge.

Students' Understanding: Self-report and Observations

The interviews conducted as part of this study confirmed the quantitative results, indicating a consistent alignment between students' conceptual and procedural knowledge in fractions. Students who demonstrated a strong conceptual understanding of fractions also exhibited proficiency in applying procedural skills, which was supported by their explanations during the interviews. As a representative example, a student expressing strong conceptual understanding and proficiency in applying procedural skills regarding fractions explained that they viewed fractions as "pieces of a whole," likening them to slices of a pizza. According to the student, each fraction (e.g., 1/8 or 1/4) represents a part of the whole, and this understanding of the whole–part relationship constitutes the conceptual aspect. The student further noted that when performing fraction operations, such as addition or subtraction, they first ensure the denominators are the same and then add the numerators, likening this process to putting the pizza slices together. This suggests that conceptual understanding may support students in organizing procedural steps.

In this example, which was representative of several similar cases, the student, who demonstrated a clear conceptual grasp of fractions, also showcased adeptness in applying procedural skills. Hence, the qualitative evidence further reinforced the significant relationship between conceptual and procedural knowledge, reinforcing the findings obtained through the quantitative part. In line with this, additional student responses similarly highlighted how conceptual imagery supported procedural fluency. For example, another student noted that "fractions make more sense when I draw them first," referring to sketches in her notebook where she shaded equal parts before performing calculations. Such self-generated visuals underscored the role of representational thinking in bridging conceptual and procedural understanding.

A cross-case coding of all student explanations (n=8) revealed that the majority of students (approximately 75%) reported that visualizing fractions helped them—either mentally or through drawings—when solving problems. Another recurring theme (around 50%) involved students stating that understanding the whole–part relationship supported their confidence in performing calculations. A smaller subset (about 25%) described relying mostly on memorized steps, indicating a more procedural orientation. These frequencies highlight both shared patterns and meaningful divergences within the student group.

The interviews further substantiated the alignment between conceptual and procedural knowledge in fractions, providing nuanced insights into students' cognitive processes. During the interviews, it was consistently noted that students with a strong conceptual understanding of fractions showed fluency in translating conceptual understanding into procedural actions. For instance, as students explained their reasoning, their gestures and expressions reflected a tactile engagement with the concept of fractions as parts of a whole, reinforcing the link between conceptualization and procedural execution. Moreover, the students' ability to seamlessly articulate the underlying principles of fraction operations while solving problems reinforced the notion that a solid conceptual foundation facilitates the application of procedural skills. These observations were not only consistent across individual interviews but also mirrored the broader trends identified in the quantitative analysis, thereby establishing a consistent association between the two facets of knowledge. Additionally, the observation notes included references to students' handwritten work produced during the interviews. Several students spontaneously drew fraction bars, circles, or number lines to support their explanations. These handwritten representations—such as partitioned circles illustrating equivalence or number line segments showing ordering of fractions—provided tangible evidence of how students externalized their conceptual understanding before initiating procedural steps. The alignment between these visual artefacts and their verbal explanations strengthened the validity of the qualitative findings.

Approximate frequencies derived from the observation notes indicated that drawing or sketching representations was the most commonly observed strategy (around 70% of students). Gestures illustrating partitioning or combining parts were noted in about 60% of the interviews. A smaller group (roughly 20%) relied almost entirely on algorithmic procedures without using visual aids or gestures, marking a clear contrast with the dominant conceptual–procedural integration pattern.

In addition, the classroom environment provided additional opportunities to observe the interplay between conceptual and procedural understanding. Students who excelled in the quantitative assessments were observed engaging in collaborative learning activities, where they not only accurately solved fraction problems but also actively participated in discussions, explaining their thought processes to peers. This peer-to-peer communication often involved the use of conceptual metaphors, such as the pizza analogy mentioned in the interview example. The collaborative nature of these interactions suggested that students were not merely memorizing procedures but were internalizing the conceptual essence of fractions, facilitating a more profound integration of conceptual and procedural knowledge. These classroom observations complemented and reinforced the findings from the interviews, painting a comprehensive picture of the intricate relationship between conceptual and procedural understanding in the context of fractions.

Across these classroom interactions, the most common recurring idea (approximately 60%) involved students explaining fraction operations using visual or object-based metaphors. Meanwhile, differing views were evident regarding the comparison of unlike denominators, with about 30% of students expressing uncertainty or difficulty. Furthermore, examples of students' shared handwritten work during group tasks—such as jointly constructed fraction models, corrected solution steps, and visual comparisons of unlike denominators—provided additional insights into how conceptual thinking emerged and evolved within collaborative settings. These artefacts showed how students discussed and refined their ideas collaboratively and refined their procedural approaches based on shared visual reasoning, thereby deepening the qualitative evidence supporting the study's results.

Discussion

The findings of this study provide insights into the relationship between conceptual knowledge and procedural knowledge in the context of fractions. The results indicate a moderate positive correlation between these two knowledge domains, suggesting that students who possess a strong conceptual understanding of fractions also generally performed better on procedural tasks. This finding aligns with previous research that has emphasized the interconnectedness of conceptual and procedural knowledge (Baki & Kartal, 2004; Hiebert & Lefevre, 1986; Li, 2025; Xu et al., 2025), and importantly, is corroborated by our own data showing a moderate correlation ($r = 0.416$, $p < 0.05$) between CK and PK scores, and a strong correlation between conceptual knowledge and mathematics grades. These findings are consistent with results observed in our study context of middle school students in Türkiye, offering localized insights into a globally relevant issue. However, ongoing discussions on the development of these knowledge types persist, with no clear consensus reached (Hallett et al., 2010; Rittle-Johnson, 2017; Schneider & Stern, 2005). This also aligns with the integrated framework proposed by Izsak and colleagues (2019), who emphasize that learning is most effective when students coordinate conceptual and procedural knowledge, rather than treating them as separate domains.

One line of thought posits that conceptual knowledge and procedural knowledge develop sequentially, with one type of knowledge preceding the other (Rittle-Johnson et al., 2015). Conversely, another perspective suggests that conceptual and procedural knowledge continually influence and improve each other, with advancements in one area leading to gains in the other (Schneider & Stern, 2005; Stovner & Klette, 2022). Supporting this reciprocal view, Sullivan et al. (2023) underline the importance of instructional strategies that integrate reasoning with skill-building, arguing that developing conceptual understanding alongside procedures is essential for deep mathematical learning.

Moreover, differences in learning abilities among individuals contribute to the divergent viewpoints on conceptual and procedural knowledge (Hallett et al., 2010; Majeed & AlRikabi, 2022; Prediger et al., 2023). Such differences are echoed in the findings of Siegler et al. (2011, 2013), who demonstrated that early variations in numerical understanding significantly predict later differences in students' capacity to coordinate procedural and conceptual knowledge. Mathematics learning is not solely reliant on algorithms and rules but also encompasses relationships, reasoning, and concepts. Without a solid conceptual foundation, students struggle to grasp the meaning behind mathematical principles and associated procedures (Bach et al., 2025; Birgin & Gürbüz, 2009; Lamon, 2020). Likewise, without procedural knowledge, students are unable to effectively apply rules and algorithms to solve problems (Silver, 1986). As found over years, incomplete mastery of a topic arises when conceptual knowledge and procedural knowledge are not aligned (Aydın & Soylu, 2006; Chan et al., 2022). To promote such alignment, Lewis and Perry (2017) advocate for collaborative teaching models like lesson study, which enable teachers to design instruction that gives equal weight to both conceptual understanding and procedural fluency. Therefore, it is crucial to strike a balance between conceptual knowledge and procedural knowledge to achieve success in mathematics. This balance is particularly emphasized in the Turkish curriculum reforms based on the updated mathematics curriculum (MoNE, 2024), where both knowledge types are treated as complementary learning goals rather than hierarchical stages.

Notably, the prevalence of procedure-based teaching may contribute to an overemphasis on procedural knowledge at the expense of conceptual understanding (Ncube & Luneta, 2025). However, procedural knowledge alone is insufficient for students to excel in mathematics; it must be supported by a strong conceptual foundation. Relying solely on procedural knowledge can lead to rote-learning and computational errors over time (Aksu, 1997). Therefore, students may benefit from understanding the concepts underlying algorithms to internalize procedural knowledge effectively. In line with these findings, it is evident that conceptual knowledge and procedural knowledge should be developed in tandem to foster success in mathematics. Available research also emphasizes that teacher beliefs and assessment practices may influence how this balance is enacted in the classroom (Baird, 2010; Saralar-Aras & Tütüncü, 2024, 2025). For instance, conceptual depth is often sacrificed in high-stakes exam environments, which tend to reward procedural speed and accuracy.

The present study identified variations in performance between conceptual knowledge and procedural knowledge among individual students. For instance, some students exhibited scores that deviated from the overall pattern. Several factors could contribute to these results, such as students losing focus or having better interpretive skills compared to operational skills. Additionally, the finding that some students were more adept at procedural tasks further supports this observation. This highlights the need for students to interweave and establish rich relationships between conceptual knowledge components. As Izsak et al. (2019) note, fostering such integration can help bridge student inconsistencies in performance and promote more robust mathematical thinking. Similar divergences have been reported in earlier studies (Hallett et al., 2010), suggesting that mismatches between instructional emphasis and students' cognitive profiles may lead to inconsistencies in their knowledge development.

Conclusion

In conclusion, the study provides evidence that middle school students (i.e., sixth and seventh graders) with stronger conceptual understanding of fractions also tend to show higher procedural performance, indicating a relationship between these knowledge domains. These point to a clear pedagogical implication that curricula and instructional practices should deliberately connect conceptual models with procedural strategies to enhance students' fractional competence and broader mathematical achievement. Educators across diverse contexts should prioritize conceptual frameworks before introducing procedural techniques, ensuring that students grasp the significance of concepts when applying procedural knowledge. By implementing evidence-based instructional strategies, differentiating support according to individual learning profiles, and promoting active, meaningful engagement with content, educators can promote the integration of conceptual and procedural knowledge, which may contribute to stronger mathematical performance. Future research should investigate cross-cultural variations, longitudinal impacts, and innovative pedagogical approaches that support the seamless integration of these critical knowledge domains, ultimately contributing to the global advancement of mathematics education.

Suggestions

It is important to acknowledge the limitations of this study together with the suggestions. Firstly, the research was conducted in a specific school in Istanbul, which limits the generalizability of the findings to all sixth- and seventh-grade students in Türkiye. Future research should aim to replicate the study in multiple schools and regions to obtain a more representative sample and validate the results across different contexts.

Additionally, data collection relied solely on a paper-and-pencil test, along with interviews and observations based on it, which may have introduced limitations. Some items may not have aligned perfectly with all students' reading or reasoning abilities. Furthermore, the level of difficulty of the test may not have aligned perfectly with the abilities of all students. To mitigate these issues, future studies could consider employing alternative assessment methods to gather a more comprehensive understanding of students' conceptual and procedural knowledge.

Furthermore, the convenience sampling method used in this study may have introduced biases. Future research should employ more rigorous sampling techniques to ensure a diverse and representative sample. Moreover, involving multiple researchers and conducting the study across various schools would enhance the reliability and validity of the findings.

Implications for Practice

The findings of this study have significant implications for instructional practices in mathematics education, particularly in the teaching of fractions. The interview results of this study highlight the connection between conceptual and procedural knowledge in fractions. The qualitative insights gained from interviews reveal that students who showed a strong conceptual grasp and tended to perform well on procedural tasks. The example provided by a student, emphasizing the understanding of fractions as parts of a whole and connecting it to procedural steps in operations, serves as a vivid illustration of the symbiotic relationship between conceptualization and procedural execution. These interview findings align with the observations made during classroom activities, where students often integrated conceptual metaphors into procedural problem-solving, highlighting the organic connection between the two knowledge domains.

The classroom observations further support the implications for instructional practices in mathematics education. Students who excelled in quantitative assessments were observed engaging in collaborative learning activities, where they not only accurately solved fraction problems but also actively communicated their conceptual understanding to peers. This peer-to-peer interaction demonstrated the practical application of conceptual knowledge in a collaborative context, reinforcing the argument that fostering a comprehensive understanding of fractions involves not only individual conceptual mastery but also effective communication and sharing of knowledge. These findings collectively emphasize the need for instructional practices that integrate conceptual and procedural knowledge.

The implications for teaching fractions outlined in this study are reinforced by the qualitative results, providing a nuanced understanding of how these instructional practices can be implemented effectively. The strong correlation between conceptual knowledge and overall mathematical performance, as well as the connection between conceptual understanding and previous mathematics grades, underlines the pivotal role of conceptual knowledge as a predictor of academic success. The study's findings advocate for a balanced instructional approach that prioritizes the development of conceptual knowledge before introducing procedural skills, aligning with the recommendations of Byrnes and Wasik (1991). Moreover, the suggestion to design strategies connecting understanding with application, incorporating real-world contexts and problem-solving, aligns with the observed classroom practices where students naturally applied conceptual understanding in procedural tasks.

In conclusion, the integration of interview results, observation outcomes, and qualitative findings support the implications for instructional practices in mathematics education. The study underscores the necessity of teaching conceptual and procedural knowledge in tandem, providing a foundation for educators to design strategies that connect understanding with application, promote collaborative learning, and address diverse learning needs.

Ethics Committee Approval

We received ethical permission from MEF University Ethics Committee, numbered: MEF-13052019.

Author Contribution

Both authors contributed to the study equally.

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Conflict of Interest

There is not any conflict of interest.

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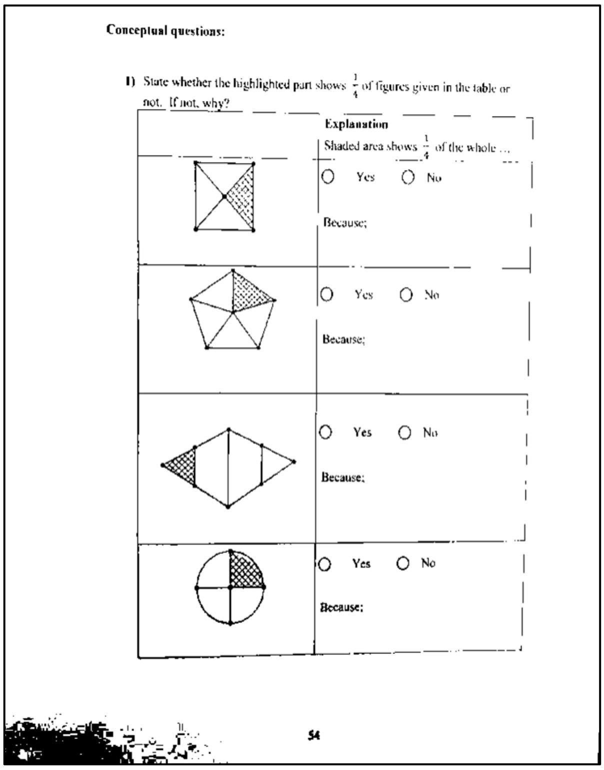
Appendices

Appendix A. Conceptual and Procedural knowledge test questions (Alacacı, 2010, pp.54-57)



Conceptual questions:

1) State whether the highlighted part shows $\frac{1}{4}$ of figures given in the table or not. If not, why?

	Explanation Shaded area shows $\frac{1}{4}$ of the whole ...
	O Yes O No Because;
	O Yes O No Because;
	O Yes O No Because;
	O Yes O No Because;

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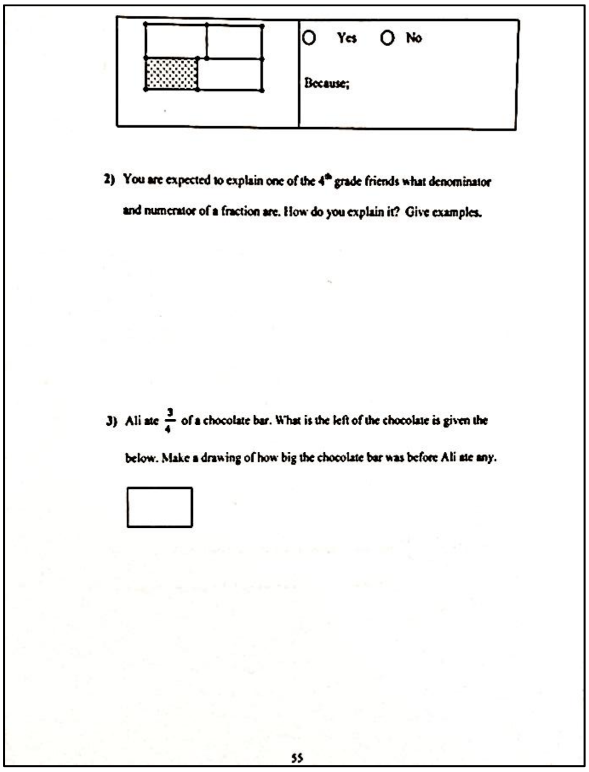

	O Yes O No Because;

2) You are expected to explain one of the 4th grade friends what denominator and numerator of a fraction are. How do you explain it? Give examples.

3) Ali ate $\frac{3}{4}$ of a chocolate bar. What is the left of the chocolate is given the below. Make a drawing of how big the chocolate bar was before Ali ate any.

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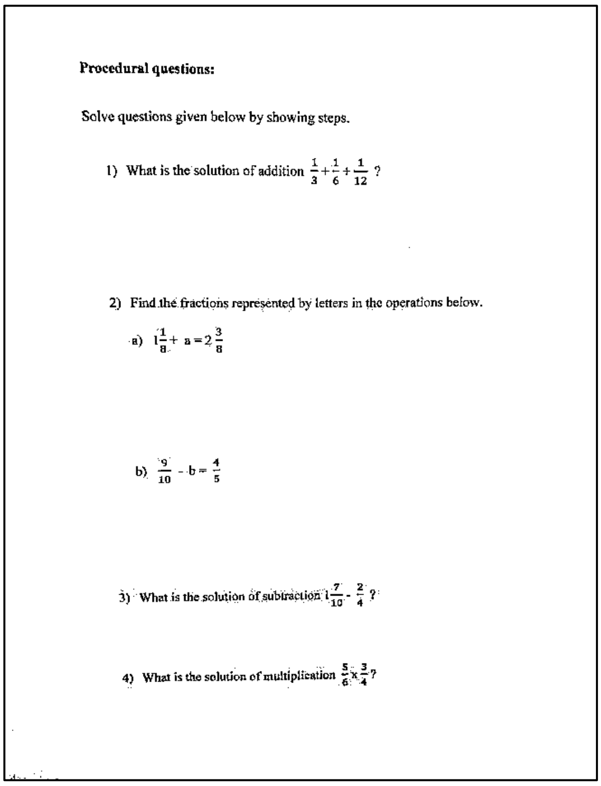


Procedural questions:

Solve questions given below by showing steps.

1) What is the solution of addition $\frac{1}{3}+\frac{1}{6}+\frac{1}{12}$?

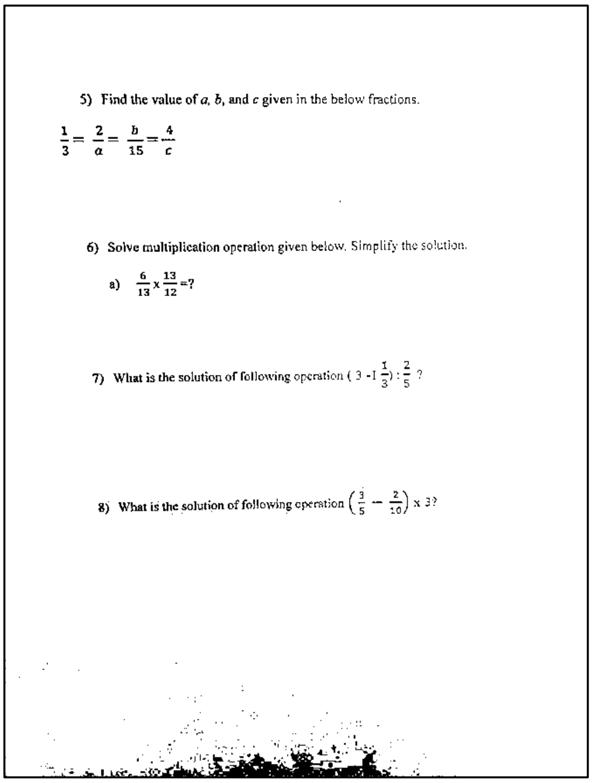
2) Find the fractions represented by letters in the operations below.

a) $1\frac{1}{8} + a = 2\frac{3}{8}$

b) $\frac{9}{10} - b = \frac{4}{5}$

3) What is the solution of subtraction $1\frac{7}{10} - \frac{2}{4}$?

4) What is the solution of multiplication $\frac{5}{6} \times \frac{3}{4}$?



5) Find the value of *a*, *b*, and *c* given in the below fractions.

$\frac{1}{3} = \frac{2}{a} = \frac{b}{15} = \frac{4}{c}$

6) Solve multiplication operation given below. Simplify the solution.

a) $\frac{6}{13} \times \frac{13}{12} = ?$

7) What is the solution of following operation $(3 - 1\frac{1}{3}) : \frac{2}{5}$?

8) What is the solution of following operation $\left(\frac{3}{5} - \frac{2}{10}\right) \times 3$?

Genişletilmiş Özet

Amaç

Bu makale, altıncı ve yedinci sınıf öğrencileri arasında kesirlerdeki kavramsal ve işlemsel bilginin nasıl etkileşimde bulunduğunu incelemeyi, değerlendirmeler, görüşmeler, gözlemler ve öğrencilerin yazılı çalışmaları aracılığıyla daha güçlü işlemsel becerilerin daha derin kavramsal anlayışla ilişkili olup olmadığını araştırmayı amaçlamaktadır. Literatürde uzun yıllardır tartışılan kavramsal bilgi ("ne olduğunu bilme") ve işlemsel bilgi ("nasıl yapılacağını bilme") arasındaki etkileşim, öğrencilerin matematiksel düşünme biçimlerini doğrudan etkilemektedir. Çalışma, altıncı ve yedinci sınıf öğrencilerinin kesirler konusundaki performanslarını hem ölçme araçları hem de gözlem ve görüşmeler aracılığıyla analiz ederek, bu etkileşimin ortaokul düzeyinde nasıl şekillendiğine dair kapsamlı bir bakış sağlamayı amaçlamaktadır. Bu bağlamda araştırma, yalnızca kavramsal ve işlemsel bilgi arasındaki korelasyonu değil, aynı zamanda bu bilginin öğrencilerin matematik başarısı ile ilişkisini de incelemektedir.

Yöntem

Araştırma, karma yöntem deseninde yürütülmüştür. İlk aşamada, İstanbul'da bir özel okulda öğrenim gören 30 altıncı ve yedinci sınıf öğrencisine (12–13 yaş aralığında) kesirler konusuna yönelik geçerliliği kanıtlanmış bir test uygulanmıştır. Bu test, kavramsal bilgiye yönelik 22 puanlık açık uçlu sorular ve işlemsel bilgiye yönelik 19 puanlık uygulama temelli sorulardan oluşmuştur. Nicel veriler SPSS aracılığıyla analiz edilerek kavramsal ve işlemsel bilgi arasındaki korelasyon hesaplanmıştır. İkinci aşamada, seçilen sekiz öğrenciyle yarı yapılandırılmış görüşmeler yapılmış; ayrıca sınıf içi gözlem notları toplanmıştır. Böylece, öğrencilerin test performansları, kendi ifadeleri ve gözlemlenen öğrenme davranışlarıyla karşılaştırılarak bulgular derinleştirilmiştir.

Verilerin analizinde tanımlayıcı istatistiklerin yanı sıra Pearson korelasyon katsayıları hesaplanmış; ayrıca görüşme kayıtları nitel olarak çözümlenmiştir. Bu şekilde, öğrencilerin yalnızca "doğru işlem yapma" becerileri değil, aynı zamanda kavramsal temelleri anlama ve açıklama yetileri de değerlendirilmiştir. Bu çalışmada nitel veriler, öğrenci görüşleri, gözlem notları ve yazılı çalışmalar üzerinden açık kodlama yöntemiyle analiz edilmiştir. Ortaya çıkan temalar belirlenmiş ve her temaya katkıda bulunan öğrenci sayısı yaklaşık değerlerle raporlanarak bulguların yorumlanmasına yansıtılmıştır.

Bulgular

Çalışma sonucunda, kavramsal ve işlemsel bilgi arasında anlamlı ve pozitif yönde bir korelasyon ($r = 0.416$, $p = 0.046$) bulunmuştur. Yüksek kavramsal bilgiye sahip öğrenciler, aynı zamanda işlemsel becerilerde de görece yüksek performans göstermiştir. Ancak, bireysel farklılıklar dikkat çekicidir: bazı öğrenciler kavramsal bilgide güçlü olup işlemsel sorularda zayıf kalırken, bazıları da işlemsel bilgide daha başarılı olmuştur.

Öğrencilerin okul matematik notları ile kesirler testindeki başarıları arasında da güçlü bir ilişki ortaya çıkmıştır. Kavramsal bilgi ile matematik notları arasındaki korelasyon oldukça yüksektir ($r = 0.902$, $p = 0.007$); işlemsel bilgi ile matematik notları arasındaki korelasyon ise orta düzeydedir ($r = 0.622$, $p = 0.006$). Bu bulgu, kavramsal bilginin öğrencilerin genel matematik başarısının en güçlü yordayıcısı olduğunu göstermektedir.

Nitel bulgular da nicel verileri desteklemiştir. Öğrencilerle yapılan görüşmelerde, kavramsal olarak kesirleri bütünün parçaları şeklinde açıklayabilen öğrencilerin aynı zamanda işlemleri daha bilinçli ve anlamlı yaptıkları görülmüştür. Ancak, bazı öğrenciler yalnızca işlemsel adımları ezberlediklerini, nedenini açıklamakta zorlandıklarını ifade etmiştir. Sınıf gözlemleri, kavramsal bilgisi yüksek öğrencilerin grup çalışmalarında daha çok akranlarına rehberlik ettiğini ve problem çözme süreçlerinde daha yaratıcı stratejiler geliştirdiklerini ortaya koymuştur.

Tartışma

Bulgular, kavramsal bilgi ile işlemsel bilginin birbirini destekleyen, ancak her zaman aynı oranda gelişmeyen bilgi türleri olduğunu göstermektedir. Öğrencilerin yalnızca işlemsel bilgiye dayalı öğrenmeleri, uzun vadede hata yapma eğilimlerini artırabilir ve anlamlı öğrenmeyi engelleyebilir. Literatürde de vurgulandığı üzere (Hiebert & Lefevre, 1986; Rittle-Johnson & Siegler, 2001), kavramsal ve işlemsel bilgi arasında karşılıklı ve sürekli bir etkileşim vardır. Bu çalışmada da bazı öğrencilerin yalnızca kuralları uyguladıkları, ancak işlemin nedenini açıklayamadıkları gözlenmiştir.

Ayrıca, kavramsal bilginin matematik başarısıyla daha yüksek düzeyde ilişkili bulunması, öğretim süreçlerinde kavramsal temellere öncelik verilmesi gerektiğini ortaya koymaktadır. Bu bağlamda, Türkiye'de güncellenen matematik öğretim programının kavramsal öğrenmeye verdiği önem, bulgularla uyumlu bir yaklaşımı yansıtmaktadır.

Sonuç

Araştırma, ortaokul öğrencilerinin kesirlerde kavramsal ve işlemsel bilgi düzeylerini bütüncül biçimde ele alarak, bu iki bilgi türünün öğrencilerin genel matematik başarısı için kritik önemde olduğunu ortaya koymuştur. Bulgular, kavramsal bilginin daha güçlü bir yordayıcı olduğunu gösterse de, işlemsel bilginin de kavramsal bilgiyle bütünleşmediği durumlarda öğrenmenin yüzeysel kaldığını göstermektedir.

Öneriler

1. Öğretim süreçleri kavramsal bilgiyi önceliklendirmeli, ardından işlemsel bilgiyi bu temeller üzerine inşa etmelidir.
2. Öğretmenler, öğrencilerin yalnızca doğru cevaba ulaşmalarını değil, aynı zamanda neden-sonuç ilişkilerini açıklamalarını da teşvik etmelidir.
3. Program geliştirme çalışmalarında, kesirler konusunun çoklu temsillerle (model, sayı doğrusu, problem bağlamları) ele alınmasına ağırlık verilmelidir.
4. Gelecek araştırmalar, farklı okul türleri ve bölgelerde yapılmalı; örneklem genişletilerek bulguların genellenebilirliği artırılmalıdır.
5. Alternatif ölçme araçları (performans görevleri, dijital ortamda etkileşimli testler) kullanılarak öğrencilerin kesir bilgisinin daha bütüncül biçimde değerlendirilmesi önerilmektedir.