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Soft Topology Generated by L-Soft Sets

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Abstaract − In this paper, the theory of soft topological space generated by L-soft sets is introduced. As a continuation of the study of operations on L-soft sets, the aim of this paper is to introduce new soft topologies using restricted and extended intersections on L-soft sets and to study the differences of these soft topologies.

Keywords − *Soft sets, L-soft sets, soft topology, L-soft topology, $\tilde{L}$-soft topology.*

1 Introduction

A common extension of rough sets and fuzzy sets is named as L-sets in [17] as a generalization of [37]. The basic ideas of L-set theory and its extensions were worked in [7].

The first interference to soft set theory was done by Molodtsov [26] in 1999 as a general mathematical tool for dealing with uncertain objects. Hereupon, the properties and applications of soft sets have been studied increasingly ([3, 9, 19, 23, 35, 36, 38, 40]). Soft set theory has also been studied in terms of the algebraic structure ([1, 6, 16, 20, 27]). To obtain a new view and applications, the fuzzy set theory has been embedding in the soft set theory ([2, 4, 15, 21]).

The operations of soft sets are firstly defined by Maji *et al.* in [22] to work on detailed theoretical study of soft sets. These soft operations have opened a new domain. In this regard, Shabir and Naz [30] defined soft topological space over an initial universe with a fixed set of parameters. Also, they presented the theory of soft topological spaces and soft separation axioms. Çağman et al. [8] redefined the operations of soft sets to bring up new results on soft set theory.

The notion of soft set in L-set theory is studied and introduced several operators of L-soft set theory in [12]. In addition to this, the rough operators on the set of all L-soft sets induced by the rough operators on L^x is investigated in [13]. The main work is [14] that researches on generalizations of soft sets in fuzzy settings and several operators for L-soft set theory.

By using soft set operations, Çağman et al.[10], introduced a topology on a soft set called "soft topology". In this work foundations of the theory of soft topological spaces are studied. The theory of soft topology has become an important area of research in recent years ([5, 18, 25, 30, 33, 34, 39]).

In the study of operations on L-soft sets it is proved that the union of L-soft sets is an L-soft sets and the intersection of L-soft sets is also an L-soft set. So, it is natural to ask how a soft topology can be defined on a L-soft set? This study presents a complete answer to this question. This new study area enables us to see clearly the motivations for developing L-soft set theory and appropriate applications, leading to an appreciation for the theory of L-soft topology.

The aim of this paper is to derive a relationship between L-soft sets and soft topology using the operations defined for L-soft set theory. For this purpose, the foundations of the theory of L-soft topology is presented with defining L-soft open set, L-soft closed set, L-soft closure, L-soft interior and more than L-soft properties in soft topological space that are not studied before. There is a difference on the definitions of restricted soft intersection and extended soft intersection of L-soft sets that will be proved extremely useful in the section 4.

In the view of extended intersection of L-soft sets, a new soft topology named as $\tilde{L}$-soft topology ($\tilde{L}$-ST) is introduced and the differences of L-soft topology and $\tilde{L}$-soft topology are studied. In most of the occasions, the assumptions are illustrated and substantiated by suitable examples.

2 Background in Works

In this section, an overview of soft sets, L-sets, fuzzy soft sets, rough sets, L-soft sets and the properties of L-soft sets are studied which can be named as preliminaries devoted to some main notions for each area, i.e., soft sets [26], L-sets [14], fuzzy soft sets [24], rough sets [28] and L-soft sets [14]. Apart form definitions and theorems are numbered, known concepts are mentioned in the text along with the reference [14].

Firstly, let us introduce definition of soft set that is given by Molodtsov in [26] as follows;

Let U be an initial universe set and let E be a set of parameters.

Definition 2.1. [10, 26] A pair (f, E) is called a soft set (over U) if and only if f is a mapping or E into the set of all subsets of the set U.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $f(e)$, $e \in E$, from this family may be considered as the set of ε-elements of the soft set (f, E), or as the set of e-approximate elements of the soft set.

From now on, we will use definitions and operations of soft sets which are more suitable for pure mathematics based on study of ([10]).

Definition 2.2. [10] A soft set f on the universe U is defined by the set of ordered pairs

$$f = \Big\{ \big(e, f(e)\big) : e \in E \Big\}$$

where $f : E \to \mathcal{P}(U)$.

We will identify any soft set f with the function $f(e)$ and and we shall use that concept as interchangeable. Soft sets are denoted by the letters f, g, h, ... and the corresponding functions by $f(e)$, $g(e)$, $h(e)$, ...

From now on, the set of all soft sets over U will be denoted by $\mathbb{S}$.

Definition 2.3. [11] Let $f \in \mathbb{S}$. Then,
If $f(e) = \emptyset$ for all $e \in E$, then f is called an empty set, denoted by Φ.
If $f(e) = U$ for all $e \in E$, then f is called universal soft set, denoted by $\tilde{E}$.

Definition 2.4. [11] Let $f, g \in \mathbb{S}$. Then,
f is a soft subset of g, denoted by $f \tilde{\subseteq} g$, if $f \subseteq g$ for all $e \in E$.
f and g are soft equal, denoted by $f = g$, if and only if $f(e) = g(e)$ for all $e \in E$.

Definition 2.5. [11] $f, g \in \mathbb{S}$. Then, the intersection of f and g, denoted $f \tilde{\cap} g$, is defined by

$$(f \tilde{\cap} g)(e) = f(e) \cap g(e) \text{ for all } e \in E$$

and the union of f and g, denoted $f \tilde{\cup} g$, is defined by

$$(f \tilde{\cup} g)(e) = f(e) \cup g(e) \text{ for all } e \in E$$

Definition 2.6. [11] $f \in \mathbb{S}$. Then, the soft complement of f, denoted $f^{\tilde{c}}$, is defined by

$$f^{\tilde{c}}(e) = U \setminus f(e), \text{ for all } e \in E$$

Definition 2.7. [10] Let $f \in \mathbb{S}$. The power soft set of f is defined by

$$\mathcal{P}(f) = \{ f_i \tilde{\subseteq} f : i \in I \}$$

and its cardinality is defined by

$$|\mathcal{P}(f)| = 2^{\sum_{e \in E} |f(e)|}$$

where $|f(e)|$ is the cardinality of $f(e)$.

Example 2.8. Let $U = \{u_1, u_2, u_3\}$ and $E = \{e_1, e_2\}$. $f \in \mathbb{S}$ and

$$f = \big\{ (e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\}) \big\}$$

Then,

$$\begin{array}{rcl}
f_1 &=& \{(e_1, \{u_1\})\}, \\
f_2 &=& \{(e_1, \{u_2\})\}, \\
f_3 &=& \{(e_1, \{u_1, u_2\})\}, \\
f_4 &=& \{(e_2, \{u_2\})\}, \\
f_5 &=& \{(e_2, \{u_3\})\}, \\
f_6 &=& \{(e_2, \{u_2, u_3\})\}, \\
f_7 &=& \{(e_1, \{u_1\}), (e_2, \{u_2\})\}, \\
f_8 &=& \{(e_1, \{u_1\}), (e_2, \{u_3\})\}, \\
f_9 &=& \{(e_1, \{u_1\}), (e_2, \{u_2, u_3\})\}, \\
f_{10} &=& \{(e_1, \{u_2\}), (e_2, \{u_2\})\},
\end{array}$$

$$
\begin{aligned}
f_{11} &= \{(e_1, \{u_2\}), (e_2, \{u_3\})\},\\
f_{12} &= \{(e_1, \{u_2\}), (e_2, \{u_2, u_3\})\},\\
f_{13} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_2\})\},\\
f_{14} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_3\})\},\\
f_{15} &= f,\\
f_{16} &= \Phi
\end{aligned}
$$

are all soft subsets of f. So $|\tilde{P}(f)| = 2^4 = 16$.

Definition 2.9. [14] (L-sets)
For a universe set X, an L-set in X is a mapping $\widetilde{A} : X \to L$. $\widetilde{A}(x)$ indicates the truth degree of "x belongs to $\widetilde{A}$". We use the symbol L^x to denote the set of all L-sets in X.

Definition 2.10. [24] (Fuzzy Soft Sets)
Let X be a universe and E a set of attributes. Then the pair $\widetilde{(X, E)}$ denotes the collection of all fuzzy soft sets on X with attributes from E and is called a fuzzy soft class.

Definition 2.11. [28] (Rough Sets)
Let X be a universe and E a set of attributes. Then the pair $\widetilde{(X, E)}$ denotes the collection of all fuzzy soft sets on X with attributes from E and is called a fuzzy soft class.

Definition 2.12. [12] (L-soft set)
A pair (F, A) is called an L-soft set over X, if $A \subseteq E$ and $F \to L^X$, denoted by $\theta = (F, A)$.

Example 2.13. Suppose $X = \{x_1, x_2, x_3\}$, and $L = [0, 1]$ equipped with Gödel Structure. Let $E = \{t_1, t_2, t_3, t_4\}$, $A_1 = \{t_1, t_2, t_3\}$ and let $F_1 : A_1 \to L^X$, where $F_1(t_1) = \{0.7/x_1\}$, $F_1(t_2) = \{1/x_1, 0.5/x_2\}$, $F_1(t_3) = \{0.6/x_1, 0.2/x_2, 0.7/x_3\}$. Then clearly, (F_1, A_1) is an L-soft set.

Definition 2.14. [14] Let $LS(X)$ be the set of all L-soft sets over X. On which, there exist two kinds of special elements: one is called a absolute soft set $(1_A, A)$, $\forall t \in A, 1_A(t) = \tilde{1}_X$, denoted by $\Gamma_A = (1_A, A)$; the other is called a null soft set $(0_A; A)$, $\forall t \in A, 0_A(t) = \tilde{0}_X$, denoted by $\Phi_A = (0_A, A)$.

Second, I give the relation L-order $\preceq$, and L-equivalence relation $\approx$ are defined in [14] which correspond the relations $\tilde{\subseteq}; =$ in classical case. For two L-soft sets $\theta_1 = (F, A)$, $\theta_2 = (G, B) \in LS(X)$,

$$(\theta_1 \preceq \theta_2) = S(\theta_1, \theta_2) = \bigwedge_{t \in A} S(F(t), G(t)),$$

$$(\theta_1 \approx \theta_2) = S(\theta_1, \theta_2) \wedge S(\theta_2, \theta_1).$$

Example 2.15. [14] Refer Example 2.13, (F_1, A_1) is an L-soft set. Let $A_2 = \{t_1, t_2, t_3, t_4\}$ and let $F_2 : A_2 \to L^X$, where $F_2(t_1) = \{0.4/x_1\}$, $F_2(t_2) = \{0.9/x_1, 0.5/x_2, 0.3/x_3\}$, $F_2(t_3) = \{0.4/x_1, 0.2/x_2, 0.5/x_3\}$, $F_2(t_4) = \{1/x_1, 0.7/x_2, 0.6/x_3\}$. Then (F_2, A_2) is also an L-soft set. Thus we obtain

$$\begin{aligned}
S((F_1, A_1), (F_2, A_2)) &= \bigwedge_{t\in A_1} S(F_1(t), F_2(t)) \\
&= S(F_1(t_1), F_2(t_1)) \wedge S(F_1(t_2), F_2(t_2)) \wedge S(F_1(t_3), F_2(t_3)) = 0.4, \\
S((F_2, A_2), (F_1, A_1)) &= \bigwedge_{t\in A_2} S(F_2(t), F_2(t)) = 0.
\end{aligned}$$

Clearly, we have

$$\begin{aligned}
&\theta_1 \tilde{\subseteq} \theta_2 \Leftrightarrow S(\theta_1, \theta_2) = 1 \Leftrightarrow A \subseteq B, \text{ and } \forall t \in A, F(t) \subseteq G(t), \\
&\theta_1 = \theta_2 \Leftrightarrow S(\theta_1, \theta_2) = 1, S(\theta_2, \theta_1) = 1 \Leftrightarrow A = B, \text{ and } \forall t \in A, F(t) = G(t),
\end{aligned}$$

So $\langle\langle LS(X), \approx\rangle, \preceq\rangle$ is an L-order set (See[14]). When $L = 2$, the above definitions coincide with [26], when $L = [0, 1]$, the above definition coincides with [24].

Example 2.16. [14] Follows Example 2.13, (F_1, A_1) is a L-soft set. Let $A_3 = A_1$ and let $F_3 : A_3 \to L^X$, where $F_3(t_1) = \{0.4/x_1\}$, $F_3(t_2) = \{0.9/x_1, 0.5/x_2\}$, $F_3(t_3) = \{0.4/x_1, 0.2/x_2, 0.5/x_3\}$. Then (F_3, A_3) is also an L-soft set and $(F_3, A_3)\tilde{\subseteq}(F_1, A_1)$.

The most important properties of L-soft sets are the soft union and the soft intersection properties are defined in [14]. There is a difference on the definitions of restricted soft intersection and extended soft intersection of L-soft sets that will be proved extremely useful in the next section as follows:

Definition 2.17. [14] Suppose $(F, A), (G, B) \in LS(X)$ are two L-soft sets. Then the soft union of (F, A) and (G, B) is an L-soft set (H, C), where $C = A \cup B$, and for $t \in C$,

$$H(t) = \begin{cases} F(t) & \text{if } t \in A - B \\ G(t) & \text{if } t \in B - A \\ F(t) \vee G(t) & \text{if } t \in A \cap B \end{cases}$$

and written as $(F, A) \tilde{\bigcup} (G, B) = (H, C)$.

Definition 2.18. [14] Suppose $(F, A), (G, B) \in LS(X)$ are two L-soft sets such that $A \cap B \neq \emptyset$. Then the restricted soft intersection of (F, A) and (G, B) is also an L-soft set (K, D), where $D = A \cap B$, and for $t \in D, K(t) = F(t) \wedge G(t)$. It is denoted by $(F, A)\tilde{\bigcap}(G, B) = (K, D)$.

Definition 2.19. [14] Suppose $(F, A), (G, B) \in LS(X)$ are two L-soft sets. Then the extended soft intersection of (F, A) and (G, B) is also an L-soft set (J, C), where $C = A \cup B$, and for $t \in C$,

$$J(t) = \begin{cases} F(t) & \text{if } t \in A - B \\ G(t) & \text{if } t \in B - A \\ F(t) \wedge G(t) & \text{if } t \in A \cap B \end{cases}$$

and written as $(F, A) \tilde{\sqcap} (G, B) = (J, C)$.

Definition 2.20. [14] Let $E = \{t_1, t_2, \ldots, t_n\}$ be the set of parameters. The NOT set of E, denoted by $\rceil E$, is defined by $\rceil E = \{\neg t_1, \neg t_2, \ldots, \neg t_n\}$, where $\rceil t_i =$ not $t_i, \forall i$. (It may be noted that $\rceil$ and $\neg$ are different operators).

About the NOT SET OF E, the following proposition holds:

Proposition 2.21. [14]

1. $\rceil(\rceil A) = A$,
2. $\rceil(A \cup B) = \rceil A \cup \rceil B$,
3. $\rceil(A \cap B) = \rceil A \cap \rceil B$.

Definition 2.22. [14] The complement of an L-soft set (F, A) is denoted by $(F, A)^c$, and is defined by $(F, A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to L^X$, for every $-t \in \rceil A$,

$$F^c(-t) = F^*(t) = F(t) \to 0.$$

3 L-Soft Topological Spaces

In this section, the notion of L-Soft topological (L-ST) space on a L-soft set is introduced and its related properties are studied. A relationship between L-soft sets and soft topology using the operations defined for L-soft set theory is derived. For this purpose, the foundations of the theory of L-soft topology is presented with defining L-soft open set, L-soft closed set, L-soft closure, L-soft interior and more than L-soft properties in L-soft topological space that are not studied before. In the view of extended intersection of L-soft sets, a new soft topology named as $\tilde{L}$-soft topology ($\tilde{L}$-ST) is introduced and the differences of L-soft topology and $\tilde{L}$-soft topology are studied.

Definition 3.1. Let $\theta \in LS(X)$ and E be a collection of all possible parameters with respect to X. A L-soft topology on θ, denoted by τ_{LS}, is a collection of L-soft subsets of θ having following properties:

i $\theta, \Phi \in \tau_{LS}$,

ii $\{\theta_i\}_{i \in I} \subseteq \tau_{LS} \Rightarrow \tilde{\bigcup}_{i \in I} \theta_i \in \tau_{LS}$,

iii $\{\theta_i\}_{i=1}^n \subseteq \tau_{LS} \Rightarrow \tilde{\bigcap}_{i=1}^n \theta_i \in \tau_{LS}$.

The triple (θ, τ_{LS}, E) is called a L-soft topological space.

Definition 3.2. Let (θ, τ_{LS}, E) be a L-soft topological space. Then, every element of τ_{LS} is called L-soft open set. Clearly, Φ and θ are L-soft open sets.

Theorem 3.3. Every L-soft power set of a L-soft set is a L-soft topological space over the L-soft set.

Proof: Let $\mathcal{P}(\theta)$ be the soft power set of θ. Then,

i $\Phi, \theta \in \mathcal{P}(\theta)$

ii Since $\theta_i \subseteq \theta$ then, $\theta_i \in \mathcal{P}(\theta)$, for all $i \in I$. Moreover, $\tilde{\bigcup}_{i\in I}\theta_i \subseteq \theta$ it follows that, $\tilde{\bigcup}_{i\in I}\theta_i \in \mathcal{P}(\theta)$.

iii Since $\theta_i \subseteq \theta$, for all $i \in I'$, I' finite soft set, so that $\tilde{\bigcap}_{i\in I'}\theta_i \subseteq \theta$. Thus $\tilde{\bigcap}_{i\in I'}\theta_i \in \mathcal{P}(\theta)$.

Hence $(\theta, \mathcal{P}(\theta), E)$ is a L-soft topological space which is called the L-soft discrete topology on θ and is denoted by τ^1_{LS} contains all soft sets over θ.

Theorem 3.4. $(\theta, \{\Phi, \theta\}, E)$ is a L-soft topological space over θ.

Proof: For satisfing the following axioms:

i $\Phi, \theta \in \{\Phi, \theta\}$

ii $\Phi \cup \theta = \theta \in \{\Phi, \theta\}$

iii $\Phi \cap \theta = \Phi \in \{\Phi, \theta\}$

$(\theta, \{\Phi, \theta\}, E)$ is a L-soft topological space. Then, it is called the indiscrete L-soft topology denoted by τ^0_{LS} contains only Φ and θ.

Definition 3.5. Let (θ, τ_{LS}, E) and $\theta_1 = (F, A) \in LS(X)$. Then, (F, A) is L-soft closed in τ_{LS} if $(F, A)^c \in \tau_{LS}$.

Throughout this work, the collection of L-soft closed sets in (θ, τ_{LS}, E) is denoted by $\digamma_{LS}$

Theorem 3.6. If $\digamma_{LS}$ is a collection of L-soft closed sets in a L-soft topological space (θ, τ_{LS}, E), then

i E and θ^c are L-soft closed.

ii Arbitrary soft intersections of members of $\digamma_{LS}$ belongs to $\digamma_{LS}$.

iii Finitely many unions of members of $\digamma_{LS}$ belongs to $\digamma_{LS}$.

Proof:

i. $E^c = \Phi$ and $(\theta^c)^c = \theta$ are L-soft open sets.

ii. If $\{\theta_i : \theta_i^{\tilde{c}} \in \tau_{LS}, i \in I \subseteq \mathbb{N}\}$ is a given collection of L-soft closed sets, then

$$\left(\tilde{\bigcap}_{i\in I}\theta_i\right)^c = \tilde{\bigcup}_{i\in I}\theta_i^c$$

is L-soft open. Therefore $\tilde{\bigcap}_{i\in I}\theta_i$ is a L-soft closed set.

iii. Similarly, if θ_i is L-soft closed for $i = 1, 2, \ldots, n$, then

$$\left(\tilde{\bigcup}_{i=1}^{n} \theta_i\right)^c = \tilde{\bigcap}_{i=1}^{n} \theta_i^c$$

is L-soft open. Hence $\tilde{\bigcup}_{i=1}^{n} \theta_i$ is a L-soft closed set. □

Definition 3.7. Let $\theta_1 = (F, A) \in LS(X)$ be a L-soft subset θ. Then, τ_{LS} interior of (F, A), denoted by $((F, A))^{\circ}_{\tau_{LS}}$, is defined by

$$((F, A))^{\circ}_{\tau_{LS}} = \tilde{\bigcup}\{(F, A) : (G, B)\tilde{\subset}(F, A), (G, B) \text{ is } L\text{-soft open}\}$$

The τ_{LS} closure of (F, A), denoted by $(\overline{(F, A)})_{\tau_{LS}}$, is defined by

$$(\overline{(F, A)})_{\tau_{LS}} = \tilde{\bigcap}\{(G, B) : (F, A)\tilde{\subset}(G, B), (G, B) \text{ is } L\text{-soft closed}\}$$

Theorem 3.8. Let (θ, τ_{LS}, E) be a L-ST space and $\theta_1 = (F, A) \subseteq \theta$. Then, (F, A) is a L-soft open set if and only if $(F, A) = ((F, A))^{\circ}_{\tau_{LS}}$.

Proof: If (F, A) is L-soft open set, then the biggest L-soft open set that contained by (F, A) is equal to (F, A). Therefore, $(F, A) = ((F, A))^{\circ}_{\tau_{LS}}$.

Conversely, it is known that $((F, A))^{\circ}_{\tau_{LS}}$ is L-soft open set, and if $(F, A) = ((F, A))^{\circ}_{\tau_{LS}}$, then (F, A) is L-soft open set. □

Theorem 3.9. Let (θ, τ_{LS}, E) be a L-ST space and $\theta_1, \theta_2, \theta_3 \subseteq \theta$. Then,

i. $((\theta_1)^{\circ}_{\tau_{LS}})^{\circ}_{\tau_{LS}} = (\theta_1)^{\circ}_{\tau_{LS}}$

ii. $\theta_1 \tilde{\subseteq} \theta_2 \Rightarrow (\theta_1)^{\circ}_{\tau_{LS}} \tilde{\subseteq} (\theta_2)^{\circ}_{\tau_{LS}}$

Proof:

i. Let $(\theta_1)^{\circ}_{\tau_{LS}} = \theta_3$. Then $\theta_3 \in \tau_{LS}$ if and only if $\theta_3 = (\theta_3)^{\circ}_{\tau_{LS}}$. Therefore $((\theta_1)^{\circ}_{\tau_{LS}})^{\circ}_{\tau_{LS}} = (\theta_1)^{\circ}_{\tau_{LS}}$.

ii. Let $\theta_1 \tilde{\subseteq} \theta_2$. From the definition of L-soft interior, $(\theta_1)^{\circ}_{\tau_{LS}} \tilde{\subset} \theta_1$, $(\theta_2)^{\circ}_{\tau_{LS}} \tilde{\subset} \theta_2$. $(\theta_2)^{\circ}_{\tau_{LS}} \tilde{\subset} \theta_2$ is the biggest L-soft open set that contained by θ_2. Hence $\theta_1 \tilde{\subseteq} \theta_2 \Rightarrow (\theta_1)^{\circ}_{\tau_{LS}} \tilde{\subseteq} (\theta_2)^{\circ}_{\tau_{LS}}$.

□

Theorem 3.10. Let (θ, τ_{LS}, E) be a L-ST space and $\theta_1 \subseteq \theta$. Then, θ_1 is a τ_{LS}-closed soft set if and only if $\theta_1 = (\overline{\theta_1})_{\tau_{LS}}$.

Proof: The proof is trivial. □

It is time to introduce a new soft topology by L-soft sets, with restricted soft intersection replaced by extended soft intersection, to obtain $\tilde{L}$-soft topology ($\tilde{L}$-ST).

The main difference from the case of L-ST is the presence of extended soft intersection. For cases of L-soft topology, case 1 and 2 this makes no difference, but for case 3 it considerably generates $\tilde{L}$-ST is considered below:

Definition 3.11. Let $\theta \in LS(X)$ and E be a collection of all possible parameters with respect to X. A $\tilde{L}$-soft topology on θ, denoted by $\tilde{\tau}_{LS}$, is a collection of L-soft subsets of θ having following properties:

i $\theta, \Phi \in \tilde{\tau}_{LS}$,

ii $\{\theta_i\}_{i\in I} \subseteq \tilde{\tau}_{LS} \Rightarrow \tilde{\bigcup}_{i\in I}\theta_i \in \tilde{\tau}_{LS}$,

iii $\{\theta_i\}_{i=1}^{n} \subseteq \tilde{\tau}_{LS} \Rightarrow \tilde{\sqcap}_{i=1}^{n}\theta_i \in \tilde{\tau}_{LS}$.

The triple $(\theta, \tilde{\tau}_{LS}, E)$ is called a $\tilde{L}$-soft topological space.

4 The Differences Between L-ST and $\tilde{L}$-ST Spaces with Examples

In this section, in the view of extended intersection of L-soft sets, the differences of L-soft topology and $\tilde{L}$-soft topology are studied. In most of the occasions, the assumptions are illustrated and substantiated by suitable examples.

These idempotents provide a useful tool for analyzing the structure of L-soft topological and $\tilde{L}$-soft topological spaces.

Example 4.1. Suppose $X = \{x_1, x_2, x_3\}$, and $L = [0, 1]$ equipped with Gödel Structure. Let $E = \{t_1, t_2, t_3, t_4\}$, $A_1 = \{t_1, t_2, t_3\}$ and let $F_1 : A_1 \to L^X$, where $F_1(t_1) = \{0.7/x_1\}$, $F_1(t_2) = \{1/x_1, 0.5/x_2\}$, $F_1(t_3) = \{0.6/x_1, 0.2/x_2, 0.7/x_3\}$. Then, $\theta_1 = (F_1, A_1)$ is a L-soft set.

Let $A_2 = \{t_1, t_2, t_3, t_4\}$ and let $F_2 : A_2 \to L^X$, where

$F_2(t_1) = \{0.4/x_1\}$,
$F_2(t_2) = \{0.9/x_1, 0.5/x_2, 0.3/x_3\}$,
$F_2(t_3) = \{0.4/x_1, 0.2/x_2, 0.5/x_3\}$,
$F_2(t_4) = \{1/x_1, 0.7/x_2, 0.6/x_3\}$.

Then $\theta_2 = (F_2, A_2)$ is a L-soft set.

Let $\theta_3 = (H, C) = (F_1, A_1) \tilde{\bigcup} (F_2, A_2)$, where $C = A_1 \cup A_2 = \{t_1, t_2, t_3, t_4\}$ and

$H(t_1) = F_1(t_1) \cup F_2(t_1) = \{0.7/x_1\}$
$H(t_2) = F_1(t_2) \vee F_2(t_2) = \{1/x_1, 0.5/x_2, 0.3/x_3\}$,
$H(t_3) = F_1(t_3) \vee F_2(t_3) = \{0.6/x_1, 0.2/x_2, 0.7/x_3\}$,
$H(t_4) = F_2(t_4) = \{1/x_1, 0.7/x_2, 0.6/x_3\}$.

Then $\theta_3 = (H, C)$ is a L-soft set.

Let $\theta_4 = (K, D) = (F_1, A_1) \tilde{\bigcap} (F_2, A_2)$, where $D = A_1 \cap A_2 = \{t_1, t_2, t_3\}$, and

$K(t_1) = F_1(t_1) \wedge F_2(t_1) = \{0.4/x_1\}$,

$K(t_2) = F_1(t_2) \wedge F_2(t_2) = \{0.9/x1; 0.5/x_2\}$,
$K(t_3) = F_1(t_3) \wedge F_2(t_3) = \{0.4/x_1, 0.2/x_2, 0.5/x_3\}$.

Then $\theta_4 = (K, D)$ is a L-soft set.

From the definition L-soft topology is generated in Definition 3.1, we get the L-soft topology on θ;

$$\tau_{LS} = \{\Phi, \theta,\ \theta_1, \theta_2, \theta_3, \theta_4\}.$$

Then, (θ, τ_{LS}, E) is called a L-soft topological space.

All our results can be extended by using same L-soft sets, but considering extended intersection rather than restricted intersection.

We can make this clear with the following example:

Example 4.2. Follows Example 4.1 and the Definition 3.11 we obtain,

$\theta_5 = (J, C) = (F_1, A_1) \sqcap (F_2, A_2)$, where $C = A_1 \cup A_2 = \{t_1, t_2, t_3, t_4\}$, and

$J(t_1) = F_1(t_1) \wedge F_2(t_1) = \{0.4/x_1\}$
$J(t2) = F_1(t_2) \wedge F_2(t_2) = \{0.9/x_1, 0.5/x_2\}$,
$J(t_3) = F_1(t_3) \wedge F_2(t_3) = \{0.4/x_1, 0.2/x_2, 0.5/x_3\}$,
$J(t_4) = F_2(t_4) = \{1/x_1, 0.7/x_2, 0.6/x_3\}$.

$\tilde{L}$-soft topology is generated in Definition 3.11, we get the $\tilde{L}$-soft topology on θ;

$$\tilde{\tau}_{LS} = \{\Phi, \theta,\ \theta_1, \theta_2, \theta_3, \theta_5\}.$$

Then, $(\theta, \tilde{\tau}_{LS}, E)$ is called a $\tilde{L}$-soft topological space.

5 Conclusion

In the study of operations on L-soft sets it is proved that the union of L-soft sets is an L-soft sets and the intersection of L-soft sets is also an L-soft set. So, it is natural to ask how a soft topology can be defined on a L-soft set? This study presents a complete answer to this question. In this work, different soft topologies on a L-soft set are defined and the differences of them are showed by examples. It considers the topics of a new area of soft sets that were not studied at all and presents the original result which may be the starting point for soft mathematical concepts and structures based on L-soft set and L-soft topology and $\tilde{L}$-soft topology.

References

[1] Acar, U., Koyuncu, F., Tanay, B., Soft sets and soft rings, Computers and Mathematics with Applications **59**, 3458-3463, 2010.

[2] Ahmad, B., Kharal, A., On fuzzy soft sets, Advances in Fuzzy Systems, 1-6, 2009.

[3] Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M., On some new operations in soft set theory, Computers and Mathematics with Applications **57**, 1547-1553, 2009.

[4] Aygünoglu, A. Aygün, H., Introduction to fuzzy soft groups, Computers and Mathematics with Applications **58**, 1279-1286, 2009.

[5] Aygünoglu, A. Aygün, H., Some notes on soft topological spaces, Neural Computation and Application, 2011.

[6] Aktaş, H. ve Çağman, N., Soft sets and soft groups, Information Sciences, 177(1), 2726-2735, 2007.

[7] Buelohlsavek, R., Fuzzy relational systems, Foundations and Principles, Kluwer, New York 2002.

[8] Çağman, N., Enginoğlu, S., Soft set theory and uni-int decision making, European Journal of Operational Research **207**, 848-855, 2010.

[9] Çağman, N., Enginoğlu, S., Soft matrix theory and its decision making, Computers and Mathematics with Applications **59**, 3308-3314, 2010.

[10] Çağman, N., Karataş, S., Enginoğlu, S., Soft Topology, Computers and Mathematics with Applications **62** (1), 351-358, 2011.

[11] N. Çağman, Contributions to the Theory of Soft Sets, Journal of New Result in Science, 4 (2014) 33-41.

[12] Xueyou Chen and Qingguo Li, Construction of Rough Approximations in Fuzzy Setting, Fuzzy Sets and Systems 158 (2007) 2641-2653.

[13] Chen, X., Rough Soft Sets in Fuzzy Setting, Lecture Notes in Computer Science 7928 (2013) 530-539.

[14] Chen, X., Soft sets in fuzzy setting, Annals of Fuzzy Mathematics and Informatics, in press.

[15] Feng F., Jun, Y. B., Liu, X., Li, L., An adjustable approach to fuzzy soft set based decision making, Journal of Computational and Applied Mathematics **234**, 10-20, 2010.

[16] Feng, F., Jun, Y. B., Zhao, X., Soft semirings, Fuzzy Sets and Systems: Theory and Applications **56** (10), 2621-2628, 2008.

[17] Goguen, J. L-fuzzy sets, J. Math. Anal. Appl. 18 (1967), 145-174.

[18] Hussain, S., Ahmad, B., Some properties of soft topological spaces, Computers and Mathematics with Applications (62), 4058-4067, 2011.

[19] Jiang, Y., Tang, Y., Chen, Q., Wang, J., Tang, s., Extending soft sets with description logics, Computers and Mathematics with Applications **59**, 2087-2096, 2010.

[20] Jun, Y. B., Lee, K. J., Khan, A., Soft ordered semigroups, Mathematical Logic Quarterly. **56** (1), 42-50, 2010.

[21] Kalayathankal, S. J., Singh, G. S., A fuzzy soft flood alarm model, Mathematics and Computers in Simulation **80**, 887-893, 2010.

[22] Maji, P.K., Biswas, R., Roy, A.R., Soft set theory, Computers and Mathematics with Applications **45**, 555-562, 2003.

[23] Majumdar, P., Samanta, S. K., Similarity measure of soft sets, New Mathematics and Natural Computation **4** (1), 1-12, 2008.

[24] Maji, P. K., Biswas, R., and Roy, A. R., Fuzzy soft sets, J. Fuzzy Math. 9 (3) (2001) 589-602.

[25] Min, W., K., A note on soft topological spaces, Computers and Mathematics with Applications 62, 3524-3528, 2011.

[26] Molodtsov, D.A., Soft set theory-first results, Computers and Mathematics with Applications **37**, 19-31, 1999.

[27] Park, C.H., Jun, Y.B., Öztürk, M.A., Soft WS-algebras. Communications of the Korean Mathematical Society. **23** (3), 313-324, 2008.

[28] Pawlak, Z., Rough sets, International Journal of Computer and Information Science 11 (1982) 341-356.

[29] Pei, D., Miao D., From soft sets to information systems, Proceedings of Granular Computing **2**, 617- 621, 2005.

[30] Shabir, M., Naz, M., On soft topological spaces, Computers and Mathematics with Applications **61**, 1786-1799, 2011.

[31] Som, T., On the theory of soft sets soft relation and fuzzy soft relation, Proc. of the National Conference On Uncertainty: A Mathematical Approach, 1-9, 2006.

[32] Sun, Q.M., Zhang, Z.L, Liu, J., Soft sets and soft modules, Proceedings of Rough Sets and Knowledge Technology, Third International Conference China, 403-409, 2008.

[33] Şenel, G., Soft Metric Spaces, Ph.D. Thesis, Gaziosmanpasa University Graduate School of Natural and Applied Sciences Department of Mathematics, 92, 2013.

[34] Şenel, G., A New Approach to Hausdorff Space Theory via the Soft Sets, Mathematical Problems in Engineering 2016 (9), DOI:10.1155/2016/2196743.

[35] Şenel, G., A Comparative Research on the Definition of Soft Point, International Journal of Computer Applications 163 (2), DOI:10.5120/ijca2017913471.

[36] Şenel, G., The Parameterization Reduction of Soft Point and its Applications with Soft Matrix, International Journal of Computer Applications 164 (1), DOI:10.5120/ijca2017913564.

[37] Zadeh, L. A., Fuzzy sets, Information and Control 8 (1965) 338-353.

[38] Xiao, Z., Gong, K., Xia, S., Zou, Y., Exclusive disjunctive soft sets, Computers and Mathematics with Applications **59**, 2128-2137, 2010.

[39] Zorlutuna, I., Akdağ, M., Min, W. K., Atmaca, S., Remarks on Soft Topological Spaces, Annals of Fuzzy Mathematics and Informatics, 3 (2) (2012), 171-185.

[40] Zou, Y., Xiao, Z., Data analysis approaches of soft sets under incomplete information. Knowledge-Based Systems **21**, 941-945, 2008.