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Some Types of Null Hypersurfaces

*[1]Bilal Eftal Acet
[1] Adıyaman University, Faculty of Art and Science,
Department of Mathematics, Adıyaman, TURKEY
eacet@adiyaman.edu.tr,

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Abstract

In this article, we introduce and study three types of null hypersurfaces of a para-Sasakian manifold which are called re-current, Lie re-current and Hopf null hypersurfaces. Also, we obtain some results on such hypersurfaces.

Keywords: Para-Sasakian manifold, Screen semi invariant null hypersurfaces, Screen totally geodesic null hypersurfaces

1. Introduction

In term of differential geometry, submanifolds theory has an attraction for geometers. One of the most important topics is the theory of null (lightlike) submanifolds. A submanifold of a semi-Riemann manifold is called a null submanifold if the induced metric is degenerate. So, geometry of null submanifold is very different from the non-degenerate submanifold.

The general view of null submanifold has been introduced in [1]. Later, K. L. Duggal and B. Şahin have developed many new classes of null submanifolds such as indefinite Kaehler manifolds [2], indefinite Sasakian manifolds [3] and different applications of null submanifolds [4]. On this subject, some applications of the theory of mathematical physics is inspired, especially electromagnetisms[1], black hole theory [4] and general relativity [5]. Many studies on null submanifolds have been reported by many geometers (see [6], [7], [9]).

On a semi-Riemannian manifold, S. Kaneyuki and M. Konzai [10] introduced a structure which is known the almost para-contact structure and then they characterized the almost para-complex structure on. Later, S. Zamkovoy [11] studied para-contact metric manifolds. The study of para-contact geometry has been continued by several papers ([12], [13], [14], [15], [16]]) which are contained role of para-contact geometry about semi-Riemannian geometry, mathematical physics and relationships with the para-Kaehler manifolds.

The purpose of this article is to examine three types of null submanifolds which are called re-current, Lie re-current and Hopf null hypersurface of a para-Sasakian manifold. Also some new results on this types of null submanifolds are given.

2. Preliminaries

A $(2n+1)-$dimension semi-Riemannian manifold $\tilde{M}$ has an almost para-contact structure if it is equipped with a tensor field $\bar{\phi}$ of type (1,1), a 1-form η, a vector field ξ satisfying the following conditions [10]:

$$\begin{aligned} &i)\ \phi^2 = I - \eta \otimes \xi, \\ &ii)\ \eta(\xi) = 1, \\ &iii)\ \phi\xi = 0,\ \eta \circ \bar{\phi} = 0, \end{aligned} \tag{2.1}$$

where I is the identity transformation.

If a manifold $\tilde{M}$ with an almost para-contact structure $(\bar{\phi}, \xi, \eta)$ admits a semi- Riemannian metric $\bar{g}$ such that [11]

$$\bar{g}(\bar{\phi}W, \bar{\phi}Y) = -\bar{g}(W,Y) + \eta(W)\eta(Y), \tag{2,2}$$

for all $Y, W \in \Gamma(T\tilde{M})$, then we say that $\tilde{M}$ has an almost para-contact metric structure an $\bar{g}$ is called compatible metric of signature.

Setting Y = ξ in (2.2), we get

$$\eta(W) = \bar{g}(W, \xi). \tag{2.3}$$

*[1]Corresponding author: Adıyaman University, Faculty of Art and Science, Department of Mathematics, Adıyaman, TURKEY. eacet@adiyaman.edu.tr,

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Definition 2.1 If $\bar{g}(W,\bar{\phi}Y)=d\eta(W,Y)$ (where $d\eta(W,Y)=\frac{1}{2}\{W\eta(Y)-Y\eta(W)-\eta([W,Y])\})$ then η is a para-contact form and the almost para-contact metric manifold $\tilde{M}$ is said to be para-contact metric manifold.

An almost para-contact metric manifold $\tilde{M}$ is a para-Sasakian manifold if and only if [13]

$$(\bar{\nabla}_W\bar{\phi})Y=-\bar{g}(W,Y)\xi+\eta(Y)W, \quad (2.4)$$

where $\bar{\nabla}$ is a Levi-Civita connection on $\tilde{M}$.

From (2.4), we get

$$\bar{\nabla}_W\xi=-\bar{\phi}W.$$

Let $\tilde{M}$ be a semi-Riemannian manifold with index q, $0<q<2n+1$, and M be a hypersurface of $\tilde{M}$, with $g=\bar{g}|_{\tilde{M}}$. M is a null hypersurface of $\tilde{M}$ if the metric g is of rank $2n-1$ and the orthogonal complement $TM^{\perp}$ of tangent space TM, given as

$$TM^{\perp}=\{Y_P\in T_P\tilde{M}:g_P(W_P,Y_P)=0,\forall W\in\Gamma(T_PM)\}$$

is a distribution of rank 1 on M [1]. $TM^{\perp}\subset TM$ and then coincides with the radical distribution $RadTM=TM\cap TM^{\perp}$.

A complementary bundle of $TM^{\perp}$ in TM is a non-degenerate distribution of constant rank $2n-1$ over M. It is known a screen distribution and demonstrated with $S(TM)$.

Theorem 2.1 [1] Let $(M,g,S(TM))$ be a null hypersurface of a semi-Riemannian manifold $\tilde{M}$. Then there exists a unique rank one vector subbundle $ltr(TM)$ of $T\tilde{M}$, with base space M, such that for any non-zero section E of $RadTM$ on a coordinate neighbourhood $U\subset M$, there exists a unique section N of $ltr(TM)$ on U satisfying for $W\in\Gamma(S(TM))|_U$:

$$\bar{g}(N,N)=0, \quad \bar{g}(N,W)=0, \quad \bar{g}(N,E)=1,$$

$ltr(TM)$ is called the null transversal vector bundle of M with respect to $S(TM)$.

By the previous theorem, we can state:

$$TM = S(TM)\perp RadTM, \quad (2.5)$$

$$T\tilde{M}= TM\oplus ltr(TM)$$

$$= S(TM)\perp\{RadTM\oplus ltr(TM)\}. \quad (2.6)$$

Let $\bar{\nabla}$ be the Levi-Civita connection of $\tilde{M}$ and $P:\Gamma(TM)\to\Gamma(S(TM))$ be the projection morphism with respect to the orthogonal decomposition of TM. Then the local Gauss and Weingarten formulas are given by

$$\bar{\nabla}_W Y=\nabla_W Y+h(W,Y), \quad (2.7)$$

$$\bar{\nabla}_W N=-A_N W+\nabla^{lt}_W N, \quad (2.8)$$

$$\nabla_W PY=\nabla^*_W PY+C(W,PY)E, \quad (2.9)$$

$$\bar{\nabla}_W E=-A^*_E W-\tau(W)E, \quad (2.10)$$

for any $Y,W\in\Gamma(TM)$, where ∇ is a linear connection on M and ∇^* is a linear connection on $S(TM)$ and B, A_N and τ are called the local second fundamental form, the local shape operator, the transversal differential 1-form, respectively.

The induced linear connection ∇ is not a metric connection and we get

$$(\nabla_W g)(Y,Z)=B(W,Z)\theta(Y)+B(W,Y)\theta(Z) \quad (2.11)$$

where θ is a differential 1-form such that $\theta(W)=\bar{g}(N,W)$.

Also the second fundamental form B is independent of the choice of $S(TM)$ and

$$B(W,E)=0. \quad (2.12)$$

The local second fundamental forms are related to their shape operators by

$$g(A^*_E W,PY)=B(W,PY),\quad g(A^*_E W,N)=0 \quad (2.13)$$

$$g(A_N W,PY)=C(W,PY),\quad g(A_N W,N)=0. \quad (2.14)$$

3. Screen Semi-Invariant Null Hypersurfaces of a Para-Sasakian Manifold

Let M be a null hypersurface of a para-Sasakian manifold $\tilde{M}$ with $\xi\in\Gamma(TM)$. If E is a local section of $\Gamma(RadTM)$, then

$$\bar{g}(\bar{\phi}E,E)=0,$$

and $\bar{\phi}E$ is tangent to M. So, we obtain a distribution $\bar{\phi}(RadTM)$ of dimension 1 on M.

If $\bar{\phi}(\mathrm{ltr}(TM)) \subset S(TM)$ and $\bar{\phi}(\mathrm{Rad}TM) \subset S(TM)$ then null hypersurface M is called a screen semi-invariant null hypersurface of $\tilde{M}$ [17].

Since M is a screen semi-invariant null hypersurface then we can state

$$\begin{aligned} \bar{g}(\bar{\phi}N, N) &= 0 \\ \bar{g}(\bar{\phi}N, E) &= 0 = -\bar{g}(N, \bar{\phi}E), \\ \bar{g}(N, E) &= 1 \end{aligned} \tag{3.1}$$

and from (2.2), we obtain

$$\bar{g}(\bar{\phi}E, \bar{\phi}N) = -1. \tag{3.2}$$

Therefore $\bar{\phi}(\mathrm{Rad}TM) \oplus \bar{\phi}(\mathrm{ltr}(TM))$ is a non-degenerate vector subbundle of screen distribution $S(TM)$.

Now, since $S(TM)$ and $\bar{\phi}(\mathrm{Rad}TM) \oplus \bar{\phi}(\mathrm{ltr}(TM))$ are non-degenerate, we can describe a non-degenerate distribution $\tilde{D}_0$ such that [4]

$$S(TM) = \tilde{D}_0 \perp \{\bar{\phi}(\mathrm{Rad}TM) \oplus \bar{\phi}(\mathrm{ltr}(TM))\}. \tag{3.3}$$

In that case $\bar{\phi}(\tilde{D}_0) = \tilde{D}_0$ and $\xi \in \tilde{D}_0$.

In view of (2.5), (2.6) and (3.3), we arrive at followings:

$$TM = \tilde{D}_0 \perp \{\bar{\phi}(\mathrm{Rad}TM) \oplus \bar{\phi}(\mathrm{ltr}(TM))\} \perp \mathrm{Rad}TM \tag{3.4}$$

$$\begin{aligned} T\tilde{M} = \; & \tilde{D}_0 \perp \{\bar{\phi}(\mathrm{Rad}TM) \oplus \bar{\phi}(\mathrm{ltr}(TM))\} \\ & \perp \{\mathrm{Rad}TM \oplus \mathrm{ltr}(TM)\}. \end{aligned} \tag{3.5}$$

If we take $\hat{D} = \mathrm{Rad}TM \perp \bar{\phi}(\mathrm{Rad}TM) \perp \tilde{D}_0$ and $\overset{0}{D} = \bar{\phi}(\mathrm{ltr}(TM))$ on M, we get

$$TM = \hat{D} \oplus \overset{0}{D}. \tag{3.6}$$

Consider the local null vector fields $\tilde{V} = \bar{\phi}E$ and $\tilde{U} = \bar{\phi}N$. Let us denote the projection morphism of TM into $\hat{D}$ and $\overset{0}{D}$, by S and Q, respectively. So, for $X \in \Gamma(TM)$, we have

$$X = SX + QX, \quad QX = u(X)\tilde{U},$$

where u is a differential 1-form locally dfined by

$$u(X) = -g(\bar{\phi}E, X), \tag{3.7}$$

with

$$v(X) = -g(\bar{\phi}N, X). \tag{3.8}$$

Applying $\bar{\phi}$ to X, we find

$$\bar{\phi}X = \bar{\phi}(SX) + u(X)N.$$

If we put $\varphi X = \bar{\phi}(SX)$ in above equation, we arrive at

$$\bar{\phi}X = \varphi X + u(X)N, \tag{3.9}$$

where φ is a tensor field defined by $\varphi = \bar{\phi} \circ S$ of type (1,1).

Again applying φ to (3.9), we arrive at

$$\varphi^2 X = X - \eta(X)\xi - u(X)U, \; u(U) = 1 \tag{3.10}$$

Then from (2.5) comparing the components, we get

$$\begin{aligned} (\nabla_X \varphi)Y &= B(X,Y)\tilde{U} - g(X,Y)\xi \\ &+ \eta(Y)X + u(Y)A_N X \end{aligned}, \tag{3.11}$$

$$(\nabla_X u)Y = u(Y)\tau(X) - B(X, \varphi Y), \tag{3.12}$$

$$(\nabla_X v)Y = g(A_N X, \varphi Y) + v(Y)\tau(X), \tag{3.13}$$

$$\nabla_X \tilde{U} = \varphi(A_N X) - \tau(X)\tilde{U}, \tag{3.14}$$

$$\nabla_X \tilde{V} = \varphi(A_E^{*} X) - \tau(X)\tilde{V}, \tag{3.15}$$

$$\varpi(X) = B(X, \tilde{U}) = C(X, \tilde{V}). \tag{3.16}$$

4. Main Results

In this part, we examine our basic results. Firstly we give the following:

Definition 4.1 Let M be a screen semi-invariant null hypersurface of a para-Sasakian manifold $\tilde{M}$ and λ be a 1-form on M. If M admits a re-current tensor field φ such that

$$(\nabla_X \varphi)Y = \lambda(X)\varphi Y, \tag{4.1}$$

then it is called re-current [8].

Proposition 4.1 Assume that M is a re-current screen semi-invariant null hypersurface of a para-Sasakian manifold. Then we get

i) φ is parallel with respect to ∇,

ii) $$A_N X = -v(X)\xi - \lambda(X)\tilde{U}, \tag{4.2}$$

iii) $$A_E^{*} X = -u(X)\xi - \lambda(X)\tilde{V}. \tag{4.3}$$

Proof. If M is a re-current screen semi-invariant null hypersurface, we can state

$$\begin{aligned} \lambda(X)\varphi Y &= B(X, Y)\tilde{U} - g(X,Y)\xi \\ &+ \eta(Y)X + u(Y)A_N X. \end{aligned} \tag{4.4}$$

Putting $Y = \xi$ and in view of (2.13) with (3.7), we get

$$\lambda(X)\tilde{V} = -g(X, E)\xi. \tag{4.5}$$

Taking inner product to $\tilde{U}$, we have

$$\lambda(X) = 0. \tag{4.6}$$

Using this result in (4.1) we arrive at (i).

Now taking $Y=\tilde{U}$ in (4.4) and by use of (4.6), we find (ii).

Similarly taking inner product $\tilde{V}$ to (4.4), we get (iii).

Definition 4.2 A null hypersurface of semi-Riemannian manifold is said to be screen conformal [4] if there exists a non-zero smooth function ζ such that

$$A_N=\zeta A_E^*$$

or

$$C(X, PY)=\zeta B(X,Y).$$

Theorem 4.1 Let M be a re-current screen semi-invariant null hypersurface of a para-Sasakian manifold $\tilde{M}$. Suppose that M is a screen conformal null hypersurface. Then M is either totally geodesic or screen totally geodesic if and only if $X\in\Gamma(\tilde{D}_0)$.

Proof. Since M is screen conformal, from (4.2) with (4.3), we get

$$\lambda(X)\tilde{U}+v(X)\xi=\zeta(\lambda(X)\tilde{V}+u(X)\xi).$$

Taking inner product with $\tilde{V}$ to above equation, we have

$$\lambda(X)=0.$$

So, by using (4.2) and (4.3), we arrive at the proof of the our assertion.

Theorem 4.2 Let M be a re-current screen semi-invariant null hypersurface of a para-Sasakian manifold $\tilde{M}$. Then $\hat{D}$ is a parallel distribution on M.

Proof. Using (4.1) with (3.11), we can write

$$\lambda(X)\varphi Y = B(X,Y)\tilde{U}-g(X,Y)\xi +\eta(Y)X+u(Y)A_N X. \quad (4.7)$$

Taking inner product with $\tilde{V}$ to (4.7) and using (4.6), we have

$$B(X,Y)=-u(Y)u(A_N X)-u(X)\eta(Y). \quad (4.8)$$

Putting $Y=\tilde{V}$ and $Y=\varphi Z$ in (4.8), we arrive at

$$B(X,\tilde{V})=0 \text{ and } B(X,\varphi Z)=0 \quad (4.9)$$

respectively.

Now, from (3.9) and (3.15), we find, for all $Z\in\Gamma(\tilde{D}_0)$,

$$g(\nabla_X E,\tilde{V})=B(X,\tilde{V}), \quad (4.10)$$

$$g(\nabla_X Z,\tilde{V})=B(X,\varphi Z), \quad (4.11)$$

$$g(\nabla_X \tilde{V},\tilde{V})=0. \quad (4.12)$$

So by use of (4.10) □ (4.12) with (4.9), we arrive at for $X\in\Gamma(TM)$ and $Y\in\Gamma(\hat{D})$,

$$\nabla_X Y\in\Gamma(\hat{D}),$$

from which we see that $\hat{D}$ is a parallel distribution.

Definition 4.3 Let M be a screen semi-invariant null hypersurface of a para-Sasakian manifold $\tilde{M}$ and μ be a 1-form on M. Then M is called Lie re-current [8] if it is admits a Lie re-current tensor field φ such that

$$(\Im_X\varphi)Y=\mu(X)\varphi Y, \quad (4.13)$$

where $\Im$ denotes the Lie derivative, that is,

$$(\Im_X\varphi)Y=[X,\varphi Y]-\varphi[X,Y].$$

If the structure tensor field φ satisfies the condition

$$\Im_X\varphi=0, \quad (4.14)$$

then φ is called Lie parallel.

Thus, a screen semi-invariant null hypersurface M of a para-Sasakian manifold $\tilde{M}$ is called Lie re-current if it admits Lie re-current structure tensor field φ.

Theorem 4.3 Let M Lie re-current screen semi-invariant null hypersurface of a para-Sasakian manifold $\tilde{M}$. Then the structure tensor field φ is Lie parallel.

Proof. In view of above definition with (3.11), we get

$$\begin{aligned}\mu(X)\varphi Y &= (\Im_X\varphi)Y\\ &=[X,\varphi Y]-\varphi[X,Y]\\ &=B(X,Y)\tilde{U}+u(Y)A_N X-g(X,Y)\xi\\ &+\eta(Y)X-\nabla_{\varphi Y}X+\varphi\nabla_Y X. \quad (4.15)\end{aligned}$$

Putting $Y=E$ in (4.15) and by use of (2.12), we find

$$\mu(X)\tilde{V}=-\nabla_{\tilde{V}}X+\varphi\nabla_E X-g(X,E)\xi. \quad (4.16)$$

Taking inner product with $\tilde{V}$ to (4.16), we obtain

$$g(\nabla_{\tilde{V}}X,\tilde{V})=u(\nabla_{\tilde{V}}X)=0. \quad (4.17)$$

In equation (4.15), replacing Y by $\tilde{V}$, we get

$$\mu(X)E=-\nabla_E X+\varphi\nabla_{\tilde{V}}X+B(X,\tilde{V})\tilde{U} +u(X)\xi. \quad (4.18)$$

Applying φ to (4.18), using (3.10) with (4.17) and comparing (4.18), we arrive at φ is Lie parallel.

Definition 4.4 Let M be a screen semi-invariant null hypersurface of a para-Sasakian manifold $\tilde{M}$ and $\tilde{U}$ be a structure tensor field on M. The structure tensor field $\tilde{U}$ is called principal if there exists a smooth function σ such that

$$A_E^{*}X = \sigma\tilde{U}. \qquad (4.19)$$

A screen semi-invariant null hypersurface M of a para-Sasakian manifold $\tilde{M}$ is a called Hopf null hypersurface if it admits principal vector field $\tilde{U}$.

If we consider equation (4.19), from (2.13) and (3.8), we arrive at

$$B(X,\tilde{U}) = -\sigma v(X). \qquad (4.20)$$

From this equation, we get

$$C(X,\tilde{V}) = -\sigma u(X). \qquad (4.21)$$

Theorem 4.4 Let M be a screen semi-invariant Hopf null hypersurface of a para-Sasakian manifold $\tilde{M}$. If M is screen totally umbilical then $\varpi = 0$ and M is a screen totally geodesic null hypersurface.

Proof. We know that, M is a screen totally umbilical null hypersurface if there exists a smooth function $\tilde{f}$ such that $A_N X = \tilde{f}g(X,Y)$ or

$$C(X,PY) = \tilde{f}g(X,Y), \qquad (4.22)$$

and $\tilde{f} = 0$ we say that M is a screen totally geodesic null hypersurface.

So, in (4.22) replacing PY with $\tilde{V}$ and by use of (3.8) and (4.21), we find

$$\tilde{f}v(X) = \tilde{f}u(X).$$

Putting $X = \tilde{U}$ in above equation we obtain $\tilde{f} = 0$. So, we get $A_N = 0 = C$ and $\varpi = 0 = g(A_N X,\tilde{V})$. Therefore $\varpi = 0$ and M is a screen totally geodesic null hypersurface.

Theorem 4.5 Let M be a screen semi-invariant null hypersurface of a para-Sasakian manifold $\tilde{M}$. If $\tilde{V}$ is a parallel null vector field then M is a Hopf null hypersurface such that $\varpi = 0$.

Proof. WeIf we consider $\tilde{V}$ is parallel null vector field, from (3.9) and (3.15), we find

$$\bar{\phi}(A_E^{*}X) - u(A_E^{*}X)N + \tau(X)\tilde{V} = 0. \qquad (4.23)$$

Applying $\bar{\phi}$ to (4.23) and in view of (2.1), we have

$$A_E^{*}X - u(A_E^{*}X)\tilde{U} + \tau(X)E = 0.$$

Taking inner product with N to above equation, we arrive at $\tau = 0$, which yields

$$A_E^{*}X = u(A_E^{*}X)\tilde{U}. \qquad (4.24)$$

So, we can say M is a Hopf null hypersurface. If we take inner product with $\tilde{U}$ to (4.24), we find $\varpi(X) = 0 = B(X,\tilde{U})$.

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