The Eurasia Proceedings of
Educational & Social Sciences (EPESS)
ISSN: 2587-1730

The Eurasia Proceedings of Educational & Social Sciences (EPESS), 2018

Volume 10, Pages 163-167

ICRES 2018: International Conference on Research in Education and Science

Pre-service Teachers' Absolute Value Equations and Inequalities Solving Strategies and Errors

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Abstract: Many students in different grade levels face difficulties while solving absolute value equations and inequalities. These difficulties may be attributed to the strategies that the math teacher used in solving absolute value equations and inequalities. This study investigated the strategies that pre-service teachers used and the errors they made while solving absolute value equations and inequalities. Fifty-one pre-service teachers enrolled in an introductory mathematics course participated in this study. Data were collected from a test that consisted of four absolute value equations and inequalities. Participants were asked to solve these absolute value equations and inequalities and show their work. Participants' errors while solving each absolute value equations and inequalities were recorded. The results of the study indicated that many pre-service teachers made few errors when solving absolute value equations and inequalities.

Keywords: Absolute value equations and inequalities, Pre-service teachers, Solving strategies, Math education

Introduction

Solving absolute value equations and inequalities, understanding their solutions and being able to explain their meaning are all important not only in Calculus courses but also in many other courses. Students face major obstacles and struggle with solving absolute value equations and inequalities because they do not have a good understanding of the basic concepts. To develop the understanding and skills to solve these equations, students should have a good understanding of mathematical equations and inequalities. Equations and inequalities are fundamental in understanding many concepts in any mathematical course including Algebra, Geometry and Calculus. Research studies have shown that students have difficulties developing proper conceptual understanding of equations (Capraro & Joffrion, 2006; Knuth, Stephens, McNeil, & Alibali, 2006; Rittle-Johnson & Alibali, 1999; Serhan, 2015; Siegler, 2003; Star, 2005), while other research studies have shown that students have difficulties with the concept of inequality (Almog & Ilany, 2012; Dreyfus & Hoch, 2004; El-Shara & Al-Abed (2010); Kieran, 2004; Tsamir & Bazzini, 2004; Vaiyavutjamai & Clements, 2006). There are few studies that focused on the understanding and solving of the absolute value (Curtis, 2016; Gagatsis & Panaoura, 2014; Thomaidis & Tzanakis, 2007; Wilhelmi, Godino, & Lacasta, 2007).

In a study that aimed at investigating students' conceptions of the notion of absolute value and their performance in solving absolute value equations and inequalities, Gagatsis and Panaoura (2014) constructed and administered a questionnaire to 135 students enrolled in eleventh grade in five different high schools in Cyprus . The researchers found that the majority of the students had not acquired a sufficient conceptual understanding of the absolute value notion**.**

In a study that examined high school students' methods of approaching absolute value inequalities, their common mistakes, misconceptions, and the possible sources of these mistakes and misconceptions, Almong and Ilany (2012) used a questionnaire that consisted of eight absolute value inequality tasks and personal interviews to collect their data. Before completing these tasks students were taught how to solve absolute value equations and inequalities. The results indicated that students faced significant difficulties solving absolute

- Selection and peer-review under responsibility of the Organizing Committee of the Conference

value inequalities. The researchers recommended that teachers use their findings to understand students' thought processes to enhance mathematics instruction.

Curtis (2016) conducted a study to test if students would develop a conceptual understanding of the absolute value if they used the distance concept on a number line. She addressed multiple questions including the following question: "How and to what extent can using a number line develop a conceptual understanding of absolute value equations and inequalities?" The participants in her study completed both a pre- and post-assessment, and some of them were selected to participate in a 15-minute interview based on their responses. Following a qualitative method analysis, she found that students struggled with remembering the procedure for solving absolute value equations and inequalities. In addition, students were able to use the concept of distance on a number line to solve absolute value equations and inequalities.

There is limited research on the strategies that students' use in solving absolute value equations and inequalities and the errors they make while doing so. The purpose of the present study is to investigate the strategies that pre-service teachers use to solve absolute value equations and inequalities and to study the errors that they may make while solving such equations and inequalities.

Research Questions

The aim of this study was to answer the following two questions:

1. What strategies do pre-service teachers use when solving absolute value equations and inequalities?
2. What errors do pre-service teachers make while solving absolute value equations and inequalities?

Method

This research study explored pre-service teachers' strategies in solving absolute value equations and inequalities and the errors that they made while doing so. The participants in this study were fifty-one first year university students who were enrolled in an introductory mathematics course. To provide an answer to the research questions, data were collected using a test that the researchers developed based on review of the literature and the aim of this study. The test consisted of solving 4 absolute value problems: two equations and two inequalities: the first equation $|x| = 9$; the second equation $|3x - 9| = 12$; the third inequality $|x + 9| < 7$ and the fourth inequality $|3x - 7| \geq x$. These four problems were designed to check pre-service teachers' strategies and the errors that they may make while solving these questions. The questions were designed to measure students' abilities to accurately solve absolute value equations and inequalities; the errors that they make and the strategies they use to solve these problems. To gain insight into the participants' knowledge and the strategies that they used, they were asked to solve these equations and inequalities, show their work and give explanations.

Analysis

Participants' responses to each of the four questions were analyzed. Their reported errors were summarized and tabulated. In addition to that, descriptive analysis was used to answer the question investigating the errors that pre-service teachers made while solving these questions.

Results

In this section we discuss the findings to provide a response to the first and second research questions regarding the strategies used by pre-service teachers when solving absolute value equations and inequalities and the errors that they make while solving such equations and inequalities. While solving absolute value equations and inequalities, all the participants used the same strategy with different variations. Table (1) shows the strategy that was used by the participants.

Table 1. Strategies used by students to solve the test questions

Question	Strategy
$\lvert x\rvert = 9$	$x = 9$ or $x = -9$
$\lvert 3x-9\rvert = 12$	$3x - 9 = 12$ or $-(3x - 9) = 12$
$\lvert x+9\rvert < 7$	$x+9 < 7$ and $-(x+9) < 7$
$\lvert 3x-7\rvert \geq x$	$3x-7 \geq x$ or $-(3x-7) \geq x$

To analyze students' errors, the researchers looked at the solutions provided by each student for each question. For question 1, solving the equation $\lvert x\rvert = 9$, the majority of the students (46 of the 51 participants) were able to provide the solution set correctly. Only five students were not able to provide the complete solution set; they stated that the solution was 9 only by solving $x = 9$; they ignored the second option which is $x = -9$ in this case. Similarly, some students ignored the second possibility while solving the second question as well; they provided the solution for $3x-9=12$ and ignored the second possibility $-(3x-9)=12$. Similar errors were made while solving the inequality questions. To answer the second research question, the errors that the pre-service teachers made while solving the given questions were summarized as follows: a) writing the interval notation incorrectly by interchanging the endpoints of the interval, b) multiplying the inequality by a negative number, c) treating the inequalities as equations, see (Table 2).

Table 2. Error types and examples

Error Type	Error Example
Multiplying the inequality by a negative number	1. Simplifying $-(3x-7) \geq x$ to $-4x \geq -7$ and then $x \geq 7/4$ 2. $-(x+9) < 7$ $-x-9 < 7$ $-x < 16$ $x > 16$
Confusing the "or" with the "and"	For the third question, instead of using the "and" connector, the "or" connecter was used.
Writing the interval notation incorrectly by interchanging the endpoints of the interval.	Expressing the solution as (-2,-16) instead of (-16,-2).
Treating the inequalities as equations	Solving questions 3 and 4 as equations: $x+9=7$
Arithmetic calculation errors	Expressing $-(3x-7) \geq x$ as $-3x+7 \geq x$
Guessing the solutions & unclear statements.	10 < 7, 7 + 9 =16
Dealing with one option only	Solving $3x-9=12$ and ignoring $-(3x-9)=12$ Solving $x = 9$ and ignoring $x = -9$
Unable to express the solution set using interval notation or the number line.	

Table (3) shows the number of participants who solved the given questions correctly. It also includes the number of participants who solved the questions correctly but did not provide a solution set in any form including interval notation or a line representation of the solution per question. Table (3) shows that only 52.5% of the participants provided a detailed solution of the given questions.

Table 3. Detailed Solution

Question	Correct process with correct answer	Correct process but without representing the solution with a set notation, an interval notation or on the number line.
1	46	0
2	41	0
3	15	9
4	5	5

Table (4) shows the number of errors for each question. The errors occurred mostly when solving the fourth absolute value inequality, out of the total errors students made while solving the given questions, almost 50% of the total errors are attributed to the fourth question. In addition, most of the students were able to answer the absolute value equations correctly which leads to the conclusion that students had more difficulties dealing with and understanding the inequality concept.

Table 4. Errors per question

Question	# of Errors	Percent of Errors
1	5	06.0%
2	10	12.1%
3	27	32.5%
4	41	49.4%

In addition, 15.7% of the students did not make any errors solving the given absolute value equalities and inequalities and 7.8% of the students made four errors while solving the given four absolute value questions (Table 5).

Table 5. Number of errors per student

# Errors	# of students	Percent of students
0	8	15.7%
1	18	35.3%
2	14	27.5%
3	7	13.7%
4	4	07.8%

For the students who made one error only (18 students), these errors occurred in solving the inequalities, to be more specific; 16 of these errors were in solving the fourth inequality. Similarly, the students who made two errors only (14 students) all made an error while solving the fourth inequality. This means those students were able to solve the absolute value equalities but had difficulties dealing with the absolute value inequalities. Only 4 students made errors in solving all the questions.

Discussion and Conclusion

This study was designed to contribute to the advancement of the mathematics curriculum and in particular to gain insight into the errors that pre-service teachers made and the strategies they used while solving absolute value equations and inequalities. The participants in this study relied on one strategy for solving absolute value equations and inequalities and encountered different types of errors. The most dominant error was treating absolute value inequalities as one sided equations and also multiplying the inequality by a negative number. Participants used the solving strategy correctly, but when they multiplied by a negative number, they did not reverse the inequality sign. In addition to that, the participants used the distributive method incorrectly and they made other arithmetic errors. These results indicate that the participants may have a conceptual misunderstanding of absolute value inequalities. Based on the results of this study, further research is needed to explore ways that may help students overcome the difficulties they face when solving absolute value equations and inequalities.

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