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Semi-Invariant Submanifolds of a Trans Sasakian Manifold with Ricci Soliton

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Abstract

In this paper, we investigate Ricci solitions on semi-invariant submanifolds of trans Sasakian manifold. We obtain the conditions for the soliton to be steady, shrinking and expanding. We give an example for semi-invariant submanifolds of trans Sasakian manifold with ricci soliton.

1 Introduction

The curvature of a Riemannian manifold is very important in determining its geometric properties. Riemannian curvature tensor and Ricci tensor are two basic elements used in calculating the curvature of a manifold. In 1988*(Hamilton, 1988)*, Hamilton defined Ricci flows using these concepts to obtain a canonical metric on a Riemannian manifold such that

$$\frac{\partial g(t)}{\partial t} = -2S.$$

On the other hand Ricci solitons, a special solution of Ricci flows, are closely related to knot theory and special case of Einstein's equation, thus enabling theoretical physicists and mathematicians to focus more on this subject. A Ricci soliton (g, V, λ) on a Riemannian manifold is defined

$$\pounds_V g + 2S + 2\lambda g = 0,$$

where $\pounds_V$ denotes the Lie derivative operator along the vector field, S is Ricci tensor and λ is a reel constant.

In recent years, the geometry of Ricci solitons has been of interest to many mathematicians. In particular, it became more important after Grigori Perelman applied Ricci solitons to solve the long-standing Poincare conjecture introduced in 1904 *Morgan and Tian (2007)*. Thus, Ricci solitons have become a popular topic

of study in differential geometry in recent years and have been investigated in different spaces having a Riemannian metric *(Ingalahalli & Bagewadi, 2012)*.
The application of Ricci solitons to submanifolds has made important contributions to the study of the geometric properties of submanifolds. The problem of studying the necessary conditions for the existence of a Ricci soliton on a submanifold of a Riemannian manifold has led to important geometric results. In particular, if a submanifold of Euclidean space has a Ricci soliton, the properties of these submanifolds are analyzed and examples of Ricci solitons are given *(Chen & Deshmukh, 2014)*. Many author studied on ricci solitons *(Chen & Deshmukh, 2014a), (Deshmukh et al., 2011), (Nagaraja & Premalatta, 2012)*.
In the Gray-Hervella classification of almost Hermitian manifolds *(Gray & Hervella, 1980)*, there appears a class W_4 of Hermitian manifolds which are closely related to locally conformal Kaehler manifolds. An almost contact metric structure on a manifold M is called a trans-Sasakian structure *(Oubina, 1985)* if the product manifold $M \times \mathbb{R}$ belongs to the class W_4. The class $C_5 \oplus C_6$ (Marrero, 1992) coincides with the class of trans-Sasakian structures of (α, β).
In *(Turgut Vanlı & Sarı, 2010)*, Turgut Vanli and Sari studied invariant submanifolds of a trans-Sasakian manifold. Necessary and sufficient conditions are created to make a submanifold of a trans-Sasakian manifold an invariant submanifold. Also,Vanli and Sari studied some theorems related to an invariant submanifold of a trans-Sasakian manifold.
Recently, C. Gherghe *(Gherghe, 2000)* introduced a nearly trans-Sasakian structure of type (α, β), which generalizes trans-Sasakian structures in the same sense as nearly Sasakian structures generalize Sasakian ones.

2 Preliminaries

Let $\widetilde{M}$ be $(2n+1)$ dimensional, an almost contact manifold with an almost contact metric structure (φ, ξ, η, g), where φ is a $(1,1)$ tensor field, ξ is a vector field, η is 1-form and g is a compatible Riemannian metric such that

$$\varphi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \varphi(\xi) = 0, \quad \eta \circ \varphi = 0, \tag{1}$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2}$$

$$g(\varphi X, Y) = -g(X, \varphi Y), \qquad g(X, \xi) = \eta(X)$$

for all $X, Y \in \Gamma(T\widetilde{M})$.
An almost contact metric structure (φ, ξ, η, g) on $\widetilde{M}$ is called a trans-Sasakian structure of type (α, β) if

$$(\nabla_X \varphi)Y = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\varphi X, Y)\xi - \eta(Y)\varphi X) \tag{3}$$

for some smooth functions α and β on $\widetilde{M}$ where ∇ is the Riemannian connection with respect to g.
From the formula (3) it follows that

$$\nabla_X \xi = -\alpha\varphi X + \beta(X - \eta(X)\xi) = -\alpha\varphi X - \beta\varphi^2 X. \tag{4}$$

If trans-Sasakian structure of type $(1,0)$ is *Sasakian,* trans-Sasakian structure of type $(0,1)$ is *Kenmotsu*, trans-Sasakian structure of type $(\alpha, 0)$ are *α-Sasakian*, trans-Sasakian structure of type $(0, \beta)$ are *β-Kenmotsu* and trans-Sasakian structure of type $(0,0)$ are *cosymplectic.*
Ricci solition which generalization of an Einstein metric was introduced by Hamilton *(Hamilton, 1988)*.

A Riemannian metric g on a Riemann manifolds are called Ricci solition such that following equation

$$L_X g + 2S + 2\lambda g = 0 \tag{5}$$

where S is Ricci tensor, L is the Lie derivative and λ a reel number. A Ricci solition is said to be expanding, steady and shrinking if $\lambda > 0, \lambda = 0$ and $\lambda < 0$.

3 Semi Invariant Submanifold of Trans Sasakian Manifold

In this section, we define semi-invariant submanifolds of trans Sasakian manifold. We give an example for semi-invaiant submanifold.

Definition 3.1. An $(2m+1)-$dimensional Riemannian submanifold M of a trans Sasakian manifold $\widetilde{M}$ is called a semi invariant submanifold if ξ is tangent to M and there exists two differentiable distributions D and $D^{\perp}$ on M satisfying:

(i) $TM = D \oplus D^{\perp} \oplus sp\{\xi\}$;

(ii) The distribution D is invariant under φ, that is $\varphi D_x = D_x$ for any $x \in M$;

(iii) The distribution $D^{\perp}$ is anti-invariant under φ, that is $\varphi D_x^{\perp} \subseteq T_x^{\perp} M$ for any $x \in M$.

Now, we choose a local field of orthonormal frame $\{E_1, ..., E_{2p}, E_{2p+1}, ..., E_{2m}, \xi\}$ on M. Then we have,

$$D = sp\{E_1, ..., E_{2p}\}, \qquad D^{\perp} = sp\{E_{2p+1}, ..., E_{2m}\} \tag{6}$$

where $dimD = 2p$ and $dimD^{\perp} = 2q$.
Then if $p = 0$ we have an anti-invariant submanifold tangent to ξ and if $q = 0$, we have an invariant submanifold. Now, we give the following example.

Example 3.2. In what follows, $(\mathbb{R}^{2n+1}, \varphi, \eta, \xi, g)$ will denote the manifold $\mathbb{R}^{2n+1}$ with its usual almost contact metric structure given by

$$\eta = e^{-z}dz, \qquad \xi = e^{z}\frac{\partial}{\partial z}$$

$$\varphi(\sum_{i=1}^{n}(X_i\frac{\partial}{\partial x_i} + Y_i\frac{\partial}{\partial y_i}) + Z\frac{\partial}{\partial z}) = \sum_{i=1}^{n}(Y_i\frac{\partial}{\partial x_i} - X_i\frac{\partial}{\partial y_i}) + \sum_{i=1}^{n}Y_i y_i\frac{\partial}{\partial z}$$

$$g = \eta \otimes \eta + e^{-2z}(\sum_{i=1}^{n}dx_i \otimes dx_i + dy_i \otimes dy_i),$$

$(x_1, ..., x_n, y_1, ..., y_n, z)$ denoting the Cartesian coordinates on $\mathbb{R}^{2n+1}$. We consider $M = \{(x_1, x_2, x_3, x_{4,}y_1, y_2, y_3, y_4, z) \in \mathbb{R}^9 : z \neq 0\}$. We determine the vector fields

$$\begin{aligned} e_1 &= e^{z}\frac{\partial}{\partial x_1}, e_2 = e^{z}\frac{\partial}{\partial x_2}, e_3 = e^{z}\frac{\partial}{\partial x_3}, e_4 = e^{z}\frac{\partial}{\partial x_4}, \\ e_5 &= e^{z}\frac{\partial}{\partial y_1}, e_6 = e^{z}\frac{\partial}{\partial y_2}, e_7 = e^{z}\frac{\partial}{\partial y_3}, e_8 = e^{z}\frac{\partial}{\partial y_4}, e_9 = e^{z}\frac{\partial}{\partial z}. \end{aligned}$$

Therefore $\{e_1, ..., e_9\}$ is an orthonormal basis on M. On the other hand

$$\varphi(e_1) = -e_5, \varphi(e_2) = -e_6, \varphi(e_3) = -e_7, \varphi(e_4) = -e_8, \varphi(e_9) = 0.$$

Then for all $X, Y \in \Gamma(TM)$, we have

$$\varphi^2(X) = -X + \eta(X)\xi, \qquad \eta(e_9) = 1$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y).$$

Then we obtain $(M, \varphi, \eta, \xi, g)$ is an almost contact manifold. A submanifold of $\mathbb{R}^9$ defined by

$$M = X(u, v, k, l, t) = (u, k, 0, 0, v, 0, l, 0, t).$$

Then local frame of TM

$$\begin{aligned} e_1 &= e^z \frac{\partial}{\partial x_1}, \qquad e_2 = e^z \frac{\partial}{\partial y_1}, \\ e_3 &= e^z \frac{\partial}{\partial x_2}, \qquad e_4 = e^z \frac{\partial}{\partial y_3}, \\ e_5 &= \frac{\partial}{\partial z_1} = \xi \end{aligned}$$

and

$$e_1^* = e^z \frac{\partial}{\partial x_3}, \qquad e_2^* = e^z \frac{\partial}{\partial y_2}$$

from a basis of $T^{\perp}M$. We determine $D_1 = sp\{e_1, e_2\}$ and $D_2 = sp\{e_3, e_4\}$. Then D_1, D_2 are invariant and anti-invariant distribution, respectively. Thus $TM = D_1 \oplus D_2 \oplus sp\{\xi\}$ is a semi invariant submanifold of $\mathbb{R}^9$.

Let $\widetilde{\nabla}$ be the Levi-Civita connection of $\widetilde{M}$ with respect to the induced metric g. Then Gauss and Weingarten formulas are given by

$$\widetilde{\nabla}_X Y = \nabla^*_X Y + h(X, Y) \tag{7}$$

$$\widetilde{\nabla}_X N = \nabla^{*\perp}_X N - A_N X \tag{8}$$

for any $X, Y \in \Gamma(TM)$ and $N \in \Gamma(T^{\perp}M)$. $\nabla^{*\perp}$ is the connection in the normal bundle, h is the second fundamental from of $\widetilde{M}$ and A_N is the Weingarten endomorphism associated with N. The second fundamental form h and the shape operator A related by

$$g(h(X, Y), N) = g(A_N X, Y). \tag{9}$$

Let M be semi invariant submanifold of $\widetilde{M}$. M is said to be totally geodesic if $h(X, Y) = 0$, for any $X, Y \in \Gamma(TM)$.
We denote by $\widetilde{R}$ and R the curvature tensor fields associated with $\widetilde{\nabla}$ and ∇^* respectively. The Gauss equation is given by

$$\widetilde{R}(X, Y, Z, W) = R^*(X, Y, Z, W) + g(h(X, Z), h(Y, W)) - g(h(X, W), h(Y, Z)) \tag{10}$$

for all $X, Y, Z, W \in \Gamma(TM)$.

4 Ricci Solitons on Semi-Invariant Submanifolds of Trans Sasakian Manifold

In this section, we study ricci solitons on semi-invaraint submanifold of a trans Sasakian manifold. We obtain expanding ,steady and shrinking of invariant and anti-invariant distributions.

Theorem 4.1. *Let (g, ξ, λ) is a Ricci soliton on a semi-invariant submanifolds M of a trans Sasakian manifold*

$\widetilde{M}$. *Then, invariant distribution D is $\eta-$Einstein.*

Proof. For all $X \in \Gamma(D)$, using (4) we have

$$\begin{aligned} (L_\xi g)(Y, Z) &= g(\nabla_Y \xi, Z) + g(Y, \nabla_Z \xi) \\ &= 2\beta g(\varphi Y, \varphi Z). \end{aligned} \tag{11}$$

We know that

$$(L_\xi g)(Y, Z) + 2S(Y, Z) + 2\lambda g(Y, Z) = 0$$

Then from (11) we obtain that

$$S(Y, Z) = -(\beta + \lambda) g(Y, Z) + \beta \eta(Y)\eta(Z). \tag{12}$$

□

We consider $\varphi(grad\alpha) = (2q-1)grad\beta$ for $(2q+1)-$dimansional semi-invariant submanifold. Then

$$\xi_\beta = g(\xi, grad\beta) = \frac{1}{2q+1} g(X, \varphi grad\alpha) = 0,$$

$$X_\beta = \frac{1}{2q-1} g(X, \varphi grad\alpha)$$

and

$$(\varphi X)_\alpha = g(\varphi X, grad\alpha)$$

$$S(X, \xi) = 2q(\alpha^2 - \beta^2)\eta(X). \tag{13}$$

Further using (12) we have

$$S(X, \xi) = -\lambda \eta(X). \tag{14}$$

Therfore from (13) and (14) we obtain

$$\lambda = 2q(\beta^2 - \alpha^2).$$

Theorem 4.2. *A Ricci soliton (g, ξ, λ) on invariant distribution D of semi-invariant submanifold of a trans sasakian manifold is expanding ,steady and shrinking according as $\beta^2 - \alpha^2 > 0$, $\beta^2 - \alpha^2 = 0$ and $\beta^2 - \alpha^2 < 0$, respectively.*

Let M is semi-invariant submanifold of α-Sasakian manifold. For all $X \in \Gamma(D^\perp)$, using (7) we have

$$\nabla_X \xi = 0 \text{ and } h(X, \xi) = \alpha \varphi X. \tag{15}$$

Then, using (4), we get

$$(L_\xi g)(Y, Z) = g(\nabla_Y \xi, Z) + g(Y, \nabla_Z \xi) = 0.$$

Therefore from (5) we obtain

$$S(Y, Z) = -\lambda g(Y, Z). \tag{16}$$

Theorem 4.3. *Let (g, ξ, λ) is a Ricci soliton on a semi-invariant submanifolds of $\alpha-$Sasakian manifold. Then anti-invariant distirbution $D^\perp$ is Einstein.*

On the other hand, for all $X \in \Gamma(D^\perp)$, using (15), we have

$$S(X, \xi) = 0.$$

Then from (16), we conclude that

$$\lambda = 0.$$

Theorem 4.4. *A Ricci soliton (g, ξ, λ) on anti-invariant distribution $D^{\perp}$ of semi-invariant submanifold of $\alpha-$Sasakian manifold is always steady.*

On the other hand, let M is semi-invariant submanifold of β-Kenmotsu manifold.
For all $Y \in \Gamma(D^{\perp})$, from (7) we get

$$\nabla_Y \xi = \beta \varphi^2 X \text{ and } h(Y, \xi) = 0. \tag{17}$$

Then, using (17) we have

$$\begin{aligned}(L_\xi g)(Y, Z) &= g(\nabla_Y \xi, Z) + g(Y, \nabla_Z \xi) \\ &= -2\beta g(\varphi Y, \varphi Z).\end{aligned}$$

Therefore using (5) we obtain

$$S(Y, Z) = -(\beta + \lambda) g(Y, Z) + \beta \eta(Y) \eta(Z). \tag{18}$$

Theorem 4.5. *Let (g, ξ, λ) is a Ricci soliton on a semi-invariant submanifolds of β-Kenmotsu manifold. Then anti-invariant distirbution $D^{\perp}$ is $\eta-$Einstein.*

On the other hand, using (17) we have

$$S(X, \xi) = 2p\beta\eta(X). \tag{19a}$$

Moreover from (18) we get

$$S(X, \xi) = -\lambda \eta(Y). \tag{20}$$

Using (19a) and (20) we obtain

$$\lambda = -2p\beta.$$

Theorem 4.6. *A Ricci soliton (g, ξ, λ) on invariant distribution $D^{\perp}$ of semi-invariant submanifold of β-Kenmotsu manifold is expanding ,steady and shrinking according as $\beta > 0$, $\beta = 0$ and $\beta < 0$, respectively.*

Example 4.7. We consider the 3-dimensional $M = \{(x, y, z) \neq (0, 0, 0) \in \mathbb{R}^3\}$ where (x, y, z) standart coordinates of $\mathbb{R}^3$. The vector fields

$$e_1 = z\frac{\partial}{\partial x}, \ e_2 = z\frac{\partial}{\partial y}, \ e_3 = -z\frac{\partial}{\partial z}$$

are linearlyindependent at each point of M. Let g be the Riemannian metric defined by

$$g(e_i, e_j) = 0 \text{ and } g(e_i, e_i) = 1.$$

Let η be the 1-form defined by $\eta(W) = g(W, e_3)$ for any $W \in \Gamma(TM)$. Let φ be the $(1, 1)$ tensor field defined by

$$\varphi(\frac{\partial}{\partial x}) = -\frac{\partial}{\partial y}, \ \ \varphi(\frac{\partial}{\partial y}) = \frac{\partial}{\partial x}, \ \ \varphi e_3 = 0.$$

Thus we have

$$\varphi e_1 = -e_2, \ \ \varphi e_2 = e_1, \ \ \varphi e_3 = 0.$$

For any vector field $W = a_1\frac{\partial}{\partial x} + a_2\frac{\partial}{\partial y} + a_3\frac{\partial}{\partial z} \in \Gamma(\mathbb{R}^3)$ we have

$$g(W,W) = a_1^2 + a_2^2 + a_3^2 \text{ and } g(\varphi W, \varphi W) = a_1^2 + a_2^2$$

and

$$\varphi^2 W = -a_1\frac{\partial}{\partial x} - a_2\frac{\partial}{\partial y} = -W + \eta(W)\xi,$$

$$\eta(\xi) = 1,\ g(\varphi W, \varphi W) = g(W,W) - \eta^2(W).$$

Then for $e_3 = \xi$, (φ, ξ, η, g) defines on almost contact metric structure on M. Now by direct computations we obtain

$$[e_3, e_1] = -e_1, \quad [e_1, e_2] = 0, \quad [e_3, e_2] = -e_2.$$

The Riemannian connnection ∇ ofthe metric tensor g is given by the Koszul's formula which is

$$2g(\nabla_X Y, Z) = Xg(Y,Z) + Yg(Z,X) - Zg(X,Y) - g(X,[Y,Z]) - g(Y,[X,Z]) + g(Z,[X,Y]).$$

By using the above formula, we obtain

$$\nabla_{e_1}e_1 = -e_3, \qquad \nabla_{e_1}e_2 = 0, \qquad \nabla_{e_1}e_3 = e_1$$

$$\nabla_{e_2}e_1 = 0, \qquad \nabla_{e_2}e_2 = -e_3, \qquad \nabla_{e_2}e_3 = e_2$$

$$\nabla_{e_3}e_1 = 0, \qquad \nabla_{e_3}e_2 = 0, \qquad \nabla_{e_3}e_3 = 0.$$

We see that

$$\begin{aligned} (\nabla_{e_1}\varphi)e_1 &= -\nabla_{e_1}e_2 - \varphi(-e_3) \\ &= 0. \end{aligned} \tag{21}$$

So,

$$\begin{aligned} 0 &= \alpha(g(e_1,e_1)e_3 - \eta(e_1)e_1) + \beta(g(\varphi e_1, e_1)e_3 - \eta(e_1)\varphi e_1) \\ &= \alpha e_3 + \beta. \end{aligned}$$

$$\begin{aligned} (\nabla_{e_1}\varphi)e_2 &= \nabla_{e_1}\varphi e_2 - \varphi\nabla_{e_1}e_2 \\ &= \nabla_{e_1}e_1 \\ &= -e_3 \end{aligned} \tag{22}$$

So,

$$\begin{aligned} -e_3 &= \alpha(g(e_1,e_2)e_3 - \eta(e_2)e_1) + \beta(g(\varphi e_1, e_2)e_3 - \eta(e_2)\varphi e_1) \\ &= \beta(-g(e_2,e_2)e_3) \\ &= -\beta e_3 \end{aligned}$$

$$\nabla_{e_1}e_3 = -\alpha\varphi e_1 + \beta(e_1 - \eta(e_1)e_3) \tag{23}$$

$$e_1 = \alpha e_2 + \beta e_1$$

by (21,22,23) we see that the manifold satisfies (1) and (3) for $X = e_1, \alpha = 0, \beta = 1$ and $e_3 = \xi$. It can be show that for $X = e_2, e_3$ the manifolds also satisfies (1) and (3) for $\alpha = 0, \beta = 1$ and $e_3 = \xi$, so the manifold is a trans Sasakian manifold of type $(0,1)$. Using the formula Riemannian curvature, we conclude the following expressions:

$$R(e_1,e_2)e_1 = 0, \qquad R(e_1,e_2)e_2 = -e_1, \qquad R(e_1,e_2)e_1 = e_2$$

$$R(e_2,e_3)e_1 = 0, \qquad R(e_2,e_3)e_2 = -e_3, \qquad R(e_2,e_3)e_3 = -e_2$$

$$R(e_1,e_3)e_1 = -e_3, \qquad R(e_1,e_3)e_2 = 0, \qquad R(e_1,e_3)e_3 = -e_1$$

$$R(e_2,e_1)e_1 = -e_2, \qquad R(e_3,e_1)e_1 = -e_3, \qquad R(e_3,e_2)e_2 = -e_3$$

from the above expressions of the curvature tensor R, we obtain that

$$S(e_1,e_1) = -2, \qquad S(e_2,e_2) = -2, \qquad S(e_3,e_3) = -2.$$

The potential vector field on M is given by

$$V = \frac{x}{z}e_1 + \frac{y}{z}e_2 + ze_3.$$

Therefor we get

$$[V,e_1] = 0, \qquad [V,e_2] = 0, \qquad [V,e_3] = e_3.$$

Then, we obtain

$$(L_V g)(e_i,e_j) + 2S(e_i,e_j) + 2\lambda g(e_i,e_j) = 0$$

for $\lambda = 2$, where $i,j = 1,2,3$. As a result, M is an steady Ricci soliton.

5 Conclusion

The Ricci soliton is an important concept in differential geometry and geometric analysis, and plays a particularly significant role in Ricci flow theory. A Ricci soliton is a metric solution that transforms itself under the Ricci flow only by a diffeomorphism and scaling. That is, the metric evolving under the Ricci flow actually evolves by changing shape (shrinking, expanding or steady) without altering the geometry. In this paper, the idea of examining semi-invariant submanifold with ricci solitons are emphasized. The works on this subject will be useful tools for the applications of submanifold with different manifolds.

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