The Eurasia Proceedings of Educational & Social Sciences (EPESS)
ISSN: 2587-1730

The Eurasia Proceedings of Educational & Social Sciences (EPESS), 2018

Volume 10, Pages 248-253

ICRES 2018: International Conference on Research in Education and Science

Investigating Economy as a Mean to Rejuvenate Fossilized Mathematical Knowledge

Pierre JOB
Ichec Brussels Management School

Jean Yves GANTOIS
Ichec Brussels Management School

Abstract: Teaching to first years at university level, we invariably had to deal with students having trouble with what we considered to be basic knowledge, like first degree equations. This issue was of paramount importance to us because most of the subsequent knowledge we had to teach relied, to some extent, on this basic knowledge. At some point we had to recognize the fact that recalling what a first degree equation was, was a completely inefficient strategy: students were bored to hear about the same concepts over and over again. Teaching in a business and management school led us to investigate the possibility of using economy as a mean to give these "old" mathematical concepts a second life. From a didactical point of view, our approach was to, sort of, reverse the connection between mathematics and economy. We went from "mathematics as a tool for economy" to "economy as a semiotic model of mathematics". Our investigation is still at a preliminary stage. However, what we have found so far hints at the possibility of using this approach to have students gain a new and fresh interest in what they believed to be well-known mathematical concepts and moreover have them create, manipulate and reflect upon mathematics through the lenses of economy, so reshaping the very meaning of some mathematical concepts and letting them have the opportunity to experience a dual relationship between mathematics and economy, each one being in turn modeled by the other one.

Keywords: Duality, Semiotic model, Fossilized knowledge, Relationship with knowledge, Transition between secondary school and university

Introduction

In Job & Gantois (2017) we addressed the issue of the high failure rates encountered in our institution, in a first-year mathematics course. We used an anthropological approach initiated by Chevallard (1992), namely the anthropological theory of the didactic (ATD). This theoretical framework allowed us to uncover "hidden" institutional constraints compelling the possible shapes a mathematical knowledge can take on in the mathematics course we are in charge of. We showed these possible shapes all bore within themselves some sort of intrinsic incoherence, impacting the logical structure of the course. This helped us understand why we failed improving the success rates despite our many efforts. This is not to say nothing can be changed for the better, as will be shown here, only that the range of motion one can have within an institution can sometimes be restricted at a scale that is not visible without being endowed with theoretical tools. Not seeing those limitations can sometimes lead us to invest time and energy in enhancing a course, in a direction that has few chances to succeed. This type of paper comes under what could be termed a "passive" high-level approach to didactic, because of the high-level constraints used that helped us see the big picture.

In this communication, we use a different approach, we focus on a more specific issue, using different tools. First our study is restricted to the study of didactic phenomena pertaining to linearity. We let aside problems related to analysis (derivatives, etc.). Secondly, this time, we are not so much interested in putting forward high level-constraints but in devising an engineering that could allow our students to get past their recurring problems with linearity, at least the way it is found in our course. More precisely, this paper's main intent is to explore the idea that the linear aspects found in our course can be addressed right from the start using an economic context. The interest of this approach is double.

Our institution is an economy school. So, a priori, it is an interesting task to reinforce the connections between mathematics and economy for, our goal as teachers in this institution, is to endow our students with the necessary mathematical knowledge that will allow them to dive into economic theories relying on mathematics.

Our approach also deals with a teaching strategy commonly found in Belgium. This strategy is that of "reminders". It amounts to simply repeat the mathematical theories students learned in previous years that are required in the course but for which they struggle with. The interesting point, from our didactic perspective, is that, reminders' effectiveness is rather dubious. Overall, students have as much problems with linearity before and after reminders. So, a natural question arose as to the possibility of bypassing reminders.

Combining these two aspects, economy and reminders, we asked ourselves about the feasibility of reinforcing the connection between our course and economy without relying on time consuming and ineffective reminders. Could the time spared from getting rid of reminders be used to create a more organic connection between economy and mathematics within a mathematics course?

Linearity and its Wealth of Difficulties

The linear aspects found in our course are rather straightforward. Roughly speaking they can be divided into two categories. Those related to the so-called "budget line theory" were a line like $p_1q_1 + p_2q_2 = B$ is used to represent all the expenditures exhausting a given budget. And those found in linear programming restricted to two variables and thus solvable using a geometric technique which amounts to study how a sheet of parallel lines representing a function of the form $f(x, y) = ax + by$ intersects a polygon representing a set of constraints of the form $cx + dy + f \leq 0$. The linearity found in those two categories are similar. It means students only need to have an understanding of the algebra behind half-planes and lines in the Euclidean plane to tackle them. These are precisely aspects of linearity they have studied multiple times in secondary school. For instance, lines in the plane are studied starting from the third year until the sixth year[1]. This is precisely one reason why we chose these two categories. They involve nothing new for students in terms of linearity. Our course does not involve higher concepts of linear algebra such as matrixes, linear forms, etc.

So, what is the big deal with linearity? Let us give a few examples that will illustrate the kind of problems students have with these linear aspects. For most of them it is a challenge to just calculate an equation of a line knowing the coordinates of two of its points. At best they are able to apply a formula by heart, not even knowing it "by heart". Calculating the intersection between two lines is also often a challenge. They don't really know how to proceed and when then do, they are not even able to verify their calculation. The slope of a line concept is understood in a rather loose way, as would anybody in the street. There is no real connection between the idea of a slope and the algebraic implementation. They don't understand the proportionality behind it. We could go on with many other surrealistic situations. But this is not our goal, unlike in the reminders strategy, to make a list of all the misunderstandings about linearity. Suffice it to say that most situations were linearity is involved rises problems among students. It depicts the skinny background our students have with respect to linearity and the seemingly unavoidable problems that arise as a consequence. Let us now turn to teachers' reactions to that issue.

Reminders as a Teaching Strategy

The dominant strategy used in our school, until this research was started at the beginning of this year, was to list all the prerequisites our students were supposed to be proficient with, including linearity, and offer them reminders on these topics (second degree, derivatives, etc.). This way of doing seemed unavoidable, because first-year students had huge problems with those prerequisites, on which the course relied.

This strategy is not specific to our institution. It is also used in other schools and universities in Belgium. One idea behind this way of doing is related to a certain epistemological view of knowledge. This view takes for granted that mathematical knowledge is structured in a deductive fashion, and so should the teaching of the discipline be structured. In the context of this article, it means that linearity being used in our mathematics

[1] In our educational system they are 6 year in secondary school labelled from 1 (12 years old) to 6 (18 years old).

course, students should first master linearity before being able to understand other part of the course relying on it.

This idea may at first seem rather sound and even natural. Nevertheless, we were forced to envision it in a different way based on the following facts.

Despite the many reminders punctuating the course, students were not much better at linearity before and after attending the reminders lessons. Not being proficient at linearity they were nonetheless able to pass the exam. Those facts contradict the dominant strategy's validity and the soundness of the underlying idea. The same facts happened at a different level in our school. Some students were able to pass a second-year mathematics exam and still have to pass our first-year exam, although the second-year course is supposed to rely, to a certain extent, on the first-year course.

It means that not mastering prerequisites is not mandatory to succeed at our exam, nor is it to succeed to more advanced courses, even if our course and others are at a certain level structured in a deductive fashion. It is not our direct purpose to explain at length the reasons behind this situation. Anyway, a few salient points might be worth emphasizing that will better delineate the spirit in which we dealt with the surrounding of reminders.

First, we may evoke institutional constraints, some of which were discussed in a previous article (Job & Gantois, 2017). To put it in a very straightforward manner, if teachers have all students not mastering linearity fail the first-year exam, almost no one would succeed. This would be unacceptable for our institution for several reasons. One of them is that our school gets money according to the number of students enrolled: the more students, the more money and the latitudes it allows. Another one is that as a mathematics teacher, in our course, our main point is not linearity considered from the secondary school point of view, but linearity as a tool to deal with, for instance, linear programming. It means linear programming is the main focus and not equations of lines, although these equations are used in solving linear programs. It is thus more important to have students be able to do something about linear programming than 'mastering' linearity. It wouldn't be acceptable to bluntly stop a student because he is not able to calculate lines when he is able to do something valuable with linear programs.

Secondly, we noticed that students tend to develop their own knowledge to solve, for instance, linear programs. But this happens at a very specific level. They do so at a cultural level and not a mathematical one. It means they devise tricks to get the correct answer without having to understand the mathematics behind. They become masters at pretending to understand, at doing the technical calculations they need to do to get the "correct" answer. Their cultural strategy being, to some extent, successful, the view they have about mathematical knowledge becomes fossilized. They do not reason so much at a mathematical level, at least not as much as they do at a more cultural one. This prevents them from having their semiotic relationship to mathematics evolve.

So reminders as a teaching strategy is a failure. We thus tried to tackle linearity in a different way, starting from economy.

The Problematic of Getting away from Reminders and Diving into Economy

One of our main goals as teachers in a school of economy is to teach something to our students they can use in their economy courses: mathematics and economy are, at least in principle, closely intertwined. Thus, if reminders about linearity remains largely ineffective, why not dive right away into problems taking place in an economical background? That is what we did and tried. As insignificant as it may look from an outside perspective, it was no small decision to make, because of the following reasons.

The first reason was that, our urge to try something different from reminders, went against the reminders trend that had been in favor for years in our school. We were issued a strong warning by some of our colleagues that can be summed up as: "If you completely remove reminders about linearity from students already struggling with solving linear equations with one unknown, they stand even less chances to succeed. It is not a very responsible decision to make to get rid of reminders". In other words, our approach was considered by some colleagues as sort of playing with fire. Their warning bears some wisdom. As noted in Job & Gantois (2017), modifications in an institution can have disastrous consequences if they outgrow the institution's capabilities to change.

The second reason completes the previous one. The design of an engineering not relying on reminders, had not been possible so far because this is the first year that the two of us are the only teachers in charge of at least a significant part of the course, including the "linear part" of the course. Previously we were up to 5 teachers in charge of this course, teachers having different inclination towards mathematics. It was not easy to come to an agreement as to what should or should not be changed in the structure of the course, in particular in relationship with reminders. This partly explains the "passive" high-level approach used in Job & Gantois (2017) were we tried to understand our course from an "outside" perspective until we were able to change something in the deeper structure of the course.

The third reason is related to economy. The connection between mathematics and economy already existed in our course but in a specific form. Economic appeared mostly as an application of mathematics instead of a more organic interaction were at the onset an economic problem is used on which some mathematics are developed to address it. In other words, the meaning of mathematical concepts involved in the course were mainly derived from mathematics themselves and then applied as an illustration of those concepts that could very well have been taught without any reference to economy. They are deep reasons behind this specific way of articulating mathematics with economy. We will not address all of them here, trying to keep in line with the main objective of this paper. Instead we will evoke the following one. Creating a more organic interaction between mathematics and economy was a risky business because we couldn't cross the borders of economy courses. This might have been considered as getting out of what is allowed for a mathematics teacher, sort of stepping into the lands of economy teachers[2]. We had to choose our problems wisely, close enough to economy to be economically meaningful (and not just flavored with economy) and still far enough not to be considered an act of piracy. These three reasons delineate the rather fragile framework into which we had to fit an engineering.

Experimental Results about an Embryonic Engineering

Let us now turn to experimental results. The engineering we designed is still at an early stage of development. Given the aforementioned framework we chose to modify the course in a step by step fashion and make them not appear as dramatic modifications. The basis of the engineering is very simple in principle but nevertheless attractive from a didactic perspective. Let us have a look at aspects of one context studied with our students. The context is to deal with a budget that allows to buy two goods. For instance, 400 € are given to buy two types of tea. The first type T_1 costs 5€/100g and the other one T_2 costs 4€/100g. Through a set of questions, students are led to elaborate an algebraic representation of the possible expenditures. We shall not detail all these questions. The interesting point for our purpose is to discuss the consequences of the fact that the representation depends on the budget's status: does it have to be spent entirely or not?

Indeed, if it must be spent in its entirety, then an algebraic representation will be given by

$$5q_1 + 4q_2 = 400$$

Where q_i denotes the quantity of T_i that is bought.
If the budget does not need to be spent entirely then it gives rise to the following representation

$$5q_1 + 4q_2 \leq 400$$

This is where the interesting didactic phenomena takes place. When asked to represent $5q_1 + 4q_2 = 400$ geometrically, students are not surprised that it gives rise to a line, even if they may have trouble drawing that line, simply because they are culturally accustomed to manipulate those notations.

On the contrary, students are not well accustomed with the variation $5q_1 + 4q_2 \leq 400$. In previous versions of the course, the reminders one, we had a section on the geometric representation of such notations ($ax + by \leq c$). Students had a really hard time understanding why it gave rise to a half-plane, not to say that most of them never understood it: one argument given to students was, for instance, to decompose $ax + by \leq c$ into an infinite set of lines $ax + by = k$ ($k \in] \leftarrow, c]$).

What is new and interesting is that with this new version of the course, some students were able to give a geometric meaning to $5q_1 + 4q_2 \leq 400$ based on an economic reasoning. The trail of such reasoning is the following.

- Some students note, for instance, that $q_1 = 40$ and $q_2 = 50$ exhausts the 400 € budget.
- It thus means that any increase of q_1 or q_2 will exceed the budget. And any decrease will no exhaust the budget.

[2] The situation can be quite different when someone is teaching both economy and mathematics.

- The geometric consequence is that starting from a point on the line representing $5q_1 + 4q_2 = 400$ like $q_1 = 40$ and $q_2 = 50$, increasing wether q_1 or q_2 or both at the same time, will give birth to a point in the plane (q_1, q_2) that will be located "above" the line. A similar conclusion can be drowned while decreasing those quantities. Such points will all be located "below" the line.
- From these considerations, students are able to give an economic meaning to the interplay between algebra and economy. Points not exhausting the budget verify $5q_1 + 4q_2 < 400$ and are geometrically located "below" the line represented by $5q_1 + 4q_2 = 400$. Points exceeding the budget verify $5q_1 + 4q_2 > 400$ and are geometrically located "above" that same line.
- Thus geometrically, $5q_1 + 4q_2 \leq 400$ can be divided into points on the line $5q_1 + 4q_2 = 400$ and points "below" it $5q_1 + 4q_2 < 400$.

This is remarkable in our institutional context. Even tough embryonic, the way these students reasoned about the geometric meaning of $5q_1 + 4q_2 \leq 400$ was to use economy as a semiotic model of algebra. This semiotic tool allowed the mapping of algebra onto geometry. This result needs to be emphasized because, to the best of our knowledge, it is the first time that students could handle a piece of notation of the form $ax + by \leq c$ by actually reasoning about it and not just learning something by heart, because it can definitely not be understood.

Conclusion

We have given strong hints as to the possibility of rejuvenating fossilized mathematical knowledge revolving around linearity by using economy as a semiotic model acting as a glue between algebra and geometry. In a way it reminds us about the way physics has been interacting with mathematics in the genesis of analysis. Physics, algebra and geometry were in turn all used as models of one another leading to the present situation where the epistemological thickness of these has been dramatically reduced.

Our work can be envisioned in that epistemological vein, of exploring the richness and density of interactions between different fields where one can be logically subdued to another and yet, semiotically, none completely prevails on another.

The peculiar use of economy, within a mathematics course, discussed in this paper also allows us to shed a new light on the reminders teaching strategy. It shows that its ubiquity is not as evident as one might think. It seems an interesting track to explore to spend less time on reminders in favor of activities that would give new semiotic content to older knowledge, using fields in closer proximity to the orientation of our students, namely economy in our case. It is an interesting issue to investigate similar lines in other fields like for instance biology or chemistry.

To conclude, we should emphasize again the fact that our engineering is still at an early stage of development. We have given strong hints as to its didactic potential but it still needs to be further developed and tested with a stronger methodology. This will be addressed in yet another paper at the time we will be have further experimental material at hand.

Recommendations

If we had any recommendation to issue, it might be for teachers to pay close attention to the dynamic of teaching a subject already known to students for which they had and still have troubles with. Paying close attention means, broadly speaking, studying the relationship of students *and teachers alike* with respect to reminders about the target knowledge.

Being more specific, teachers should be able to detect at what point precisely in a lesson, students do give up or exhibit an ineffective thought patterns whether they already displayed it or not in the past. And more over and more importantly, what makes them behave this way? Specifically, those "unwanted"[3] patterns, are they unavoidable or do they arise *because of the way students are taught, because of the reminders setting*?

This last question appears to us of paramount importance. If we recall[4] the warning issued by colleagues when trying to step away from reminders we were told it was a potential threat to students' understanding of the target

[3] "Unwanted" is quoted because some behaviours might be unwanted but at the same time be somewhat unavoidable as a stepping stone towards a deeper understanding.
[4] See above.

knowledge. Despite this warning, it appears, although it needs to be further investigated, that the reminders themselves were preventing students from creating a new relationship with the target knowledge.

We thus might advise teachers to follow lines similar to those we followed here, not for the sake of repeating what has already been done, but as a tool to interrogate the hidden parts of a teaching. To what extent do obstacles faced by students lie in the shadow of the "hidden" features of a teaching. Using Brousseau's terminology it sums up to questioning the didactic nature of obstacles (Brousseau, XXX). Such a study is definitely a requirement when studying the deeper nature of a knowledge to be taught and may help qualify a related obstacle as an epistemological obstacle (Schneider, 2010).

References

Brousseau, G. (2002), *Theory of Didactical Situations in Mathematics*, Netherlands: Springer.

Chevallard, Y. (1992). Concepts fondamentaux de la didactique : perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques*, 12(1), 73-112.

Job, P., & Gantois, J.-Y. (2017). High level constraints weighting on the possible shapes knowledge can take on. In M. Shelley M., M. Pehlivan (Ed.), *Education Research Highlights in Mathematics in Science and Technology 2017* (pp. 14-20). Ames, IA: ISRES Publishing.

Schneider, M. (2010), *Traité de didactique des mathématiques : la didactique par des exemples et contre-exemples*, Liège : Presses Universitaires de Liège.

Author Information

Pierre Job	**Jean-Yves Gantois**
Ichec Brussels Management School	Ichec Brussels Management School
Boulevard Brand Whitlock 4,	Boulevard Brand Whitlock 4,
1150 Woluwe-Saint-Pierre	1150 Woluwe-Saint-Pierre
Belgium	Belgium
Contact e-mail: *pierre.job@ichec.be*	