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PREDICTION OF GROSS DOMESTIC PRODUCT (GDP) BY USING GREY COBB-DOUGLAS PRODUCTION FUNCTION

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ABSTRACT

Purpose- In this paper, we investigate the Grey Cobb-Douglas production model applicable to estimation of economical indicators.
Methodology- In the multi regression model, explanatory variables for estimation of future value of indicators is estimated by using Grey Cobb-Douglas model.
Findings- GDP is an indicator for economic growth. We are used the annual data of United State of American economy for 1951 to 2008 and estimated the 2009-2018 years. The sum of the contributions of factors is 1.497 and greater than one, so it shows increasing return to scale.
Conclusion- The percentage of the increase in GDP is greater than that of the increase in capital stock and labor.

Keywords: Cobb-Douglas production function, economic growth, Grey-predicting model.
JEL Codes: E23, E37, C53

1. INTRODUCTION

The production level in general economy or firm environment is described by production function. One of the main problems for the economics authorities is to choice the functional relationship between the economic inputs and production value. In the literature generally four different production functions is used. In literature, very often used production functions are linear production function, Cobb-Douglas production function(C-D), Constant Elasticity of Substitution production function (CES), Variable Elasticity Substitution production function (VES), Leontief Production function and Translog production function (Cheng, M. and Han, Y., 2017; Godin, A. and Kinsella, S., 2013). In this study, we used Cobb-Douglas production function to establish model impact of capital investments and labor on economic growth.

Christensen, Jorgenson and Lau (1973) consider an extension of Cobb-Douglas function that is called *translog production* function. Important contributes made by Romer (1986), Lucas (1988), Mankiw, Romer and Weil (1992) and Benhabib and Spiegel (1994).

The error term in Cobb-Douglas production function is modeled as either additive or multiplicative. In this study, we will use additive form. Bahatti (1993), Hossain et al. (2010), Prajneshu (2008), Golfeld and Quandt (1970) are used classic regression model estimate the Cobb-Douglas function.

There are a lot of factor have contributed to production level, such that capital, technology optimal allocation of sources, innovations etc. In this study, the future economic production level is forecasted based on GM (1, 1) Grey Cobb-Douglas production model. In addition, we make an empirical analysis on the elasticity of substitution, direction of technical change and the contribution rate of USA economic growth factors to total factor productivity.

2. PRODUCTION FUNCTIONS

Cobb-Douglas function has a constant cost share of capital and strong comovement in labor productivity and capital productivity. The growth of economics generally has measured by Gross Domestic Production (GDP) rate in current price. Economic growth is only based on asset and employment. In substance, economics production is effected from various environmental factors such as capital, labor, agricultural activities, technology, industry, energy, raw materials etc.

In economics, the production function describes the empirical relationship between given the quantity of economics inputs and specified outputs. One of the basic problems for economics governance in production process is determining the functional relationship between the production output and input factors. The general form of production function is described by $f: D \rightarrow R_+$, $D \in R_+^n$ and

$$Q = f(X_1, X_2, \dots, X_n) \tag{1}$$

where $X_1, X_2, \dots, X_n$ are inputs and Q is production level. A production function with n input factors is called $h-$ homogeneous, $h > 0$, if

$$f(kX_1, kX_2, \dots, kX_n) = k^h f(X_1, X_2, \dots, X_n) \tag{2}$$

where $k\epsilon R$, if $h > 1$, per percent increase in input levels would result greater than per percent increase in the output level, if $h < 1$, per percent increase in input levels would result less than per percent increase in output and $h = 1$ represent the constant return to scale.

Traditionally assumed that the most important factors that affect the production are capital and labor inputs (Cobb, C. W. and Douglas, P. H.,1928). We can write the production function with two input factors as follows,

$$Q = f(L, K) = AK^\alpha L^\beta \tag{3}$$

where Q is the quantity of total output (production), L is the quantity of labor and K is the capital used.

The output Q is usually measured by physical units produced or by their values, Labor is typically measured in man-hours or number of employees. Capital represents aggregations of different components. Determination of its value is difficult. The historical evolution of production function see also refer to Misra, S. K. (2010). The generalized form of Cobb-Douglas function is given as,

$$Q(\boldsymbol{X}) = AX_1^{\beta_1} X_2^{\beta_2} \dots X_n^{\beta_n}. \tag{4}$$

If the logarithms of both sides are taken, we obtained that

$$\ln Q = \beta_0 + \beta_1 \ln X_1 + \dots + \beta_n \ln X_n \tag{5}$$

where $\beta_0 = lnA$, $\boldsymbol{X} = (X_1, X_2, \dots, X_n) \in R_+^n$, $A > 0$ is a constant, Q is total production level, X_i are input factors, $i = 1, 2, \dots, n$ and β_1, $\beta_2, \dots, \beta_n$ are parameters, n is the number of factors which are used in the production function (Vîlcu, G. E. ,2018; Wang, X., 2016).

2.1.Parameter Estimations of Generalized Cobb-Douglas Production Function

Cobb-Douglas Production function defined as in Eq. (4) is used to estimate the following regression equation,

$$Q_i = AX_{1i}^{\beta_1} X_{2i}^{\beta_2} \dots X_{ni}^{\beta_n} + \varepsilon_i$$

Shifting the ε_i left side of equation then taking the logarithm on both side of equation,

$$log\ (Q_i - \varepsilon_i) = \log A + \beta_1 \log X_{1i} + \beta_2 \log X_{2i} + \dots + \beta_n \log X_{ni} \tag{6}$$

Let $m_i = \varepsilon_i / Q_i$ (the relative error of the observations of Q_i) (Mahaboob, B. et al., 2017),

We consider the m_i is small. In this case, we can write,

$$\log(Q_i - \varepsilon_i) = \log Q_i + \log(1 - m_i) \cong \log Q_i - m_i\ . \tag{7}$$

Hence,

$$\log Q_i = \log A + \beta_1 \log X_{1i} + \beta_2 \log X_{2i} + \dots + \beta_n \log X_{ni} + m_i \tag{8}$$

where, $m_i \sim N(0\ , \sigma^2 / Q_i^2)$ so it is not satisfy the assumption of linear regression. Now we multiply by Q_i on both of side equation $logQ_i$ then, we obtain new equation as,

$$Q_i\ \log Q_i = Q_i \log A + Q_i \beta_1 \log X_{1i} + Q_i \beta_2 \log X_{2i} + \dots + Q_i \beta_n \log X_{ni} + Q_i m_i$$

$$Q_i \log Q_i = Q_i \log A + Q_i \beta_1 \log X_{1i} + Q_i \beta_2 \log X_{2i} + \dots + Q_i \beta_n \log X_{ni} + \varepsilon_i \tag{9}$$

where $\varepsilon_i \sim N(0, \sigma^2)$.

In the matrix notation,

$$\boldsymbol{Q} = \begin{pmatrix} Q_1 log\ \ Q_1 \\ Q_2 \log Q_2 \\ \vdots \\ Q_n \log Q_n \end{pmatrix} \quad , \quad \widehat{\boldsymbol{\beta}} = \begin{pmatrix} \log \hat{A} \\ \widehat{\beta_1} \\ \vdots \\ \widehat{\beta_n} \end{pmatrix} \quad , \quad A = \begin{pmatrix} Q_1 & Q_1 \log X_{11} & \cdots & Q_1 \log X_{n1} \\ Q_2 & Q_2 \log X_{12} & \cdots & Q_1 \log X_{n1} \\ \vdots & \vdots & \vdots & \vdots \\ Q_n & Q_n \log X_{1n} & \cdots & Q_1 \log X_{nn} \end{pmatrix} \tag{10}$$

The least squares estimation of the parameters obtains as follows (Qi, W., Yingsheng, S. and Pengfei, J., 2010),

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{A}^T \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{Q}. \tag{11}$$

3. GREY PREDICTION MODEL GM(1,1)

In the analysis of empirical data is widely used the multiple regression models, based on multiple regression models to prediction of the future value of response variable need to know the future value of the independent random variables. Grey theory proposed by J. L. Deng (1982) can provide a more flexible approximation to fitting a model to observed data. Grey model was building on the differential equation. Solution of differential equation has an exponential function.

In the grey system theory, the basic model is GM(1,1). It is used for predict a single variable. This model eliminates the randomness of data by accumulating the original time series (Honghong Liu, 2010).

We can summarize the model calculation as following,

Step 1: Original data:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\right) \tag{12}$$

We assume that the original data non-negative and generally assumed that $n \geq 4$.

Step 2: Accumulate generating data (1-AGO):

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\right) \tag{13}$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \qquad k = 1,2,\dots,n \tag{14}$$

this sequence is monotone increasing.

Step 3: The generated mean sequence $Z^{(1)}$ is described as,

$$Z^{(1)} = \left(z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n)\right) \tag{14}$$

where

$$z^{(1)}(k) = 0.5\left(x^{(1)}(k) + x^{(1)}(k-1)\right), \quad k = 2,3,\dots,n \tag{15}$$

$X^{(1)}$ represent the monotonic increasing sequence, so it is similar to the solution of first order linear differential equation. This differential equation is called whitening equation for $X^{(1)}$ and is given by

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \tag{16}$$

where a is called developing parameter and parameter b is called grey input coefficient.

Step 4: a and b parameters are determined by discrete form of above differential equation. The difference equation is written as,

$$X^{(0)}(k) + aZ^{(1)}(k) = b, \qquad \mathrm{k} = 1, 2, 3, \dots \tag{17}$$

The model parameters a and b are estimated as follows,

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = (B^T B)^{-1} B^T Y \tag{18}$$

where

$$B = \begin{pmatrix} -\frac{1}{2}\left(x^{(1)}(1) + x^{(1)}(2)\right) & 1 \\ -\frac{1}{2}\left(x^{(1)}(2) + x^{(1)}(3)\right) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}\left(x^{(1)}(n-1) + x^{(1)}(n)\right) & 1 \end{pmatrix}, \quad Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix} \tag{19}$$

Step 5: Solving the differential equation with an initial condition $x^{(1)}(1) = x^{(0)}(1)$, we obtain a prediction model as follows,

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{\hat{b}}{\hat{a}}\right] e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} \quad , \quad k = 1,2,3,\dots \tag{20}$$

We estimate the original data by inverse accumulated generating operation (IAGO) which is defined as

$$\hat{x}^{(0)}(k+1) = \left(1 - e^{-\hat{a}}\right) \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-\hat{a}k} \ , \qquad k = 1,2,3,\dots \tag{21}$$

4. EMPIRICAL ANALYSIS

In this study, we analysis the economic growth of USA, first we describe the behavior pattern of economic factors and then we measure the input factor's contribution rate to economic growth.

We are used the annual data as an output and input factors,

- Gross Domestic Product (GDP - Billion US dollar) (Q_t) as a output,
- Labor (the number of employees) (10.000 people) (L_t),
- Capital (fixed asset investment) (Billion US dollar) (K_t) as a input factors series

for United state of American economy for 1951 to 2008. Data obtained from the United States Department of Commerce and the NIPA National Income Product Accounts (Thompson, H., 2016).

4.1. Statistical Analysis of Parameter Estimation

Cobb-Douglas production function in Eq. (6) can be converted to linear form as follows,

$$\ln Q = \ln A + \alpha \ln K + \beta \ln L.$$

The regression results obtained as Table 2.

Table 2: The Regression Results

Adjusted R Square	0.9955
F statistics	6306.6823

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-5.7761	1.1151	-5.1799	3.252E-06
ln K	0.4030	0.0677	5.9545	1.905E-07
Ln L	1.0936	0.1281	8.5377	1.170E-11

The model coefficient of determination R^2 shows that the model has a very high fitting precision. Cobb-Douglas model parameters is obtained as follows,

Table 3: Cobb-Douglas Model Parameters

$\hat{A}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha} + \hat{\beta}$
0.0031	0.403	1.094	1.497

Finally, Cobb-Douglas production function regression equation can be obtained as follows,

$$\hat{Q}_{regression} = 0.0031\, K^{0.403}\, L^{1.094}\ . \tag{22}$$

Result of the regression show that regression equation is significant. Then, the explained ratio of GDP by changes in capital and labour is 99.55%.

Figure 2: Actual GDP and Estimated GDP According to Cobb - Douglas Production Function for $\hat{A} = 0.0031$, $\hat{\alpha} = 0.403$ and $\hat{\beta} = 1.097$.

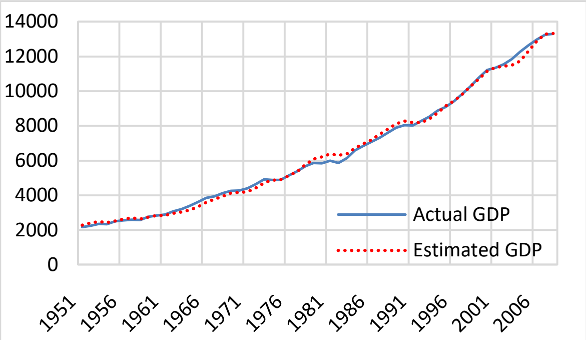
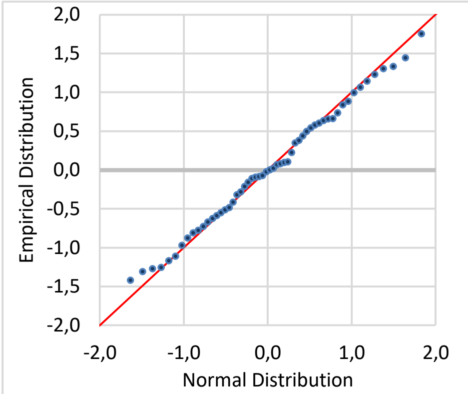


Figure 3: Q - Q Plot of Residuals Between Real GDP and Estimated GDP



The points in the Q - Q plot lie on straightforward line. This shows that the residuals are based on normal distribution. Errors between the observed real GDP values and the values which estimated by Cobb-Douglas production function are distributed as a normal. This situation is shown by using the Q - Q plot in Fig. 3.

Production function in Eq. (22) show that level of production technology is 0.0031. The elasticity of capital α is 0.403. This value shows that 1% increase in capital lead to 0.403% increase in GDP. The elasticity of labour β is also 1.094. This value shows that a 1% increase in labour lead to a 1.094% increase in GDP. The sum of elasticities of input factors is

$$\alpha + \beta = 0.403 + 1.094 = 1.497 > 1.$$

This shows that per percent of increase in GDP is greater than that of the increase in capital and number of employees, i.e. it shows increasing return to scale.

4.2. Grey Cobb-Douglas Production Function

Predictions for the values of K and L input factors obtained by using GM(1,1) model is shown in Table 4.

Table 4: Prediction of Production Factors by Using GM(1,1) Model

Year	GM(1,1) prediction for L	GM(1,1) prediction for K	Year	GM(1,1) prediction for L	GM(1,1) prediction for K
2009	141277.17	467.76	2014	147525.08	452.55
2010	137815.06	397.71	2015	150057.65	467.40
2011	140180.94	410.77	2016	152633.70	482.74
2012	142587.44	424.25	2017	155253.97	498.58
2013	145035.25	438.17	2018	157919.23	514.94

We determine the Grey Cobb-Douglas production function as (Zhu, S., Wu, Q. J. and Wang, Y., 2011),

$$\hat{Q}_t = \hat{A}\,\left(\hat{K}_t^{Grey}\right)^{\hat{\alpha}}\left(\hat{L}_t^{Grey}\right)^{\hat{\beta}}. \tag{23}$$

Table 5: The GM(1,1) forecasting and Grey Cobb-Douglas forecasting for GDP

Year	GM(1,1) forecasting for GDP	Grey Cobb-Douglas forecasting for GDP	Year	GM(1,1) forecasting for GDP	Grey Cobb-Douglas forecasting for GDP
2009	16570.35	15836.51	2014	16417.59	16384.25
2010	14481.48	14437.28	2015	16940.78	16910.72
2011	14942.97	14901.18	2016	17480.64	17454.09
2012	15419.17	15379.99	2017	18037.72	18014.93
2013	15910.55	15874.18	2018	18612.54	18593.79

4.3. ERROR ANALYSIS

Error Analysis is needed for examining the precision of forecasted results. The Mean Absolute Percentage Error (MAPE) is one of the most widely used methods that is evaluation of forecasting error. The MAPE is calculated as:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Q_t - \hat{Q}_t}{Q_t}\right|, \qquad (24)$$

where Q_t is actual value, $\hat{Q}_t$ is also forecasting value at time t, n is the number of periods forecasted (Makridakis, S., Wheelwright, S. C. and Hyndman, R. J., 2008).

MAPE value for Cobb-Douglas production function estimation is calculated as,

$$MAPE = 0.0282.$$

This value is less than 10%, so we can say that Cobb-Douglas production function is suitable model for forecasting of GDP values.

5. CONCLUSION

The percent change growth of production is proportional to percentage change growth in the quantities of input factors without changing factor usage shares. That is the constant return to scale form of the production function. Cobb-Douglas production function model is applied to capital, labor, Gross Domestic Product (GDP) time series for United States of American economy for 1951 to 2008 and is obtained that marginal contribution to GDP of capital input is 0.403 and marginal contribution to GDP on labor input is 1.094. Findings show that United States economic (GDP) is labor intensive. In addition, it is confirmed that the labor force and capital has a positive effect on economic growth. Besides, GM(1,1) model is used to prediction of future labor and capital values. Predicted values for K and L putting into grey Cobb-Douglas production model is forecasted GDP values. Finally, improvement of the labor quality can help to increase of the GDP values.

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