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Hartley-Ross type unbiased estimators using the stratified random sampling

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Abstract

This study mentions Hartley-Ross type unbiased ratio estimators of the finite population mean in the stratified random sampling using the auxiliary variable. We propose the unbiased estimators using the estimators in Kadilar and Cingi [5],[6]. We derive the variance equations, up to the first degree of approximation, for all proposed estimators. The proposed estimators have been compared with the mentioned estimators in theory. Finally, we also demonstrate theoretical findings by the support of numerical illustrations.

Keywords: ratio estimator; unbiased estimator; auxiliary information; efficiency; stratified random sampling; variance.

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1. Introduction and Notations

In the simple random sampling, Hartley and Ross [3] firstly defined the unbiased ratio estimator. Then, the unbiased ratio estimators in the stratified random sampling were presented by Pascual [10]. Singh et al. [11] and Kadilar and Cekim [4] proposed Hartley-Ross type unbiased estimators for the simple random sampling using various auxiliary information. Recently, Khan and Shabbir [7], [8] and Khan et al. [9] have also suggested several Hartley-Ross type unbiased estimators under the ranked set sampling and the stratified ranked set sampling.

A finite population $U = (U_1, U_2, ..., U_N)$ of size N is assumed that the population of N units be divided into L strata with N_h elements in the $h-$th stratum $(h = 1, 2, ..., L)$. Let n_h be the size of the sample drawn by using the Simple Random Sampling without Replacement from a population of size N_h. Suppose that values y_{hi} and x_{hi} be on the

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study and auxiliary variables in the stratum h, respectively, where $i = 1, 2, ..., n_h$. Let the $h-$th stratum sample means be $\overline{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ and $\overline{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$, respectively. Let the stratified mean estimator for y and x be , respectively, $\overline{y}_{st} = \sum_{h=1}^{L} W_h \overline{y}_h$ and $\overline{x}_{st} = \sum_{h=1}^{L} W_h \overline{x}_h$. Here $W_h = (N_h/N)$ is the known stratum weight. The population means of the study and auxiliary variables are supposed that $\overline{Y} = \overline{Y}_{st} = \sum_{h=1}^{L} W_h \overline{Y}_h$ and $\overline{X} = \overline{X}_{st} = \sum_{h=1}^{L} W_h \overline{X}_h$, where $\overline{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} Y_{hi}$ and $\overline{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} X_{hi}$, respectively.

The well-known ratio estimator of the population mean, $\overline{Y}$, is given by Cochran [1] as

$$y_{C1} = \frac{\overline{y}}{\overline{x}}\overline{X}. \tag{1.1}$$

Hartley and Ross [3] consider this ratio estimator of the population mean proposed by Cochran [1] as

$$y_{C2} = \overline{r}\,\overline{X}, \tag{1.2}$$

where $\overline{r} = \frac{1}{n}\sum_{i=1}^{n} r_i,\ r_i = \frac{y_i}{x_i}$. Later, the bias of this estimator is estimated unbiasedly by Hartley and Ross [3] as

$$B(y_{C2}) = -\frac{n(N-1)}{N(n-1)}\left(\overline{y} - \overline{r}\ \overline{x}\right)$$

and they obtain the unbiased ratio estimator

$$y_{HR} = \overline{r}\,\overline{X} + \frac{n(N-1)}{N(n-1)}\left(\overline{y} - \overline{r}\ \overline{x}\right) \tag{1.3}$$

for the population mean in the simple random sampling.

Kadilar and Cingi [5], [6] define some estimators using the coefficient of kurtosis (β_2) and the coefficient of variation (C_x) of the auxiliary variable under the stratified random sampling as

$$t_1 = \overline{y}_{st}\frac{\overline{X}_{st} + C_{xst}}{\overline{x}_{st} + C_{xst}}, \tag{1.4}$$

$$t_2 = \overline{y}_{st}\frac{\overline{X}_{st} + \beta_{2st}(x)}{\overline{x}_{st} + \beta_{2st}(x)}, \tag{1.5}$$

$$t_3 = \overline{y}_{st}\frac{\left(\overline{X}\beta_2(x)\right)_{st} + C_{xst}}{\left(\overline{x}\beta_2(x)\right)_{st} + C_{xst}}, \tag{1.6}$$

$$t_4 = \overline{y}_{st}\frac{\left(\overline{X}C_x\right)_{st} + \beta_{2st}(x)}{\left(\overline{x}C_x\right)_{st} + \beta_{2st}(x)}, \tag{1.7}$$

and

$$t_5 = k\frac{\overline{y}_{st}}{\overline{x}_{st}}\overline{X}, \tag{1.8}$$

where

$$\begin{aligned}
C_{xst} &= \sum_{h=1}^{L} W_h C_{xh},\ \beta_{2st}(x) = \sum_{h=1}^{L} W_h \beta_{2h}(x), \\
\left(\overline{X}\beta_2(x)\right)_{st} &= \sum_{h=1}^{L} W_h \overline{X}_h \beta_{2h}(x),\ \ (\overline{x}\beta_2(x))_{st} = \sum_{h=1}^{L} W_h \overline{x}_h \beta_{2h}(x), \\
\left(\overline{X}C_x\right)_{st} &= \sum_{h=1}^{L} W_h \overline{X}_h C_{xh},\ \ (\overline{x}C_x)_{st} = \sum_{h=1}^{L} W_h \overline{x}_h C_{xh},
\end{aligned}$$

and k is a constant that makes the mean squared error (MSE) of t_5 minimum.

The biases of the estimators, in (1.4)-(1.8), are obtained, to the first degree of approximation, respectively, as follows:

$$B(t_j) = \frac{1}{X_{Sj}}\left[\sum_{h=1}^{L} W_h^2 \gamma_h \left(\frac{\overline{Y}_{st}}{X_{Sj}} S_{xh}^2 - S_{yxh}\right)\right], j = 1, 2, 3, 4$$

and

$$B(t_5) = (k-1)\,\overline{Y} + \frac{1}{\overline{X}}\left[\sum_{h=1}^{L} W_h^2 \gamma_h \left(\frac{\overline{Y}}{\overline{X}} S_{xh}^2 - kS_{yxh}\right)\right],$$

such that

$$\begin{aligned}
S_{yh}^2 &= \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \overline{Y}_h)^2,\ S_{xh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(x_{hi} - \overline{X}_h)^2, \\
S_{yxh} &= \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \overline{Y}_h)(x_{hi} - \overline{X}_h) \text{ and } \gamma_h = \frac{N_h - n_h}{N_h n_h}
\end{aligned}$$

where

$$\begin{aligned}
X_{S1} &= \overline{X}_{st} + C_{xst},\ X_{S2} = \overline{X}_{st} + \beta_{2st}(x), \\
X_{S3} &= \left(\overline{X}\beta_2(x)\right)_{st} + C_{xst} \text{ and } X_{S4} = \left(\overline{X}C_x\right)_{st} + \beta_{2st}(x).
\end{aligned}$$

2. Proposed Estimators

We improve Hartley-Ross estimators using the proposed estimators by Kadilar and Cingi [5], [6] with their unbiased biases, and in this way, we obtain the following estimators:

$$\begin{aligned} y_{New1} &= \overline{y}_{st}\frac{X_{S1}}{\overline{x}_{st}+C_{xst}} \\ &\quad -\frac{1}{X_{S1}}\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(\frac{\overline{y}_{st}}{X_{S1}}S_{xh}^2-s_{yxh}\right)\right], \end{aligned} \tag{2.1}$$

$$\begin{aligned} y_{New2} &= \overline{y}_{st}\frac{X_{S2}}{\overline{x}_{st}+\beta_{2st}(x)} \\ &\quad -\frac{1}{X_{S2}}\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(\frac{\overline{y}_{st}}{X_{S2}}S_{xh}^2-s_{yxh}\right)\right], \end{aligned} \tag{2.2}$$

$$\begin{aligned} y_{New3} &= \overline{y}_{st}\frac{X_{S3}}{\left(\overline{x}\beta_2(x)\right)_{st}+C_{xst}} \\ &\quad -\frac{1}{X_{S3}}\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(\frac{\overline{y}_{st}}{X_{S3}}S_{xh}^2-s_{yxh}\right)\right], \end{aligned} \tag{2.3}$$

$$\begin{aligned} y_{New4} &= \overline{y}_{st}\frac{X_{S4}}{\left(\overline{x}C_x\right)_{st}+\beta_{2st}(x)} \\ &\quad -\frac{1}{X_{S4}}\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(\frac{\overline{y}_{st}}{X_{S4}}S_{xh}^2-s_{yxh}\right)\right], \end{aligned} \tag{2.4}$$

and

$$y_{New5}=k\frac{\overline{y}_{st}}{\overline{x}_{st}}\overline{X}-(k-1)\,\overline{y}_{st}-\frac{1}{\overline{X}}\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(\frac{\overline{y}_{st}}{\overline{X}}S_{xh}^2-ks_{yxh}\right)\right], \tag{2.5}$$

where $\overline{y}_{st}$ and s_{yxh} are unbiased estimators of $\overline{Y}_{st}$ and S_{yx}, respectively.

To obtain the variance of the suggested estimators, we define

$$\overline{y}_{st}=\overline{Y}(1+\vartheta_0),\ \overline{x}_{st}=\overline{X}(1+\vartheta_1),\ \text{and}\ \sum_{h=1}^{L}W_h^2\gamma_h s_{yxh}=\sum_{h=1}^{L}W_h^2\gamma_h S_{yx}(1+\vartheta_2)$$

such that

$$\begin{aligned} E(\vartheta_0) &= E(\vartheta_1)=E(\vartheta_2)=0, \\ E(\vartheta_0^2) &= V_{0,2},\ E(\vartheta_1^2)=V_{2,0},\ E(\vartheta_2^2)=D_{0,0}, \\ E(\vartheta_0\vartheta_1) &= V_{1,1},\ E(\vartheta_0\vartheta_2)=D_{0,1}\ \text{and}\ E(\vartheta_1\vartheta_2)=D_{1,0}, \end{aligned}$$

where

$$V_{r,s}=\sum_{h=1}^{L}W_h^{r+s}\frac{E\left[\left(\overline{x}_h-\overline{X}_h\right)^r\left(\overline{y}_h-\overline{Y}_h\right)^s\right]}{\overline{X}^r\overline{Y}^s},$$

$$D_{r,s}=\frac{\sum\limits_{h=1}^{L}W_h^{r+s+1}\gamma_h\left(\frac{S_{yxh}^2}{W_h\gamma_h\left(\mu_{22h}-S_{yxh}^2\right)}\right)^{r+s-1}\sum\limits_{h=1}^{L}W_h^{r+s}\gamma_h^{r+s}\mu_{12h}^{r}\mu_{21h}^{s}}{\overline{X}^r\overline{Y}^s\left(\sum\limits_{h=1}^{L}W_h\gamma_hS_{yxh}\right)^{2-(r+s)}},$$

and

$$\mu_{jkh}=\frac{1}{N_h-1}\sum_{i=1}^{N_h}(Y_{hi}-\overline{Y}_h)^j(X_{hi}-\overline{X}_h)^k,\ h=1,2,...,L.$$

We express the proposed estimators y_{Newi}, $i=1,2,...,5$ with regard to ϑ's as:

$$\begin{aligned}
y_{New1} &= \overline{Y}(1+\vartheta_0)\left(1+\alpha\vartheta_1\right)^{-1}\\
&\quad-\frac{1}{X_{S1}}\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(\frac{\overline{Y}(1+\vartheta_0)}{X_{S1}}S_{xh}^2-S_{yxh}\left(1+\vartheta_2\right)\right)\right],\\
y_{New2} &= \overline{Y}(1+\vartheta_0)\left(1+\delta\vartheta_1\right)^{-1}\\
&\quad-\frac{1}{X_{S2}}\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(\frac{\overline{Y}(1+\vartheta_0)}{X_{S2}}S_{xh}^2-S_{yxh}\left(1+\vartheta_2\right)\right)\right],\\
y_{New3} &= \overline{Y}(1+\vartheta_0)\left(1+\varphi\vartheta_1\right)^{-1}\\
&\quad-\frac{1}{X_{S3}}\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(\frac{\overline{Y}(1+\vartheta_0)}{X_{S3}}S_{xh}^2-S_{yxh}\left(1+\vartheta_2\right)\right)\right],\\
y_{New4} &= \overline{Y}(1+\vartheta_0)\left(1+w\vartheta_1\right)^{-1}\\
&\quad-\frac{1}{X_{S4}}\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(\frac{\overline{Y}(1+\vartheta_0)}{X_{S4}}S_{xh}^2-S_{yxh}\left(1+\vartheta_2\right)\right)\right],
\end{aligned}$$

and

$$\begin{aligned}
y_{New5} &= k\overline{Y}(1+\vartheta_0)(1+\vartheta_1)^{-1}-(k-1)\,\overline{Y}(1+\vartheta_0)\\
&\quad-\frac{1}{\overline{X}}\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(RS_{xh}^2(1+\vartheta_0)-kS_{yxh}\left(1+\vartheta_2\right)\right)\right],
\end{aligned}$$

where

$$\alpha=\frac{\overline{X}_{st}}{X_{S1}}\ ,\ \delta=\frac{\overline{X}_{st}}{X_{S2}}\ ,\ \varphi=\frac{\left(\overline{X}\beta_2(x)\right)_{st}}{X_{S3}}\ ,\text{ and }w=\frac{\left(\overline{X}C_x\right)_{st}}{X_{S4}}.$$

In this way, we obtain the variance equations of the proposed estimators that are given in (2.1)-(2.5), respectively, as follows:

$$V\left(y_{Newj}\right) \cong \overline{Y}^2 A_\theta + \frac{1}{X_{Sj}^2}\left[\sum_{h=1}^{L} W_h^2\gamma_h\left(\frac{\overline{Y}S_{xh}^2}{X_{Sj}} - S_{yxh}\right)\right]^2$$

$$-\frac{2\overline{Y}}{X_{Sj}}\left[\sum_{h=1}^{L} W_h^2\gamma_h\left(\frac{\overline{Y}S_{xh}^2}{X_{Sj}}A_\theta - S_{yxh}B_\theta\right)\right],\ j=1,2,3,4 \tag{2.6}$$

and

$$V\left(y_{New5}\right) \cong \overline{Y}^2 A_\theta + \frac{1}{\overline{X}^2}\left[\sum_{h=1}^{L} W_h^2\gamma_h\left(RS_{xh}^2 - kS_{yxh}\right)\right]^2$$

$$-2R\left[\sum_{h=1}^{L} W_h^2\gamma_h\left(RS_{xh}^2\left(A_\theta + k\left(1-k\right)V_{2,0}\right)\right.\right.$$

$$\left.\left.-kS_{yxh}\left(B_\theta + k\left(1-k\right)V_{2,0}\right)\right)\right], \tag{2.7}$$

where

$$A_\theta = V_{0,2} + \theta^2 V_{2,0} - 2\theta V_{1,1},\ R = \frac{\overline{Y}}{\overline{X}},$$

and

$$B_\theta = -\theta V_{1,1} + D_{0,1} - \theta D_{1,0} + \theta^2 V_{2,0},\ \theta = \alpha,\ \delta,\ \varphi,\ w \text{ and } k.$$

Note that the term of γ_h^3 is ignored, because it is equal to approximately zero. For minimizing the variance, given in (2.7), we obtain the optimum value of k by

$$k_{opt} = \frac{\Delta}{\Pi}, \tag{2.8}$$

where

$$\Delta = \overline{Y}^2 V_{1,1} + \frac{1}{\overline{X}^2}\left[\sum_{h=1}^{L} W_h^4\gamma_h^2 RS_{xh}^2 S_{yxh} + \sum_{h=1}^{L}\sum_{t=1}^{L} W_h^2\gamma_h W_t^2\gamma_t R\left(S_{xh}^2 S_{yxt} + S_{xt}^2 S_{yxh}\right)\right]$$

$$+R\left[\sum_{h=1}^{L} W_h^2\gamma_h\left(RS_{xh}^2\left(-2V_{1,1} + V_{2,0}\right) - S_{yxh}D_{0,1}\right)\right]$$

and

$$\Pi = \overline{Y}^2 V_{2,0} + \frac{1}{\overline{X}^2}\left[\sum_{h=1}^{L} W_h^2\gamma_h S_{yxh}\right]^2$$

$$+2R\sum_{h=1}^{L} W_h^2\gamma_h S_{yxh}(-D_{1,0} - V_{1,1} + V_{2,0}).$$

Replacing this optimum value in (2.7), to make the $V\left(y_{New5}\right)$ minimum, we get

$$V_{\min}\left(y_{New5}\right) \cong \Gamma - \frac{\Delta^2}{\Pi}, \tag{2.9}$$

where

$$\Gamma = V_{0,2}\left[\overline{Y}^2 - 2R^2\sum_{h=1}^{L} W_h^2\gamma_h S_{xh}^2\right] + \frac{R^2}{\overline{X}^2}\left[\sum_{h=1}^{L} W_h^2\gamma_h S_{xh}^2\right]^2.$$

3. Efficiency Comparisons

In this section, we compare proposed unbiased estimators given in (2.1)-(2.5), with the mentioned estimators, given in (1.4)-(1.8). Firstly, comparing the variance of the proposed estimators in (2.6) with the MSE of the estimators given in Kadilar and Cingi [5], we have the following inequality

$$V\left(y_{Newj}\right) < MSE(t_j) = \overline{Y}^2 A_\theta, \text{ where } \theta = \alpha,\ \delta,\ \varphi,\ w \text{ and } j = 1,2,3,4,$$

if

$$-\frac{2\overline{Y}}{X_{Sj}}\left[\sum_{h=1}^{L} W_h^2\gamma_h\left(\frac{\overline{Y}S_{xh}^2}{X_{Sj}}A_\theta - S_{yxh}B_\theta\right)\right]$$

$$+\frac{1}{X_{Sj}^2}\left[\sum_{h=1}^{L} W_h^2\gamma_h\left(\frac{\overline{Y}S_{xh}^2}{X_{Sj}} - S_{yxh}\right)\right]^2 < 0,\ j = 1,2,3,4. \tag{3.1}$$

Secondly, comparing the variance of the proposed estimator in (2.7) with the MSE of the estimators given in Kadilar and Cingi [6], we have

$$V\left(y_{New5}\right) < MSE(t_5) = \overline{Y}^2\left\{k^{*2}C + \left(k^* - 1\right)^2\right\},$$

where

$$C = V_{2,0} - 2V_{1,1} + V_{0,2} \text{ and } \theta = k,$$

if

$$\begin{aligned}
&-\overline{Y}^2\left[k^{*2}C - \left(k^* - 1\right)^2 + A_\theta\right]\\
&-2R\left[\sum_{h=1}^{L} W_h^2\gamma_h\left(RS_{xh}^2\left(A_\theta + k\left(1-k\right)V_{2,0}\right)\right.\right.\\
&\left.\left.-kS_{yxh}\left(B_\theta + k\left(1-k\right)V_{2,0}\right)\right)\right]\\
&+\tfrac{1}{\overline{X}^2}\left[\sum_{h=1}^{L} W_h^2\gamma_h\left(RS_{xh}^2 - kS_{yxh}\right)\right]^2 < 0.
\end{aligned} \tag{3.2}$$

Finally, we also compare the minimum variance of the proposed estimator in (2.9) with the minimum MSE of the estimators given in Kadilar and Cingi [6]. For this reason, it can be written as

$$V_{\min}\left(y_{New5}\right) < MSE_{\min}(t_5) = \overline{Y}^2\frac{C}{C + \overline{Y}^2}$$

if

$$\left[\Gamma - \frac{\Delta^2}{\Pi}\right] - \overline{Y}^2\frac{C}{C + \overline{Y}^2} < 0, \tag{3.3}$$

where the optimum value of k^* is

$$k_{opt}^* = \frac{\overline{Y}^2}{C + \overline{Y}^2}\ .$$

If the conditions (3.1)-(3.3) are satisfied, the proposed estimators are more efficient than the mentioned estimators t_i, $i = 1, 2, ..., 5$, under the determined conditions.

4. Empirical Study

To show the merits of the proposed estimators among the other estimators, two data sets previously used by Kadilar and Cingi [5] and Cingi et al. [2] are considered. First data set consists of 854 districts in Turkey. Summaries of the Population I are shown in Table 1.

Population I (Source: Institute of Statistics, Republic of Turkey [5]):
Y; the apple production amount in 1999, X; the number of apple trees in 1999. Stratum: Regions in Turkey (as 1: Marmara; 2: Aegean; 3: Mediterranean; 4: Central Anatolia; 5: Black Sea; 6: East and Southeast Anatolia).

Table 1. Descriptive Statistics for the Population I

Stratum	1	2	3	4	5	6
N_h	106	106	94	171	204	173
n_h	9	17	38	67	7	2
W_h	0.12	0.12	0.11	0.20	0.24	0.20
$\overline{Y}_h$	1536.77	2212.59	9384.31	5588.01	966.96	404.40
$\overline{X}_h$	24375.59	27421.70	72409.95	74365.68	26441.72	9843.83
C_{xh}	2.02	2.10	2.22	3.84	1.72	1.91
$\beta_{2h}(x)$	26.68	34.57	26.14	97.6	27.47	28.11
S_{yh}	6425.09	11551.53	29907.48	28643.42	2389.77	945.75
S_{xh}	49189.08	57461.62	160757.31	285603.13	45402.78	18793.95

Second data set consists of 923 districts in Turkey. Similarly, summaries of the Population II are shown in Table 2.

Population II (Source: Ministry of Education, Republic of Turkey [2]):
Y; the number of students in 2007, X; the number of schools in 2007. Stratum: Regions in Turkey (as 1: Marmara; 2: Aegean; 3: Mediterranean; 4: Central Anatolia; 5: Black Sea; 6: East and Southeast Anatolia).

Table 2. Descriptive Statistics for the Population II

Stratum	1	2	3	4	5	6
N_h	127	117	103	170	205	201
n_h	31	21	29	38	22	39
W_h	$0,14$	$0,13$	$0,11$	$0,18$	$0,22$	$0,22$
$\overline{Y}_h$	20804.59	9211.79	14309.30	9478.85	5569.95	12997.59
$\overline{X}_h$	30.81	30.29	43.19	30.21	29.50	57.54
C_{xh}	0.85	0.83	1.09	1.01	0.99	0.84
$\beta_{2h}(x)$	2.51	2.09	8.42	3.49	4.07	8.2
S_{yh}	30486.75	15180.77	27549.70	18218.93	8497.78	23094.14
S_{xh}	26.05	25.08	47.12	30.40	29.33	48.26

The sample sizes of each stratum are selected with the help of the Neyman allocation method for two data sets. From Table 3, we infer that proposed estimators have the smaller MSE values than the corresponding estimators in literature, and therefore, the proposed estimators are more efficient than the estimators existed in literature for two population data sets I and II.

Table 3. MSE and Variance values of t_j and y_{Newj} ratio estimators

Population I			
Estimators	MSE	Estimators	Var
t_1	213983.25	y_{New1}	191806.32
t_2	214105.63	y_{New2}	191936.66
t_3	213976.29	y_{New3}	191798.91
t_4	214023.46	y_{New4}	191849.14
t_5	208772.05	y_{New5}	184456.47
Population II			
Estimators	MSE	Estimators	Var
t_1	814512.10	y_{New1}	806132.37
t_2	852070.00	y_{New2}	844601.86
t_3	807570.20	y_{New3}	799000.85
t_4	806065.00	y_{New4}	797453.31
t_5	801131.20	y_{New5}	747600.45

5. Conclusion

In this article, we study on the estimators given by Kadilar and Cingi [5], [6] to obtain the unbiased estimation of the population mean in the stratified random sampling. Both the theoretical and empirical results show that the suggested unbiased estimators have smaller variance values than the compared estimators under the determined conditions. Moreover, the results in Table 3 clearly indicate that the suggested estimator of y_{New5} is the best estimator for the data sets used in Section 4.

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