http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 30.03.2018 *Year*: **2018**, *Number*: **25**, *Pages*: **8-15**
Published: 06.10.2018 *Original Article*

N-Fuzzy BI-Topological Space and Separation Axioms

Faisal Khan[1,*] <faisalmaths@hu.edu.pk>
Saleem Abdullah[2] <saleemabdullah81@yahoo.com>
Muhammad Rahim[1] <rrahimmaths@gmail.com>
Muhammad Shahzad[1] <shahzadmaths@hu.edu.pk>

[1]*Department of Mathematics, Hazara University Mansehra, 21300 KP, Pakistan*
[2]*Department of Mathematics, Abdul Wali Khan University Mardan, 23200 KP,Pakistan*

Abstaract − In this article, we introduced N-fuzzy bi-topological space by using the concepts of fuzzy bi-topological space. We further define some basic properties of N-fuzzy bi-topological spaces, secondly we study the concepts of natural separation axioms of bi-topological in N-fuzzy bi-topological space which is pair wise separation Axioms mixed topology with the help of two N-fuzzy topologies of a N-fuzzy bi-topological space. Relation between such pairwise separation axioms and natural fuzzy separation axioms of the mixed fuzzy topological space are investigated.

Keywords − *N-fuzzy set, N-fuzzy bi-topological space, natural separation axioms, natural fuzzy separation axioms.*

1 Introduction

The concept of fuzzy sets was introduced by Zadeh in [1] and thereafter the paper of Chang [3] give the way for the farther growth of new direction. The notion of fuzzyness has been applied for studying different aspects of mathematics by Tripathy and Baruah [20], Tripathy and Borgoha in [21, 22], Tripathy and Tripathy and Sarma [23] and many research on sequence spaces in recent years. The notion of bi-topological spaces has been discuss from different aspects by Tripathy and Acharjee [24], Tripathy and Debnath [25] and others. Kandil introduced the concept of fuzzy bi-topological spaces. Later on several researcher were attracted by the notion of fuzzy bi-topological spaces. An N-fuzzy bi-topological space is a non-empty Set X together with two N-fuzzy topologies on it. We will apply the N-structure to fuzzy bi-topological space and pairwise, separation axioms. Different pair wise separation

*Corresponding Author.

Axioms are defend which is the generalization of natural Separation axioms in the sense that such a notion reduces to the natural separation axioms of a N-fuzzy topological space when two topological spaces coincide. In this paper, pairwise, separation axioms are introduced and a mixed topology is introduced with the help of two N-fuzzy topologies of a N-fuzzy bi-topological space. Relation between such pairwise separation axioms and natural fuzzy separation axioms of the mixed fuzzy topological space are investigated. Finally, pairwise N-fuzzy normal bi-topological space, pairwise weakly and pairwise strongly separated space are introduced and investigated their properties with the mixed topology.

2 Preliminary

In this section we deals with basic concept of fuzzy set, fuzzy topological spaces N-fuzzy set and N-fuzzy topological and bi- topological spaces.

Definition 2.1. [1] Let **X** be a non-empty set. A fuzzy set **A** in **X** is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \epsilon X$. It is clear that A is completely determined by the set of tuples $A = \{(x, \mu_A(x)) : x \epsilon X\}$.

Definition 2.2. [2] A fuzzy topological space on a set **X** is a family τ of fuzzy sets in **X** which satisfies the following condition:
i For all $0, 1 \ \epsilon \ \tau$.
ii For all $A, B \epsilon \tau \Rightarrow A \wedge B \epsilon \tau$.
iii For all $(A_j)_{j \epsilon J} \epsilon \tau \Rightarrow \vee_{j \epsilon J} A_j \epsilon \tau$.
The pair (X, τ) is called fuzzy topological space and the members of τ are called fuzzy open sets.

Definition 2.3. [5] A fuzzy bi-topological space on a set **X** is a families (τ_1, τ_2) of fuzzy sets in **X** which satisfies the following condition:
i $0, 1 \ \epsilon \ \tau_1, \tau_2$.
ii For all $A, B \epsilon \tau_1, \tau_2 \Rightarrow A \wedge B \epsilon \tau_1 \tau_2$.
iii For all $(A_j)_{j \epsilon J} \epsilon \tau_1, \tau_2 \Rightarrow \vee_{j \epsilon J} A_j \epsilon \tau_1, \tau_2$.
The pair (X, τ_1, τ_2) is called fuzzy bi-topological space and the members of τ_1, τ_2 are called fuzzy open sets.

3 N-Fuzzy Set and its Properties

Definition 3.1. N-Structure:(Negative-valued function): A mapping from $f : X \to [-1, 0]$ is the collection of functions from a set X to [-1,0],(we say that an elements of $f : X \to [-1, 0]$) is a negative-valued function from X to [-1,0] (briefly,N-function on X). (By an N-structure we mean an ordered pair (X,f)) of X and an N-function f on X.

Note: Through out this paper we apply N-Structure to fuzzy bi-topological space and pairwise, separation axioms.

Definition 3.2. Let **X** be a set and **I** be the interval $\mathbf{I} = [-1, 0]$ a N-fuzzy set **A** is characterized by a membership function μ_A which associate with each point $x \in X$ its grade of membership $\mu_A(x) \in [-1, 0]$.

Definition 3.3. N-fuzzy topological space: A N-fuzzy topological space on a set **X** is a family τ of N-fuzzy sets in **X** which satisfies the following condition:
i $-1, 0 \in \tau$.
ii if $\mathbf{A_j} \in \tau$ then $\forall\ j \in J, \bigcap_{j \in J} \mathbf{A_j} \in \tau$.
iii if **A** and $\mathbf{B} \in \tau$ then $\mathbf{A} \bigcup \mathbf{B} \in \tau$.
The pair (X, τ) is called N-fuzzy topological space and the members of τ are called N-fuzzy open sets.

Definition 3.4. A N-fuzzy set λ is called N-fuzzy closed set if $\lambda^c \in \tau$. i.e the complement of N-fuzzy set λ of τ belongs to τ.

4 N-Fuzzy BI-Topological Space and Separation Axioms

Definition 4.1. A N-fuzzy bi-topological space on a set **X** is a families τ_1, τ_2 of N-fuzzy sets in **X** which satisfies the following properties:
i $-1, 0 \in \tau_1, \tau_2$.
ii if $\mathbf{A_j} \in \tau_1, \tau_2$ then $\forall\ j \in J, \bigcap_{j \in J} \mathbf{A_j} \in \tau_1, \tau_2$.
iii if **A** and $\mathbf{B} \in \tau_1, \tau_2$ then $\mathbf{A} \bigcup \mathbf{B} \in \tau_1, \tau_2$.
The pair (X, τ_1, τ_2) is called N-fuzzy bi-topological space, and the members of τ_1, τ_2 are called N-fuzzy open sets.

Definition 4.2. A N-fuzzy set λ is called N-fuzzy closed set if $\lambda^c \in (\tau_1, \tau_2)$, the complement belongs to (τ_1, τ_2).

Definition 4.3. A N-fuzzy bi-topological space $(\mathbf{X}, \tau_1, \tau_2)$ is said to be pairwise $\mathbf{T}_1$ if every pair of distinct N-fuzzy points x and y in X, there exits τ_1 open set R and a τ_2 open set S such that $R(x) = 0$, $y \notin R$ and $x \notin S$ then $S(y) = 0$.

Definition 4.4. A N-fuzzy bi-topological space $(\mathbf{X}, \tau_1, \tau_2)$ is said to be pairwise N-fuzzy Hausdorff space if for each pair of distinct points x and y, there are τ_1 open set R and a τ_2 open set S such that $R(x) = 0$, $S(y) = 0$, and $R \cup S = -1$.

Definition 4.5. A N-fuzzy bi-topological space $(\mathbf{X}, \tau_1, \tau_2)$ is said to be pairwise N-fuzzy regular w.r.t.τ_2 iff $\mu \in (-1, 0)$, $R \in \tau_1^c, x \in X$ and $\lambda < -1 - R(x)$ imply that there exists $S \in \tau_2$ and $T \in \tau_2$ with $\lambda < S(x) \subseteq S$ and $S \subseteq -1 - T$. (X, τ_1, τ_2) is called pairwise N-fuzzy regular if it is N-fuzzy regular w.r.t.τ_1 and τ_2 N-fuzzy regular w.r.t.τ_1.
The following theorem play a fundamental role in this work. The relation between compactness and closeness of a subject of a pairwise Hausdorff bi-topological space. The ordinary subset Y is regarded as a N-fuzzy subsets.

Theorem 4.6. If (X, τ_1, τ_2) is a pairwise Hausdorff N-fuzzy bi-topological space and Y is an ordinary-1 compact N-fuzzy set in X, then Y is closed.

Proof. To show that x_λ is not in Y implies x_λ is not an accumulation point of Y. Means that $x_\lambda \notin Y$ implies that $-1 > \lambda > y(x)$ and so, $x \notin Y$. Then $x \neq Y$ for all $y \epsilon Y$, by the pairwise Hausdorff property of (X, τ_1, τ_2) there exists a τ_1 open set R and a τ_2 open set S such that $R(x) = S(y)$ and $R \cup S = -1$. Thus, for $x_\lambda \notin Y$ and $y \epsilon Y$. $R(x) = (R \cup S) = 0$ with $R \cup S = -1$. Varying y over all points belonging to Y, the collection $\{Y \cup S\}$ is a 1^* shading of Y, then so, it reduces to a finite 1^* subshading say, $\{Y \cup S_{y1,} Y \cup S_{y2,} .... Y \cup S_{yn}\}$. Simply write $S = S_{y1}$. Since $R_x^{y1}(x) = 0$ for all $0 \leq i \leq n$, we have $R(x) = 0$ and $x_\lambda \epsilon R$. Now $R \cup S = (R_x^{y1} \cup ..... R_x^{yn}) \cup (S_{y1} \cap S_{y2} \cap ..... \cap S_{yn}) = -1$ for $y \epsilon Y$, there exists $Y \cup S_{y1}$ such that $Y \cup S_{y1}(y) = 0$ implies $Y \cup S(y) = 0 = Y(y)$ so $Y \cup S = Y$. Also, $Y \cup R = Y \cup R = -1$ implies that $Y(x) = -1$ or $R(x) = -1$. Therefore, $Y(x) + U(x) > 0$. Hence Y and R are not quasi-coincident, and therefore x_λ is not a τ_1 accumulation point of Y. This proves that Y is τ_1 closed.

Definition 4.7. In a pairwise N-fuzzy Hausdorff space (X, τ_1, τ_2) is τ_1-1^* compact subset is τ_2 closed.

Definition 4.8. With the help of two N-fuzzy topologies of a N-fuzzy bi-topological space a third N-fuzzy topology is defined on it. This topology is named as mixed N-fuzzy topology. We then relate separation axioms and there property relative to the mixed topology with pairwise separation axioms of the N-fuzzy bi-topological space.

Theorem 4.9. Let (X, τ_1, τ_2) be a N-fuzzy bi-topological space, $\{Y_\mu\}$ be a collection of ordinary subsets of X which are τ_2 1^* compact as N-fuzzy subsets.

Proof. Let $\tau = \{i_{\mu,} : Y \to X\}$ and (τ_ν) be the collection of N-fuzzy topologies on X such that $E_\mu : (Y_\mu, \tau_1) \to (X, \tau_\nu)$ are continuous, where (Y_μ, τ_1) means subspace topology on X, then they are continuous. That is, $\tau_1(\tau_2)$ is topology such that

(a): $\tau_1(\tau_2) \supseteq \tau_\nu$ for all ν

$E_\mu : (Y_\mu, \tau_1) \to (X, \tau_1)(\tau_2)$ are continuous.

(b): If $(\tau_o) \supseteq \tau_\nu$ for all ν s. t.

$E_\mu : (Y_\mu, \tau_1) \to (X, \tau_o$ are continuous then $\tau_o \supseteq \tau_1(\tau_2)$ the N-fuzzy topology $\tau_1(\tau_2)$ is called a mixed N-fuzzy topology on X. Clearly, $\tau_1 \epsilon \{\tau_\nu\}$ and therefore, $\tau_1 \subseteq \tau_1(\tau_2)$. Although we have used the symbol $\tau_1(\tau_2)$ for the mixed topology arising out of N-fuzzy topologies τ_1 and τ_2. This theorem is applied to the rest of this paper. The following theorem shows the relation between Hausdorff property of the mixed topology and the pairwise Hausdorff property of the bi-topological space.

Theorem 4.10. If (X, τ_1, τ_2) is a pairwise N-fuzzy Hausdorff space then the mixed N-fuzzy topology $\tau_1(\tau_2)$ is a N-fuzzy Hausdorff topology.

Proof. The pair (X, τ_1, τ_2) is a pairwise N-fuzzy Hausdorff space, $x, y \epsilon X$ and $x \neq y$ there exist $R \epsilon \tau_1$ and $S \epsilon \tau_2$,$R(x) = S(x)$ such that $R \cup S = -1$ To show that $(X, \tau_1, (\tau_2))$ is a N-fuzzy Hausdorff space, we claim that both R and S are $\tau_1, (\tau_2)$ open. Let, Y_μ be ordinary subsets of X which are 1^* compact w.r.t. N-fuzzy topology

τ_2, let $A = \{i_\mu : Y \rightarrow X\}$ and $\{\tau_\mu$ be the collection of inclusion mappings and N-fuzzy topologies on X such that $i_\mu : (X, \tau_1, (\tau_2)) \rightarrow (X, \tau_\nu)$ are continuous. For each $z\epsilon Y_\mu$,$i_\mu^{-1}(R)(z) = R((i_\mu(z) = max\{Y_\mu(z), R(z)\}) = (Y_\mu \cup R)(z)$. Therefore, $i_\mu^{-1}(R)$ is $(Y_{\mu,}\tau_1)$ open since τ_1 is coarser then $\tau_1(\tau_2)$ $\tau_1 - open$ set R is $\tau_1(\tau_2)$ open. In the similar way, $B = i_\mu^{-1}(s) = (Y_\mu \cup S)$ is open in $(Y_\mu \cup \tau_2)$ and therefore its complement in Y_μ, Y_μ B is closes Y_μ, τ_2 from the result of 1.7.9 show that Y_μ-B is $(Y_{\mu,}\tau_2)$-1* compact. Also $(Y_{\mu,}\tau_1\tau_2)$ inherits pairwise Hausdorff property from (X, τ_1, τ_2). From the theorem 4.2, Y_μ B is $(Y_{\mu,}\tau_1)$ closed and $i_\mu^{-1}(s) = Y_\mu \cup s = B$ is $(Y_{\mu,}\tau_1)$ open for every Y_μ. We say that $s\epsilon\tau_1, (\tau_2)$. Now let $\tau_o = \{s|i_\mu^{-1}(s)\epsilon(Y_{\mu,}\tau_1)\}$ open for every Y_μ we have $s\epsilon\tau_1(\tau_2)$. Let $\tau_o = \{s|i_\mu^{-1}(s)\epsilon(Y_{\mu,}\tau_1)\}$ for all Y_μ so τ_o is a topology on X such that $i_\mu((X, Y_\mu, \tau_1)) \rightarrow (X, \tau_o)$ are continuous So, τ_o is one of the members of $\{\tau_\nu\}$ and hence τ_o is $\subseteq \tau_1(\tau_2)$. Now $B = Y_\mu \cup S = i_\mu^{-1}(s)\epsilon(Y_{\mu,}\tau_1)$ for all Y_μ then so $s\epsilon\tau_o \subseteq \tau_1(\tau_2)$, and $B = Y_\mu \cup S = i_\mu^{-1}(s)(Y_{\mu,}\tau_1)$ for all Y_μ so,$s\epsilon\tau_o \subseteq \tau_1(\tau_2)$. This prove that $S\epsilon\tau_1(\tau_2)$. Thus $x, y\epsilon X$ and $x \neq y$ implies that there exists $R\epsilon\tau_1(\tau_2)$ and $S\epsilon\tau_1(\tau_2)$ with the $R(x) = S(x) = 0$ and $r \cup s = -1$ then, $(X, \tau_1, (\tau_2))$ is a N-fuzzy Hausdorff space.
Note: Some authors have studied fuzzy regularity in different ways. Some of them are equivalent and others are independent as shown by Dewam M.ali. The following lemmas are related with the theorems of pairwise fuzzy regular bi-topological spaces.

Theorem 4.11. If R is a closed relative to subspace topology on y induced from τ then, $R = Y \cup R_o$ R_o is τ closed.

Proof. Here the subspace topology $\tau_{1y} = \{Y \cup H|G\epsilon\tau\}$. R is τ_{1y}-closed. Then $1 - R$ is τ_{1y}-open. $-1 - R = Y \cup S$ where S is τ-open. therefore $-1 - R(y) = max\{Y(y), S(y)\} = max\{-1, S(y)\} = S(y)$ implies that $R(y) = -1 - S(y) = max\{Y(y), -1 - S(y)\} = (Y \cup S^c)(y)$. Then $R = (Y \cup S^c) = Y \cup R_o$. Since S is τ open, $S^c = R_o$ is τ closed. Here is the representation of open subsets of the supremum topology $S\tau_\mu$.

Theorem 4.12. An the supremum topology $(S\tau_\mu)$ there open set are the intersections of finite unions of different τ_μ open set.

Proof. Let τ be the collection of intersection of finite unions of elements $R\tau_{\mu,}$ that is $\tau = \{\cap(U_{i=1}^n R_i), R_i\epsilon R\tau_\mu$ it clear that τ is a N-fuzzy topology and (i) $\tau \supseteq \tau_\mu$ for all μ. (ii) If τ_o is N-fuzzy in $\tau_o \supseteq \tau_\mu$ for all μ Then so, $\tau_o \supseteq \tau \supseteq \tau_\mu$. Now let $\cap(U_{i=1}^n R_i)\epsilon\tau$,Where $R_i\epsilon R\tau_\mu$ then $\cap_\mu(U_{i=1}^n R_i) \notin \tau$,$\tau_o$ for some $i = i_o$ implies that $R \notin \tau_\mu$ for all μ which contradicts that $R\epsilon R\tau_\mu$. Hence we have $\cap_n(U_{i=1}^n R_i)\epsilon\tau_o$ and so, $\tau \subseteq \tau_o$ and $\tau = l.u.b.\tau_o$.

Theorem 4.13. A function $f : (X, \tau) \rightarrow (Y, \tau_\mu)$ is continuous for all if $f : (X, \tau) \rightarrow (Y, S\tau_\mu)$ is continuous.

Proof. (i) Let the function $f : (X, \tau) \rightarrow (Y, \tau_\mu)$ is continuous for all μ and $R\epsilon S\tau_\mu$, then $R = \cap(U_{i=1}^n R_i)$. Now the inverse function is $f^{-1} : (\cap(U_{i=1}^n R_i)) = \cap(U_{i=1}^n f^{-1}R_i)\epsilon\tau$. Hence the function f is continuous from X, τ to (Y, τ_μ) for all μ.
(ii) Let the function $f : (X, \tau) \rightarrow (Y, S\tau_\mu)$ be continuous in every R in τ_μ is in $S\tau_\mu$ and we conclude that the function $f^{-1}(R)\epsilon\tau$. And hence the function $f : (X, \tau) \rightarrow (Y, \tau_\mu)$ is continuous.

Theorem 4.14. Let (X, τ_1, τ_2) be pairwise N-fuzzy Hausdorff and pairwise N-fuzzy regular space,Y_k be a τ_2-1* compact ordinary subsets of X means N-fuzzy subsets and $\tau_1(\tau_2)_{1yk}$ is N-fuzzy regular for each Y_k.

Proof. Let us consider $x\epsilon Y_k$ and $R\epsilon(\tau_1(\tau_2)_{1yk})^c$ and $\mu\epsilon(-1, 0)$,such that $\mu < -1 - R(x)$. Then $R = Y_k \cup R_1$ where R_1 is $\tau_1(\tau_2)$ closed. From the continuity of i_{yk} : $(Yk, \tau_1) \rightarrow (X, \tau_1, \tau_2)$, and $i_{yk}^{-1}(R_1)\tau_{1\ 1yk}$ is closed, that is $R = Y_k \cup R_1$ is $\tau_{1\ 1yk}$-closed. Since Y_k is τ_1-1* compact set in the pairwise Hausdorff space (X, τ_1, τ_2) and Y_k is τ_1 closed. So by lemma above, R is τ_1 closed. The pair (X, τ_1, τ_2) is pairwise N-fuzzy regular there exists $S\epsilon\tau_1$ and $U\epsilon\tau_2$ with the condition $\mu < S(x)$,$R \subseteq U$ and $S \subseteq -1 - U$ also $S\epsilon\tau_1 \subseteq \tau_1(\tau_2)$ implies that $S\epsilon\tau_1 \subseteq \tau_1(\tau_2)$. It can be shown that $U\epsilon\tau_1(\tau_2)$ that $(R \subseteq Y_k \cup U)$, $\mu < (Y_k \cup S)(x)$ and $Y_k \cup S \subseteq -1 - Y_k \cup U$. Therefore (Y_k, τ_1, τ_2), then i_{yk}^{-1} is N-fuzzy regular.

Theorem 4.15. Let (X, τ_1, τ_2) be a pairwise N-fuzzy Hausdorff space in which every τ_2-1* compact sets are τ_2-1* compact. Let $Y_k\tau_2$-1* compact ordinary sets and $\tau_1(\tau_2)$ be the mixed topology on X. If (X, τ_1, τ_2) is a N-fuzzy regular space then $(Y_k, \tau_1 i_{yk}^{-1}, \tau_2 i_{yk}^{-1})$ is pairwise N-fuzzy regular for each Y_k.

Proof. Let us consider $x\epsilon Y_k$ and R is a $\tau_1 i_{yk}^{-1}$ closed set and $R = Y_k \cup R_o$ where R_o is τ_1 closed and Y_k is τ_1 closed. Then R is $\tau_1(\tau_2)$ closed. Since (X, τ_1, τ_2) is N-fuzzy regular, for $\mu\epsilon(-1, 0)$, $R\epsilon(\tau_1(\tau_2))^c$, $x\epsilon X$ and $\mu < -1 - R(x)$ then there exist $(S, U\epsilon\tau_1, \tau_2)$ with the condition $\mu < -1 - S(x)$, $R \subseteq U$ and $S \subseteq -1 - U$. Now $i_{yk}^{-1}(s) = Y_k \cup S$ is $\tau_{1\ 1yk}$-open and so $[-1 - Y_k \cup S]$ is $\tau_{1\ 1yk}$ closed. Since Y_k is τ_1-1* compact, $[-1 - (Y_k \cup S)]$ is τ_{21yk} closed and hence $Y_k \cup S$ is τ_{21yk} open and $R \subseteq Y_k \cup U$. Now $i_{yk}^{-1}(U) = Y_k \cup U$ is $\tau_{2\ 1yk}$ open. Therefore $Y_k \cup S\epsilon\tau_{2\ 1yk}$ and $Y_k \cup U\epsilon\tau_{2\ 1yk}$, $Y_k \cup S \subseteq -1 - (Y_k \cup U)$. Hence, $(Y_k, \tau_{1\ 1yk}, \tau_{2\ 1yk})\tau_{2\ 1yk}$ is regular. So, $(Y_k, \tau_{1\ 1yk}, \tau_{2\ 1yk})$ is pairwise N-fuzzy regular for each Y_k. Which completes the theorem.

5 Conclusion

The results in this paper give us the structural properties of a N-Fuzzy bi-topological space and pairwise separation axioms which is the generalization of natural separation axioms. Many more results and its structural properties and applications can be expected.

Acknowledgement

The authors are grateful to the Editor and reviewers comments.

References

[1] L. A. Zadeh, *Fuzzy sets*, Information and Control 8 (1965) 338-353.

[2] N. Palaniappan, *Fuzzy topology*, Narosa Publications, (2002).

[3] C. L. Chang, *Fuzzy topological spaces*, J Math Anal Appl 24 (1968) 182-192.

[4] S. Ganguly and S. Saha, *on separation axioms and separation of connected sets in Fuzzy topological space*, Bull cal.Math Soc 79 (1987) 215-225.

[5] N. R. Das and P. C. Baishya, *Fuzzy bi-topological space and separation Axioms*, The Jour, Fuzzy Math 2 (1994) 3890-386.

[6] R. L. Srivastava and A. K. Srivastava, *Fuzzy T1 topological spaces*, J Math Anal. Appl 102 (1984) 442-448.

[7] A. K. Srivastava, M. Ali, Dewan, *A note on K K Azad's Fuzzy Hausdorffness conceptsn*, Fuzzy sets and systems 42 (1991) 363-367.

[8] J. C. Kelley, *Bi-topological Spaces*, Inoc London Math. Soc 3 (1963) 781-89.

[9] J. Mahanta and P. K. Das, *Results on Fuzzy soft topological space*, Math. G.M March -3 (2012) PP1-11.

[10] T. J. Neog, D. K. Sut and G. C. Hazarika, *Fuzzy soft topological Space*, Int. J. Latest Trend Math Vol.2 No.1 March (2012).

[11] R. Srivastava, *On separation axioms in a newly defined fuzzy topology*, Fuzzy Sets and Systems, 62 (1994) 341-346.

[12] R. Srivastava, M. Srivastava, *On pairwise Hausdorff fuzzy bitopological spaces5*, J. of Fuzzy Math, 5 (1997) 553-564.

[13] R. Srivastava, M. Srivastava, *On certain separation axioms in fuzzy bitopological spaces*, Far East Journal of Math. Sciences, 27 (2007) 579-587.

[14] A. K. Srivastava, R. Srivastava, *Sierpinski object in fuzzy topology*, in: Proc. Int. Symp. on Modern Anal. and Appls. (Conf. Proc.), Prentice-Hall of India, (1985) 17- 25.

[15] R. Srivastava, S. N. Lal and A. K. Srivastava, *On fuzzy T_0 and R_0 topological spaces*, Journal of Mathematical Analysis and Applications, 136 (1988) 6673.

[16] R. Srivastava, S. N. Lal and A. K. Srivastava, *Fuzzy Hausdorff topological spaces*, Journal of Mathematical Analysis and Applications, 81 (1981) 497506.

[17] R. Lowen, *Fuzzy topological spaces and fuzzy compactness*, Journal of Mathematical Analysis and Applications, 56 (1976) 621633.

[18] S. Carlson, *Fuzzy topological spaces, Part 1: Fuzzy sets and fuzzy topologies*, Early ideas and obstacles, Rose-Hulman Institute of Technology.

[19] S. Willard, *General Topology*, Dover Publications, New York, 2004.

[20] B. C. Tripathy and A. Baruah, *Lacunary statically convergent and lacunary strongly convergent generalized difference sequences of fuzzy real numbers.* Kyungpook Mathematical Journal, 50(4),(2010) 565-574.

[21] B. C. Tripathy, A. Baruah, M. Et and M. Gungor, *On almost statistical convergence of new type of generalized difference sequence of fuzzy numbers*. Iranian Journal of Science and Technology (Sciences), 36(2), (2012) 147-155.

[22] B. C. Tripathy and A. J. Dutta, *Lacunary bounded variation sequence of fuzzy real numbers*. Journal of Intelligent and Fuzzy Systems, 24(1), (2013) 185-189.

[23] B. C. Tripathy, J. D. Sarma, *On weakly b-continuous functions in bitopological spaces*. Acta Scientiarum. Technology, 35(3), (2013).

[24] B. C. Tripathy ans S. Acharjee, *On (Y, d)-Bitopological semi-closed set via topological ideal. Proyecciones*. Journal of Mathematics, 33(3), (2014) 245-257.

[25] B. C. Tripathy and S. Debnath, *Y-open sets and Y-continuous mappings in fuzzy bitopological spaces*. Journal of Intelligent and Fuzzy Systems, 24(3), (2013) 631-635.