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A study on new methods of ratio exponential type imputation in sample surveys

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Abstract

In this article, we have suggested new methods of ratio exponential type imputation and proposed their corresponding point estimators to deal with the problems of non-response in sample surveys for the prior outlay of an auxiliary variable x. The expression of the biases and their mean square errors of the proposed estimators have been derived, upto the first order of large sample approximation under SRSWOR scheme and compared with the mean method of imputation, ratio method of imputation, regression method of imputation and the estimators of Singh and Horn (Metrika [16]), Singh and Deo (Statistical Papers [15]), Toutenburg *et al.* (Statistical Papers [18]), Singh (Statistics [17]) and Gira (Applied Mathematical Sciences [5]). After comparison, the condition which makes the proposed forty four estimators more efficient than others are found. To verify the theoretical results, simulation studies are performed on five real data sets.

Keywords: Efficiency, Imputation methods, Bias, Mean square error (MSE), Auxiliary variable.

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1. Introduction

The non-response is a well known major problem, which is encountered by practitioners in the trade of sample surveys. Repeated surveys are as a matter of course more likely to this setback than single-occasion surveys. For examples, in case of milk yield surveys the animal may be sold or may die during the survey period. Thus, the observations may be missing for some of the time stages. Such type of non-response (missingness) may have different patterns and causes.

Determining the efficient analytical approach in the continuation of incomplete survey report due to non-response is a major question for the analysts and researchers. The hypothesis concerning population parameters can be corrupt if the sufficient information approximately the nature of non-response is not known. A impulsive question arises what

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such needs to look to justify ignoring the incomplete mechanism. Rubin [12] addressed three concepts: missing at random (MAR), observed at random (OAR) and parameter distribution (PD). Rubin marked "the data are MAR if the probability of the observed missingness pattern, subject to the observed and unobserved data, does not calculate on the worth of the unobserved data". Heitzan and Basu [6] have remarked the meaning of missing at random (MAR) and missing completely at random (MCAR).

Imputation, the pursue of "filling in" missing data mutually plausible values, is an attractive concern to correlate incomplete data. It apparently solves the missing data problem at the different methods of the analysis. To deal the problem of missing values ultimately, Sande [13] and Kalton *et al.* [10] intended imputation methods that collect incomplete data sets structurally and complete its analysis simply. Imputation method is also be driven out by all of the assist of an auxiliary variable, if one is available. Later for the MCAR response mechanism, Singh and Horn [16] indicated a compromised approach of imputation. Singh and Deo [15], Ahmed *et al.* [1], Toutenburg*et al.* [18], Kadilar and Cingi [9], Singh [17], Singh *et al.* [14], Diana and Perri [4] and Gira [5] have indicated several new imputation based methods by all of the assist of an auxiliary variable.

For missing completely at random (MCAR) response mechanism, we have proposed the ratio exponential type imputation methods to tackle with the problems of non-response in sample surveys. Following the indicated imputation methods, estimators have been proposed for estimating the population mean and consequently their behaviors are studied. Performances of the proposed estimators are compared with some existing estimators.

2. Structures and Notations

Let y and x be denoted by the positively correlated study variable and auxiliary variable respectively. A simple random sample (without replacement) s_n of n units is drawn from a finite population $U = (U_1, U_2, ..., U_N)$ of N units to estimate population mean $\bar{Y}$ which uses the known values of population parameters such as Standard Deviation (S_x), Coefficient of Kurtosis ($\beta_2(x)$), Coefficient of Variation (C_x) and Correlation Coefficient (ρ_{yx}). Let r be the number of responding units out of sampled n units, the set of responding units by R and the set of non-responding units by R^c. If the units involve the responding unit set, the values on the study variable y_i are observed anyway. If they involve the non-responding unit set, the values on the study variable y_i are missing and thereafter the imputed values are derived for a well known units.

$$(2.1) \qquad y_{.i} = \begin{cases} y_i & \text{if } i \epsilon R \\ \hat{y}_i & \text{if } i \epsilon R^c \end{cases}$$

where $\hat{y}_i$ denotes the imputed value for the i^{th} non-responding unit.
The general point estimator of population mean $\bar{Y}$ takes the form

$$(2.2) \qquad \bar{y}_s = \frac{1}{n}\sum_{i \epsilon s_n} y_{.i} = \frac{1}{n}\left[\sum_{i \epsilon R} y_{.i} + \sum_{i \epsilon R^c} y_{.i}\right] = \frac{1}{n}\left[\sum_{i \epsilon R} y_i + \sum_{i \epsilon R^c} \hat{y}_i\right]$$

Here, the value $\hat{y}_i$ is different for each imputation method.
The consequently notations have been approaching in this work:
$\bar{Y}$, $\bar{X}$: The population means of the variables y and x respectively.
$\bar{y}_r$, $\bar{x}_r$: The response means of the respective variables for the sample sizes shown in suffices.
$\bar{x}_n$: The sample mean of the variable x.
ρ_{yx} : The correlation coefficient between the variables y and x.
$\beta_2(x)$: The population coefficient of kurtosis of the variable x .

$S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$: The population variance of the variable x.
S_y^2 : The population variance of the variable y .
C_y and C_x : The coefficients of variation of the variables shown in suffices.

3. Brief of Some Existing Estimators

In this section, we act several estimators for estimating the population mean under non-response.

3.1. Mean Method of Imputation. Under the mean method of imputation, the point estimator (2.2) of population mean $\bar{Y}$ is derived as

$$\bar{y}_m = \frac{1}{r}\sum_{i=1}^{r} y_i = \bar{y}_r \tag{3.1}$$

which is known as the response mean estimator $\bar{y}_r$ of population mean $\bar{Y}$. The variance of the response sample mean $\bar{y}_r$, is given by

$$Var(\bar{y}_r) = \left(\frac{1}{r} - \frac{1}{N}\right)\bar{Y}^2 C_y^2 \tag{3.2}$$

3.2. Ratio Method of Imputation. Under the ratio method of imputation, the point estimator (2.2) of population mean $\bar{Y}$ is derived as

$$\bar{y}_{RAT} = \bar{y}_r \frac{\bar{x}_n}{\bar{x}_r} \tag{3.3}$$

which is known as the ratio estimator $\bar{y}_{RAT}$ of population mean $\bar{Y}$. The MSE of estimator $\bar{y}_{RAT}$ is obtained under MCAR mechanism upto first order of large approximation, is given by

$$MSE(\bar{y}_{RAT}) = Var(\bar{y}_r) + \left(\frac{1}{r} - \frac{1}{n}\right)\bar{Y}^2\left(C_x^2 - 2\rho_{yx}C_yC_x\right) \tag{3.4}$$

3.3. Regression Method of Imputation. Under the regression method of imputation, the point estimator (2.2) of population mean $\bar{Y}$ is given by is given by

$$\bar{y}_{REG} = \bar{y}_r + \hat{b}(\bar{x}_n - \bar{x}_r) \tag{3.5}$$

which is known as regression estimator $\bar{y}_{REG}$ of population mean $\bar{Y}$. where $\hat{b} = \frac{s_{yx}(r)}{s_x^2(r)}$.
The MSE of $\bar{y}_{REG}$ is obtained under MCAR mechanism upto first order of approximation, is given by

$$MSE(\bar{y}_{REG}) = \bar{Y}^2 C_y^2\left[\left(\frac{1}{r} - \frac{1}{N}\right) - \left(\frac{1}{r} - \frac{1}{n}\right)\rho_{yx}^2\right] \tag{3.6}$$

3.4. Singh and Horn [16] Estimator. Singh and Horn [16] suggested a compromised imputation in survey sampling. Under this method of imputation, the point estimator (2.2) of population mean $\bar{Y}$ is derived as

$$\bar{y}_{SH} = \alpha\bar{y}_r + (1-\alpha)\bar{y}_r\frac{\bar{x}_n}{\bar{x}_r} \tag{3.7}$$

where α is a suitable constant. The optimum value of α is $1 - \rho_{yx}\frac{C_y}{C_x}$.
Now, taking the optimum value of α in (3.7) , we obtained the minimum MSE of $\bar{y}_{SH}$ under MCAR mechanism up-to the first order of approximation, is given by

$$MSE_{min}(\bar{y}_{SH}) = MSE(\bar{y}_{RAT}) - \left(\frac{1}{r} - \frac{1}{n}\right)\left(C_x - \rho_{yx}C_y\right)^2\bar{Y}^2 \tag{3.8}$$

3.5. Singh and Deo [15] **Estimator.** Singh and Deo [15] suggested imputation by power transformation in survey sampling. Under this method of imputation, the point estimator (2.2) of population mean $\bar{Y}$ becomes

$$\bar{y}_{SD} = \bar{y}_r \left(\frac{\bar{x}_n}{\bar{x}_r}\right)^{\beta} \tag{3.9}$$

where β is a suitable constant. The optimum value of β is $\rho_{yx}\frac{C_y}{C_x}$.
Now, taking the optimum value of β in (3.9), we obtained the minimum MSE of $\bar{y}_{SD}$ under MCAR mechanism up-to first order of approximation, given by

$$MSE_{min}(\bar{y}_{SD}) = MSE(\bar{y}_{RAT}) - \left(\frac{1}{r} - \frac{1}{n}\right) S_x^2 \left(\frac{S_{yx}}{S_x^2} - \frac{\bar{Y}}{\bar{X}}\right)^2 \tag{3.10}$$

3.6. Toutenburg *et al.* [18] **Estimators.** We discuss two estimators of Toutenburg *et al.* [18] for the population mean $\bar{Y}$, are given by

$$\bar{y}_{TSS_1} = \bar{y}\left[\frac{(n-p)\bar{x} + p\bar{x}^*}{n\bar{x}}\right] \tag{3.11}$$

$$\bar{y}_{TSS_2} = \bar{y}\left[\frac{(n-p)^2\bar{x} + (2n-p)p\bar{x}^*}{(n-p)n\bar{x} + np\bar{x}^*}\right] \tag{3.12}$$

where $\bar{x}^* = \frac{1}{n-r}\sum_{i=1}^{n-r} x_i$.
In the present paper, the estimators $\bar{y}_{TSS_1}$ and $\bar{y}_{TSS_2}$ in equation (3.11) and (3.12) respectively can be written as

$$\bar{y}_{TSS_1} = \bar{y}_r \frac{\bar{x}_n}{\bar{x}_r} \tag{3.13}$$

$$\bar{y}_{TSS_2} = \bar{y}_r + \left(\frac{r\bar{y}_r}{n\bar{x}_n}\right)(\bar{x}_n - \bar{x}_r) \tag{3.14}$$

Thus, the estimator $\bar{y}_{TSS_1}$ is same as the ratio estimator.
The MSEs of estimators $\bar{y}_{TSS_1}$ and $\bar{y}_{TSS_2}$ are obtained under MCAR mechanism, up-to first order of large approximation, are given by

$$MSE(\bar{y}_{TSS_1}) = Var(\bar{y}_r) + \left(\frac{1}{r} - \frac{1}{n}\right) \bar{Y}^2 \left(C_x^2 - 2\rho_{yx}C_yC_x\right) \tag{3.15}$$

$$MSE(\bar{y}_{TSS_2}) = Var(\bar{y}_r) + \left(\frac{1}{r} - \frac{1}{n}\right)\left(\frac{r}{n}\right) \bar{Y}^2 \left[\left(\frac{r}{n}\right) C_x^2 - 2\rho_{yx}C_yC_x\right] \tag{3.16}$$

3.7. Singh [17] **Estimator.** Singh [17] suggested a new method of imputation in survey sampling. Under this method of imputation, The point estimator (2.2) of the population mean $\bar{Y}$ becomes

$$\bar{y}_{SINGH} = \frac{\bar{y}_r\bar{x}_n}{\gamma\bar{x}_r + (1-\gamma)\bar{x}_n} \tag{3.17}$$

where γ is a suitable constant. The optimum value of γ is $\rho_{yx}\frac{C_y}{C_x}$.
Now, using the optimum value of γ in equation (3.17), we obtained the minimum MSE of the estimator $\bar{y}_{SINGH}$ under MCAR mechanism up-to the first order of large approximation, given by

$$MSE_{min}(\bar{y}_{SINGH}) = MSE(\bar{y}_{RAT}) - \left(\frac{1}{r} - \frac{1}{n}\right)\left(C_x - \rho_{yx}C_y\right)^2 \bar{Y}^2 \tag{3.18}$$

3.8. Gira [5] Estimator. Gira [5] suggested a new method of ratio type imputation in sample surveys. Under this method of imputation, The point estimator (2.2) of the population mean $\bar{Y}$ becomes

$$(3.19) \qquad \bar{y}_{GIRA} = \bar{y}_r \frac{\phi - \bar{x}_r}{\phi - \bar{x}_n}$$

where ϕ is a suitably chosen constant. The optimum value of ϕ is $\bar{X}\left(\frac{C_x}{\rho_{yx} C_y} - 1\right)$.
Now, using the optimum value of ϕ in equation (3.19), we obtained the minimum MSE of the estimator $\bar{y}_{GIRA}$ under MCAR mechanism up-to the first order of large approximation, given by

$$(3.20) \qquad MSE_{min}(\bar{y}_{GIRA}) = Var(\bar{y}_r) - \left(\frac{1}{r} - \frac{1}{n}\right)\rho_{yx}^2 \bar{Y}^2 C_y^2$$

4. Mathematical Formulation of the Problem

In this section, the suggested methods are structured to tackle the problems of non-response in sample surveys. The missing values are replaced by calibrate imputed values of the ratio exponential type imputation methods. This methods of imputation have been suggested to impute the missing values under MCAR response mechanism.
After imputation, the first suggested method of imputation becomes

$$(4.1) \qquad (i) y_{.i} = \begin{cases} y_i \exp\left[\frac{p(\bar{X} - \bar{x}_r)}{p(\bar{X} + \bar{x}_r) + 2q}\right] & \text{if } i \epsilon R \\ \frac{\bar{y}_r}{\bar{x}_r}\left(x_i - \frac{n}{n-r}(\bar{x}_n - \bar{x}_r)\right) \exp\left[\frac{p(\bar{X} - \bar{x}_r)}{p(\bar{X} + \bar{x}_r) + 2q}\right] & \text{if } i \epsilon R^c \end{cases}$$

Under the first suggested method of imputation given in equation (4.1), the point estimators (2.2) of the population mean $\bar{Y}$ become

$$(4.2) \qquad \tau(p, q) = \bar{y}_r \exp\left[\frac{p(\bar{X} - \bar{x}_r)}{p(\bar{X} + \bar{x}_r) + 2q}\right]$$

where $p \neq 0$ and q are either some real number or the known values of population parameters of the auxiliary variable x such as Standard Deviation (S_x), Coefficient of Kurtosis ($\beta_2(x)$), Coefficient of Variation (C_x) and Correlation Coefficient (ρ_{yx}). In Table 1, we get efficient twenty two ratio exponential type estimators $\tau_i (i = 1, 2, ..., 22)$ as the new family of $\tau(p, q)$ for different suitable values of p and q.
After imputation, the second suggested method of imputation takes the form:

$$(4.3) \qquad (ii) y_{.i} = \begin{cases} y_i \exp\left[\frac{p(\bar{X} - \bar{x}_n)}{p(\bar{X} + \bar{x}_n) + 2q}\right] & \text{if } i \epsilon R \\ \frac{\bar{y}_r}{\bar{x}_r}\left(x_i - \frac{n}{n-r}(\bar{x}_n - \bar{x}_r)\right) \exp\left[\frac{p(\bar{X} - \bar{x}_n)}{p(\bar{X} + \bar{x}_n) + 2q}\right] & \text{if } i \epsilon R^c \end{cases}$$

Under the second suggested method of imputation given in equation (4.3), the point estimators (2.2) of the population mean $\bar{Y}$ become

$$(4.4) \qquad \eta(p, q) = \bar{y}_r \exp\left[\frac{p(\bar{X} - \bar{x}_n)}{p(\bar{X} + \bar{x}_n) + 2q}\right]$$

where $p \neq 0$ and q are either some real number or the known values of population parameters of the auxiliary variable x such as Standard Deviation (S_x), Coefficient of Kurtosis ($\beta_2(x)$), Coefficient of Variation (C_x) and Correlation Coefficient (ρ_{yx}). In Table

Table 1. Some members of the suggested ratio exponential type estimators $\tau_i (i = 1, 2, ..., 22)$

Estimators	p	q
$\tau_1 = \bar{y}_r \exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right)$	1	0
$\tau_2 = \bar{y}_r \exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r+2}\right)$	1	1
$\tau_3 = \bar{y}_r \exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r+2S_x}\right)$	1	S_x
$\tau_4 = \bar{y}_r \exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r+2\beta_2(x)}\right)$	1	$\beta_2(x)$
$\tau_5 = \bar{y}_r \exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r+2C_x}\right)$	1	C_x
$\tau_6 = \bar{y}_r \exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r+2\rho_{yx}}\right)$	1	ρ_{yx}
$\tau_7 = \bar{y}_r \exp\left[\frac{S_x(\bar{X}-\bar{x}_r)}{S_x(\bar{X}+\bar{x}_r)+2}\right]$	S_x	1
$\tau_8 = \bar{y}_r \exp\left[\frac{S_x(\bar{X}-\bar{x}_r)}{S_x(\bar{X}+\bar{x}_r)+2\beta_2(x)}\right]$	S_x	$\beta_2(x)$
$\tau_9 = \bar{y}_r \exp\left[\frac{S_x(\bar{X}-\bar{x}_r)}{S_x(\bar{X}+\bar{x}_r)+2C_x}\right]$	S_x	C_x
$\tau_{10} = \bar{y}_r \exp\left[\frac{S_x(\bar{X}-\bar{x}_r)}{S_x(\bar{X}+\bar{x}_r)+2\rho_{yx}}\right]$	S_x	ρ_{yx}
$\tau_{11} = \bar{y}_r \exp\left[\frac{\beta_2(x)(\bar{X}-\bar{x}_r)}{\beta_2(x)(\bar{X}+\bar{x}_r)+2}\right]$	$\beta_2(x)$	1
$\tau_{12} = \bar{y}_r \exp\left[\frac{\beta_2(x)(\bar{X}-\bar{x}_r)}{\beta_2(x)(\bar{X}+\bar{x}_r)+2S_x}\right]$	$\beta_2(x)$	S_x
$\tau_{13} = \bar{y}_r \exp\left[\frac{\beta_2(x)(\bar{X}-\bar{x}_r)}{\beta_2(x)(\bar{X}+\bar{x}_r)+2C_x}\right]$	$\beta_2(x)$	C_x
$\tau_{14} = \bar{y}_r \exp\left[\frac{\beta_2(x)(\bar{X}-\bar{x}_r)}{\beta_2(x)(\bar{X}+\bar{x}_r)+2\rho_{yx}}\right]$	$\beta_2(x)$	ρ_{yx}
$\tau_{15} = \bar{y}_r \exp\left[\frac{C_x(\bar{X}-\bar{x}_r)}{C_x(\bar{X}+\bar{x}_r)+2}\right]$	C_x	1
$\tau_{16} = \bar{y}_r \exp\left[\frac{C_x(\bar{X}-\bar{x}_r)}{C_x(\bar{X}+\bar{x}_r)+2S_x}\right]$	C_x	S_x
$\tau_{17} = \bar{y}_r \exp\left[\frac{C_x(\bar{X}-\bar{x}_r)}{C_x(\bar{X}+\bar{x}_r)+2\beta_2(x)}\right]$	C_x	$\beta_2(x)$
$\tau_{18} = \bar{y}_r \exp\left[\frac{C_x(\bar{X}-\bar{x}_r)}{C_x(\bar{X}+\bar{x}_r)+2\rho_{yx}}\right]$	C_x	ρ_{yx}
$\tau_{19} = \bar{y}_r \exp\left[\frac{\rho_{yx}(\bar{X}-\bar{x}_r)}{\rho_{yx}(\bar{X}+\bar{x}_r)+2}\right]$	ρ_{yx}	1
$\tau_{20} = \bar{y}_r \exp\left[\frac{\rho_{yx}(\bar{X}-\bar{x}_r)}{\rho_{yx}(\bar{X}+\bar{x}_r)+2S_x}\right]$	ρ_{yx}	S_x
$\tau_{21} = \bar{y}_r \exp\left[\frac{\rho_{yx}(\bar{X}-\bar{x}_r)}{\rho_{yx}(\bar{X}+\bar{x}_r)+2\beta_2(x)}\right]$	ρ_{yx}	$\beta_2(x)$
$\tau_{22} = \bar{y}_r \exp\left[\frac{\rho_{yx}(\bar{X}-\bar{x}_r)}{\rho_{yx}(\bar{X}+\bar{x}_r)+2C_x}\right]$	ρ_{yx}	C_x

2, we get efficient twenty two ratio exponential type estimators $\eta_i (i = 1, 2, ..., 22)$ as the new family of $\eta(p, q)$ for different suitable values of p and q.

Table 2. Some members of the suggested ratio exponential type estimators $\eta_i (i = 1, 2, ..., 22)$

Estimators	p	q
$\eta_1 = \bar{y_r} \exp\left(\frac{\bar{X}-\bar{x}_n}{\bar{X}+\bar{x}_n}\right)$	1	0
$\eta_2 = \bar{y_r} \exp\left(\frac{\bar{X}-\bar{x}_n}{\bar{X}+\bar{x}_n+2}\right)$	1	1
$\eta_3 = \bar{y_r} \exp\left(\frac{\bar{X}-\bar{x}_n}{\bar{X}+\bar{x}_n+2S_x}\right)$	1	S_x
$\eta_4 = \bar{y_r} \exp\left(\frac{\bar{X}-\bar{x}_n}{\bar{X}+\bar{x}_n+2\beta_2(x)}\right)$	1	$\beta_2(x)$
$\eta_5 = \bar{y_r} \exp\left(\frac{\bar{X}-\bar{x}_n}{\bar{X}+\bar{x}_n+2C_x}\right)$	1	C_x
$\eta_6 = \bar{y_r} \exp\left(\frac{\bar{X}-\bar{x}_n}{\bar{X}+\bar{x}_n+2\rho_{yx}}\right)$	1	ρ_{yx}
$\eta_7 = \bar{y_r} \exp\left[\frac{S_x\left(\bar{X}-\bar{x}_n\right)}{S_x\left(\bar{X}+\bar{x}_n\right)+2}\right]$	S_x	1
$\eta_8 = \bar{y_r} \exp\left[\frac{S_x\left(\bar{X}-\bar{x}_n\right)}{S_x\left(\bar{X}+\bar{x}_n\right)+2\beta_2(x)}\right]$	S_x	$\beta_2(x)$
$\eta_9 = \bar{y_r} \exp\left[\frac{S_x\left(\bar{X}-\bar{x}_n\right)}{S_x\left(\bar{X}+\bar{x}_n\right)+2C_x}\right]$	S_x	C_x
$\eta_{10} = \bar{y_r} \exp\left[\frac{S_x\left(\bar{X}-\bar{x}_n\right)}{S_x\left(\bar{X}+\bar{x}_n\right)+2\rho_{yx}}\right]$	S_x	ρ_{yx}
$\eta_{11} = \bar{y_r} \exp\left[\frac{\beta_2(x)\left(\bar{X}-\bar{x}_n\right)}{\beta_2(x)\left(\bar{X}+\bar{x}_n\right)+2}\right]$	$\beta_2(x)$	1
$\eta_{12} = \bar{y_r} \exp\left[\frac{\beta_2(x)\left(\bar{X}-\bar{x}_n\right)}{\beta_2(x)\left(\bar{X}+\bar{x}_n\right)+2S_x}\right]$	$\beta_2(x)$	S_x
$\eta_{13} = \bar{y_r} \exp\left[\frac{\beta_2(x)\left(\bar{X}-\bar{x}_n\right)}{\beta_2(x)\left(\bar{X}+\bar{x}_n\right)+2C_x}\right]$	$\beta_2(x)$	C_x
$\eta_{14} = \bar{y_r} \exp\left[\frac{\beta_2(x)\left(\bar{X}-\bar{x}_n\right)}{\beta_2(x)\left(\bar{X}+\bar{x}_n\right)+2\rho_{yx}}\right]$	$\beta_2(x)$	ρ_{yx}
$\eta_{15} = \bar{y_r} \exp\left[\frac{C_x\left(\bar{X}-\bar{x}_n\right)}{C_x\left(\bar{X}+\bar{x}_n\right)+2}\right]$	C_x	1
$\eta_{16} = \bar{y_r} \exp\left[\frac{C_x\left(\bar{X}-\bar{x}_n\right)}{C_x\left(\bar{X}+\bar{x}_n\right)+2S_x}\right]$	C_x	S_x
$\eta_{17} = \bar{y_r} \exp\left[\frac{C_x\left(\bar{X}-\bar{x}_n\right)}{C_x\left(\bar{X}+\bar{x}_n\right)+2\beta_2(x)}\right]$	C_x	$\beta_2(x)$
$\eta_{18} = \bar{y_r} \exp\left[\frac{C_x\left(\bar{X}-\bar{x}_n\right)}{C_x\left(\bar{X}+\bar{x}_n\right)+2\rho_{yx}}\right]$	C_x	ρ_{yx}
$\eta_{19} = \bar{y_r} \exp\left[\frac{\rho_{yx}\left(\bar{X}-\bar{x}_n\right)}{\rho_{yx}\left(\bar{X}+\bar{x}_n\right)+2}\right]$	ρ_{yx}	1
$\eta_{20} = \bar{y_r} \exp\left[\frac{\rho_{yx}\left(\bar{X}-\bar{x}_n\right)}{\rho_{yx}\left(\bar{X}+\bar{x}_n\right)+2S_x}\right]$	ρ_{yx}	S_x
$\eta_{21} = \bar{y_r} \exp\left[\frac{\rho_{yx}\left(\bar{X}-\bar{x}_n\right)}{\rho_{yx}\left(\bar{X}+\bar{x}_n\right)+2\beta_2(x)}\right]$	ρ_{yx}	$\beta_2(x)$
$\eta_{22} = \bar{y_r} \exp\left[\frac{\rho_{yx}\left(\bar{X}-\bar{x}_n\right)}{\rho_{yx}\left(\bar{X}+\bar{x}_n\right)+2C_x}\right]$	ρ_{yx}	C_x

5. Properties of the suggested estimators $\tau_i (i = 1, 2, ..., 22)$ and $\eta_i (i = 1, 2, ..., 22)$

To obtain the biases and mean square errors (MSEs) of the suggested estimators τ_i and η_i (where $i = 1, 2, ..., 22$) for different suitable choices of p and q up-to the first order of large sample approximations are derived under the following transformations:
$\bar{y}_r = \bar{Y}(1 + e_1)$, $\bar{x}_r = \bar{X}(1 + e_2)$ and $\bar{x}_n = \bar{X}(1 + e_3)$ such that $E(e_i) = 0$, $|e_i| < 1\forall$

$i = 1, 2, 3$.
Under the above large transformations, the estimators $\tau_i(i = 1, 2, ..., 22)$ and $\eta_i(i = 1, 2, ..., 22)$ take the following forms:

$$(5.1) \qquad \tau_i = \bar{Y}\left(1 + e_1\right) exp\left[-\frac{1}{2}\theta_i e_2\left(1 + \frac{1}{2}\theta_i e_2\right)^{-1}\right]$$

$$(5.2) \qquad \eta_i = \bar{Y}\left(1 + e_1\right) exp\left[-\frac{1}{2}\theta_i e_3\left(1 + \frac{1}{2}\theta_i e_3\right)^{-1}\right]$$

where $\theta_i = \frac{p\bar{X}}{p\bar{X}+q}(i = 1, 2, ..., 22)$ for different suitable choices of p and q.

$\theta_1 = 1$, $\theta_2 = \frac{\bar{X}}{\bar{X}+1}$, $\theta_3 = \frac{\bar{X}}{\bar{X}+S_x}$, $\theta_4 = \frac{\bar{X}}{\bar{X}+\beta_2(x)}$, $\theta_5 = \frac{\bar{X}}{\bar{X}+C_x}$, $\theta_6 = \frac{\bar{X}}{\bar{X}+\rho_{yx}}$, $\theta_7 = \frac{S_x\bar{X}}{S_x\bar{X}+1}$, $\theta_8 = \frac{S_x\bar{X}}{S_x\bar{X}+\beta_2(x)}$, $\theta_9 = \frac{S_x\bar{X}}{S_x\bar{X}+C_x}$, $\theta_{10} = \frac{S_x\bar{X}}{S_x\bar{X}+\rho_{yx}}$, $\theta_{11} = \frac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X}+1}$, $\theta_{12} = \frac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X}+S_x}$, $\theta_{13} = \frac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X}+C_x}$, $\theta_{14} = \frac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X}+\rho_{yx}}$, $\theta_{15} = \frac{C_x\bar{X}}{C_x\bar{X}+1}$, $\theta_{16} = \frac{C_x\bar{X}}{C_x\bar{X}+S_x}$, $\theta_{17} = \frac{C_x\bar{X}}{C_x\bar{X}+\beta_2(x)}$, $\theta_{18} = \frac{C_x\bar{X}}{C_x\bar{X}+\rho_{yx}}$, $\theta_{19} = \frac{\rho_{yx}\bar{X}}{\rho_{yx}\bar{X}+1}$, $\theta_{20} = \frac{\rho_{yx}\bar{X}}{\rho_{yx}\bar{X}+S_x}$, $\theta_{21} = \frac{\rho_{yx}\bar{X}}{\rho_{yx}\bar{X}+\beta_2(x)}$ and $\theta_{22} = \frac{\rho_{yx}\bar{X}}{\rho_{yx}\bar{X}+C_x}$.
The equations (5.1) and (5.2) can be written neglecting the terms of $e's$ having power greater than two, we get

$$(5.3) \qquad \tau_i - \bar{Y} \cong \bar{Y}\left[e_1 - \frac{1}{2}\theta_i e_2 - \frac{1}{2}\theta_i e_1 e_2 + \frac{3}{8}\theta_i^2 e_2^2\right]$$

$$(5.4) \qquad \eta_i - \bar{Y} \cong \bar{Y}\left[e_1 - \frac{1}{2}\theta_i e_3 - \frac{1}{2}\theta_i e_1 e_3 + \frac{3}{8}\theta_i^2 e_3^2\right]$$

Taking expectation of both sides of equations (5.3) and (5.4) respectively, we get the biases of the proposed estimators up-to first order of large approximations as

$$(5.5) \qquad Bias(\tau_i) = \bar{Y}\frac{1}{8}\theta_i\left[\left(\frac{1}{r} - \frac{1}{N}\right)\left(3\theta_i C_x^2 - 4\rho_{yx}C_y C_x\right)\right]$$

$$(5.6) \qquad Bias(\eta_i) = \bar{Y}\frac{1}{8}\theta_i\left[\left(\frac{1}{n} - \frac{1}{N}\right)\left(3\theta_i C_x^2 - 4\rho_{yx}C_y C_x\right)\right]$$

Now, after squaring both sides of equations (5.3) and (5.4) and neglecting the terms of $e's$ having power greater than two, we have

$$(5.7) \qquad \left(\tau_i - \bar{Y}\right)^2 \cong \bar{Y}^2\left[e_1 - \frac{1}{2}\theta_i e_2\right]^2$$

$$(5.8) \qquad \left(\eta_i - \bar{Y}\right) \cong \bar{Y}^2\left[e_1 - \frac{1}{2}\theta_i e_3\right]^2$$

Taking expectation of both sides of equations (5.7) and (5.8) respectively, we get the MSEs of the proposed estimators τ_i and η_i (where $i = 1, 2, ..., 22$) for the first order of large approximations as

$$(5.9) \qquad MSE(\tau_i) = \bar{Y}^2\left[\left(\frac{1}{r} - \frac{1}{N}\right)\left(C_y^2 + \frac{1}{4}\theta_i^2 C_x^2 - \theta_i\rho_{yx}C_y C_x\right)\right]$$

$$(5.10) \qquad MSE(\eta_i) = \bar{Y}^2\left[\left(\frac{1}{r} - \frac{1}{N}\right)C_y^2 + \left(\frac{1}{n} - \frac{1}{N}\right)\left(\frac{1}{4}\theta_i^2 C_x^2 - \theta_i\rho_{yx}C_y C_x\right)\right]$$

6. Simulation Study

In this section, the proposed estimators are compared with respect to the some existing estimators. The percent relative efficiencies of the proposed estimators τ_i and η_i (where $i = 1, 2, ..., 22$) with respect to the mean method of imputation, ratio method of imputation, regression method of imputation, Singh and Horn [16] estimator, Singh and Deo [15] estimator, Toutenburg *et al.* [18] estimators, Singh [17] estimator and Gira [5] estimator respectively and computed as the followings:

$$E_1 = PRE(t_i, \bar{y}_r) = \frac{V(\bar{y}_r)}{MSE(t_i)} \times 100$$

$$E_2 = PRE(t_i, \bar{y}_{RAT}) = \frac{MSE(\bar{y}_{RAT})}{MSE(t_i)} \times 100$$

$$E_3 = PRE(t_i, \bar{y}_{REG}) = \frac{MSE(\bar{y}_{REG})}{MSE(t_i)} \times 100$$

$$E_4 = PRE(t_i, \bar{y}_{SH}) = \frac{MSE_{min}(\bar{y}_{SH})}{MSE(t_i)} \times 100$$

$$E_5 = PRE(t_i, \bar{y}_{SD}) = \frac{MSE_{min}(\bar{y}_{SD})}{MSE(t_i)} \times 100$$

$$E_6 = PRE(t_i, \bar{y}_{TSS1}) = \frac{MSE(\bar{y}_{TSS1})}{MSE(t_i)} \times 100$$

$$E_7 = PRE(t_i, \bar{y}_{TSS2}) = \frac{MSE(\bar{y}_{TSS2})}{MSE(t_i)} \times 100$$

$$E_8 = PRE(t_i, \bar{y}_{SINGH}) = \frac{MSE_{min}(\bar{y}_{SINGH})}{MSE(t_i)} \times 100$$

$$E_9 = PRE(t_i, \bar{y}_{GIRA}) = \frac{MSE_{min}(\bar{y}_{GIRA})}{MSE(t_i)} \times 100$$

where $t_i = (\tau_i, \eta_i, (i = 1, 2, ..., 22))$. Five different types of real data set have been considered to analyze the performance of the suggested ratio exponential imputation methods over other existing methods.

Table 3. Parameters of five different data sets

Parameters	data set 1 [Source: [7]]	data set 2 [Source: [8]]	data set 3 [Source: [3]]	data set 4 [Source: [2]]	data set 5 [Source: [11]] page 228
N	106	104	278	100	80
n	20	20	30	10	20
r	18	17	24	7	16
$\bar{Y}$	15.37	6.254	39.068	101.1	51.8264
$\bar{X}$	243.76	13931.683	25.111	58.8	2.8513
$\beta_2(x)$	25.71	17.516	38.8898	2.2387	1.3005
C_y	4.1802	1.866	1.4451	0.1450	0.3542
C_x	2.0179	1.653	1.6198	0.1281	0.9484
ρ_{yx}	0.82	0.865	0.7213	0.65	0.9150

Table 4. Percent relative efficiencies of the proposed estimators τ_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 1.

$N = 106$, $n = 20$, $r = 18$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
τ_1	150.962	140.803	138.735	138.735	138.735	140.803	141.438	138.735	138.735
τ_2	150.702	140.561	138.496	138.496	138.496	140.561	141.194	138.496	138.496
τ_3	114.255	106.567	105.001	105.001	105.001	106.567	107.047	105.001	105.001
τ_4	145.012	135.254	133.267	133.267	133.267	135.254	135.863	133.267	133.267
τ_5	150.440	140.317	138.255	138.255	138.255	140.317	140.949	138.255	138.255
τ_6	150.748	140.604	138.539	138.539	138.539	140.604	141.238	138.539	138.539
τ_7	150.961	140.803	138.734	138.734	138.734	140.803	141.437	138.734	138.734
τ_8	150.948	140.791	138.722	138.722	138.722	140.791	141.425	138.722	138.722
τ_9	150.961	140.802	138.734	138.734	138.734	140.802	141.437	138.734	138.734
τ_{10}	150.961	140.803	138.734	138.734	138.734	140.803	141.437	138.734	138.734
τ_{11}	150.952	140.794	138.725	138.725	138.725	140.794	141.428	138.725	138.725
τ_{12}	146.401	136.549	134.543	134.543	134.543	136.549	137.165	134.543	134.543
τ_{13}	150.941	140.784	138.716	138.716	138.716	140.784	141.419	138.716	138.716
τ_{14}	150.953	140.795	138.727	138.727	138.727	140.795	141.430	138.727	138.727
τ_{15}	150.833	140.683	138.616	138.616	138.616	140.683	141.317	138.616	138.616
τ_{16}	122.452	114.212	112.534	112.534	112.534	114.212	114.727	112.534	112.534
τ_{17}	147.833	137.885	135.860	135.860	135.860	137.885	138.507	135.860	135.860
τ_{18}	150.856	140.704	138.637	138.637	138.637	140.704	141.339	138.637	138.637
τ_{19}	150.645	140.508	138.444	138.444	138.444	140.508	141.141	138.444	138.444
τ_{20}	112.298	104.741	103.203	103.203	103.203	104.741	105.213	103.203	103.203
τ_{21}	143.883	134.201	132.229	132.229	132.229	134.201	134.806	132.229	132.229
τ_{22}	150.327	140.211	138.151	138.151	138.151	140.211	140.843	138.151	138.151

Table 5. Percent relative efficiencies of the proposed estimators τ_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 2.

$N = 104$, $n = 20$, $r = 16$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
τ_1	232.601	191.488	191.464	191.464	191.464	191.488	192.808	191.464	191.464
τ_2	232.586	191.476	191.453	191.453	191.453	191.476	192.796	191.453	191.453
τ_3	135.310	111.394	111.380	111.380	111.380	111.394	112.161	111.380	111.380
τ_4	232.347	191.279	191.255	191.255	191.255	191.279	192.597	191.255	191.255
τ_5	232.577	191.469	191.445	191.445	191.445	191.469	192.788	191.445	191.445
τ_6	232.588	191.478	191.454	191.454	191.454	191.478	192.797	191.454	191.454
τ_7	232.601	191.488	191.464	191.464	191.464	191.488	192.808	191.464	191.464
τ_8	232.601	191.488	191.464	191.464	191.464	191.488	192.808	191.464	191.464
τ_9	232.601	191.488	191.464	191.464	191.464	191.488	192.808	191.464	191.464
τ_{10}	232.601	191.488	191.464	191.464	191.464	191.488	192.808	191.464	191.464
τ_{11}	232.600	191.488	191.464	191.464	191.464	191.488	192.807	191.464	191.464
τ_{12}	215.693	177.569	177.547	177.547	177.547	177.569	178.793	177.547	177.547
τ_{13}	232.599	191.487	191.463	191.463	191.463	191.487	192.807	191.463	191.463
τ_{14}	232.600	191.488	191.464	191.464	191.464	191.488	192.807	191.464	191.464
τ_{15}	232.592	191.481	191.457	191.457	191.457	191.481	192.801	191.457	191.457
τ_{16}	150.169	123.627	123.611	123.611	123.611	123.627	124.479	123.611	123.611
τ_{17}	232.447	191.362	191.338	191.338	191.338	191.362	192.680	191.338	191.338
τ_{18}	232.593	191.482	191.458	191.458	191.458	191.482	192.802	191.458	191.458
τ_{19}	232.584	191.475	191.451	191.451	191.451	191.475	192.794	191.451	191.451
τ_{20}	131.593	108.334	108.320	108.320	108.320	108.334	109.080	108.320	108.320
τ_{21}	232.307	191.247	191.223	191.223	191.223	191.247	192.565	191.223	191.223
τ_{22}	232.573	191.466	191.442	191.442	191.442	191.466	192.785	191.442	191.442

Table 6. Percent relative efficiencies of the proposed estimators τ_i ($i = 1, 2, ..., 22$) over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 2.

$N = 104$, $n = 20$, $r = 18$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
τ_1	232.601	211.566	211.554	211.554	211.554	211.566	211.683	211.554	211.554
τ_2	232.586	211.553	211.541	211.541	211.541	211.553	211.67	211.541	211.541
τ_3	135.31	123.074	123.067	123.067	123.067	123.074	123.142	123.067	123.067
τ_4	232.347	211.336	211.323	211.323	211.323	211.336	211.452	211.323	211.323
τ_5	232.577	211.545	211.532	211.532	211.532	211.545	211.661	211.532	211.532
τ_6	232.588	211.555	211.543	211.543	211.543	211.555	211.672	211.543	211.543
τ_7	232.601	211.566	211.554	211.554	211.554	211.566	211.683	211.554	211.554
τ_8	232.601	211.566	211.554	211.554	211.554	211.566	211.683	211.554	211.554
τ_9	232.601	211.566	211.554	211.554	211.554	211.566	211.683	211.554	211.554
τ_{10}	232.601	211.566	211.554	211.554	211.554	211.566	211.683	211.554	211.554
τ_{11}	232.6	211.566	211.553	211.553	211.553	211.566	211.683	211.553	211.553
τ_{12}	215.693	196.188	196.176	196.176	196.176	196.188	196.296	196.176	196.176
τ_{13}	232.599	211.565	211.553	211.553	211.553	211.565	211.682	211.553	211.553
τ_{14}	232.600	211.566	211.554	211.554	211.554	211.566	211.683	211.554	211.554
τ_{15}	232.592	211.558	211.546	211.546	211.546	211.558	211.675	211.546	211.546
τ_{16}	150.169	136.589	136.582	136.582	136.582	136.589	136.665	136.582	136.582
τ_{17}	232.447	211.427	211.414	211.414	211.414	211.427	211.543	211.414	211.414
τ_{18}	232.593	211.560	211.547	211.547	211.547	211.560	211.676	211.547	211.547
τ_{19}	232.584	211.551	211.539	211.539	211.539	211.551	211.668	211.539	211.539
τ_{20}	131.593	119.693	119.686	119.686	119.686	119.693	119.759	119.686	119.686
τ_{21}	232.307	211.300	211.287	211.287	211.287	211.300	211.416	211.287	211.287
τ_{22}	232.573	211.541	211.529	211.529	211.529	211.541	211.658	211.529	211.529

Table 7. Percent relative efficiencies of the proposed estimators τ_i ($i = 1, 2, ..., 22$) over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 3.

$N = 278$, $n = 30$, $r = 21$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
τ_1	197.785	174.64	164.391	164.391	164.391	174.64	164.649	164.391	164.391
τ_2	194.945	172.132	162.031	162.031	162.031	172.132	162.285	162.031	162.031
τ_3	135.657	119.782	112.753	112.753	112.753	119.782	112.93	112.753	112.753
τ_4	136.774	120.768	113.681	113.681	113.681	120.768	113.859	113.681	113.681
τ_5	193.17	170.565	160.556	160.556	160.556	170.565	160.807	160.556	160.556
τ_6	195.741	172.835	162.693	162.693	162.693	172.835	162.947	162.693	162.693
τ_7	197.716	174.579	164.334	164.334	164.334	174.579	164.591	164.334	164.334
τ_8	195.070	172.243	162.135	162.135	162.135	172.243	162.389	162.135	162.135
τ_9	197.673	174.541	164.298	164.298	164.298	174.541	164.556	164.298	164.298
τ_{10}	197.735	174.596	164.350	164.350	164.350	174.596	164.607	164.350	164.350
τ_{11}	197.712	174.576	164.331	164.331	164.331	174.576	164.589	164.331	164.331
τ_{12}	194.813	172.016	161.922	161.922	161.922	172.016	162.175	161.922	161.922
τ_{13}	197.668	174.537	164.294	164.294	164.294	174.537	164.551	164.294	164.294
τ_{14}	197.733	174.594	164.348	164.348	164.348	174.594	164.605	164.348	164.348
τ_{15}	196.037	173.097	162.939	162.939	162.939	173.097	163.194	162.939	162.939
τ_{16}	148.307	130.952	123.268	123.268	123.268	130.952	123.461	123.268	123.268
τ_{17}	149.528	132.031	124.283	124.283	124.283	132.031	124.477	124.283	124.283
τ_{18}	196.526	173.529	163.346	163.346	163.346	173.529	163.601	163.346	163.346
τ_{19}	193.838	171.155	161.111	161.111	161.111	171.155	161.364	161.111	161.111
τ_{20}	128.088	113.099	106.462	106.462	106.462	113.099	106.628	106.462	106.462
τ_{21}	129.064	113.961	107.273	107.273	107.273	113.961	107.441	107.273	107.273
τ_{22}	191.381	168.986	159.069	159.069	159.069	168.986	159.318	159.069	159.069

Table 8. Percent relative efficiencies of the proposed estimators τ_i ($i = 1, 2, ..., 22$) over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 3.

$N = 278$, $n = 30$, $r = 27$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
τ_1	197.785	189.885	186.388	186.388	186.388	189.885	188.198	186.388	186.388
τ_2	194.945	187.159	183.711	183.711	183.711	187.159	185.496	183.711	183.711
τ_3	135.657	130.239	127.84	127.84	127.84	130.239	129.082	127.84	127.84
τ_4	136.774	131.311	128.892	128.892	128.892	131.311	130.144	128.892	128.892
τ_5	193.170	185.455	182.039	182.039	182.039	185.455	183.807	182.039	182.039
τ_6	195.741	187.923	184.461	184.461	184.461	187.923	186.253	184.461	184.461
τ_7	197.716	189.819	186.322	186.322	186.322	189.819	188.132	186.322	186.322
τ_8	195.070	187.279	183.829	183.829	183.829	187.279	185.615	183.829	183.829
τ_9	197.673	189.778	186.282	186.282	186.282	189.778	188.092	186.282	186.282
τ_{10}	197.735	189.837	186.341	186.341	186.341	189.837	188.151	186.341	186.341
τ_{11}	197.712	189.816	186.319	186.319	186.319	189.816	188.129	186.319	186.319
τ_{12}	194.813	187.033	183.587	183.587	183.587	187.033	185.371	183.587	183.587
τ_{13}	197.668	189.773	186.277	186.277	186.277	189.773	188.087	186.277	186.277
τ_{14}	197.733	189.835	186.338	186.338	186.338	189.835	188.149	186.338	186.338
τ_{15}	196.037	188.208	184.741	184.741	184.741	188.208	186.535	184.741	184.741
τ_{16}	148.307	142.384	139.761	139.761	139.761	142.384	141.119	139.761	139.761
τ_{17}	149.528	143.556	140.912	140.912	140.912	143.556	142.281	140.912	140.912
τ_{18}	196.526	188.677	185.202	185.202	185.202	188.677	187.001	185.202	185.202
τ_{19}	193.838	186.097	182.669	182.669	182.669	186.097	184.443	182.669	182.669
τ_{20}	128.088	122.972	120.707	120.707	120.707	122.972	121.879	120.707	120.707
τ_{21}	129.064	123.909	121.626	121.626	121.626	123.909	122.808	121.626	121.626
τ_{22}	191.381	183.738	180.353	180.353	180.353	183.738	182.105	180.353	180.353

Table 9. Percent relative efficiencies of the proposed estimators τ_i ($i = 1, 2, ..., 22$) over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 4.

$N = 100$, $n = 10$, $r = 7$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
τ_1	161.062	141.942	139.111	139.111	139.111	141.942	139.163	139.111	139.111
τ_2	160.254	141.230	138.413	138.413	138.413	141.23	138.464	138.413	138.413
τ_3	155.210	136.785	134.056	134.056	134.056	136.785	134.106	134.056	134.056
τ_4	159.264	140.357	137.558	137.558	137.558	140.357	137.609	137.558	137.558
τ_5	160.958	141.851	139.021	139.021	139.021	141.851	139.073	139.021	139.021
τ_6	160.536	141.479	138.656	138.656	138.656	141.479	138.708	138.656	138.656
τ_7	160.954	141.847	139.018	139.018	139.018	141.847	139.070	139.018	139.018
τ_8	160.821	141.730	138.903	138.903	138.903	141.730	138.954	138.903	138.903
τ_9	161.048	141.930	139.099	139.099	139.099	141.930	139.151	139.099	139.099
τ_{10}	160.992	141.881	139.050	139.050	139.050	141.881	139.102	139.050	139.050
τ_{11}	160.700	141.623	138.798	138.798	138.798	141.623	138.850	138.798	138.798
τ_{12}	158.376	139.575	136.791	136.791	136.791	139.575	136.842	136.791	136.791
τ_{13}	161.016	141.901	139.071	139.071	139.071	141.901	139.123	139.071	139.071
τ_{14}	160.827	141.735	138.907	138.907	138.907	141.735	138.959	138.907	138.907
τ_{15}	155.009	136.608	133.883	133.883	133.883	136.608	133.933	133.883	133.883
τ_{16}	131.292	115.707	113.398	113.398	113.398	115.707	113.441	113.398	113.398
τ_{17}	148.527	130.895	128.284	128.284	128.284	130.895	128.332	128.284	128.284
τ_{18}	157.053	138.409	135.648	135.648	135.648	138.409	135.699	135.648	135.648
τ_{19}	159.822	140.849	138.040	138.040	138.040	140.849	138.091	138.040	138.040
τ_{20}	152.332	134.249	131.571	131.571	131.571	134.249	131.620	131.571	131.571
τ_{21}	158.314	139.520	136.737	136.737	136.737	139.520	136.788	136.737	136.737
τ_{22}	160.902	141.801	138.973	138.973	138.973	141.801	139.025	138.973	138.973

Table 10. Percent relative efficiencies of the proposed estimators τ_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 4.

$N = 100$, $n = 10$, $r = 9$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
τ_1	161.062	154.549	153.584	153.584	153.584	154.549	153.957	153.584	153.584
τ_2	160.254	153.773	152.813	152.813	152.813	153.773	153.184	152.813	152.813
τ_3	155.210	148.933	148.004	148.004	148.004	148.933	148.363	148.004	148.004
τ_4	159.264	152.823	151.869	151.869	151.869	152.823	152.238	151.869	151.869
τ_5	160.958	154.449	153.485	153.485	153.485	154.449	153.857	153.485	153.485
τ_6	160.536	154.044	153.082	153.082	153.082	154.044	153.454	153.082	153.082
τ_7	160.954	154.445	153.481	153.481	153.481	154.445	153.854	153.481	153.481
τ_8	160.821	154.317	153.354	153.354	153.354	154.317	153.726	153.354	153.354
τ_9	161.048	154.535	153.571	153.571	153.571	154.535	153.944	153.571	153.571
τ_{10}	160.992	154.482	153.517	153.517	153.517	154.482	153.890	153.517	153.517
τ_{11}	160.700	154.201	153.239	153.239	153.239	154.201	153.611	153.239	153.239
τ_{12}	158.376	151.971	151.023	151.023	151.023	151.971	151.389	151.023	151.023
τ_{13}	161.016	154.504	153.540	153.540	153.540	154.504	153.912	153.540	153.540
τ_{14}	160.827	154.323	153.360	153.360	153.360	154.323	153.732	153.360	153.360
τ_{15}	155.009	148.740	147.812	147.812	147.812	148.740	148.171	147.812	147.812
τ_{16}	131.292	125.983	125.197	125.197	125.197	125.983	125.500	125.197	125.197
τ_{17}	148.527	142.520	141.631	141.631	141.631	142.520	141.975	141.631	141.631
τ_{18}	157.053	150.702	149.761	149.761	149.761	150.702	150.125	149.761	149.761
τ_{19}	159.822	153.359	152.401	152.401	152.401	153.359	152.771	152.401	152.401
τ_{20}	152.332	146.172	145.260	145.260	145.260	146.172	145.612	145.260	145.260
τ_{21}	158.314	151.911	150.963	150.963	150.963	151.911	151.330	150.963	150.963
τ_{22}	160.902	154.395	153.432	153.432	153.432	154.395	153.804	153.432	153.432

Table 11. Percent relative efficiencies of the proposed estimators τ_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 5.

$N = 80$, $n = 20$, $r = 14$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
τ_1	292.078	533.119	203.156	203.156	203.156	533.119	300.898	203.156	203.156
τ_2	593.201	1082.75	412.603	412.603	412.603	1082.75	611.115	412.603	412.603
τ_3	465.76	850.136	323.962	323.962	323.962	850.136	479.826	323.962	323.962
τ_4	614.271	1121.21	427.259	427.259	427.259	1121.21	632.821	427.259	427.259
τ_5	585.45	1068.6	407.213	407.213	407.213	1068.6	603.13	407.213	407.213
τ_6	579.76	1058.21	403.255	403.255	403.255	1058.21	597.268	403.255	403.255
τ_7	424.224	774.321	295.071	295.071	295.071	774.321	437.035	295.071	295.071
τ_8	463.096	845.273	322.109	322.109	322.109	845.273	477.082	322.109	322.109
τ_9	417.381	761.831	290.311	290.311	290.311	761.831	429.986	290.311	290.311
τ_{10}	412.934	753.714	287.218	287.218	287.218	753.714	425.404	287.218	287.218
τ_{11}	548.797	1001.7	381.718	381.718	381.718	1001.7	565.37	381.718	381.718
τ_{12}	547.646	999.599	380.918	380.918	380.918	999.599	564.185	380.918	380.918
τ_{13}	538.759	983.378	374.737	374.737	374.737	983.378	555.029	374.737	374.737
τ_{14}	531.918	970.891	369.978	369.978	369.978	970.891	547.982	369.978	369.978
τ_{15}	600.009	1095.17	417.339	417.339	417.339	1095.17	618.128	417.339	417.339
τ_{16}	448.237	818.15	311.773	311.773	311.773	818.15	461.773	311.773	311.773
τ_{17}	613.891	1120.51	426.994	426.994	426.994	1120.51	632.43	426.994	426.994
τ_{18}	588.048	1073.340	409.019	409.019	409.019	1073.34	605.806	409.019	409.019
τ_{19}	603.99	1102.44	420.108	420.108	420.108	1102.44	622.23	420.108	420.108
τ_{20}	436.426	796.592	303.558	303.558	303.558	796.592	449.605	303.558	303.558
τ_{21}	612.595	1118.15	426.093	426.093	426.093	1118.15	631.094	426.093	426.093
τ_{22}	597.922	1091.37	415.887	415.887	415.887	1091.37	615.979	415.887	415.887

Table 12. Percent relative efficiencies of the proposed estimators τ_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 5.

$N = 80$, $n = 20$, $r = 18$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
τ_1	292.078	377.609	260.525	260.525	260.525	377.609	344.738	260.525	260.525
τ_2	593.201	766.911	529.118	529.118	529.118	766.911	700.151	529.118	529.118
τ_3	465.760	602.152	415.445	415.445	415.445	602.152	549.734	415.445	415.445
τ_4	614.271	794.151	547.912	547.912	547.912	794.151	725.020	547.912	547.912
τ_5	585.450	756.891	522.205	522.205	522.205	756.891	691.003	522.205	522.205
τ_6	579.760	749.534	517.129	517.129	517.129	749.534	684.287	517.129	517.129
τ_7	424.224	548.452	378.396	378.396	378.396	548.452	500.709	378.396	378.396
τ_8	463.096	598.708	413.069	413.069	413.069	598.708	546.590	413.069	413.069
τ_9	417.381	539.605	372.292	372.292	372.292	539.605	492.632	372.292	372.292
τ_{10}	412.934	533.856	368.325	368.325	368.325	533.856	487.384	368.325	368.325
τ_{11}	548.797	709.505	489.511	489.511	489.511	709.505	647.742	489.511	489.511
τ_{12}	547.646	708.017	488.485	488.485	488.485	708.017	646.383	488.485	488.485
τ_{13}	538.759	696.527	480.558	480.558	480.558	696.527	635.894	480.558	480.558
τ_{14}	531.918	687.683	474.456	474.456	474.456	687.683	627.820	474.456	474.456
τ_{15}	600.009	775.713	535.190	535.190	535.190	775.713	708.187	535.190	535.190
τ_{16}	448.237	579.496	399.814	399.814	399.814	579.496	529.051	399.814	399.814
τ_{17}	613.891	793.660	547.573	547.573	547.573	793.660	724.571	547.573	547.573
τ_{18}	588.048	760.249	524.522	524.522	524.522	760.249	694.069	524.522	524.522
τ_{19}	603.990	780.860	538.742	538.742	538.742	780.860	712.886	538.742	538.742
τ_{20}	436.426	564.227	389.279	389.279	389.279	564.227	515.111	389.279	389.279
τ_{21}	612.595	791.984	546.417	546.417	546.417	791.984	723.042	546.417	546.417
τ_{22}	597.922	773.015	533.329	533.329	533.329	773.015	705.724	533.329	533.329

Table 13. Percent relative efficiencies of the proposed estimators η_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 1.

$N = 106$, $n = 20$, $r = 18$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
η_1	142.231	132.660	130.711	130.711	130.711	132.660	133.258	130.711	130.711
η_2	142.028	132.471	130.524	130.524	130.524	132.471	133.067	130.524	130.524
η_3	112.326	104.768	103.229	103.229	103.229	104.768	105.240	103.229	103.229
η_4	137.554	128.297	126.413	126.413	126.413	128.297	128.876	126.413	126.413
η_5	141.823	132.280	130.336	130.336	130.336	132.280	132.876	130.336	130.336
η_6	142.064	132.505	130.558	130.558	130.558	132.505	133.102	130.558	130.558
η_7	142.230	132.659	130.711	130.711	130.711	132.659	133.257	130.711	130.711
η_8	142.220	132.650	130.701	130.701	130.701	132.650	133.248	130.701	130.701
η_9	142.230	132.659	130.710	130.710	130.710	132.659	133.257	130.710	130.710
η_{10}	142.230	132.660	130.711	130.711	130.711	132.660	133.257	130.711	130.711
η_{11}	142.223	132.652	130.704	130.704	130.704	132.652	133.250	130.704	130.704
η_{12}	138.651	129.321	127.422	127.422	127.422	129.321	129.904	127.422	127.422
η_{13}	142.215	132.645	130.696	130.696	130.696	132.645	133.243	130.696	130.696
η_{14}	142.224	132.654	130.705	130.705	130.705	132.654	133.252	130.705	130.705
η_{15}	142.130	132.566	130.618	130.618	130.618	132.566	133.163	130.618	130.618
η_{16}	119.228	111.205	109.571	109.571	109.571	111.205	111.706	109.571	109.571
η_{17}	139.780	130.374	128.458	128.458	128.458	130.374	130.961	128.458	128.458
η_{18}	142.148	132.583	130.635	130.635	130.635	132.583	133.180	130.635	130.635
η_{19}	141.984	132.429	130.484	130.484	130.484	132.429	133.026	130.484	130.484
η_{20}	110.659	103.212	101.696	101.696	101.696	103.212	103.677	101.696	101.696
η_{21}	136.659	127.463	125.591	125.591	125.591	127.463	128.038	125.591	125.591
η_{22}	141.735	132.197	130.255	130.255	130.255	132.197	132.793	130.255	130.255

Table 14. Percent relative efficiencies of the proposed estimators η_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 2.

$N = 104$, $n = 20$, $r = 17$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
η_1	187.920	162.722	162.707	162.707	162.707	162.722	163.130	162.707	162.707
η_2	187.912	162.715	162.701	162.701	162.701	162.715	163.124	162.701	162.701
η_3	127.253	110.190	110.180	110.180	110.180	110.190	110.466	110.180	110.180
η_4	187.784	162.604	162.590	162.590	162.590	162.604	163.012	162.590	162.590
η_5	187.907	162.711	162.696	162.696	162.696	162.711	163.119	162.696	162.696
η_6	187.913	162.716	162.702	162.702	162.702	162.716	163.125	162.702	162.702
η_7	187.920	162.722	162.707	162.707	162.707	162.722	163.130	162.707	162.707
η_8	187.920	162.722	162.707	162.707	162.707	162.722	163.130	162.707	162.707
η_9	187.920	162.722	162.707	162.707	162.707	162.722	163.130	162.707	162.707
η_{10}	187.920	162.722	162.707	162.707	162.707	162.722	163.130	162.707	162.707
η_{10}	187.919	162.722	162.707	162.707	162.707	162.722	163.130	162.707	162.707
η_{12}	178.635	154.683	154.669	154.669	154.669	154.683	155.071	154.669	154.669
η_{13}	187.919	162.721	162.707	162.707	162.707	162.721	163.130	162.707	162.707
η_{14}	187.919	162.722	162.707	162.707	162.707	162.722	163.130	162.707	162.707
η_{15}	187.915	162.718	162.703	162.703	162.703	162.718	163.126	162.703	162.703
η_{16}	137.775	119.301	119.291	119.291	119.291	119.301	119.601	119.291	119.291
η_{17}	187.837	162.651	162.636	162.636	162.636	162.651	163.059	162.636	162.636
η_{18}	187.916	162.719	162.704	162.704	162.704	162.719	163.127	162.704	162.704
η_{19}	187.911	162.714	162.700	162.700	162.700	162.714	163.123	162.700	162.700
η_{20}	124.538	107.839	107.829	107.829	107.829	107.839	108.109	107.829	107.829
η_{21}	187.762	162.586	162.571	162.571	162.571	162.586	162.994	162.571	162.571
η_{22}	187.905	162.709	162.695	162.695	162.695	162.709	163.117	162.695	162.695

Table 15. Percent relative efficiencies of the proposed estimators η_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 2.

$N = 104$, $n = 20$, $r = 18$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
η_1	200.457	182.329	182.319	182.319	182.319	182.329	182.430	182.319	182.319
η_2	200.447	182.321	182.310	182.310	182.310	182.321	182.421	182.310	182.310
η_3	129.769	118.034	118.027	118.027	118.027	118.034	118.099	118.027	118.027
η_4	200.291	182.178	182.168	182.168	182.168	182.178	182.279	182.168	182.168
η_5	200.441	182.315	182.304	182.304	182.304	182.315	182.416	182.304	182.304
η_6	200.448	182.322	182.311	182.311	182.311	182.322	182.422	182.311	182.311
η_7	200.457	182.329	182.319	182.319	182.319	182.329	182.430	182.319	182.319
η_8	200.457	182.329	182.319	182.319	182.319	182.329	182.430	182.319	182.319
η_9	200.457	182.329	182.319	182.319	182.319	182.329	182.430	182.319	182.319
η_{10}	200.457	182.329	182.319	182.319	182.319	182.329	182.430	182.319	182.319
η_{11}	200.456	182.329	182.318	182.318	182.318	182.329	182.429	182.318	182.318
η_{12}	189.220	172.108	172.098	172.098	172.098	172.108	172.203	172.098	172.098
η_{13}	200.456	182.328	182.318	182.318	182.318	182.328	182.429	182.318	182.318
η_{14}	200.456	182.329	182.318	182.318	182.318	182.329	182.429	182.318	182.318
η_{15}	200.451	182.324	182.313	182.313	182.313	182.324	182.425	182.313	182.313
η_{16}	141.580	128.777	128.769	128.769	128.769	128.777	128.848	128.769	128.769
η_{17}	200.356	182.238	182.227	182.227	182.227	182.238	182.339	182.227	182.227
η_{18}	200.452	182.325	182.314	182.314	182.314	182.325	182.425	182.314	182.314
η_{19}	200.446	182.319	182.309	182.309	182.309	182.319	182.420	182.309	182.309
η_{20}	126.750	115.288	115.281	115.281	115.281	115.288	115.352	115.281	115.281
η_{21}	200.265	182.155	182.144	182.144	182.144	182.155	182.255	182.144	182.144
η_{22}	200.438	182.313	182.302	182.302	182.302	182.313	182.413	182.302	182.302

Table 17. Percent relative efficiencies of the proposed estimators η_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 3.

$N = 278$, $n = 30$, $r = 27$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
η_1	178.457	171.330	168.174	168.174	168.174	171.330	169.807	168.174	168.174
η_2	176.395	169.350	166.231	166.231	166.231	169.350	167.846	166.231	166.231
η_3	130.503	125.291	122.983	122.983	122.983	125.291	124.178	122.983	122.983
η_4	131.421	126.172	123.848	123.848	123.848	126.172	125.051	123.848	123.848
η_5	175.101	168.107	165.011	165.011	165.011	168.107	166.614	165.011	165.011
η_6	176.974	169.906	166.777	166.777	166.777	169.906	168.397	166.777	166.777
η_7	178.407	171.282	168.127	168.127	168.127	171.282	169.760	168.127	168.127
η_8	176.487	169.438	166.317	166.317	166.317	169.438	167.932	166.317	166.317
η_9	178.376	171.252	168.097	168.097	168.097	171.252	169.730	168.097	168.097
η_{10}	178.421	171.295	168.140	168.140	168.140	171.295	169.773	168.140	168.140
η_{11}	178.405	171.280	168.124	168.124	168.124	171.280	169.758	168.124	168.124
η_{12}	176.300	169.258	166.141	166.141	166.141	169.258	167.755	166.141	166.141
η_{13}	178.372	171.248	168.094	168.094	168.094	171.248	169.727	168.094	168.094
η_{14}	178.419	171.294	168.138	168.138	168.138	171.294	169.772	168.138	168.138
η_{15}	177.190	170.113	166.979	166.979	166.979	170.113	168.602	166.979	166.979
η_{16}	140.775	135.153	132.663	132.663	132.663	135.153	133.952	132.663	132.663
η_{17}	141.752	136.091	133.584	133.584	133.584	136.091	134.882	133.584	133.584
η_{18}	177.545	170.454	167.314	167.314	167.314	170.454	168.940	167.314	167.314
η_{19}	175.589	168.576	165.471	165.471	165.471	168.576	167.078	165.471	165.471
η_{20}	124.223	119.262	117.065	117.065	117.065	119.262	118.202	117.065	117.065
η_{21}	125.039	120.045	117.833	117.833	117.833	120.045	118.978	117.833	117.833
η_{22}	173.792	166.850	163.777	163.777	163.777	166.850	165.368	163.777	163.777

Table 16. Percent relative efficiencies of the proposed estimators η_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 3.

$N = 278$, $n = 30$, $r = 24$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
η_1	162.913	150.054	144.360	144.360	144.360	150.054	145.457	144.360	144.360
η_2	161.400	148.660	143.019	143.019	143.019	148.660	144.106	143.019	143.019
η_3	125.835	115.902	111.504	111.504	111.504	115.902	112.352	111.504	111.504
η_4	126.584	116.592	112.168	112.168	112.168	116.592	113.020	112.168	112.168
η_5	160.447	147.782	142.174	142.174	142.174	147.782	143.255	142.174	142.174
η_6	161.826	149.052	143.396	143.396	143.396	149.052	144.486	143.396	143.396
η_7	162.877	150.020	144.327	144.327	144.327	150.020	145.424	144.327	144.327
η_8	161.468	148.722	143.079	143.079	143.079	148.722	144.166	143.079	143.079
η_9	162.854	149.999	144.307	144.307	144.307	149.999	145.404	144.307	144.307
η_{10}	162.887	150.030	144.336	144.336	144.336	150.030	145.433	144.336	144.336
η_{10}	162.875	150.019	144.326	144.326	144.326	150.019	145.423	144.326	144.326
η_{12}	161.330	148.596	142.957	142.957	142.957	148.596	144.043	142.957	142.957
η_{13}	162.851	149.997	144.305	144.305	144.305	149.997	145.402	144.305	144.305
η_{14}	162.886	150.028	144.335	144.335	144.335	150.028	145.432	144.335	144.335
η_{15}	161.984	149.198	143.536	143.536	143.536	149.198	144.627	143.536	143.536
η_{16}	134.125	123.537	118.850	118.850	118.850	123.537	119.753	118.850	118.850
η_{17}	134.903	124.254	119.539	119.539	119.539	124.254	120.448	119.539	119.539
η_{18}	162.245	149.438	143.767	143.767	143.767	149.438	144.86	143.767	143.767
η_{19}	160.807	148.114	142.493	142.493	142.493	148.114	143.576	142.493	142.493
η_{20}	120.669	111.144	106.926	106.926	106.926	111.144	107.739	106.926	106.926
η_{21}	121.344	111.766	107.524	107.524	107.524	111.766	108.342	107.524	107.524
η_{22}	159.480	146.892	141.317	141.317	141.317	146.892	142.392	141.317	141.317

Table 18. Percent relative efficiencies of the proposed estimators η_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 4.

$N = 100$, $n = 10$, $r = 7$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
η_1	134.558	118.584	116.219	116.219	116.219	118.584	116.262	116.219	116.219
η_2	134.175	118.247	115.888	115.888	115.888	118.247	115.931	115.888	115.888
η_3	131.746	116.107	113.791	113.791	113.791	116.107	113.833	113.791	113.791
η_4	133.703	117.831	115.481	115.481	115.481	117.831	115.524	115.481	115.481
η_5	134.508	118.541	116.176	116.176	116.176	118.541	116.220	116.176	116.176
η_6	134.308	118.365	116.004	116.004	116.004	118.365	116.047	116.004	116.004
η_7	134.507	118.539	116.175	116.175	116.175	118.539	116.218	116.175	116.175
η_8	134.444	118.484	116.120	116.120	116.120	118.484	116.164	116.120	116.120
η_9	134.551	118.579	116.213	116.213	116.213	118.579	116.256	116.213	116.213
η_{10}	134.525	118.555	116.19	116.190	116.190	118.555	116.233	116.190	116.190
η_{10}	134.386	118.433	116.071	116.071	116.071	118.433	116.114	116.071	116.071
η_{12}	133.278	117.457	115.114	115.114	115.114	117.457	115.157	115.114	115.114
η_{13}	134.536	118.565	116.200	116.200	116.200	118.565	116.243	116.200	116.200
η_{14}	134.446	118.486	116.123	116.123	116.123	118.486	116.166	116.123	116.123
η_{15}	131.648	116.020	113.706	113.706	113.706	116.020	113.748	113.706	113.706
η_{16}	119.254	105.098	103.001	103.001	103.001	105.098	103.040	103.001	103.001
η_{17}	128.424	113.178	110.921	110.921	110.921	113.178	110.962	110.921	110.921
η_{18}	132.641	116.896	114.564	114.564	114.564	116.896	114.606	114.564	114.564
η_{19}	133.969	118.066	115.711	115.711	115.711	118.066	115.754	115.711	115.711
η_{20}	130.331	114.859	112.568	112.568	112.568	114.859	112.610	112.568	112.568
η_{21}	133.248	117.430	115.088	115.088	115.088	117.430	115.131	115.088	115.088
η_{22}	134.482	118.518	116.153	116.153	116.153	118.518	116.197	116.153	116.153

Table 19. Percent relative efficiencies of the proposed estimators η_i $(i = 1, 2, ..., 22)$ over the existing estimators $\bar{y}_r$, $\bar{y}_{RAT}$, $\bar{y}_{REG}$, $\bar{y}_{SH}$, $\bar{y}_{SD}$, $\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$, $\bar{y}_{SINGH}$ and $\bar{y}_{GIRA}$ respectively for data set 5.

$N = 80$, $n = 20$, $r = 16$									
Estimators	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
η_1	197.324	309.279	156.023	156.023	156.023	309.279	230.300	156.023	156.023
η_2	265.652	416.374	210.049	210.049	210.049	416.374	310.047	210.049	210.049
η_3	243.293	381.330	192.370	192.370	192.370	381.330	283.951	192.370	192.370
η_4	268.748	421.227	212.497	212.497	212.497	421.227	313.66	212.497	212.497
η_5	264.476	414.531	209.119	209.119	209.119	414.531	308.674	209.119	209.119
η_6	263.599	413.157	208.426	208.426	208.426	413.157	307.651	208.426	208.426
η_7	234.305	367.243	185.264	185.264	185.264	367.243	273.462	185.264	185.264
η_8	242.746	380.472	191.938	191.938	191.938	380.472	283.313	191.938	191.938
η_9	232.725	364.766	184.014	184.014	184.014	364.766	271.617	184.014	184.014
η_{10}	231.682	363.130	183.189	183.189	183.189	363.130	270.399	183.189	183.189
η_{10}	258.623	405.358	204.492	204.492	204.492	405.358	301.844	204.492	204.492
η_{12}	258.432	405.057	204.340	204.340	204.340	405.057	301.62	204.340	204.340
η_{13}	256.932	402.707	203.154	203.154	203.154	402.707	299.869	203.154	203.154
η_{14}	255.755	400.862	202.224	202.224	202.224	400.862	298.496	202.224	202.224
η_{15}	266.668	417.967	210.853	210.853	210.853	417.967	311.233	210.853	210.853
η_{16}	239.623	375.578	189.468	189.468	189.468	375.578	279.668	189.468	189.468
η_{17}	268.693	421.141	212.454	212.454	212.454	421.141	313.596	212.454	212.454
η_{18}	264.872	415.152	209.433	209.433	209.433	415.152	309.137	209.433	209.433
η_{19}	267.255	418.887	211.317	211.317	211.317	418.887	311.918	211.317	211.317
η_{20}	237.051	371.546	187.435	187.435	187.435	371.546	276.666	187.435	187.435
η_{21}	268.507	420.849	212.307	212.307	212.307	420.849	313.379	212.307	212.307
η_{22}	266.358	417.481	210.608	210.608	210.608	417.481	310.871	210.608	210.608

7. Conclusions

The following interpretation can be read out from Tables (4 - 19):

(1) From Table 4 it is clear that for a 18% sampled population with response rate 90%, the percent relative efficiencies of the suggested estimators τ_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 112.298% to 150.962%; the ratio method of imputation remains 104.741% to 140.803%; the regression method of imputation, Singh and Horn $(\bar{y}_{SH})$ estimator, Singh and Deo $(\bar{y}_{SD})$ estimator, Singh $(\bar{y}_{SINGH})$ estimator and Gira $(\bar{y}_{GIRA})$ estimator remain 103.203% to 138.735%; Toutenburg *et al.*$(\bar{y}_{TSS1}, \bar{y}_{TSS2})$ estimators remain 104.741% to 140.803% and 105.213% to 141.438% respectively.

(2) From Table 5 it is clear that for a 19% sampled population with response rate 80%, the percent relative efficiencies of the suggested estimators τ_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 131.593% to 232.601%; the ratio method of imputation remains 108.334% to 191.488%; the regression method of imputation, Singh and Horn $(\bar{y}_{SH})$ estimator, Singh and Deo $(\bar{y}_{SD})$ estimator, Singh $(\bar{y}_{SINGH})$ estimator and Gira $(\bar{y}_{GIRA})$ estimator remain 108.320% to 191.464%; Toutenburg *et al.*$(\bar{y}_{TSS1}, \bar{y}_{TSS2})$ estimators remain 108.334% to 191.488% and 109.080% to 192.808% respectively.

(3) From Table 6 it is clear that for a 19% sampled population with response rate 90%, the percent relative efficiencies of the suggested estimators τ_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 131.593% to 232.601%; the ratio method of imputation remains 119.693% to 211.566%; the regression method of imputation, Singh and Horn $(\bar{y}_{SH})$ estimator, Singh and Deo $(\bar{y}_{SD})$ estimator, Singh $(\bar{y}_{SINGH})$ estimator and Gira $(\bar{y}_{GIRA})$ estimator remain 119.686% to 211.554%; Toutenburg *et al.*$(\bar{y}_{TSS1}, \bar{y}_{TSS2})$ estimators remain 119.693% to 211.566% and 119.759% to 211.683% respectively.

(4) From Table 7 it is clear that for a 10% sampled population with response rate 70%, the percent relative efficiencies of the suggested estimators τ_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 128.088% to 197.785%; the ratio method of imputation remains 113.099% to 174.64%; the regression method of imputation, Singh and Horn $(\bar{y}_{SH})$ estimator, Singh and Deo $(\bar{y}_{SD})$ estimator, Singh $(\bar{y}_{SINGH})$ estimator and Gira $(\bar{y}_{GIRA})$ estimator remain 106.462% to 164.391%; Toutenburg *et al.*$(\bar{y}_{TSS1}, \bar{y}_{TSS2})$ estimators remain 113.099% to 174.64% and 106.628% to 164.649% respectively.

(5) From Table 8 it is clear that for a 10% sampled population with response rate 90%, the percent relative efficiencies of the suggested estimators τ_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 128.088% to 197.785%; the ratio method of imputation remains 122.972% to 189.885%; the regression method of imputation, Singh and Horn $(\bar{y}_{SH})$ estimator, Singh and Deo $(\bar{y}_{SD})$ estimator, Singh $(\bar{y}_{SINGH})$ estimator and Gira $(\bar{y}_{GIRA})$ estimator remain 120.707% to 186.388%; Toutenburg *et al.*$(\bar{y}_{TSS1}, \bar{y}_{TSS2})$ estimators remain 122.972% to 189.885% and 121.879% to 188.198% respectively.

(6) From Table 9 it is clear that for a 10% sampled population with response rate 70%, the percent relative efficiencies of the suggested estimators τ_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 131.292% to 161.062%; the ratio method of imputation remains 115.707% to 141.942%; the regression method of imputation, Singh and Horn $(\bar{y}_{SH})$ estimator, Singh and Deo $(\bar{y}_{SD})$ estimator, Singh $(\bar{y}_{SINGH})$ estimator and Gira $(\bar{y}_{GIRA})$ estimator remain 113.398% to 139.111%; Toutenburg *et al.*$(\bar{y}_{TSS1}, \bar{y}_{TSS2})$ estimators remain 115.707% to 141.942% and 113.441% to 139.163% respectively.

(7) From Table 10 it is clear that for a 10% sampled population with response rate 90%, the percent relative efficiencies of the suggested estimators τ_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 131.292% to 161.062%; the ratio method of imputation remains 125.983% to 154.549%;

the regression method of imputation, Singh and Horn ($\bar{y}_{SH}$) estimator, Singh and Deo ($\bar{y}_{SD}$) estimator, Singh ($\bar{y}_{SINGH}$) estimator and Gira ($\bar{y}_{GIRA}$) estimator remain 125.197% to 153.584%; Toutenburg *et al.*($\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$) estimators remain 125.983% to 154.549% and 125.500% to 153.957% respectively.

(8) From Table 11 it is clear that for a 25% sampled population with response rate 70%, the percent relative efficiencies of the suggested estimators τ_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 292.078% to 614.271%; the ratio method of imputation remains 533.119% to 1121.21%; the regression method of imputation, Singh and Horn ($\bar{y}_{SH}$) estimator, Singh and Deo ($\bar{y}_{SD}$) estimator, Singh ($\bar{y}_{SINGH}$) estimator and Gira ($\bar{y}_{GIRA}$) estimator remain 203.156% to 427.259%; Toutenburg *et al.*($\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$) estimators remain 533.119% to 1121.21% and 300.898% to 632.821% respectively.

(9) From Table 12 it is clear that for a 25% sampled population with response rate 90%, the percent relative efficiencies of the suggested estimators τ_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 292.078% to 614.271%; the ratio method of imputation remains 377.609% to 794.151%; the regression method of imputation, Singh and Horn ($\bar{y}_{SH}$) estimator, Singh and Deo ($\bar{y}_{SD}$) estimator, Singh ($\bar{y}_{SINGH}$) estimator and Gira ($\bar{y}_{GIRA}$) estimator remain 260.525% to 547.912%; Toutenburg *et al.*($\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$) estimators remain 377.609% to 794.151% and 344.738% to 725.020% respectively.

(10) From Table 13 it is clear that for a 18% sampled population with response rate 90%, the percent relative efficiencies of the suggested estimators η_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 110.659% to 142.231%; the ratio method of imputation remains 103.212% to 132.660%; the regression method of imputation, Singh and Horn ($\bar{y}_{SH}$) estimator, Singh and Deo ($\bar{y}_{SD}$) estimator, Singh ($\bar{y}_{SINGH}$) estimator and Gira ($\bar{y}_{GIRA}$) estimator remain 101.696% to 130.711%; Toutenburg *et al.*($\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$) estimators remain 103.212% to 132.660% and 103.677% to 133.258% respectively.

(11) From Table 14 it is clear that for a 19% sampled population with response rate 85%, the percent relative efficiencies of the suggested estimators η_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 124.538% to 187.920%; the ratio method of imputation remains 107.839% to 162.722%; the regression method of imputation, Singh and Horn ($\bar{y}_{SH}$) estimator, Singh and Deo ($\bar{y}_{SD}$) estimator, Singh ($\bar{y}_{SINGH}$) estimator and Gira ($\bar{y}_{GIRA}$) estimator remain 107.829% to 162.707%; Toutenburg *et al.*($\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$) estimators remain 107.839% to 162.722% and 108.109% to 163.130% respectively.

(12) From Table 15 it is clear that for a 19% sampled population with response rate 90%, the percent relative efficiencies of the suggested estimators η_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 126.750% to 200.457%; the ratio method of imputation remains 115.288% to 182.329%; the regression method of imputation, Singh and Horn ($\bar{y}_{SH}$) estimator, Singh and Deo ($\bar{y}_{SD}$) estimator, Singh ($\bar{y}_{SINGH}$) estimator and Gira ($\bar{y}_{GIRA}$) estimator remain 115.281% to 182.319%; Toutenburg *et al.*($\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$) estimators remain 115.288% to 182.329% and 115.352% to 182.430% respectively.

(13) From Table 16 it is clear that for a 10% sampled population with response rate 80%, the percent relative efficiencies of the suggested estimators η_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 120.669% to 162.913%; the ratio method of imputation remains 111.144% to 150.054%; the regression method of imputation, Singh and Horn ($\bar{y}_{SH}$) estimator, Singh and Deo ($\bar{y}_{SD}$) estimator, Singh ($\bar{y}_{SINGH}$) estimator and Gira ($\bar{y}_{GIRA}$) estimator remain 106.926% to 144.360%; Toutenburg *et al.*($\bar{y}_{TSS1}$, $\bar{y}_{TSS2}$) estimators remain 111.144% to 150.054%

and 107.739% to 145.457% respectively.
(14) From Table 17 it is clear that for a 10% sampled population with response rate 90%, the percent relative efficiencies of the suggested estimators η_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 124.223% to 178.457%; the ratio method of imputation remains 119.262% to 171.330%; the regression method of imputation, Singh and Horn $(\bar{y}_{SH})$ estimator, Singh and Deo $(\bar{y}_{SD})$ estimator, Singh $(\bar{y}_{SINGH})$ estimator and Gira $(\bar{y}_{GIRA})$ estimator remain 117.065% to 168.174%; Toutenburg *et al.*$(\bar{y}_{TSS1}, \bar{y}_{TSS2})$ estimators remain 119.262% to 171.330% and 118.202% to 169.807% respectively.
(15) From Table 18 it is clear that for a 10% sampled population with response rate 70%, the percent relative efficiencies of the suggested estimators η_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 130.331% to 134.558%; the ratio method of imputation remains 114.859% to 118.584%; the regression method of imputation, Singh and Horn $(\bar{y}_{SH})$ estimator, Singh and Deo $(\bar{y}_{SD})$ estimator, Singh $(\bar{y}_{SINGH})$ estimator and Gira $(\bar{y}_{GIRA})$ estimator remain 112.568% to 116.219%; Toutenburg *et al.*$(\bar{y}_{TSS1}, \bar{y}_{TSS2})$ estimators remain 114.855% to 118.584% and 112.610% to 116.262% respectively.
(16) From Table 19 it is clear that for a 25% sampled population with response rate 80%, the percent relative efficiencies of the suggested estimators η_i $(i = 1, 2, ..., 22)$ with respect to the other existing estimators like as the mean method of imputation remains 197.324% to 268.748%; the ratio method of imputation remains 309.279% to 421.227%; the regression method of imputation, Singh and Horn $(\bar{y}_{SH})$ estimator, Singh and Deo $(\bar{y}_{SD})$ estimator, Singh $(\bar{y}_{SINGH})$ estimator and Gira $(\bar{y}_{GIRA})$ estimator remain 156.023% to 212.497%; Toutenburg *et al.*$(\bar{y}_{TSS1}, \bar{y}_{TSS2})$ estimators remain 309.279% to 421.227% and 230.300% to 313.660% respectively.

In this paper, we have presented the five real data sets of the different parameters in Table 3. Here, the response rate is considered between 70% to 90%. From Tables [4-19], it is noticed that the percent relative efficiencies of the suggested estimators involved in simulation studies for given the real data sets . It is found that our suggested imputation methods are more efficient than the mean method of imputation, ratio method of imputation, regression method of imputation, Singh and Horn [16] estimator, Singh and Deo [15] estimator, Toutenburg *et al.* [18] estimators, Singh [17] estimator and Gira [5] estimator. Hence, the performances of the suggested methods and their corresponding estimators are highly justified in simulation studies which are shown in Tables [4-19] that may be recommended for further use.

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