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Analysis of Angle of Twist of Axially Layered Functionally Graded Circular Hollow Shafts

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Finite Element Method

Abstract: In this study, the angle of twist of axially layered functionally graded circular hollow shafts subjected to a twisting torque at the free end was analyzed under clamped-free boundary conditions using finite element software ANSYS. The hollow shafts were made using three layers including various mixtures of ceramic and metal materials. Layer locations on the shafts were performed using L_9 orthogonal array based on Taguchi Method. The layer combination with optimum levels was obtained using analysis of the signal-to-noise (S/N) ratio. Significant layers and their percent effects on the angles of twist were analyzed using analysis of variance (ANOVA). According to results obtained, the increase of the ceramic material in layers leads to the decrease of the angle of twist of the beams. The most meaningful layers on response were obtained as first layer with 52.83 % effect ratio, second layer with 29.41% effect ratio, and third layer with 17.76 % effect ratio.

Eksenel Yönde Tabakalı Fonksiyonel Derecelendirilmiş Dairesel İçi Boş Şaftların Burulma Açısı Analizi

Anahtar Kelimeler
Burulma açısı,
Fonksiyonel derecelendirilmiş malzemeler (FDM)
Dairesel içi boş şaft,
Sonlu Elemanlar Metodu

Özet: Bu çalışmada, serbest uçundan burulma torkuna maruz kalmış eksenel yönde tabakalı fonksiyonel derecelendirilmiş dairesel içi boş şaftların tutulu-serbest sınır şartı altında sonlu elemanlar yazılımı ANSYS kullanarak burulma açısı analiz edilmiştir. Dairesel içi boş şaftlar seramik ve metal malzemelerin değişik karışımlarını içeren üç tabaka kullanarak yapıldı. Şaftlar üzerinde tabaka konumları Taguchi metoduna bağlı L_9 ortogonal dizi kullanılarak gerçekleştirildi. Optimum seviyeli tabaka kombinasyonu sinyal-gürültü oran analizi kullanılarak elde edildi. Burulma açısı üzerinde önemli tabakalar ve onların yüzde etkileri varyans analizi (ANOVA) kullanılarak analiz edildi. Elde edilen sonuçlara göre, tabakalardaki seramik malzeme artışı kirişlerin burulma açısı azalmasını sağlamıştır. Yanıtlar üzerinde en anlamlı tabakalar % 52.83 etki oranı ile birinci tabaka, % 29.41 etki oranı ile ikinci tabaka ve % 17.76 etki oranı ile üçüncü tabaka olarak elde edilmiştir.

1. Introduction

In the different applications of the engineering fields, various materials such as metal and ceramic have been used for long years. Each material has different characteristics on account of usage areas. For example, ceramic materials have excellent properties in terms of heat resistance but their use is limited since they have low toughness [1]. In addition, metal materials have excellent strength and toughness [1]. Thus, the new materials can be designed using the excellent properties of these materials such as ceramic and metal. An example of these materials is functionally graded materials (FGMs). FGMs can be termed as a type of the composite materials occurred using two or more constituent phases consisting of a continuously changing composition [2]. The concept of FGMs was explained by scientists who carry out studies in material science in 1984, Japan [3]. In the literature, there are several studies including torsion. Batra [4] presented a research including torsion of a cylinder and the cylinder was designed using functionally graded materials. Arghavan and Hematiyan [5] investigated the torsion of hollow

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tubes formed using functionally graded materials. Horgan and Chan [6] carried out the torsion of isotropic linearly elastic bars prepared using functionally graded materials. Rahaeifard [7] analyzed the size-dependent torsion of bars designed using functionally graded materials. As can be described from this literature, there are a lot of various studies including torsion. In this study, the angle of twist of axially layered functionally graded circular hollow shafts subjected to a twisting torque at the free end were investigated for clamped-free boundary conditions numerically. The locations of layers on the shafts and levels of layers were determined depending on Taguchi's L_9 orthogonal array.

2. Materials and Methods

The Young's modulus of the materials on the angle of twist for shaft is of great importance directly. In addition, FGMs can be generally produced using ceramic and metal materials. Because of that, the shaft configurations were designed using various combinations of the layers made of ceramic (Zirconia) and metal (Aluminium) materials. The material properties of these materials were illustrated in Table 1.

Table 1. Material properties of metal and ceramic [8]

Material	Type	Young's modulus (E)	Poisson's ratio (ν)
Aluminium	Metal	70 (GPa)	0.3
Zirconia	Ceramic	151 (GPa)	0.3

In order to analyze the angle of twist of the shafts, shear modulus of ceramic and metal materials are needed. The shear modulus (G) of the materials in Table 1 were calculated using Equation 1 [9].

$$G = \frac{E}{2(1+\nu)} \quad (1)$$

Numerical set-up was done using Taguchi's L_9 orthogonal array. The layers of the shafts were determined to be control factors. The levels of the layers were performed based on different volume fractions of ceramic and metal materials and thus layers with various levels were designed. The layers and their levels for axially layered functionally graded circular hollow shafts are demonstrated in Table 2.

Table 2. Layers and Levels for Shafts

Layers of Shafts	Symbol	Levels for Each Layer of Shafts		
		First Level	Second Level	Third Level
First Layer	FL	$(FL)_1$	$(FL)_2$	$(FL)_3$
Second Layer	SL	$(SL)_1$	$(SL)_2$	$(SL)_3$
Third Layer	TL	$(TL)_1$	$(TL)_2$	$(TL)_3$

The increase of levels of layers were done based on 5% ceramic contents and so the layers with various materials properties were occurred. For layers with different levels, effective material properties (P_{ef}) can be explained by the following Equation 2 [1].

$$P_{ef} = \sum_{j=1} P_j V_{f_j} \quad (2)$$

here, P_j and V_{fj} are explained to be the material properties and volume fraction of the constituent material j respectively and so the total volume fractions of all the constituent materials can be described to be one depending on Equation 3 [1].

$$\sum_{j=1} V_{f_j} = 1 \quad (3)$$

The locations of the layers with different material properties calculated according to Equation 2 and 3 were designed Taguchi's L_9 orthogonal array for the shafts. Numerical data obtained for each shaft configuration were analyzed using Minitab 15 software [13] based on "smaller is better" quality characteristic as described in Equation 4 [10].

$$(S/N)_{SB} \text{ for } \alpha = -10.\log\left(n^{-1}\sum_{i=1}^{n}(y_i)^2\right) \quad (4)$$

here, n signifies the number of analyses for angle of twist in a trial and y_i describes ith data analyzed.

3. Numerical Approach

The axially layered functionally graded circular hollow shafts consisting of three layers for analysis of the angles of twist were modelled using finite element software ANSYS parametric design language [12]. In the numerical analyses, the element type called as BEAM189 based on Timoshenko beam theory was used and it is a quadratic 3-node beam element depending on three dimension [11]. The axially layered functionally graded circular hollow shafts has clamped-free boundary conditions and the twisting torque with 600 Nm value was applied at the free end of the shafts. L = 0.3 is length of the shaft in meter and so length of each layer was designed as 0.1 meter. The axially layered functionally graded circular hollow shafts have an outer diameter of 0.03 meter and an inner diameters of 0.01 meter along the shafts, homogeneously. The circular hollow shaft with clamped-free boundary conditions is demonstrated in Figure 1.

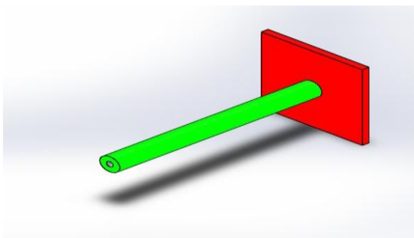

Figure 1. The circular hollow shafts with clamped-free boundary conditions

4. Results and Discussions

The objective of this study is to find out the angles of twist of axially layered functionally graded circular hollow shafts subjected to a twisting torque at the free end for clamped-free boundary conditions using finite element software ANSYS. Numerical results analyzed and their S/N ratio values calculated based on L_9 orthogonal array were tabulated in Table 3. The angle of twist in radian for the axially layered functionally graded circular hollow shafts in Table 3 were shown in Figure 2.

Table 3. Numerical Results and Their S/N ratio for Shafts

Run	Layer Combinations of The Shafts			Results	
				α in degree	η in dB
1	$(FL)_1$	$(SL)_1$	$(TL)_1$	4.023	-12.0910
2	$(FL)_1$	$(SL)_2$	$(TL)_2$	3.917	-11.8591
3	$(FL)_1$	$(SL)_3$	$(TL)_3$	3.821	-11.6435
4	$(FL)_2$	$(SL)_1$	$(TL)_2$	3.897	-11.8146
5	$(FL)_2$	$(SL)_2$	$(TL)_3$	3.795	-11.5842
6	$(FL)_2$	$(SL)_3$	$(TL)_1$	3.829	-11.6617
7	$(FL)_3$	$(SL)_1$	$(TL)_3$	3.782	-11.5544
8	$(FL)_3$	$(SL)_2$	$(TL)_1$	3.811	-11.6208
9	$(FL)_3$	$(SL)_3$	$(TL)_2$	3.711	-11.3898
Overall Mean for Angle of Twist ($\overline{T_\alpha}$)				3.843	

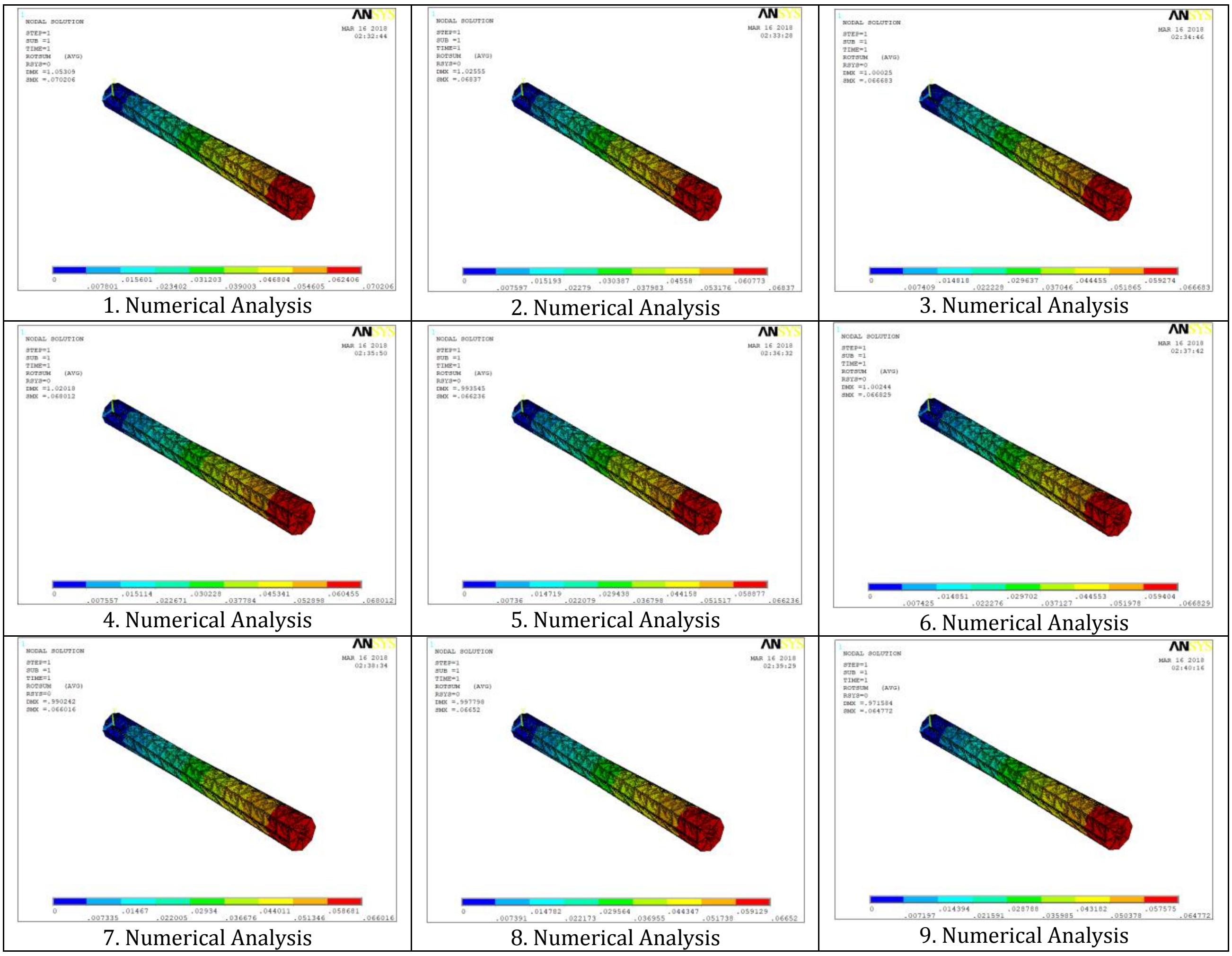

Figure 2. Shapes and results in radian for shafts

Table 4. Response Table for S/N and Raw Data based on the Angle of Twist

Level	S/N ratio values in dB			Means in Degree		
	FL	SL	TL	FL	SL	TL
1	-11.860	-11.820	-11.790	3.920	3.901	3.888
2	-11.690	-11.690	-11.690	3.840	3.841	3.842
3	-11.520	-11.570	-11.590	3.768	3.787	3.799
Delta	0.340	0.250	0.200	0.152	0.114	0.088
Rank	1	2	3	1	2	3

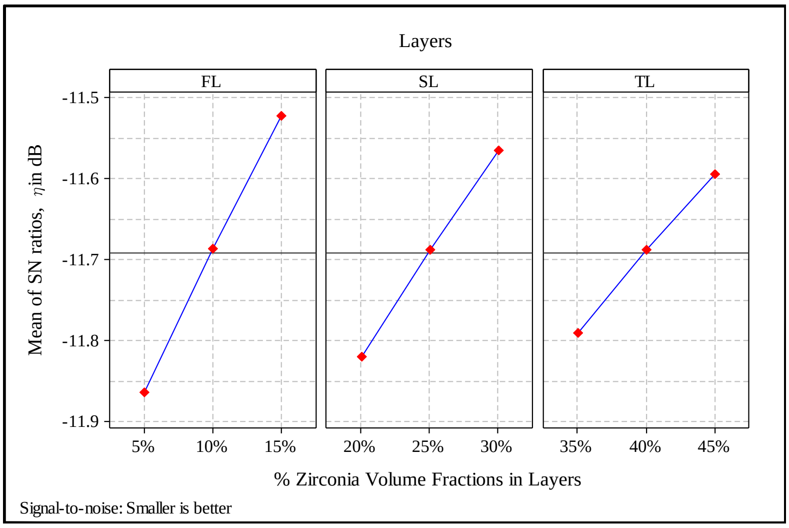

Figure 3. Effects of Layers on Responses for S/N data

4.1. Decision of optimum levels of layers

In order to select the optimum levels of layers, numerical results calculated of the axially layered functionally graded circular hollow shafts designed based on L_9 orthogonal array were analyzed according to "smaller is better" quality characteristic. Each shaft has different mechanical properties. In addition, ceramics ratio between neighboring layers is different. Average numerical results for raw data and their S/N data for each layer at first level, second level, and third level were given in Table 4. It is seen from the Table 4 that the shaft consisting of layers with optimum levels are obtained using third levels of the layers.

4.2. Effect of layers on responses

In order to see the effects of layers on responses, average S/N data in Table 4 were used. These data were plotted in Figures 3. It is seen from the Figure 3

that the angle of the twist for the shafts decreases with the increase of the ceramic contents in layers and so third levels of the layers provide the minimum angle of the twist of the axially layered functionally graded circular hollow shafts.

4.3. Analysis of variance for angle of twist

Analysis of Variance (ANOVA) was performed in order to investigate the significance and percent effects of layers on the responses. ANOVA at 95 confidence level was determined for R-Sq = 100.00%, R-Sq (adj) = 100.00% using raw data. ANOVA results are tabulated in Table 5.

According to ANOVA results, the layers are significant control parameters on the angle of twist for axially layered functionally graded circular hollow shafts with C-F boundary conditions due to $p < 0.05$ (95% confidence level). In addition, the most effective control parameters as layers on the angle of twist are found for FL with 52.83 % effect ratio, SL with 29.41% effect ratio, and TL with 17.76 % effect ratio, respectively.

4.4. Prediction of optimum response for angle of twist

The optimum result for angle of twist is estimated using optimum levels of significant layers. In this study, the layers were determined as significant parameters according to ANOVA results. Thus estimated mean for angle of twist (μ_α) can be explained as given Equation 5 [10].

$$\mu_\alpha = \overline{FL_3} + \overline{SL_3} + \overline{TL_3} - 2\overline{T_\alpha} \quad (5)$$

where, $\overline{T_\alpha}$ signifies the overall mean for angle of twist depending on L_9 orthogonal array and it was taken to be 3.843 from in Table 3. Also, $\overline{FL_3}$ = 3.768, $\overline{SL_3}$ = 3.787, and $\overline{TL_3}$ = 3.799 for raw data describe average values for Layer 1, Layer 2 and Layer 3 at third level respectively and these data were taken from Table 4. Substituting these values determined in Equation 5, the estimated mean for angle of twist is found to be 3.668 in degree. The predicted and ANSYS results for the angle of twist are demonstrated in Table 6.

Table 5. Analysis of Variance for Raw Data

Source	DF	Seq SS	Adj SS	Variance	F	P	% Effect
FL	2	0.034838	0.034838	0.017419	156769	0	52.83
SL	2	0.019396	0.019396	0.009698	87283	0	29.41
TL	2	0.011711	0.011711	0.005855	52699	0	17.76
Error	2	0	0	0			
Total	8	0.065945					

Table 6. The Predicted and ANSYS results

Optimum Levels of Layers	Predicted Results	ANSYS Results	% Difference
FL_3-SL_3-TL_3	3.668 in Degree	3.669 in Degree	0.03
	-11.2886 in dB	-11.2910 in dB	0.02

5. Conclusions

This study deals with the angle of twist of axially layered functionally graded circular hollow shafts subjected to a twisting torque at free end. The shafts were analyzed under clamped-free boundary conditions. Layer combinations of shafts were evaluated using Taguchi's L_9 orthogonal array and numerical analyses for shafts were observed depending on finite element software ANSYS. The major results determined for this study are summarized as follows:

- The overall mean calculated for angle of twist is found as 3.843 in degree according to L_9 orthogonal array.
- The optimum result for the angle of twist was determined using layers at third level.
- The angle of twist decreases with the increase of ceramic content in layers.
- The most effective layers on the angle of twist are carried out for FL with 52.83 % effect ratio, SL with 29.41% effect ratio, and TL with 17.76 % effect ratio, respectively.
- According to ANOVA at 95% confidence level, the layers are significant control parameters on the angle of twist due to $p < 0.05$.
- The % difference between predicted and ANSYS results for the angle of twist is detected as 0.03% for mean and 0.02% for S/N ratio.

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