The Eurasia Proceedings of
Educational & Social Sciences (EPESS)
ISSN: 2587-1730

The Eurasia Proceedings of Educational & Social Sciences (EPESS), 2018

Volume 9, Pages 250 - 260

ICEMST 2018: International Conference on Education in Mathematics, Science and Technology

Laboratory Activities to Develop the Geometric Reasoning

Marcele da SILVA SANTOS
Prefeitura Municipal de Macaé / SEEDUC – RJ

Neide da FONSECA PARRACHO SANT'ANNA
Colégio Pedro II

Abstract: A set of activities designed to foster geometric thought of students in the 2nd grade of junior High School (8th grade of elementary school in Brazil) is presented here. The activities were proposed to students of a public school. The learning was supported by teaching resources as Tangram, Geoboard and Bending. The activities, interdisciplinary, realistic and playful, were carried out in the Laboratory for the Teaching of Mathematics. The activities were performed in groups organized in such a way to enhance the interaction between students of different levels of knowledge. A test to check the van Hiele level of students was applied before and afterwards. The evaluations of the results of the tests reveal advances in the van Hiele level. The proposal applied to an experimental group provided an environment conducive to meaningful learning. It must be pointed that it also increased students confidence and stimulated them to develop continuity for the acquisition of new knowledge.

Keywords: Mathematical education, Van Hiele theory, Geometry, Meaningful learning

Introduction

This research deals with teaching Geometry in Elementary School. It follows the learning of Mathematics of students of the 8th grade along a school year. Activities were designed to foster the development of geometric thinking to aid the students to progress to higher levels of reasoning according to the Van Hiele Theory of Development of Geometric Thought (Nasser, 1992).

The activities were tested in a school of the public network of the Municipality of Macaé, in the State of Rio de Janeiro, Brazil. They were supported by concrete didactic resources and play activities, in interdisciplinary contexts. They were developed in the Mathematics Laboratory (Lorenzatto, 2009) and articulated the curricular contents of Mathematics with the daily situations of the student reality. Topics of study defined following Tomaz & David (2015) and Skovsmose (1994) principles of exploring everyday classroom situations were addressed in such a way as to allow for interdisciplinarity and interaction with the diversified realities of the students.

In the next section the theoretical foundations of the adopted approach, based on Ausubel (1961) Meaningful Learning Model and van Hiele (1957) Geometric Learning Levels Model, are presented. In Section 3, the teaching proposal is developed, with the presentation of features of the teaching in a Laboratory and of the evaluation tool applied. In Section 4 the activities are presented. In Section 5, the results obtained are analyzed.

Meaningful Learning and the Van Hiele Levels

The activities were structured with a main aim of ensuring meaningful learning (Ausubel, 1961). The Meaning Learning Model (Ausubel, 1961) starts from a critique of mechanical teaching that does not generate meaning and results that the student quickly forgets the information received. The model recognizes previous knowledge as the most important variable for meaningful learning, even though, in some cases, previous information may

- Selection and peer-review under responsibility of the Organizing Committee of the Conference

block learning. The spontaneously acquired knowledge is better assimilated. Mechanical learning makes the information received volatile. Later, the lack of such information, which should serve as a foundation for other concepts to be constructed, becomes a gap and makes the student's cognitive advances impossible.

Ausubel argues that meaningful reception encompasses the acquisition of new meanings from a mechanism or material potentially meaningful to the student. The new information acquired changes the old ones, by a reorganization of concepts. In his model, the acquisition of new knowledge depends on three elements: prior knowledge, subsumers and anchor ideas.

The material used for the development of new learning must be articulated with the needs and be appropriate to the objectives that one wishes to achieve. The learner must have considerable "anchoring ideas" that allow associations to be made to the new material. The confrontation of existing ideas with new information allows the construction of true meanings.

Besides, the student must be prepared to receive new information using anchor ideas to produce meanings. This needs developing subsumers. The often monotonous classroom should be renewed through teaching methods that enable discovery. The teacher must become a facilitator and mediator of the learning process, the one that provides situations, guides and encourages the student, who is the active subject of her learning. Teacher and student share the spaces, each one performing a function and both learning and teaching from their experiences.

Three types of meaningful learning must be considered: representational learning, the most elementary, in which the symbols are concrete elements, without any meaning beyond the univocal relation between object and event; conceptual learning, marked by the presence of regularities perceived in events or objects, which begin to be represented by a linguistic symbol, the individual no longer needing an event or object to give meaning to the symbol; and propositional learning in which the meaning of the new information is expressed by a proposition. This has as prerequisite the two types of previous learning.

These successive levels of learning lead us to van Hiele's (1986) model. This model is also based on an evolution of student's ability of learning each content. As in the model of meaningful learning, the van Hiele levels model also emphasizes the importance of respecting order and not skipping stages in teaching.
Van Hiele schematically divides the geometrical thought in five levels.

Level 1: Recognition

Students identify the figures visually by their overall appearance. They recognize, describe, compare, and classify them through their forms, but do not identify properties. At this level the individual can learn a geometric vocabulary, identify forms, and reproduce specific objects, but the thinking is based on visual considerations. This is representational learning.

Level 2: Analysis

Students begin to analyze and discern the characteristics of the figures by comparison and learn the symbology appropriate to describe them, but can not correlate figures or properties. "Properties are used to conceptualize classes of configurations" (Crowley, 1987), what enables to conceptual learning. Nevertheless reasoning is still based on informal analysis from observation and experience, what is typical of representational learning.

Level 3: Abstraction

Students at this level "are able to deduce properties of a figure and recognize classes of figures."(Crowley, 1987), but they still do not understand the meaning of a deduction or the role of axioms. "Students are able to follow formal demonstrations, but they do not see how one can alter the logical order or how one can build proof from different or unfamiliar premises."(Crowley, 1987). As students establish a logical ordering of figure properties by means of short deduction sequences and understand the correlations between the figures, they are able to conceptual learning.

Level 4: Deduction

Students begin to develop longer sequences of statements and understand the significance of deduction, the role of axioms, theorems, and proofs. The person at this level "... sees the possibility of developing a demonstration in more than one way; comprises the interaction of necessary and sufficient conditions; is capable of making distinctions between an affirmation and its reciprocal "(Crowley, 1987). The realization of conjectures and initiated efforts is spontaneous. A student at this level can build evidence, not just memorize it. She is then able to propositional learning.

Level 5: Rigor

At this level, "geometry is seen as an abstract plane" (Crowley, 1987). Students have the ability to understand formal demonstrations. They are able to understand axioms even in the absence of concrete models. They are in a full propositional level where they feel the need of rigor.

The Van Hiele Model leads the student to the level of visualization of a geometric concept, then to the level of analysis, then to that of logical ordering, further to the level of deduction, and finally to the level of rigor of conceptualization. At this point, the student becomes able to understand and fully relate abstract concepts.
Four important characteristics of the levels are highlighted by de Villiers (1987):
• Fixed order: The order in which students progress respects hierarchical levels. A student who is at level n> 1 passed an earlier level, n-1. It is presumed that for a student to assimilate the contents of a next level must master the previous level.
• Adjacency: At each level of thought, what was intrinsic at the previous level becomes extrinsic at the current level. Objects from an earlier level become objects of study at a later level.
• Distinction: Each level has its own linguistic characteristics and is interconnected in a particular way through them. Nevertheless, no sudden advance from one level to another should be expected and many intermediate cases, where the individual advances to one level and subsequently returns to the previous level may occur.
• Separation: Two people with reasoning on different levels can not understand each other.
Interest in the contributions of this model has attracted the attention of many mathematical education professionals. It involves two components: the description of the different types of geometric reasoning of the students throughout their formation and a description of how a teacher can organize the activity in their classes so that students can reach a higher level of reasoning.
A sequence of phases to organize the teaching according to the Van Hiele model, presented by Jaime and (1993); Fouz & De Donosti (2005) and Vargas & Araya (2013), drives meaningful learning.

Phase 1: Information / interrogation

The teacher identifies the previous knowledge that the students have about the subject. Teachers and students make observations and raise important questions. Students should be informed about the new field of study that will be initiated, the types of problems they will be confronted, the methods and materials that will be used.

Phase 2: Guided action

The teaching is directed through concrete activities, which respect a didactic sequence. Students should be encouraged to discover, understand, assimilate, and apply ideas, concepts, properties, and relationships. The teacher should guide problem solving when necessary.

Phase 3: Explanation

This phase is based on previous experiences, students should be able to express through oral or written language the results obtained from their experiences and argue about these with the teacher and other students. For this she should be inserted by the teacher in the appropriate vocabulary.

Phase 4: Free Orientation

Students use their knowledge to solve different problems. The teacher should limit help and propose problems that promote the discovery of new relationships and properties and that may allow different solutions or none at all. With this, the student acquires experience.

Phase 5: Integration

Students review and synthesize what they have learned in order to form an overview and a new internal network of learned knowledge. The teacher can offer recovery activities for students who have some shortcomings in acquiring geometric knowledge.
By the end of the fifth phase, students are ready to progress. A new reasoning domain replaces the old one.
In phases 1 and 2, the need of concrete objects and situations to conduct explorations and investigations chracterizes representational learning. In phase 3, the symbols have meanings, a specific language begins to be acquired and improved what would be conceptual learning. Finally, phases 4 and 5 involve propositional learning.

There is some difficulty in placing certain students at a specific level. There may be individuals in transition from one level to another. This may correspond to subsumers not integrated with the prior information. Perhaps the previous indicators are not well anchored, they may have unscientific meaning, a different concept than the expected and correct one has been created, what is impeding the progression in levels.

The Work in the Laboratory of Mathematics Learning

To produce improvements in student learning, we must plan, design, and propose tasks. The two theories start from concrete, so the activities carried out in the Laboratory are supposed to help in the assimilation of new knowledge and consequently in the promotion of levels.

Students in a class may show different levels of geometric thought. This suggests that the student be treated individually. Content management should be organized in a way that enables students at different levels to advance to higher levels.The idea of implementing diversified didactic strategies gives the teaching subsidies to attract students' attention and motivation. In the Laboratory the teaching materials are selected, adapted and created according to each context in which it will be inserted, and according to the objectives previously established.

The Laboratory of Mathematics Learning (LML) is a space for the collective, collaborative and cooperative construction of mathematical knowledge. It uses didactic resources that have the function of enhancing the exploration of contents, enriching the teaching activities and making the learning process more pleasant and effective.

The LML is a challenge for both, teacher and students. Classes in the Laboratory require the teacher to have a precise and structured plan, everything must be tested. The LML modifies the behavior of the teacher, which becomes a mediator of learning. It modifies also the behavior of the students themselves, as the explorations carried out are surprising and demand immediate responses.

The class had 38 students in the 8th grade, with ages varying from 13 to 17. This level was chosen because it is at this stage that geometrical questions of daily life awaken students' interest in a natural and spontaneous way, enabling the exploration of situations that develop the capacity to argue and construct demonstrations. Thus, in an organized way, the teaching of Geometry can help the students to develop the reasoning to understand, describe, and represent the multiple situations of their world.As an initial feature, the class is agitated and unmotivated to learn the discipline. Most students, when questioned, declare that they “do not like math”.

Method

First, a pre-test, based on van Hiele's Theory, was prepared. Two classes performed the pre-test, the first with the 38 students in the experimental group and the second with 37 students, as a control group. This controller class received a traditional treatment of a Mathematics class of the school: oral presentation and textbook exercises.The pre-test included 19 objective and discursive questions addressing from the identification of forms to the more rigorous thought structuring.Using the results of the pre-test, the students were classified according to the theory of van Hiele. For the individual evaluation, the questions were approximately arranged in increasing order of difficulty, in such a way that students in each level would feel able to answer the questions until a certain point. This facilitated the analysis of the individual results.

Identifying the transition from one level of development to another is quite complex. The student can face some questions in a given level, but in a certain moment to return to an earlier level. We cannot rule out also situations of errors and accidental hits.

The questions were divided into three groups: 1) questions 1, 2, 4 and 5; 2) questions 3, 6, 7 an 11; 3) questions 8, 9, 10, 12, 13, 14, 15, 16, 17 and 18; 4) question 19.

Each item required the student to use their knowledge, from the teaching of content or experience acquired at some previous stage. Items 1, 2, 4 and 5 were designed to inform about this previous knowledge, which might affect the ability to solve the other questions. The students had to recognize figures by their overall appearance and identify their names or features associated with such names. Students in level I would be able to answer these items.

Items 3, 6, 7, 11 required the student to analyze properties of the geometric figures, but these did not directly relate the figures or properties to each other. These questions allowed the student to demonstrate the mastery of language and nomenclature corresponding to level II.

Items 8, 9, 10, 12, 13, 14, 15, 16, 17 and18 were designed to hint about entrance on level 3. They allowed to relate the figures to each other and to properties. They enabled more elaborate processes of construction, but still did not involve deductive reasoning.

Finally, item 19 asked for a deduction, which would allow the identification of level 4.
Items 8, 10 and 19 had a key role in the analysis.
8. Are there triangles that are isosceles and acutangle at the same time? Justify your answer with an example.
10. What properties of the inner and outer angles of a triangle do you know?
19. In this figure (figure 1), can we say that AB = DE. Why?

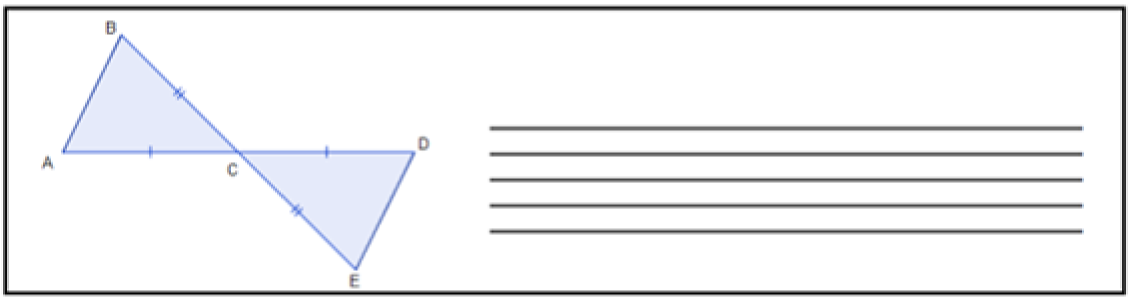

Figure 1. Proposed problem

Follows the classification rule employed, which was able to precisely classify all students.

Level I: Student correctly answered at least one question of group 1 and no question of higher level.
Transition from level I to level II Student correctly answered at least one question of group 1 and one question of group 2 and no other question.

Level II: Student answered correctly two or more questions of group 2. Answering correctly at least one question of group 3 also indicates that the student has attained at least this level.
Transition from level II to level III: Student answered correctly two or more questions of group 2 and at least one question of group 3 different from questions 8 and 10.

Level III: Student answered correctly Question 8, Question 10 or more than one question of group 3.
Transition from level III to level IV: Student answered correctly Question 8, Question 10 or more than one question of group 3 and answered correctly Question 19 but was not able to present a correct justification.

Level IV: Student answered Question 19 at Level IV, presenting a well-grounded argument.

The distributions found in the pre-test, in the experimental and control classes, along the levels were, as follows. At level 1: 1 and 7 students; at level 2: 18 and 15 students, and, at transition levels, from level 1 to 2: 18 and 13 students; from level 2 to 3: 1 student in both. Above level 3 no student was found. These are levels where greater organization and abstraction of geometric thinking is required.

The Activities

The teaching of Geometry in the 8th year, by the school curriculum, addresses the study of quadrilaterals and triangles. The activities, supported by concrete didactic resources or not, were designed with the main function of solidifying and reinforcing concepts that should have been apprehended before. Another main objective was to combine Algebra and Geometry, to provide a teaching that allows the student to identify and associate forms with algebraic calculations.

Specific materials were built, developed according to the characteristics of the class and locality, but with possibilities of being adapted to other contexts.

The class was separated into groups and the division respected some criteria, among which the variation of classification levels. The presence of different levels in the same group had the aim of stimulating mutual aid, allowing for greater individual development of the members at lower levels. To assure individual involvement, individual portfolios were used, where all the productions made during classes were gathered.

Involving real world situations, activities in the Mathematics Education Laboratory must encourage students to explore, experiment, reason in an organized way and rescue assimilated concepts in order to expand their knowledge. The 15 activities consider, in addition to the interests of the students, their individual needs and levels of cognitive development according to van Hiele theory. As registered by de Villiers (2010), van Hiele attributed the main reason for the failure of the traditional Geometry curriculum to the fact that it was presented at a higher level than the students' level.At this point, we describe three of the activities performed in the experimental class.

Activity 1: Location in the Cartesian Plane and Geographical Coordinates

Goals. The student advances toward:

1. Recognize and represent in the Cartesian coordinate system;
2. Explore maps and associate the geographical coordinates of latitude and longitude with the Cartesian plane;
3. Perceive the richness of the regional variations found in the class.

Skills / Abilities

Use the geometric knowledge to perform the reading and representation of reality and act on it. Interpret the location and movement of people or objects in three-dimensional space and their representation in two-dimensional space. Solve problem-situations involving geometric knowledge of space and form.

Procedures

1. The activity begins with a research task, where students raise characteristics of their places of origin, including latitudes and longitudes. At another time, and with a political map, students are encouraged to locate them through the coordinates surveyed.
2. The teacher can use datashow to exemplify the location of a state through latitude and longitude coordinates.
3. It is interesting for students to record all locations that arise in the class so that, at the end, they can compare distances, common characteristics, and regional differences. This activity can be performed in conjunction with the discipline of Geography.
4. The students receive a card with a Cartesian plane with some points marked. The students should represent them with the aid of elastics on the board of a Geoplane and register their ordered pairs. They should then locate some points according to ordered pairs provided.

5. The student should receive information about the Cartesian coordinate system, its nomenclature and representations to complete the task.
6. The verification of the results of the activity can be done by comparison of the Geoplane boards of the groups. The teacher should be alert to correct every shortcoming that may arise.
Educational resources: Political map and Geoplane board.

Application of the Activity

We can associate the Cartesian Plane with latitude and longitude, themes of Geography. The Global Positioning System allows finding the exact location on Earth. The Cartesian plane was explored through this path, because the students had quite different origins. The activity provided an appreciation of the cultural differences present in the classroom.

In some cases, students used two perpendicular rulers to locate their home state. There were no obstacles to the accomplishment of this activity, they showed interest in carrying out the task and were surprised by the different localities that appeared.

In the sequence, the Geoplane board with its two perpendicular axes, one horizontal and one vertical, which intersect at the origin of the coordinates. It was established how the Cartesian coordinates are ordered pairs (x; y).The names of the horizontal axis of abscissa (x) and the vertical of ordinate (y) were taught. Emphasis was given to the correspondence of the points in the axes to the numbers in the set of real numbers.

Some students confused abscissas with the ordinates, but the board of the Geoplane contained this information. This fact facilitated the location later.

The activity was adapted so that the students developed familiarity with the representations in the Cartesian plane. This activity will enable students to further assimilate the algebraic and geometric representations of a system of first-degree equations with greater ease.

Activity 2: Condition of Existence of a Triangle

Goals. The student advances toward:

1. Understand triangles with the aid of concrete materials.
2. Investigate a condition for the existence of a triangle.

Skills / Abilities

Relate information, represented in concrete and abstract forms to build consistent argumentation. Use geometric knowledge to perform the reading and representation of reality and act upon it. Identify characteristics of polygons. Use geometric notions in the selection of arguments and proposal of solution of daily problems.

Procedures

1. The student receives sets of three strips of cardboard and is informed about their length. Receives also brackets to join the strips.
2. The student should try to join the ends with the brackets. Sometimes, it will be possible to form a triangle; sometimes it will not be possible. They will then be asked to justify this impossibility.
3. The student should report in an activity sheet the conclusions reached.
Didactic resource: Strips of paper. Brackets.

Application of the Activity

This problem, if presented in the traditional way, does not raise much interest. The students do not believe that it is not possible to construct a triangle with sides of any length. The activity lays in concrete terms an abstract aspect of Mathematics.

Intentionally two situations are simulated for the construction of polygons with received strips. Because there are three "angles" the polygon is a triangle.

It was very interesting to note the students' surprise when they could not join the sides to build the polygon. The students began to inquire and comment that something was wrong.

Student: "Teacher, is this a bit of a catch-up with us? This will never close! "
Teacher: "Are you sure it does not fit? What is the cause of it? Did not the other combine? What happened to make things change? ".
After they realized the impossibility, they received the confirmation and handed a sheet of paper to record their answer to the questions "What happened in each case?" and “why it happened”, to fill spaces comparing with the signs of “<” and “>” the sums of two of the lengths and a third length. The students completed the information comparing them with signs of greater, lesser or equal. Finally, they were asked to write a justified answer the abstract question: "Given any three measures, is it possible to construct a triangle whose sides have these measures?”

Activity 3: Comparing measurements without measuring

Goals: The student advances toward:

1. Compare the measurement of the area of figures without directly measuring each of them.
2. Explore the conservation or modification of measures of the area of polygonal figures using checkered meshes.

Skills / Abilities:

Build and develop notions of size and measure to understand the reality and to solve everyday problems. Solve problem-situations involving different quantities. Select suitable units of measure. Evaluate the reasonableness of numerical result in the construction quantitative statements. Evaluate proposals of intervention on the reality, using geometric knowledge.

Procedures:

1. Students should construct an isosceles trapezoid on the Geoplane and record the measures of the bases and the height.
2. Next, draw the diagonals with the aid of a rubber band.
3. Then, in a checkered grid register the polygon built on the geoplaneGeoplane and name the vertices.
4. Finally, compare the area of triangles formed with three of the four vertices.
5. The teacher instigates the students, during the exploration, trying to correct possible hasty generalizations. Didactic resource: Geoplane.

Application of the Activity

It is customary in mathematics to compare numeric measures to determine whether some figure is larger, equal or smaller than another? But when it comes to comparing measures without measuring, there is a curiosity about what can be done. Under this perspective, this activity allowed the student to compare the areas of polygons without directly measuring them properly. The student should solve problems that appear in practice when comparing the areas of delimited regions. For the 8th-graders, it is very strange to come across a mathematical problem with no apparent numerical quantities.

Firstly, to prepare the students, a construction task is performed in the Geoplane that makes possible to compare the measures. The activity starts with the construction of a trapezoid in the Geoplane, where it is easy to identify the larger and the smaller basis. Next, the students are instructed to draw the diagonals with the aid of a rubber band. After that, they are asked to use their knowledge, about bases, heights and areas of triangles, to compare the areas of the triangles formed.

The students are led to discover that the triangles with the same basis have the same area and triangles with the larger basis have larger areas, because their heights are equal. The students solve the problem without having to measure or calculate. They may be surprised with some equalities but understand their proof.

The activity was concluded proposing a real problem situation that involved an important issue in the scenario of deforestation and environmental imbalance. In a map, a region in the form of an escalene trapezoid was selected and the students were asked to highlight sub-regions of equal area.

Results and Discussion

After these actions were implemented in the Mathematics classes, a post-test was applied to evaluate the progress of each student. The same test initially applied again to the two classes. Not all the 38 students in the experimental class and the 36 students in the control class present in the pre-test were not present for the post-test application. Only 32 students of the experimental class and 27 students of the control class appeared.

In both classes, the students presented improvements, but it could be verified which method produced greater progress in the students' performances. The results of the two classes in the pre-test and in the post-test are summarized in (Table 1).

Table 1. Pre-test and post-test levels

	Experimental class		Control class	
Level	Pre-test	Post-test	Pre-test	Post-test
I	1	0	7	0
I – II	18	0	13	6
II	18	9	15	15
II – III	1	13	1	3
III	0	6	0	3
III – IV	0	4	0	0

In the pre-test, the two classes initially presented small difference in relation to the global presence of the highest van Hiele levels. It can be observed only a greater homogeneity in the experimental class, with a larger number of students at the level II and in the transition from I to II.

The results obtained with the application of the post-test, after the interventions, in the experimental and control groups are more distanced. (Table 2) shows in the experimental class the highest number of students in transition from level II to level III, more students in level III and even four students in transition from III to IV, against none in the control class. On the other hand the control class presented a higher number of students in level II, which is now the lowest level as no student remained at the basic van Hiele level in either class.

The percentage of students above level II at the end of the school year gives also a clear indication. It can be extracted from (Table 1) that the experimental class had in this situation 23 students, or 71.8% of the total of 32 students examined, while the control class had only six, or 22.2% of its total of 27. Note that at the beginning there was only one student in each class in this situation.

The global improvement can be also compared in (Table 2), which shows that, while the control class had 51.9% of its students changing to a higher level, the experimental class had 87.5% of the students passing to a higher level.

Table 2. .Percentage of students with improvement

Class	Students improving	Total of students	%
Experimental	28	32	87.5
Control	14	27	51.9

(Tables 3) and (Tables 4) detail the changes by level, for the students present at the two tests, in the experimental and the control class. It reveals the advances obtained by each student of a different initial level.

Table 3. Performance of the experimental group in the pre-test and post-test

	Pre-test	I	I - II	II	II - III	TOTAL
Post-test						
II		1	4	4	0	9
II - III		0	7	6	0	13
III		0	2	3	1	6
III - IV		0	0	4	0	4
TOTAL		1	13	17	1	32

Table 4. Performance of the control group in the pre-test and post-test

	Pre-test	I	I - II	II	II - III	TOTAL
Post-test						
I-II		1	4	0	0	5
II		2	4	9	1	16
II - III		0	1	2	0	3
III		1	0	2	0	3
III - IV		0	0	0	0	0
TOTAL		4	9	13	1	27

In (Table 3), it can be seen that there were 14 students of the experimental class initially classified at level I and in the transition from level I to II and all of them advanced to level II. Only four of 17 students initially in level II remained at that level. The only student who was initially in the transition from level II to III advanced to level III. Finally, all four students who were classified at the transition from III to IV in the post-test were initially at level II. Thus, although progress along the year might be expected to occur only to the next level, larger jumps occurred.

The same investigation was carried out in the control group. It can be seen in Table that among the students that took the post-test, four were initially at level I and all advanced. Of those who were in the transition from level I to II, four advanced to level II and one advanced for transition from II to III. On the other hand, four, from the total of nine, remained in the original position. There were 13 at level II and nine of them remained at that level. The other four advanced to higher levels.

It can be noticed that one student who had been classified in transition from level II to III was now classified in level II. This result highlights the difficulty in the development of geometric reasoning of the student who faced a traditional teaching.

Conclusion

A methodology of development of activities involving everyday situations and interdisciplinary approaches, with active students' participation in a Laboratory of Mathematics and with application of tests to evaluate van Hiele levels was here presented. It involved a series of activities specially designed to facilitate the development of Geometric reasoning.

The proposal applied to an experimental group provided an environment conducive to meaningful learning. It must be pointed that it also increased students' confidence and stimulated them to develop continuity for the acquisition of new knowledge.

Recommendations

This experiment highlights the importance of the development of specific didactic materials, designed with a view to the construction of knowledge and to the encouragement of creativity, for the development of students' geometric thought.

Acknowledgements

We thank all those who collaborated for this research: students, parents, school, City Hall of Macaé, Colégio Pedro II, Plataforma Brasil and for the opportunity granted to share our course and research results for the international academic community.

References

Ausubel, D. P. (1961). *Meaningful Learning and Retention: Intrapersonal Cognitive Variables*. Review of Educational Research, 31, 500-510.

Crowley (1984). *The van Hiele Model of the Development of Geometry Thought In Lindquist*, M.M. & Shulte, A.P., Learning and Teaching Geometry. National Council of Teachers of Mathematics, Resbon-VA.

De Villiers, M. (2015, Jan 10). *Research evidence on hierarchical thinking, teaching strategies and the Van Hiele Theory: Some critical comments*. University of Stellenbosch: Rumeus, General format. Retrieved from http://math.kennesaw.edu/~mdevilli/VanHieleCritique-87.pdf.

Fouz, F. & de Donosti, B. (2015, May 8). *Modelo de Van Hiele para la didáctica de la geometría: Un paseo por la geometría*. General format. Retrieved from <http://cimm.ucr.ac.cr/ciaem/articulos/universitario/materiales>

Jaime, A. (1993) *Aportaciones a la interpretación y aplicación del Modelo de Van Hiele: La enseñanza de las isometrías en el plano. La evaluación del nivel de razonamiento.* D. Sc. Thesis, Universidad de Valencia, Spain.

Nasser, L.*Using the van Hiele theory to improve secondary school geometry in Brazil*. (1992). D. Sc. Thesis – London King's College, University of London, London.

Skovsmose, O. (1994). *Towards a critical mathematics education. Educational Studies in Mathematics*. (25), p. 35-57.

Tomaz, V. S.; David, M. M. *How students' everyday situations modify classroom mathematical activity: the case of water consumption*. Journal for Research in Mathematics Education, 46 (4), 455–496.

Van Hiele, P. (1986) Structure and Insight: *A Theory of Mathematical Education. Academic Press*. London.

Vargas, G. V. & Araya, R. G. (2013). *El modelo de Van Hiele y la enseñanza de la geometría*. *Uniciencia*, 27(1), p. 74-94.

Author Information

Marcele Santos
Prefeitura Municipal de Macaé/ Seeduc – RJ
Rio de Janeiro - Brasil
Contact e-mail: *rjcelly@hotmail.com*

Neide Sant'Anna
Colégio Pedro II.
Rio de Janeiro - Brasil