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Mathematical Model of Infertility Due to Obesity in Female

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Abstract - Now-a-days the issue of obesity is increasing worldwide due to food habits and has become a major problem. A primary reason for the exponential increase in obesity is due to consumption of high calorie food. Obesity has injurious effect on human health including the reproductive system in female. The risk of infertility, miscarriage rate and pregnancy complications are more in obese female. The impact of obesity is very risky in maturity phase of female. In order to study the impact of infertility in susceptible female due to obesity, the system of non-linear ordinary differential equations is formulated. The stability analysis is studied for infertility model. Numerical simulation is carried out using data to support the proposed analysis.

Keywords *- Mathematical Model, Obesity, Infertility, Basic Reproduction Number, Stability, Simulation*

1 Introduction

Healthy life and life-style of an individual are directly related with food habits. Food habits to maintain health is a challenge for an individual in fast era of life [2]. Unhealthy person may have many health problems like obesity, heart disease, diabetes, blood pressure etc.

In recent years, there has been an exponential rise in the number of obese individuals especially in developed countries like India, United States, Germany so that the World Health Organization (WHO) has declared obesity as an epidemic [3]. Obese individuals are at a much higher risk for serious medical conditions such as certain types of cancers, infertility, heart disease, diabetes, liver disease, brain stroke etc. The recent studies have proved that the rise of obesity among the world population because of high calorie intake coupled with less physical activity. Obesity occurs when someone regularly takes in more calories than it burns [4]. The average physically active man needs about 2,500 calories a day and woman need about 2,000 calories a day to maintain a Body Mass Index (BMI) [6]. Fast foods reduce the quality of diet and provide unhealthy choices especially among

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youngsters which are increasing since past four decades. Fast food contains calories, fat, sugar, carbohydrates, and salt that causes obesity [5].

Obesity has injurious effect for human health including reproductive system in female by causing hormonal imbalance and ovulation problem [7]. Obesity is considered as additional risk factor for infertility in female who have regular cycles. The female affected by obesity not only have problems with fertility, but are also at a greater risk for pregnancy complications.

In this paper, we will analyze how obesity causes infertility due to high calorie food in female using SEI model. Table 1 consisting of notations and its description along with its parametric values and mathematical model are described in Section 2. Local and global stability of the system is studied in Section 3. Section 4 validates proposed problem with numerical simulation.

2 Mathematical Model

Here, we formulate a mathematical model for the analysis of infertility due to obesity in female who is fond of high calorie food. The notations along with its parametric values are shown in Table 1.

Table 1. Notations and its Parametric Values

Notations		**Parametric Values**
$N(t)$	Sample size at any instant of time t	1000
$S(t)$	Number of susceptible female at any instant of time t	100
$C(t)$	Number of female taking calorie food at any instant of time t	35
$O_B(t)$	Number of female becoming obese at any instant of time t	25
$I(t)$	Number of Infertile female at any instant of time t	18
B	New recruitment rate	0.20
β	Rate at which susceptible female takes calorie food	0.20
δ	Rate at which susceptible female acquires obesity due to high calorie food	0.25
η	Rate at which creates infertility in female due to obesity	0.70
α	Rate at which susceptible female is naturally obese	0.05
ε	Rate at which infertile female starts consuming calorie food	0.40
μ	Natural escape rate	0.40

The transmission diagram of the female being infertile due to high calorie food is shown in Figure 1.

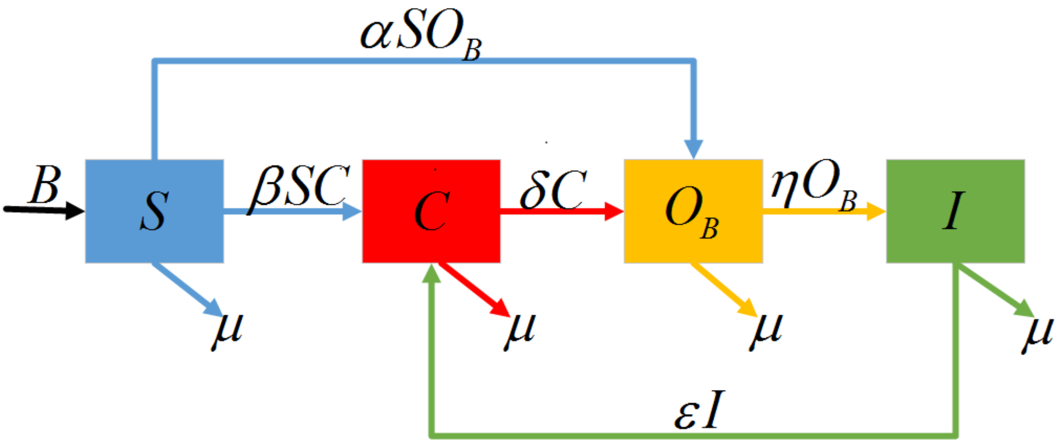

Figure 1. Transmission of infertility model

In this model, susceptible female (S) becomes obese in two ways. Firstly susceptible female becomes naturally obese at the rate α and secondly β is the rate which describes the female consuming high calorie food (C); and become obese at the rate δ. This obesity leads to infertility among female at the rate η. It may happen that this infertile may again starts to take calorie food in their diet at the rate ε. Here, B and μ describes new recruitment rate and natural escape rate respectively.

Figure 1 is described by the system of following non-linear ordinary differential equations.

$$
\begin{aligned}
\frac{dS}{dt} &= B - \alpha SO_B - \beta SC - \mu S \\
\frac{dC}{dt} &= \beta SC - \delta C + \varepsilon I - \mu C \\
\frac{dO_B}{dt} &= \alpha SO_B + \delta C - \eta O_B - \mu O_B \\
\frac{dI}{dt} &= \eta O_B - \varepsilon I - \mu I
\end{aligned}
\tag{1}
$$

Here, $S + C + O_B + I \le N$ and $S > 0, C, O_B, I \ge 0$.

Adding the above set of systems of equations (1) we get,

$$\frac{d}{dt}\left(S + C + O_B + I\right) = B - \mu\left(S + C + O_B + I\right) \ge 0 \tag{2}$$

This gives, $\lim_{t\to\infty} \sup\left(S + C + O_B + I\right) \le \frac{B}{\mu}$. (3)

Thus the feasible region for (1) is,

$$\Lambda = \left\{ \left(S + C + O_B + I \right) \big/ S + C + O_B + I \le \frac{B}{\mu}, S > 0, C, O_B, I \ge 0 \right\}. \tag{4}$$

On solving these set of equation (1) by putting equal to zero we get the equilibrium point. Therefore, the equilibrium point of the model is $E_0 = \left(\frac{B}{\mu}, 0, 0, 0 \right)$.

Now, we need to calculate the basic reproduction number to know the motion of susceptible female in the system using the next generation matrix method [1]. The next generation matrix method gives spectral radius of matrix fv^{-1} where f and v are the Jacobian matrices of F and V evaluated with respect to each compartment at an equilibrium state.

Let $X = \left(C, O_B, I, S \right)$

$$\therefore \frac{dX}{dt} = F\left(X \right) - V\left(X \right) \tag{5}$$

where $F\left(X \right)$ denotes the rate of new obese female in the compartment and $V\left(X \right)$ denotes the rate transmission of obese female from one compartment to other which is as follows

$$F\left(X \right) = \begin{bmatrix} \beta SC \\ \alpha SO_B \\ 0 \\ 0 \end{bmatrix} \text{ and } V\left(X \right) = \begin{bmatrix} \delta C - \varepsilon I + \mu C \\ -\delta C + \eta O_B + \mu O_B \\ -\eta O_B + \varepsilon I + \mu I \\ -B + \alpha SO_B + \beta SC + \mu S \end{bmatrix}$$

Now, the derivative of F and V calculated at equilibrium point $\left(E_0 \right)$ gives matrices f and v of order 4×4 defined as,

$$f = \left[\frac{\partial F_i \left(E_0 \right)}{\partial X_j} \right] \text{ and } v = \left[\frac{\partial V_i \left(E_0 \right)}{\partial X_j} \right] \text{ for } i, j = 1, 2, 3, 4$$

$$\text{So, } f = \begin{bmatrix} \frac{\beta B}{\mu} & 0 & 0 & 0 \\ 0 & \frac{\alpha B}{\mu} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } v = \begin{bmatrix} \delta + \mu & 0 & -\varepsilon & 0 \\ -\delta & \eta + \mu & 0 & 0 \\ 0 & -\eta & \varepsilon + \mu & 0 \\ \frac{\beta B}{\mu} & \frac{\alpha B}{\mu} & 0 & \mu \end{bmatrix}$$

Thus, the basic reproduction number R_0 is calculated at an equilibrium point E_0 is

$$R_0 = \frac{\beta B\left(\varepsilon(\eta+\mu)+\mu(\eta+\mu)\right)}{\mu^2\left(\varepsilon(\eta+\delta+\mu)+\eta(\delta+\mu)+\mu(\delta+\mu)\right)} \tag{6}$$

On solving the above set of equation (1) we get the other equilibrium point which $E^* = \left(S^*, C^*, O_B^*, I^*\right)$

Here,

$$S^* = A$$

$$C^* = \frac{\left[\begin{array}{l} A\alpha\left(\beta B\{\mu(2\varepsilon+\mu+\eta)+\varepsilon(\eta+\varepsilon)\}+\mu\left(\begin{array}{l}\varepsilon\{\mu(-2\mu-2\delta-2\eta)-\varepsilon(\mu+\delta)-\eta(\delta+\varepsilon)\}\\ +\mu\{-\delta(\mu+\eta)-\mu(\mu+\eta)\}\end{array}\right)\right)\\ +\mu^2\left\{\eta\left(\eta(\mu+\delta+\varepsilon)+2\delta(\varepsilon+\mu)\right)+\mu\left(\mu(2\eta+\mu+\delta)+\varepsilon(3\eta+2\delta)\right)+\varepsilon^2(\eta+\delta+\mu)\right\}\\ +B\left\{\left(\beta\mu\{\mu(-2\varepsilon-2\eta-\mu)-\eta(3\varepsilon+\eta)\}+\varepsilon^2\eta(\alpha+\beta)\right)+\eta\varepsilon(\alpha\mu-\beta\eta)\right\} \end{array}\right]}{\left[\mu\{\mu(\mu+\delta+\eta+\varepsilon)+\delta\eta+\varepsilon(\delta+\eta)\}\{\mu(\beta-\alpha)+\beta(\varepsilon+\eta)-\alpha\varepsilon\}\right]}$$

$$O_B^* = \frac{\delta\left[A\mu(\mu+\varepsilon)+B(\varepsilon+\mu)\right]}{\mu\left[\mu(-\mu+A\alpha-\delta-\eta-\varepsilon)-\eta(\delta+\varepsilon)-\varepsilon(\delta-A\alpha)\right]}$$

$$I^* = \frac{-\left[(-A\mu+B)\delta\eta\right]}{\mu\left[\mu(-\mu+A\alpha-\delta-\eta-\varepsilon)-\eta(\delta+\varepsilon)-\varepsilon(\delta-A\alpha)\right]}$$

where,

$$A = \frac{1}{2\left(\beta\alpha(\varepsilon+\mu)\right)}\left[\begin{array}{l} (\mu+\varepsilon)\left(\alpha(\delta+\mu)\right)+\beta\left(\mu(\varepsilon+\mu+\eta)+\eta\varepsilon\right)\pm\\ \left(\begin{array}{l} \mu^2\left(\begin{array}{l}\beta\left[\begin{array}{l}\alpha\{2\left(\eta(-\delta-2\varepsilon-\mu)+\delta(-2\varepsilon-\mu)\right)-\varepsilon(2\varepsilon+4\mu)\}\\ +\beta\{\varepsilon^2+\mu\left(\mu+2(\varepsilon+\eta)\right)+\eta(\eta+4\varepsilon)\}\end{array}\right]\\ +\alpha\left[\{\mu\left(\mu(\alpha-2\beta)+2\alpha\varepsilon\right)\}+\alpha\{\delta\left(\delta+2\mu+\varepsilon(\varepsilon+4\delta)\right)\}\right]\end{array}\right)\\ +\varepsilon\left[\begin{array}{l}\varepsilon\left(\alpha\{\beta\{2\mu(-\delta-\eta)+2\delta\eta\}+\alpha\delta(\delta+2\mu)\}+\beta^2\eta(\eta+2\mu)\right)\\ +2\mu\left(\delta^2\alpha^2+\beta^2\eta^2\right)\end{array}\right] \end{array}\right)^{\frac{1}{2}} \end{array}\right]$$

3. Stability Analysis

The equilibrium for the local and global stability of the infertility model is discussed here.

3.1. Local Stability

First, we calculate the local stability behaviour of equilibrium $E_0 = \left(\frac{B}{\mu}, 0, 0\right)$ by using Jacobian matrix J_0 [10][11]. The Jacobian matrix of the given model is as given below

$$J_0 = \begin{bmatrix} -\mu & -\beta\frac{B}{\mu} & -\alpha\frac{B}{\mu} & 0 \\ 0 & \beta\frac{B}{\mu}-\delta-\mu & 0 & \varepsilon \\ 0 & \delta & \alpha\frac{B}{\mu}-\eta-\mu & 0 \\ 0 & 0 & \eta & -\varepsilon-\mu \end{bmatrix}$$

$$trace(J_0) < 0 \text{ provided } (\alpha+\beta)\frac{B}{\mu} < 4\mu+\delta+\eta+\varepsilon \tag{7}$$

Hence, E_0 is locally stable.

Next, we determine the local stability behavior of equilibrium $E^* = \left(S^*, C^*, O_B^*, I^*\right)$ by using the Ruth-Hurwitz criteria [8].

The Jacobian matrix J for equilibrium E^* of the model is as follows:

$$J = \begin{bmatrix} -\alpha O_B^* - \beta C^* - \mu & -\beta S^* & -\alpha S^* & 0 \\ \beta C^* & \beta S^* - \delta - \mu & 0 & \varepsilon \\ \alpha O_B^* & \delta & \alpha S^* - \eta - \mu & 0 \\ 0 & 0 & \eta & -\varepsilon - \mu \end{bmatrix}$$

Here, we take $\alpha O_B^* + \beta C^* + \mu = a_{11}$; $-\beta S^* + \delta + \mu = a_{22}$; $-\alpha S^* + \eta + \mu = a_{33}$; $\varepsilon + \mu = a_{44}$. Then the Jacobian J is reduced as

$$J = \begin{bmatrix} -a_{11} & -\beta S^* & -\alpha S^* & 0 \\ \beta C^* & -a_{22} & 0 & \varepsilon \\ \alpha O_B^* & \delta & -a_{33} & 0 \\ 0 & 0 & \eta & -a_{44} \end{bmatrix}$$

Now, the corresponding characteristic equation of Jacobian matrix J is

$$x^4 + A_{11}x^3 + A_{22}x^2 + A_{33}x + A_{44} = 0$$

where

$$\begin{aligned}
A_{11} &= a_{44} + a_{33} + a_{22} + a_{11} \\
A_{22} &= a_{44}\left(a_{33} + a_{22} + a_{11}\right) + a_{33}\left(a_{22} + a_{11}\right) + S^*\left(\alpha^2 O^* + \beta^2 C^*\right) \\
A_{33} &= a_{44}a_{33}\left(a_{22} + a_{11}\right) + a_{22}a_{11}\left(a_{44} + a_{33}\right) + a_{44}S^*\left(\alpha^2 O^* + \beta^2 C^*\right) \\
&\quad + \delta\left(\beta\alpha C^* S^* - \eta\varepsilon\right) + S^*\left(\alpha^2 O^* + \beta^2 C^*\right) \\
A_{44} &= a_{44}a_{33}\left(\beta^2 C^* S^* + a_{22}a_{11}\right) + a_{44}S^*\left(\alpha^2 O^* a_{22} + \alpha\beta\delta C^*\right) + \eta\varepsilon\left(\alpha\beta O^* S^* - \delta a_{11}\right)
\end{aligned}$$

Here, $A_{11} > 0, A_{22} > 0, A_{33} > 0 \, and \, A_{44} > 0$ and satisfy the condition of Routh-Hurwitz criterion (Routh E.J. 1877). Then the equilibrium E^* is locally stable.

3.2. Global Stability

The infertility model is globally stable if $\det\left(I - fv^{-1}\right) > 0$ [10][11].

$$\det\left(I - fv^{-1}\right) = 1 - R_0 = 1 - 0.1752 = 0.8248 > 0 \,. \tag{8}$$

Hence, E_0 is globally stable.

Now, we discuss the global stability behavior of E^* by Lyapunov function [9].

Consider the Lyapunov function

$$\begin{aligned}
L(t) &= \frac{1}{2}\left[\left(S - S^*\right) + \left(C - C^*\right) + \left(O_B - O_B^*\right) + \left(I - I^*\right)\right]^2 \\
L'(t) &= \left[\left(S - S^*\right) + \left(C - C^*\right) + \left(O_B - O_B^*\right) + \left(I - I^*\right)\right]\left[S' + C' + O_B' + I'\right] \\
&= \left[\left(S - S^*\right) + \left(C - C^*\right) + \left(O_B - O_B^*\right) + \left(I - I^*\right)\right] \\
&\qquad \left[\mu S^* + \mu C^* + \mu O_B^* + \mu I^* - \mu S - \mu C - \mu O_B - \mu I\right] \\
&= -\mu\left[\left(S - S^*\right) + \left(C - C^*\right) + \left(O_B - O_B^*\right) + \left(I - I^*\right)\right]^2 \le 0
\end{aligned}$$

Here we used, $B = \mu S^* + \mu C^* + \mu O_B^* + \mu I^*$.

Hence, E^* is globally stable.

4 Sensitivity Analysis

In this section, the sensitivity analysis for all model parameters is discussed in Table 2. The normalised sensitivity index of the parameters is computed by using the formula: $\gamma_{\theta}^{R_0} = \frac{\partial R_0}{\partial \theta} \cdot \frac{\theta}{R_0}$ where, θ denotes the model parameter [10].

Table 1. Notations and its Parametric Values

Parameter	Value	Parameter	Value
B	+	ε	+
β	+	δ	-
η	+	μ	-

New recruitment rate, rate at which susceptible female takes calorie food, rate at which creates infertility in female due to obesity and rate at which infertile female starts consuming calorie food have positive effect on R_0 which means that these parameters help to reduce infertility. The rate at which susceptible female is naturally obese have no impact on model. Other parameters have negative impact on model

5 Numerical Simulation

In this section, we will study the numerical results of all compartments.

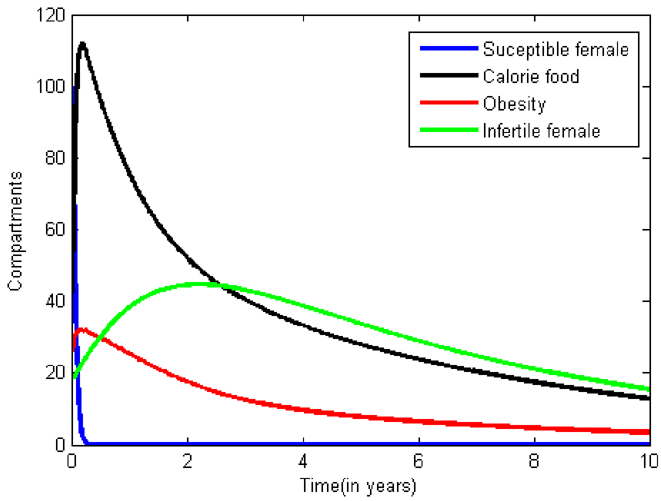

Figure 2. Movement of an individual in each compartment

Figure 2 shows that if the susceptible female who are likely to become obese starts taking more calorie food then the obesity due to it increases approximate in 1 year and this leads to increase in the infertility among female individuals.

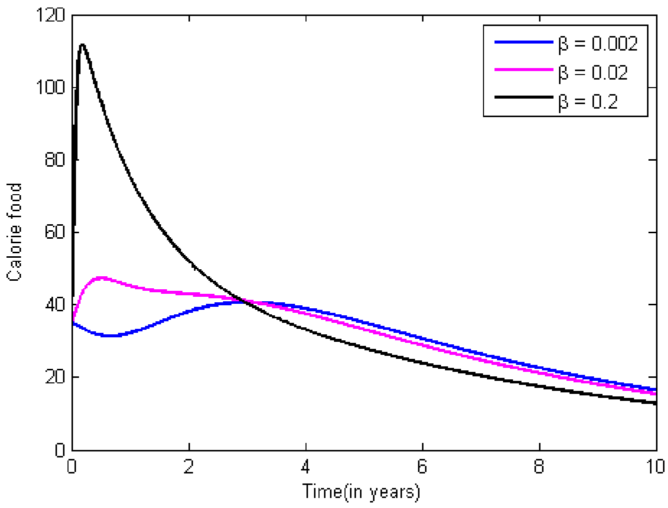

Figure 3. Effect of rate at which susceptible female takes calorie food

Figure 3 shows that if the rate at which susceptible female takes calorie food (β) is small then females are at low risk to be obese for almost 1 year and after that it increases but the opposite effect is observed for the large value of (β).

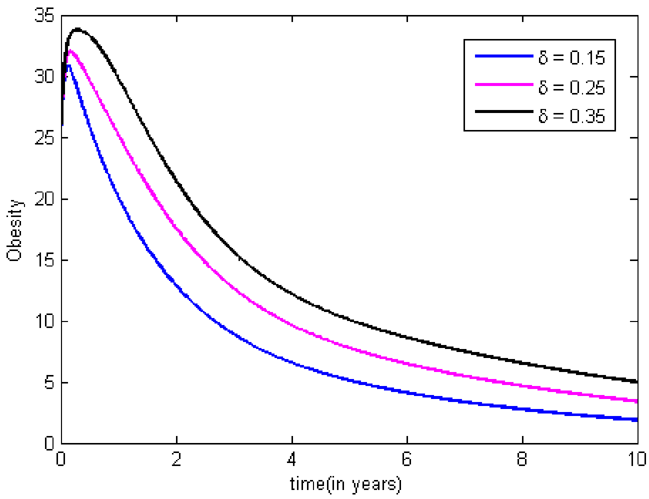

Figure 4. Effect of rate at which susceptible female acquires obesity due to high calorie food

Figure 4 indicates that if the rate of susceptible female become obese due to high calorie food (δ) is increased from 15% to 35% the, obesity in susceptible female is increased initially and there after it decreases.

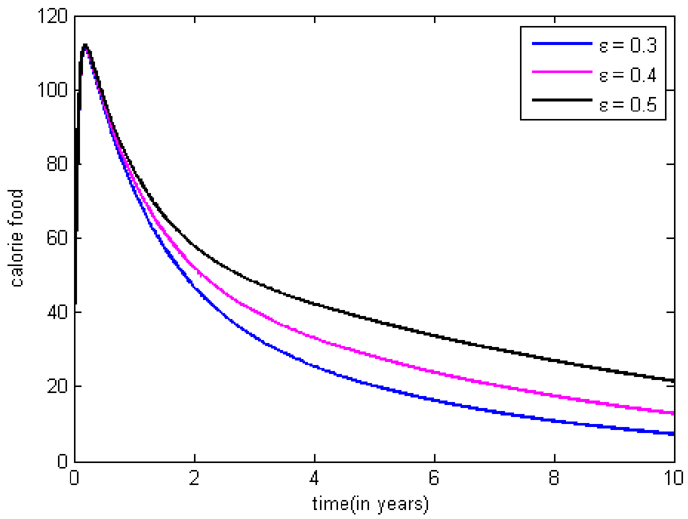

Figure 5. Effect of rate at which infertile female start consuming calorie food

Figure 5 shows that if the rate at which infertile female starts consuming calorie food (ε) is increased than number of females taking calorie food also increases almost equal in the one year after that it decreases which means individual female get conscious about their health.

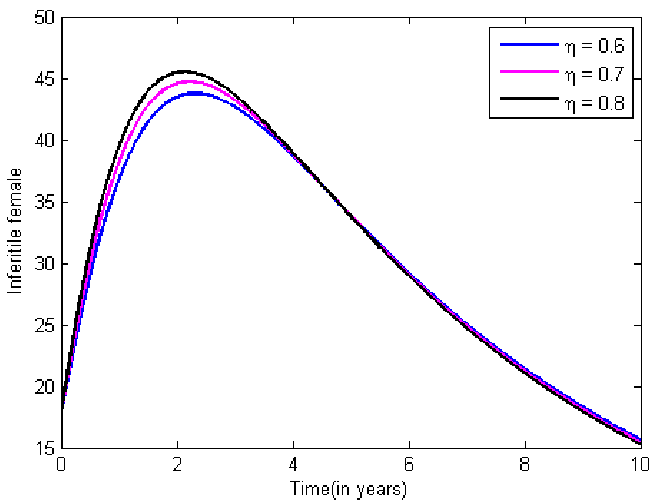

Figure 6. Effect of rate at which creates infertility in female due to obesity

Figure 6 indicates that if the rate which causes infertility in female due to obesity (η) is increased from 60% to 80% then the number of infertile female increase by approximate 43 to 46 in first two years after that it starts decreasing.

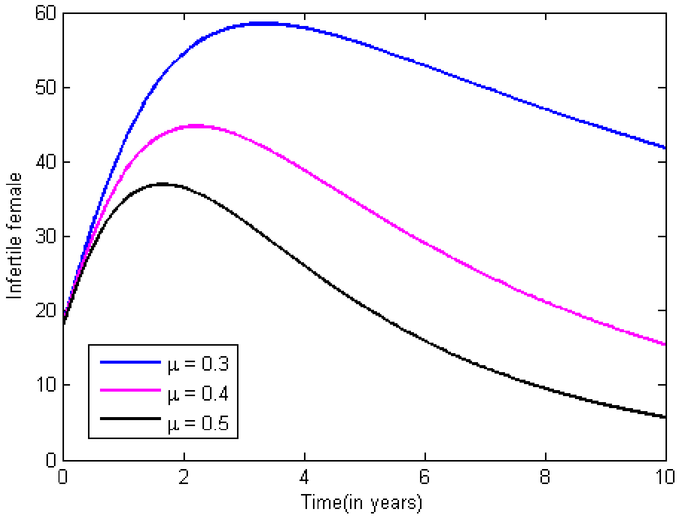

Figure 7. Effect of natural escape rate on infertility

Figure 7 shows that if one increases the natural escape rate from 30% to 50% the number of individuals increase in beginning for 3 years approximate by 36 to 60 and then after that it decreases which shows that after some years infertile female start taking healthy food and become less obese.

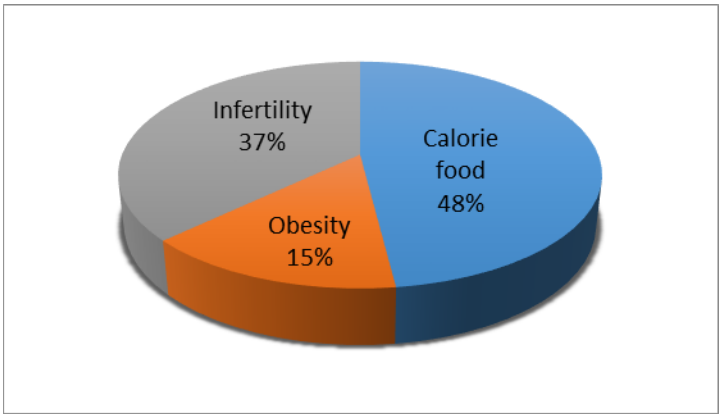

Figure 8. Percentage of calorie food, obesity and infertility in susceptible female

Figure 8 shows that if females start taking 48% calorie food in their diet, then there are 15% of chances for becoming obese, due to which 37% of infertility can prevail among these female individuals during their maturity stage.

5 Conclusion

In this paper, a mathematical model for the cause of infertility due to obesity is formulated. Also the system proves to be locally and globally stable at an equilibrium point by satisfying all its condition. The basic reproduction number is computed as 0.1752, which show that 17% of female affected by infertility due to high calorie food. The fact is infertility tends to go up in the proportion to the amount of access weight one is carrying. It is impossible to get read of this risk without losing weight, so it very important to maintain our health by regular physical activity or by any other means. Every individual should take care of their health so as to avoid risk in future. This model will help to educate society to become more health conscious.

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