Sakarya University Journal of Science, 22 (6), 1878-1885, 2018.

SAKARYA ÜNİVERSİTESİ FEN BİLİMLERİ ENSTİTÜSÜ DERGİSİ
SAKARYA UNIVERSITY JOURNAL OF SCIENCE
e-ISSN: 2147-835X
Dergi sayfası: http://www.saujs.sakarya.edu.tr

Received
09-05-2018
Accepted
01-10-2018

Doi
10.16984/saufenbilder.422168

Determining the Effect of Some Biasing Parameter Selection Methods for the Two Stage Ridge Regression Estimator

Nimet Özbay [*1], Selma Toker [2]

ABSTRACT

The use of biased estimation techniques is inevitable in connection with multicollinearity in simultaneous equations model. Two stage ridge estimator is a pioneer biased estimator which is used to recover the problems that are originated from the multicollinearity. The noteworthy issue regarding two stage ridge estimator is selection of its biasing parameter. Based on the works in the literature related to ridge estimator in a linear regression model, several methods on selection of the biasing parameter of the two stage ridge estimator are investigated in this paper. To demonstrate the best estimators of the biasing parameter, a Monte Carlo experiment is conducted. The utility of the proposed estimators of the biasing parameter for two stage ridge estimator is observed in terms of mean square error criterion.

Keywords: biasing parameter, multicollinearity, ridge estimator, two stage least squares

[*] Corresponding Author
[1] Çukurova University, Department of Statistics, nturker@cu.edu.tr
[2] Çukurova University, Department of Statistics, stoker@cu.edu.tr

1. INTRODUCTION

Let $Y_{T\times M}$ and $X_{T\times K}$ be matrices of observations, $\Gamma_{M\times M}$ and $B_{K\times M}$ be the matrices of structural coefficients and $U_{T\times M}$ be the matrix of structural disturbances. Then, simultaneous equations model is shown in the matrix form below:

$$Y\Gamma + XB = U \tag{1}$$

where the elements of X are nonstochastic and fixed with $rank(X) = K \leq T$ and the structural disturbances have zero mean and they are homoscedastic. The reduced form of the model (1) can be written as follows:

$$Y = X\Pi + V. \tag{2}$$

In equation (2)

$$\Pi = -B\Gamma^{-1} \tag{3}$$

and

$$V = U\Gamma^{-1} \tag{4}$$

are the reduced form coefficients.

$$y_1 = Y_1\gamma_1 + X_1\beta_1 + u_1 \tag{5}$$

is the first equation of the system which is derived according to zero restrictions criterion. For $m_1 + 1$ included and $m_1^* = M - m_1 - 1$ excluded jointly dependent variables and K_1 included and $K_1^* = K - K_1$ excluded predetermined variables, $Y = [y_1 \quad Y_1 \quad Y_1^*]$ and $X = [X_1 \quad X_1^*]$ are assumed to be variables with the size of $T \times m_1$, $T \times m_1^*$, $T \times K_1$ and $T \times K_1^*$ corresponding to Y_1, Y_1^*, X_1 and X_1^*. $\gamma_{.1} = [1 \quad -\gamma_1 \quad 0\,]'$ and $\beta_{.1} = [-\beta_1 \quad 0\,]'$ are assumed to be variables with the size of $m_1 \times 1$ and $K_1 \times 1$ corresponding to γ_1 and β_1 and u_1 is the first column of U.

$$[y_1 \quad Y_1 \quad Y_1^*] = [X_1 \quad X_1^*]\begin{bmatrix}\pi_{11} & \Pi_{11} & \Pi_{11}^* \\ \pi_{21} & \Pi_{21} & \Pi_{21}^*\end{bmatrix} + [v_1 \quad V_1 \quad V_1^*],$$

is the partition of the reduced form equation (2) with the variables

$$y_1 = X\pi_1 + v_1$$

and

$$Y_1 = X\Pi_1 + V_1, \tag{6}$$

where $\pi_1 = [\pi_{11} \quad \pi_{21}]'$ and $\Pi_1 = [\Pi_{11} \quad \Pi_{21}]'$ are assumed to be variables having the size of $K_1 \times 1$, $K_1^* \times 1$, $K_1 \times m_1$, $K_1^* \times m_1$, $T \times 1$ and $T \times m_1$ corresponding to π_{11}, π_{21}, Π_{11}, Π_{21}, v_1 and V_1. Three subsequent equations show the identifiability relationship between the structural parameters and the reduced form parameters for the first equation:

$$\pi_{11} = \Pi_{11}\gamma_1 + \beta_1,$$

$$\pi_{21} = \Pi_{21}\gamma_1$$

and

$$v_1 = V_1\gamma_1 + u_1 \tag{7}$$

by taking account of only the first column of Γ, B and U in the reduced form coefficients (3) and (4).

With the notations

$$Z_1 = [\,Y_1 \quad X_1]_{T\times p_1},$$

and

$$\delta_1 = [\,\gamma_1 \quad \beta_1\,]'_{p_1\times 1}$$

where $p_1 = m_1 + K_1$, the first equation of the system (5) is obtained as

$$y_1 = Z_1\delta_1 + u_1. \tag{8}$$

The structural equation (8) is formed as below by replacing the equations (6) and (7),

$$y_1 = [\,X\Pi_1 \quad X_1\,]\begin{bmatrix}\gamma_1 \\ \beta_1\end{bmatrix} + v_1. \tag{9}$$

Reconsidering the equation (9), the final form reveals as follows:

$$y_1 = \bar{Z}_1\delta_1 + v_1, \tag{10}$$

where $\bar{Z}_1 = E(Z_1) = [\,X\Pi_1 \quad X_1\,]$, $E(v_1) = 0$ and $E(v_1v_1') = \sigma^2 I$.

As a most common technique, two stage least squares (TSLS) estimation is applied to simultaneous equations model to estimate the structural parameters. The way for this purpose in the first stage is to replace explanatory endogenous variables by their instrumental variables which are ordinary least squares (OLS) estimates that are obtained by using the exogenous variables. Next for the second stage, the regression coefficients are estimated again by the OLS estimator. TSLS estimator is defined as follows

$$\delta_1^{LS} = (\bar{Z}_1'\bar{Z}_1)^{-1}\bar{Z}_1'y_1. \tag{11}$$

Since $\bar{Z}_1$ is unknown,

$$\widehat{\Pi}_1 = (X'X)^{-1}X'Y_1$$

is used at the first stage to constitute

$$\hat{\bar{Z}}_1 = [\,X\widehat{\Pi}_1 \quad X_1].$$

By substituting this estimation of $\bar{Z}_1$ in the equation (11), the operational form of the TSLS estimator is derived as follows:

$$\hat{\delta}_1^{LS} = \left(\hat{\bar{Z}}_1'\hat{\bar{Z}}_1\right)^{-1}\hat{\bar{Z}}_1'y_1.$$

Despite ease of computation of the TSLS estimator, running into multicollinearity in simultaneous equations model leads us to find alternative biased estimation methods to the TSLS estimation. Within this context, widely used estimator is ridge estimator (RE) (Hoerl and Kennard [1]) which is recommended for estimating parameters in simultaneous equations model by Vinod and Ullah [2]. Thus, two stage RE of Vinod and Ullah [2] plays a prominent role to eliminate the multicollinearity. Ordinary and operational forms of the two stage RE are

$$\delta_1^{RE} = (\bar{Z}_1'\bar{Z}_1 + kI)^{-1}\bar{Z}_1'y_1,$$

$$\hat{\delta}_1^{RE} = \left(\hat{\bar{Z}}_1'\hat{\bar{Z}}_1 + kI\right)^{-1}\hat{\bar{Z}}_1'y_1,$$

where $k > 0$.

2. MATERIAL AND METHOD

The model (10) can be written in a canonical form as follows

$$y_1 = Z\alpha_1 + v_1,$$

where $Z = \bar{Z}_1P$, $\alpha_1 = P'\delta_1$ and P is an orthogonal matrix such that $Z'Z = P'\bar{Z}_1'\bar{Z}_1P = \Lambda_1 = diag(\lambda_{11}, \dots, \lambda_{1p_1})$ where λ_{1i} are the eigenvalues of $\bar{Z}_1'\bar{Z}_1$. By using this canonical form, the TSLS estimator can be written as

$$\alpha_1^{LS} = \Lambda_1^{-1}Z'y_1.$$

In practice, this estimator is used as follows

$$\hat{\alpha}_1^{LS} = \widehat{\Lambda}_1^{-1}\hat{Z}'y_1,$$

where Λ_1 is substituted by $\widehat{\Lambda}_1 = P'\hat{\bar{Z}}_1'\hat{\bar{Z}}_1P$ for the unknown Λ_1 and $\hat{Z} = \hat{\bar{Z}}_1P$ is put in the place of Z.

Let us write the two stage RE in the canonical form as

$$\alpha_1^{RE} = (\Lambda_1 + kI)^{-1}Z'y_1.$$

This estimator is used practically as follows:

$$\hat{\alpha}_1^{RE} = \left(\widehat{\Lambda}_1 + kI\right)^{-1}\hat{Z}'y_1.$$

As for comparing the performance of the estimators, the scalar mean square error (mse) is the most practical and efficient tool of measure. The mse of the TSLS estimator and the two stage RE are

$$mse(\alpha_1^{LS}) = \sigma^2\sum_{i=1}^{p_1}\frac{1}{\lambda_{1i}},$$

$$mse(\alpha_1^{RE}) = \sigma^2\sum_{i=1}^{p_1}\frac{\lambda_{1i}}{(\lambda_{1i}+k)^2} + k^2\sum_{i=1}^{p_1}\frac{\alpha_{1i}^2}{(\lambda_{1i}+k)^2}, \quad (12)$$

where the first part of the equation (12) represents the function of variance while the second part shows the function of squared bias.

Two stage RE is preferable to the TSLS estimator with regard to more reliable calculations in the existence of multicollinearity. In the meantime, there is a difficulty in using the two stage RE depending on the selection of its biasing parameter. To eliminate this problem, we consider various methods which are also examined for linear regression model in the literature: Hoerl and Kennard [1], Hoerl et al. [3], Lawless and Wang [4], Hocking et al. [5], Kibria [6], Khalaf and Shukur [7], Alkhamisi et al. [8], Alkhamisi and Shukur [9], Muniz and Kibria [10] and Muniz et al. [11]. The aforementioned studies are broadly summarized in Mansson et al. [12], hence we follow this article to estimate the biasing parameter of the two stage RE.

We mainly investigate the estimators of the biasing parameter of the two stage RE below:

Hoerl and Kennard [1]:

$$\hat{k}_1 = \frac{\hat{\sigma}^2}{\hat{\alpha}_{1max}^2},$$

where $\hat{\sigma}^2$ is the unbiased estimator of σ^2 and $\hat{\alpha}_{1max}$ is the maximum element of $\hat{\alpha}_1$.

Hoerl et al. [3]:

$$\hat{k}_2 = \frac{p_1\hat{\sigma}^2}{\sum_{i=1}^{p_1}\hat{\alpha}_{1i}^2}.$$

Lawless and Wang [4]:

$\hat{k}_3 = \frac{p_1 \hat{\sigma}^2}{\sum_{i=1}^{p_1} \lambda_{1i} \hat{\alpha}_{1i}^2}$.

Hocking et al. [5]:

$\hat{k}_4 = \hat{\sigma}^2 \frac{\sum_{i=1}^{p_1} (\lambda_{1i} \hat{\alpha}_{1i})^2}{(\sum_{i=1}^{p_1} \lambda_{1i} \hat{\alpha}_{1i}^2)^2}$.

Kibria [6]:

$\hat{k}_5 = \frac{1}{p_1} \sum_{i=1}^{p_1} \frac{\hat{\sigma}^2}{\hat{\alpha}_{1i}^2}$, $\hat{k}_6 = \frac{\hat{\sigma}^2}{(\prod_{i=1}^{p_1} \hat{\alpha}_{1i}^2)^{\frac{1}{p_1}}}$,

$\hat{k}_7 = Median \left\{ \frac{\hat{\sigma}^2}{\hat{\alpha}_{1i}^2} \right\}$.

Khalaf and Shukur [7]:

$\hat{k}_8 = \frac{\lambda_{1max} \hat{\sigma}^2}{(T-p_1)\hat{\sigma}^2 + \lambda_{1max} \hat{\alpha}_{1max}^2}$,

where λ_{1max} is the maximum eigenvalue of $\bar{Z}_1' \bar{Z}_1$.

Alkhamisi et al. [8]:

$\hat{k}_9 = max \left(\frac{\lambda_{1i} \hat{\sigma}^2}{(T-p_1)\hat{\sigma}^2 + \lambda_{1i} \hat{\alpha}_{1i}^2} \right)$,

$\hat{k}_{10} = Median \left(\frac{\lambda_{1i} \hat{\sigma}^2}{(T-p_1)\hat{\sigma}^2 + \lambda_{1i} \hat{\alpha}_{1i}^2} \right)$.

Muniz and Kibria [10]:

$\hat{k}_{11} = \left(\prod_{i=1}^{p_1} \frac{\lambda_{1i} \hat{\sigma}^2}{(T-p_1)\hat{\sigma}^2 + \lambda_{1i} \hat{\alpha}_{1i}^2} \right)^{\frac{1}{p_1}}$,

$\hat{k}_{12} = max \left(\frac{1}{\sqrt{\hat{\sigma}^2 / \hat{\alpha}_{1i}^2}} \right)$,

$\hat{k}_{13} = \left(\prod_{i=1}^{p_1} \frac{1}{\sqrt{\hat{\sigma}^2 / \hat{\alpha}_{1i}^2}} \right)^{\frac{1}{p_1}}$, $\hat{k}_{14} = \left(\prod_{i=1}^{p_1} \sqrt{\hat{\sigma}^2 / \hat{\alpha}_{1i}^2} \right)^{\frac{1}{p_1}}$,

$\hat{k}_{15} = Median \left(\frac{1}{\sqrt{\hat{\sigma}^2 / \hat{\alpha}_{1i}^2}} \right)$.

Muniz et al. [11]:

$\hat{k}_{16} = max \left(\frac{(T-p_1)\hat{\sigma}^2 + \lambda_{1max} \hat{\alpha}_{1i}^2}{\lambda_{1max} \hat{\sigma}^2} \right)$,

$\hat{k}_{17} = max \left(\frac{\lambda_{1max} \hat{\sigma}^2}{(T-p_1)\hat{\sigma}^2 + \lambda_{1max} \hat{\alpha}_{1i}^2} \right)$,

$\hat{k}_{18} = \left(\prod_{i=1}^{p_1} \frac{(T-p_1)\hat{\sigma}^2 + \lambda_{1max} \hat{\alpha}_{1i}^2}{\lambda_{1max} \hat{\sigma}^2} \right)^{\frac{1}{p_1}}$,

$\hat{k}_{19} = \left(\prod_{i=1}^{p_1} \frac{\lambda_{1max} \hat{\sigma}^2}{(T-p_1)\hat{\sigma}^2 + \lambda_{1max} \hat{\alpha}_{1i}^2} \right)^{\frac{1}{p_1}}$,

$\hat{k}_{20} = Median \left(\frac{(T-p_1)\hat{\sigma}^2 + \lambda_{1max} \hat{\alpha}_{1i}^2}{\lambda_{1max} \hat{\sigma}^2} \right)$.

3. AN APPLICATION: A MONTE CARLO SIMULATION

A Monte Carlo simulation is a prominent way to demonstrate how the estimators of the biasing parameter effect the mastery of the two stage RE. These are the papers in which some Monte Carlo studies are handled in the simultaneous equations model: Wagnar [13], Hendry [14], Park [15], Capps Jr and Grubbs [16], Johnston and Dinardo [17], Geweke [18], Agunbiade [19,20] and Agunbiade and Iyaniwura [21].

The structural form of the model built by Agunbiade and Iyaniwura [21] corresponding to three structural equations Equation 1, Equation 2 and Equation 3 is as follows:

$y_{1t} = y_{3t}\gamma_{13} + x_{1t}\beta_{11} + x_{2t}\beta_{12} + u_{1t}$,

$y_{2t} = y_{1t}\gamma_{21} + x_{1t}\beta_{21} + x_{3t}\beta_{23} + u_{2t}$,

$y_{3t} = y_{2t}\gamma_{32} + x_{2t}\beta_{32} + x_{3t}\beta_{33} + u_{3t}$.

Arbitrary model parameters for this structural model are also given as:

$\gamma_{13} = 1.8$, $\beta_{11} = 0.2$, $\beta_{12} = 1.2$,

$\gamma_{21} = 1.5$, $\beta_{21} = 2.5$, $\beta_{23} = 2.1$,

$\gamma_{32} = 0.9$, $\beta_{32} = 0.4$, $\beta_{33} = 3.3$.

Taking account of different levels of error variance (κ) and multicollinearity degree (ρ) with sample size $T = 60$ and using the root mean square error $(rmse)$ criterion, the TSLS estimator and the two stage RE are compared empirically.

We generate predetermined variables having $N_3(0, \Sigma)$ where Σ is assumed to be as a correlation matrix with a given correlation ρ. Similarly, multivariate normal distribution is used to generate the error terms according to variance-covariance matrix below with the parameter κ:

$$\kappa \times \begin{bmatrix} 7.0 & 5.0 & 4.0 \\ 5.0 & 4.5 & 3.5 \\ 4.0 & 3.5 & 3.0 \end{bmatrix}.$$

Different choices of values that are used for the parameters are $\rho = 0.70, 0.80, 0.90, 0.99$ and $\kappa = 0.1, 1, 10, 100$. The experiment is repeated 10000 times by using MATLAB program. After the sample is generated, the estimated $rmse$ is calculated by

$$\widehat{rmse}(\hat{\theta}) = \sqrt{\frac{1}{10000}\sum_{j=1}^{10000}(\hat{\theta}_j - \theta)'(\hat{\theta}_j - \theta)},$$

where θ is the parameter vector of a given structural equation, $\hat{\theta}$ is the estimator of this parameter vector and $\hat{\theta}_j$ is the estimation of the parameter vector in the j-th replication. The results are summarized in Tables 1-3 corresponding to Equation 1, Equation 2 and Equation 3 for the TSLS estimator and the two stage RE.

4. RESULTS AND DISCUSSION

According to Tables 1-3, to outperform the TSLS estimator is a general conclusion for the two stage RE with different estimations of the biasing parameter. At the same time, for the smallest value of the error variance ($\kappa = 0.1$) the superiority of the TSLS estimator may be observed. The point need to focus on is the noteworthy influence of the proposed estimators of the biasing parameter on efficiency of the two stage RE. By virtue of this simulation study, which estimator of the biasing parameter will be preferred is demonstrated. Thus, the difficulty in the selection of the biasing parameter of the two stage RE is dispelled. In consequence, $\hat{k}_3$, $\hat{k}_6$, $\hat{k}_{11}$ and $\hat{k}_{20}$ seem to be the best performed biasing parameters for the efficiency of the two stage RE. In connection to the variation in the level of the error variance and multicollinearity, the estimation ability of the two stage RE changes through the recommended estimators of the biasing parameter. Increasing both the level of multicollinearity and the error variance has a positive effect on the estimated $rmse$ values of the two stage RE.

Based on Tables 1-3, let us comment on the Equations 1-3 separately. Considering Table 1, two stage RE with $\hat{k}_3$ gives the smallest estimated $rmse$ values in general. However, we cannot reach a certain conclusion for a quite high level of multicollinearity. When the level of multicollinearity is lower the results are convincing for $\hat{k}_{11}$ according to Table 2, in contrast $\hat{k}_6$ is preferable when the level of multicollinearity is higher. In Table 3, taking into consideration of the estimators of the biasing parameter, it is deduced that $\hat{k}_3$ and $\hat{k}_{20}$ surpass their competitors. Moreover, Equation 1 yields the smallest estimated $rmse$ values among all the equations.

5. CONCLUDING REMARKS

The usage of the two stage RE in order to estimate the exogenous variables of a structural equation compels us to concentrate on the selection of its biasing parameter. We clarify this problem in this article since there are not too many papers that are prone to this issue in the literature. We find out the best estimator of the biasing parameter within $\hat{k}_1$ to $\hat{k}_{20}$ by a comprehensive application.

As for summarizing the outcomes of the Monte Carlo experiment, it is inferred that the level of the error variance and multicollinearity is an indicator which alters the estimated $rmse$ values of the estimators. An increment in the magnitude of the error variance and multicollinearity results in an increment in the estimated $rmse$ values of the two stage RE.

Generally, each estimator of the biasing parameter provides its own effect for two stage RE to be outperform the TSLS estimator. But, especially $\hat{k}_3$, $\hat{k}_6$, $\hat{k}_{11}$ and $\hat{k}_{20}$ give the smallest estimated rmse values for two stage RE. As a result, we advise researchers, who confront with simultaneous equations model exposed to multicollinearity, that one of the proposed estimators of the biasing parameter for two stage RE can be preferred.

Table 1. Estimated $rmse$ values of the estimators in Equation 1

Equation 1												
			Two stage RE with different $\hat{k}$ values									
$\boldsymbol{\rho}$	$\boldsymbol{\kappa}$	**TSLS estimator**	$\hat{k}_1$	$\hat{k}_2$	$\hat{k}_3$	$\hat{k}_4$	$\hat{k}_5$	$\hat{k}_6$	$\hat{k}_7$	$\hat{k}_8$	$\hat{k}_9$	$\hat{k}_{10}$
0.70	0.1	0.3190	0.4580	0.3667	**0.1891**	0.2881	0.4140	0.3920	0.4580	0.4306	0.2848	0.2155
	1	1.5333	1.2525	1.0873	**0.5251**	0.9142	1.1970	1.1533	1.2455	1.0176	0.8359	0.5426
	10	246.3471	1.8468	1.4123	1.1244	1.3533	1.7830	1.5962	1.8468	1.2808	1.2654	1.1284
	100	152.8417	2.1725	1.4335	2.0220	1.4356	2.1725	2.0964	1.4359	1.3220	**1.3214**	1.9960
0.80	0.1	0.3884	0.5853	0.4748	**0.2271**	0.3706	0.5381	0.5089	0.5853	0.5533	0.3657	0.2543
	1	26.8411	1.3493	1.2031	**0.6179**	1.0455	1.3046	1.2665	1.3326	1.1541	0.9734	0.6342
	10	202.8415	2.1717	1.4345	1.2748	1.3908	2.1701	2.0605	2.0590	1.3550	1.3468	1.2762
	100	2329.8874	2.1446	1.3877	2.6943	1.4420	2.0938	1.6165	1.4424	**1.3586**	1.3591	2.6750
0.90	0.1	0.5625	0.8613	0.7015	0.3113	0.5520	0.7928	0.7522	0.8613	0.8208	0.5442	0.3375
	1	35.1412	1.4507	1.3443	**0.8101**	1.2315	1.4235	1.3972	1.4238	1.3240	1.1792	0.8226
	10	278.8939	2.1251	1.4494	1.5941	**1.4296**	2.0455	1.6333	1.5294	1.4339	1.4376	1.5832
	100	302.7489	2.0248	**1.3840**	3.8961	1.4486	1.8450	1.4593	1.4492	1.3950	1.3967	4.0281
0.99	0.1	146.6652	1.4852	1.3721	**0.8381**	1.2544	1.4689	1.4375	1.4419	1.4684	1.2480	0.8501
	1	176.0489	1.6970	1.4860	1.7629	1.4938	1.5468	1.4857	**1.4843**	1.4859	1.4989	1.7470
	10	121.6339	1.5018	1.5609	3.3718	1.4687	**1.4679**	1.4773	1.4687	1.5120	1.5278	3.5032
	100	801.3517	1.7149	1.6326	4.0039	1.4572	1.5579	**1.4094**	1.4581	1.7184	1.4364	5.6537
			Two stage RE with different $\hat{k}$ continued values									
$\boldsymbol{\rho}$	$\boldsymbol{\kappa}$	**TSLS estimator**	$\hat{k}_{11}$	$\hat{k}_{12}$	$\hat{k}_{13}$	$\hat{k}_{14}$	$\hat{k}_{15}$	$\hat{k}_{16}$	$\hat{k}_{17}$	$\hat{k}_{18}$	$\hat{k}_{19}$	$\hat{k}_{20}$
0.70	0.1	0.3190	0.2188	0.2429	0.2256	0.3076	0.2198	0.2348	0.4452	0.2115	0.3738	0.2053
	1	1.5333	0.5601	0.5337	0.5297	0.7494	0.5284	0.5277	1.0176	0.5263	0.9550	0.5258
	10	246.3471	1.1352	1.1235	1.1230	1.2786	**1.1229**	1.1230	1.2809	1.1230	1.2757	1.1230
	100	152.8417	1.7921	2.2490	2.2591	1.3348	2.2523	2.2571	1.3220	2.2573	1.3214	2.2573
0.80	0.1	0.3884	0.2624	0.3062	0.2808	0.3973	0.2727	0.2943	0.5820	0.2598	0.4859	0.2512
	1	26.8411	0.6555	0.6299	0.6239	0.8959	0.6222	0.6215	1.1541	0.6194	1.0928	0.6188
	10	202.8415	1.2792	1.2743	**1.2740**	1.3894	1.2740	1.2741	1.3550	1.2740	1.3523	1.2740
	100	2329.8874	2.2707	3.2301	3.2668	1.3646	3.2576	3.2696	1.3586	3.2712	1.3596	3.2716
0.90	0.1	0.5625	0.3545	0.4502	0.4049	0.5957	0.3888	0.4301	0.8351	0.3676	0.7220	0.3506
	1	35.1412	0.8460	0.8279	0.8173	1.1429	0.8154	0.8154	1.3240	0.8120	1.2771	0.8111
	10	278.8939	1.5567	1.6015	1.6064	1.4348	1.6057	1.6060	1.4339	1.6064	1.4352	1.6065
	100	302.7489	3.1621	5.4807	5.7750	1.4358	5.7703	5.7953	1.3950	5.8239	1.4037	5.8323
0.99	0.1	146.6652	0.8783	1.1492	1.0159	1.3452	1.0104	1.1206	1.4684	0.9390	1.4123	0.9204
	1	176.0489	1.6890	1.6493	1.7123	1.4986	1.6923	1.7115	1.4859	1.7415	1.4900	1.7453
	10	121.6339	2.8040	2.6592	3.9992	1.6897	4.2288	3.7955	1.5120	4.9547	1.5683	5.2621
	100	801.3517	5.0154	5.2434	19.9796	1.5913	23.5878	9.0615	1.4335	23.7658	1.5182	26.7838

Table 2. Estimated $rmse$ values of the estimators in Equation 2

Equation 2												
			Two stage RE with different $\hat{k}$ values									
$\boldsymbol{\rho}$	$\boldsymbol{\kappa}$	**TSLS estimator**	$\hat{k}_1$	$\hat{k}_2$	$\hat{k}_3$	$\hat{k}_4$	$\hat{k}_5$	$\hat{k}_6$	$\hat{k}_7$	$\hat{k}_8$	$\hat{k}_9$	$\hat{k}_{10}$
0.70	0.1	1.7328	2.4038	1.5714	1.0048	2.7424	2.4438	2.1268	2.4038	2.3885	2.7291	0.4568
	1	139.4882	3.2571	2.4577	4.8516	3.3082	3.2587	3.1719	3.2571	3.2423	3.2796	1.9585
	10	68.1909	3.4147	3.4153	4.7518	**3.4141**	3.4148	3.4150	3.4147	3.4340	3.4321	4.7233
	100	37.4451	**3.3788**	3.3808	4.0811	3.3853	3.3814	3.3811	3.3809	3.4435	3.4404	4.0905
0.80	0.1	35.1409	2.6655	1.8982	1.2808	2.9073	2.6926	2.4313	2.6655	2.6547	2.8968	0.5330
	1	308.9423	21.9702	9.8668	4.0395	3.3469	3.3096	3.3177	3.3079	21.9704	3.3261	3.4147
	10	161.9416	3.4166	3.4151	4.3654	3.4144	**3.4139**	3.4142	3.4143	3.4295	3.4332	4.3700
	100	218.0851	**3.3761**	3.3763	4.0589	3.3826	3.3779	3.3771	3.3791	3.4325	3.4260	4.0787
0.90	0.1	905.6709	2.9698	2.3482	2.0754	3.0989	2.9812	2.8107	2.9698	2.9644	3.0921	0.6846
	1	127.8484	3.5823	3.4065	7.5497	3.3907	3.3716	**3.3572**	3.3694	3.5856	3.3804	5.4459
	10	296.5322	3.4187	3.4131	3.9617	3.4124	3.4181	**3.4120**	3.4124	3.4358	3.4285	3.9695
	100	240.9072	3.3794	**3.3714**	4.1613	3.3788	3.3770	3.3728	3.3789	3.4209	3.4061	4.2422
0.99	0.1	169.8761	3.5599	3.3837	7.4558	3.3875	3.3623	3.3398	3.3587	3.5602	3.3859	5.2985
	1	248.7308	3.4291	3.4254	3.8590	3.4246	**3.4233**	3.4241	3.4246	3.4303	3.4257	3.8724
	10	215.3956	3.4913	3.4044	3.8136	**3.4039**	3.4383	3.4049	3.4039	3.4044	3.4050	4.0347
	100	579.4789	3.4869	3.3788	4.5570	3.3726	3.4221	**3.3655**	3.3726	3.3730	3.3741	5.5796

Table 2 continued

Equation 2												
			Two stage RE with different $\widehat{k}$ continued values									
ρ	κ	**TSLS estimator**	$\hat{k}_{11}$	$\hat{k}_{12}$	$\hat{k}_{13}$	$\hat{k}_{14}$	$\hat{k}_{15}$	$\hat{k}_{16}$	$\hat{k}_{17}$	$\hat{k}_{18}$	$\hat{k}_{19}$	$\hat{k}_{20}$
0.70	0.1	1.7328	**0.2842**	2.3125	1.6939	1.9928	1.4974	2.6415	2.7291	1.5578	2.1062	1.1447
	1	139.4882	**0.9873**	3.1098	2.4657	3.0907	1.7203	3.1891	3.2796	1.8527	3.1568	1.5822
	10	68.1909	4.4597	4.8340	4.8432	3.5721	4.8446	4.8718	3.4321	4.8730	3.4350	4.8733
	100	37.4451	4.0466	4.1006	4.1007	3.6164	4.1007	4.1010	3.4404	4.1010	3.4419	4.1010
0.80	0.1	35.1409	**0.3618**	2.6305	2.1256	2.3385	1.9354	2.8751	2.8968	2.0274	2.4154	1.6033
	1	308.9423	4.2634	3.3318	3.3151	3.3168	3.3974	3.4251	3.3261	**3.3075**	3.3251	3.6322
	10	161.9416	4.2090	4.4047	4.4139	3.5265	4.4134	4.4249	3.4295	4.4259	3.4342	4.4260
	100	218.0851	4.0054	4.1000	4.1005	3.5938	4.1006	4.1013	3.4260	4.1013	3.4284	4.1014
0.90	0.1	905.6709	**0.5167**	2.9919	2.7126	2.7803	2.5743	3.1352	3.0921	2.6865	2.8019	2.3591
	1	127.8484	5.0335	3.6549	4.1843	3.4092	4.4624	3.6782	3.3804	5.1432	3.3577	6.1487
	10	296.5322	3.9140	3.9719	3.9755	3.4821	3.9745	3.9776	3.4242	3.9779	3.4290	3.9780
	100	240.9072	4.0185	4.3300	4.3359	3.5440	4.3372	4.3403	3.4052	4.3406	3.4103	4.3408
0.99	0.1	169.8761	4.8314	3.3739	3.3399	3.3398	**3.3393**	3.4056	3.3859	3.3402	3.3395	3.3477
	1	248.7308	3.7827	3.7012	3.7746	3.4360	3.7604	3.8136	3.4238	3.8515	3.4256	3.8510
	10	215.3956	3.6011	3.8614	4.1275	3.4119	4.0974	4.1603	3.4044	4.2188	3.4058	4.2308
	100	579.4789	4.2689	7.0720	8.2100	3.4409	8.2841	8.2728	3.3730	8.4380	3.3850	8.4737

Table 3. Estimated $rmse$ values of the estimators in Equation 3

Equation 3												
			Two stage RE with different $\widehat{k}$ values									
ρ	κ	**TSLS estimator**	$\hat{k}_1$	$\hat{k}_2$	$\hat{k}_3$	$\hat{k}_4$	$\hat{k}_5$	$\hat{k}_6$	$\hat{k}_7$	$\hat{k}_8$	$\hat{k}_9$	$\hat{k}_{10}$
0.70	0.1	0.8464	3.3219	1.4327	**0.3946**	2.9240	3.2077	2.7137	2.9301	3.2828	2.9008	1.1065
	1	134.3958	3.4250	2.9649	0.9547	3.3232	3.4096	3.3662	3.3242	3.3458	3.2766	1.3593
	10	5261.5765	3.3870	3.3368	1.7763	3.3714	3.3783	3.3663	3.3717	3.3090	3.2994	1.8970
	100	77.9728	3.3743	3.3721	2.7899	3.3658	3.3748	3.3735	3.3743	3.3102	3.3092	2.7982
0.80	0.1	1.7157	3.3436	1.7013	**0.4664**	3.0263	3.2606	2.8731	3.0297	3.3145	3.0067	1.0946
	1	514.3002	3.4434	3.0861	1.0880	3.3384	3.4425	3.3898	3.3389	3.3576	3.3034	1.4187
	10	272.4579	3.3834	3.3522	1.9536	3.3730	3.3762	3.3684	3.3731	3.3243	3.3175	2.0408
	100	83.0514	3.3805	3.3770	3.0741	3.3659	3.4314	3.4007	3.3805	3.3225	3.3214	3.0748
0.90	0.1	31.9403	3.3701	2.2007	**0.6066**	3.1702	3.3279	3.0975	3.1713	3.3534	3.1570	1.1274
	1	142.9223	3.3990	3.2329	1.3311	3.3585	3.3924	3.3703	3.3587	3.3706	3.3378	1.5675
	10	362.4192	3.3803	3.3686	2.2808	3.3752	3.3756	3.3728	3.3753	3.3422	3.3388	2.3211
	100	94.7380	3.3618	3.3660	3.6151	3.3662	3.3697	3.3676	3.3662	3.3331	3.3340	3.6302
0.99	0.1	166.4569	3.4011	3.2376	**1.3090**	3.3615	3.3952	3.3839	3.3615	3.3916	3.3594	1.5551
	1	65.6271	3.3860	3.3764	2.3546	3.3823	3.3824	3.3800	3.3823	3.3809	3.3773	2.3759
	10	225.4804	3.3666	3.3738	3.5326	3.3777	3.3762	3.3752	3.3777	3.3509	3.3603	3.6387
	100	252.5844	3.2694	3.3161	4.5658	3.3663	3.3664	3.3571	3.3663	3.2648	3.3430	5.3532
			Two stage RE with different $\widehat{k}$ continued values									
ρ	κ	**TSLS estimator**	$\hat{k}_{11}$	$\hat{k}_{12}$	$\hat{k}_{13}$	$\hat{k}_{14}$	$\hat{k}_{15}$	$\hat{k}_{16}$	$\hat{k}_{17}$	$\hat{k}_{18}$	$\hat{k}_{19}$	$\hat{k}_{20}$
0.70	0.1	0.8464	1.0678	2.1807	1.1539	2.2325	1.0267	2.6362	3.2828	0.8628	2.6203	0.6755
	1	134.3958	1.6437	1.3952	0.9967	3.0480	1.0403	1.2012	3.3458	0.9593	3.1978	**0.9411**
	10	5261.5765	2.1444	1.7272	1.7031	3.0630	1.7010	1.6945	3.3090	1.6912	3.2798	**1.6904**
	100	77.9728	2.8800	2.7391	2.7389	3.2412	2.7389	2.7384	3.3107	**2.7384**	3.3100	2.7384
0.80	0.1	1.7157	1.1487	2.3837	1.3627	2.4544	1.2362	2.7747	3.3145	1.0306	2.7941	0.8249
	1	514.3002	1.7494	1.5706	1.0918	3.2532	1.1929	1.3490	3.3576	1.0933	3.2488	**1.0738**
	10	272.4579	2.2885	1.8988	1.8770	3.1254	1.8744	1.8656	3.3243	1.8628	3.3052	**1.8620**
	100	83.0514	3.0938	3.0620	3.0619	3.3017	3.0619	**3.0618**	3.3233	3.0618	3.3224	3.0618
0.90	0.1	31.9403	1.3151	2.7055	1.7665	2.8006	1.6603	2.9797	3.3534	1.3715	3.0418	1.1474
	1	142.9223	1.9512	1.8909	1.4428	3.1851	1.4791	1.6164	3.3706	1.3374	3.3098	**1.3144**
	10	362.4192	2.5452	2.2103	2.1950	3.2118	2.1926	2.1788	3.3422	2.1771	3.3345	**2.1766**
	100	94.7380	3.4682	3.7871	3.7885	**3.2850**	3.7882	3.7912	3.3348	3.7913	3.3340	3.7913
0.99	0.1	166.4569	2.0174	3.2507	2.6857	3.3476	2.9361	3.2938	3.3916	2.5141	3.3608	2.5367
	1	65.6271	2.7011	2.7847	2.6490	3.3236	2.6190	2.4656	3.3809	2.3484	3.3742	**2.3243**
	10	225.4804	3.3801	3.7603	3.8286	**3.2855**	3.8555	3.9546	3.3604	3.9657	3.3576	3.9702
	100	252.5844	4.3857	7.1006	9.1136	**3.2635**	9.4450	9.1312	3.3435	9.6483	3.3263	9.7321

ACKNOWLEDGMENTS

This paper is supported by Çukurova University Scientific Research Projects Unit Project Number: FBA-2018-9770.

REFERENCES

[1] A. E. Hoerl and R. W. Kennard. "Ridge regression: biased estimation for non-orthogonal problems". *Technometrics*. vol. 12. no. 1. pp. 55-67. 1970.

[2] H. D. Vinod and A. Ullah. "Recent advances in regression methods". Marcel Dekker. New York. Inc.. 1981.

[3] A. E. Hoerl. R. W. Kennard and K. F. Baldwin. "Ridge regression: some simulations". *Communications in Statistics-Simulation and Computation*. vol. 4. pp. 105–123. 1975.

[4] J. F. Lawless and P. A. Wang. "Simulation study of ridge and other regression estimators". *Communications in Statistics-Theory and Methods* . vol. 5. no. 4. pp. 307-323. 1976.

[5] R. R. Hocking. F. M. Speed and M. J. Lynn. "A class of biased estimators in linear regression". *Technometrics*. vol. 18. no. 4. pp. 425-437. 1976.

[6] B. M. G. Kibria. "Performance of some new ridge regression estimators". *Communications in Statistics-Simulation and Computation.* vol. 32. no. pp. 419–435. 2003.

[7] G. Khalaf and G. Shukur. "Choosing ridge parameters for regression problems". *Communications in Statistics-Theory and Methods*. vol. 34. pp. 1177-1182. 2005.

[8] M. Alkhamisi. G. Khalaf and G. Shukur. "Some modifications for choosing ridge parameters". *Communications in Statistics-Theory and Methods*. vol. 35. pp. 2005-2020. 2006.

[9] M. Alkhamisi and G. Shukur. "Developing ridge parameters for SUR model". *Communications in Statistics-Theory and Methods*. vol. 37. no. 4. pp. 544-564. 2008.

[10] G. Muniz and B. M. G. Kibria. "On some ridge regression estimators: an empirical comparisons". *Communications in Statistics-Simulation and Computation*. vol. 38. pp. 621-630. 2009.

[11] G. Muniz. B. M. G. Kibria. G. Shukur and K. Mansson. "On developing ridge regression parameters: a graphical investigation". *SORT*. vol. 36. no. 2. pp. 115-138. 2012.

[12] K. Mansson. G. Shukur and B. M. G Kibria. "A simulation study of some ridge regression estimators under different distributional assumptions". *Communications in Statistics-Simulation and Computation*. vol. 39. no. 8. pp. 1639-1670. 2010.

[13] H. M. Wagnar. "A monte carlo study of estimates of simultaneous linear structural equations". *Econometrica*. vol. 26. pp. 117-133. 1958.

[14] D. F. Hendry. "The structure of simultaneous equation estimators". *Journal of Econometrics*. vol. 4. pp. 51-88. 1976.

[15] S. B. Park. "Some sampling properties of minimum expected loss (MELO) estimates of structural coefficients". *Journal of Econometrics.* vol. 18. pp. 295–311. 1982.

[16] O. Capps Jr and W. D. Grubbs. "A monte carlo study of collinearity in linear simultaneous equation models". *Journal of Statistical Computation and Simulation*. vol. 39. pp. 139-162. 1991.

[17] J. Johnston and J. E. Dinardo. "Econometric methods". McGraw-Hill. New York. 1997.

[18] J. Geweke. "Using simulation methods for bayesian econometric models: inference. development and communication". *Econometric Reviews*. vol. 18. no. 1. pp. 1-73. 1999.

[19] D. A. Agunbiade. "A monte carlo approach to the estimation of a just identified simultaneous three-equations model with three multicollinear exogenous variables". Unpublished Ph.D. Thesis. University of Ibadan. 2007.

[20] D. A. Agunbiade. "Effect of multicollinearity and the sensitivity of the estimation methods in simultaneous equation model". *Journal of Modern Mathematics and Statistics*. vol. 5. no. 1. pp. 9-12. 2011.

[21] D. A. Agunbiade and J. O. Iyaniwura. "Estimation under multicollinearity: a comparative approach using monte carlo methods". *Journal of Mathematics and Statistics*. vol. 6 no. 2. pp. 183-192. 2010.