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Modified Holt's linear trend method

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Abstract

Exponential smoothing models are simple, accurate and robust forecasting models and because of these they are widely applied in the literature. Holt's linear trend method is a valuable extension of exponential smoothing that helps deal with trending data. In this study we propose a modified version of Holt's linear trend method that eliminates the initialization issue faced when fitting the original model and simplifies the optimization process. The proposed method is compared empirically with the most popular forecasting algorithms based on exponential smoothing and Box-Jenkins ARIMA with respect to its predictive performance on the M3-Competition data set and is shown to outperform its competitors.

Keywords: Exponential smoothing, Forecasting, Initial value, M3-Competition, Smoothing parameter.

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1. Introduction

Forecasting is an essential activity in various branches of science and in many areas of industrial, commercial and economic activity. Forecasts can be obtained by using purely judgmental, explanatory and extrapolative methods or any combination of these three but extrapolative methods are reliable, objective, inexpensive, quick, and easily automated. In recent decades, numerous time series forecasting models have been proposed. A review of the 25 year period until the year 2005 can be found in [4]. It is clear that time series forecasting methods are still dominated by two major techniques: [2] (ARIMA)

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and exponential smoothing (ES) [3]. However, ES methods are the most widely used techniques in forecasting due to their simplicity, robustness and accuracy as automatic forecasting procedures [8]. An excellent review of the literature on ES can be found in [5] which is later updated in [6] and [10]. The popularity of ES in time series analysis is not just as a result of its simplicity but also its proven record against more sophisticated approaches [15, 14, 11].

In ES models recent observations are given relatively more weight in forecasting than the older observations. ES is not a single model but rather a family of models. ES models assume that the time series has up to three underlying data components: level, trend and seasonality. The goal of an ES model is to estimate the final values of the level, trend and seasonal pattern and then to use these final values to construct forecasts. Each model consists of one of the five types of trend (none, additive, damped additive, multiplicative, and damped multiplicative) and one of the three types of seasonality (none, additive, and multiplicative). [16] proposed a taxonomy of ES methods, which was extended and modified later by [7], [11], and [17]. There are 15 different models, the best known of which are SES (no trend, no seasonality), Holt's linear model (additive trend, no seasonality) and Holt-Winters' additive model (additive trend, additive seasonality). [10] proposed the ETS state space models to provide a solid theoretical foundation for ES. In this work two possible innovative state space models for each of the 15 exponential smoothing methods are defined, one corresponding to a model with additive errors and the other to a model with multiplicative errors, resulting in 30 potential models.

There are many studies on the numerical and theoretical comparison of Box-Jenkins and ES methods. For several decades, ES has been considered an ad hoc approach to forecasting, with no proper underlying stochastic formulation. The state space framework described by [11] brings exponential smoothing into the same class as ARIMA models. ES is widely applicable and has a sound stochastic model behind the forecast therefore the "ad hoc approach" argument is no longer true. [11] introduced a state space framework that subsumes all the exponential smoothing models and allows for the computation of prediction intervals, likelihood and model selection criteria. They also proposed an automatic forecasting strategy based on this model framework.

However, the main competition between these two major forecasting methods is on their post-sample forecasting accuracy. The 1001 and 3003 time series used respectively in the M-competition [12] and the M3-competition [14] have become recognized collections of test data for the evaluation of forecasting methods. The first conclusion of these competitions is that "statistically sophisticated or complex methods did not provide more accurate forecasts than simpler ones". Some standard and simple combinations of ES methods were used in these competitions and their performances verified this result. According to statements by the participants of the M-competition, the Box-Jenkins methodology (ARIMA models) required the most time (on the average more than one hour per series). To propose a simple, accurate, robust and automatic forecasting method as an alternative to ES methods is not easy task after the results of [11]. In this study, they applied an automatic forecasting strategy to the M-competition data [12] and the M3 competition data [14]. The automatic forecasting procedure proposed in this paper tries 24 of the 30 state space models on a given time series and selects the "best" model using the AIC. They show that the methodology is particularly good at short term forecasts (up to about 6 periods ahead), and especially accurate for seasonal short-term series (beating all other methods in the competitions).

Even though ES models are well developed, they still have some important shortcomings that affect the quality of the forecasts obtained using them. The most important issues faced when building ES models are related to initialization and optimization problems. For example [13] showed that even though for other exponential smoothing models the type of initialization and loss functions that are employed did not result in significant changes in post sample forecasting accuracies, for Holt's linear trend model they were very influential especially for long term forecasting horizons. Even if this was not the case, the fact that trying to find an optimal initial value both complicates and prolongs the optimization process can not be overlooked. The modified simple exponential smoothing model proposed in [18] helps deal with these problems but still lacks the ability to deal with possible trending behavior that may be present in the data. In this study we aim to modify the Holt's linear trend model using the ideas in [18]. The performance of the proposed method will be tested empirically using the M-3 competition data set that was mentioned above.

2. Modified Holt's Linear Trend Method

Modified exponential smoothing (MES) methods can be adapted for each of the 30 different ES models classified by [10] but only one form of MES will be given in this study which can be called modified Holt's linear trend method (MHES). The goal is again to forecast future values of a time series from its own current and past values. That is, given a series of equally spaced observations, X_t for $t = 1, 2, \ldots, n$ on some quantity, forecasts for $h = 1, 2, \ldots$ should be obtained. Let us first consider the standard Holt's linear trend method [9], which is given in equations (2.1)-(2.3). The method estimates the local growth, T_t, by smoothing successive differences, $(S_t - S_{t-1})$ of the local level, S_t. The forecast function is the sum of level and projected growth:

$$S_t = \alpha X_t + (1-\alpha)(S_{t-1} + T_{t-1}), \tag{2.1}$$

$$T_t = \beta(S_t - S_{t-1}) + (1-\beta)T_{t-1}, \tag{2.2}$$

$$\hat{X}_t(h) = S_t + hT_t, \tag{2.3}$$

where $\hat{X}_t(h)$ is the h-step-ahead forecast, α and β are smoothing parameters, $0 < \alpha, \beta < 1$. There are two smoothing parameters to estimate and starting values for both the level and trend must be provided. An initial smoothed value can be chosen using various techniques. The most common techniques are least squares estimation, backcasting, using a training set, using convenient initial values (e.g. using the first data value to initialize the level or the difference between the first and the second data value to initialize the trend) and setting the initial values to zero [13]. The parameters can be estimated by minimizing the one-step-ahead MSE, MAE, MAPE or some other criterion for measuring in-sample forecast error.

Now, Holt's linear trend method will be modified as follows:

$$S_t = \left(\frac{p}{t}\right) X_t + \left(\frac{t-p}{t}\right)(S_{t-1} + T_{t-1}), \tag{2.4}$$

$$T_t = \left(\frac{q}{t}\right)(S_t - S_{t-1}) + \left(\frac{t-q}{t}\right) T_{t-1}, \; n \geq p \geq q \geq 0, \tag{2.5}$$

$$\hat{X}_t(h) = S_t + hT_t, \tag{2.6}$$

for $p \in \{1, \ldots, n\}$ and $q \in \{0, 1, \ldots, n\}$. Unlike other approaches, the proposed model avoids potential initialization problems by letting $S_t = X_t$ for $t \leq p$, $T_t = X_t - X_{t-1}$ for $t \leq q$ and $T_1 = 0$. This model will simply be notated as $MHES(p, q)$. Note that there are two smoothing parameters (p and q) to estimate but no starting values are needed for level and trend. Also notice that for $q = 0$ the model $MHES(p, 0)$ defined by equations (2.4)-(2.6) reduces to a modified simple exponential smoothing ($MSES(p)$) model [18]:

$$S(t) = \begin{cases} \left(\frac{p}{t}\right) X_t + \left(\frac{t-p}{t}\right) S_{t-1}, & \text{for } t > p, \\ X_t, & \text{for } t \leq p. \end{cases}$$

3. Application to M3-competition Data

To test the performance of the proposed modification, the method is applied to the M3-competition data [14] since this collection is the most recent and comprehensive time-series data collection available and its results are verified. The M3-competition data collection consists of 3003 time series data sets each of which have a pre-determined number of data points that are used for testing the out-sample performances of the competing methods. For example the last 6 data points for yearly data, the last 8 data points for quarterly data and the last 18 data points for the quarterly data are not used while estimating the smoothing parameters and the models are tested on these out-sample points. After the smoothing parameters are estimated using the in-sample data points in each set (as specified in the M3-competition), forecasts up to 18 steps ahead (the number of out-sample data points as specified in the M3-competition) are computed. Then, the symmetric mean absolute percentage errors (sMAPE) for all forecast horizons are computed and averaged across all 3003 series to stay consistent with the rest of the literature. The one-step-ahead sMAPE can be defined as:

$$sMAPE = 200 \times mean \left(\frac{\left| X_t - \hat{X}_t \right|}{|X_t| + \left| \hat{X}_t \right|} \right),$$

where X_t is the actual value and $\hat{X}_t$ is the one-step-ahead forecasted value.

Results from four different versions of the proposed modification will be given here. For all versions the data will be deseasonalized by the classical decomposition method of the ratio-to-moving averages, if necessary. The first version is the modified simple exponential smoothing model $MHES(p^*, 0)$ where p^* is the first parameter optimized by minimizing the in-sample one-step-ahead sMAPE holding $q = 0$.

The second version that will be considered here is the $MHES(p, 1)$ where $q = 1$ and p is optimized as before. Even though it may look like this model is simply a specific parametrization of the model, this special case is of relevance since it allows for all the trend components over time to receive equal weights which can not be achieved with other ES methods.

For the third version we perform a simple model selection of the previous two versions where the best, i. e. the one that yields to a smaller forecast error for each data set, between these two is chosen and used for prediction. This version of the model will be denoted as $MHES - select$ in the tables that follow.

The final version $MHES(p^*, q)$ is the one where both parameters are optimized in a sequential fashion. For this version first the optimal value for the parameter p^* is obtained by minimizing the in-sample one-step-ahead sMAPE holding $q = 0$ and then the optimal value for the parameter q ($q \leq p*$) is found by minimizing the in-sample one-step-ahead sMAPE again.

Reseasonalized forecasts are produced for all versions for as many steps ahead as required. It is worth noting that the out-sample data points in each data set are left out while estimating the smoothing parameters and only the errors from the out-sample data points (the forecast horizons as determined in the M3-competition) are given here. The results from the proposed models along with results from some of the methods from the M3-competition are given in Tables 1- 7[14]. To stay consistent with other research on this data set, symmetric MAPEs are averaged across the series for each forecast horizon.

Table 1 contains the results for all 3003 series of the M3-competition. Here it can be seen that $MHES(p, 0)$ performs better than its competitor the single exponential smoothing model for all forecasting horizons individually and averaged. $MHES(p^*, q)$ and $MHES - select$ both perform better than Holt's linear trend model, Box-Jenkins and ETS for all forecasting horizons again individually and averaged. It is worth noting that even the one model $MHES(p, 1)$ outperforms both ETS and $Box - Jenkins$ based models for relatively short-term forecast horizons (horizons 1-8). It is also worth underlining the fact that the model selection for $MHES - select$ is carried out of only the two parameterizations of one model compared to the full parameterizations of 24 alternative models considered in its competitor ETS. These can be seen as proofs of the strong positive effect this modification can have on forecasting accuracy.

The results can be studied closely as done in Tables 2 and 3 for seasonal and non-seasonal data. From these tables it can be seen that $MHES(p, 0)$ outperforms single exponential smoothing for both data sets for all forecasting horizons again. $MHES(p^*, q)$ and $MHES - select$ both perform better than Holt's linear trend model and Box-Jenkins for both seasonal and non-seasonal data sets. ETS performs slightly better than the proposed modifications for seasonal data due to the fact that it allows for various seasonal components (additional and multiplicative) however for non-seasonal data $MHES(p^*, q)$ and especially $MHES - select$ outperform ETS on all forecasting horizons.

Tables 4- 7 have summaries of the results for the annual, quarterly, monthly and other data sets respectively. $MHES(p, 0)$ produced comparable results to single exponential smoothing for quarterly and other data sets while single exponential smoothing performed slightly better for annual data and $MHES(p, 0)$ performed much better for monthly data. For the annual data in Table 4, the $MHES(p^*, q)$ and $MHES - select$ both outperformed Holt, Box-Jenkins and ETS for all forecasting horizons. The superiority of $MHES - select$ for annual data is not only against these three models but also for annual data it was able to outperform all competitors of the M3-competition including the best performing Theta method [1]. It is worth noticing that for the same annual data sets ETS is unable to produce better forecasts than the naive approach.

For the quarterly series in Table 5 $MHES - select$ and $MHES(p^*, q)$ produced better forecasts compared to Holt, Box-Jenkins and ETS when the averaged $sMAPE$ are studied for both short term and long term forecasting horizons. For monthly data the results can be seen in Table 6. Here $MHES(p^*, q)$ and $MHES - select$ outperform Holt and Box-Jenkins consistently while ETS performs slightly better for short-term horizons and the proposed approaches perform better for long-term horizons therefore they produce comparable results.

Finally for the other series in Table 7 the proposed modified approaches $MHES - select$ and $MHES(p^*, q)$ both perform better compared to Holt, Box-Jenkins and ETS for all forecasting horizons individually and when averaged for both short and long term horizons.

Table 1. Average symmetric MAPE across different forecast horizons: all 3003 series

	Forecasting horizons										Averages					
Method	1	2	3	4	5	6	8	12	15	18	1-4	1-6	1-8	1-12	1-15	1-18
Naive2	10.5	11.3	13.6	15.1	15.1	15.9	14.5	16.0	19.3	20.7	12.62	13.57	13.76	14.24	14.81	15.47
Single	9.5	10.6	12.7	14.1	14.3	15.0	13.3	14.5	18.3	19.4	11.73	12.71	12.84	13.13	13.67	14.32
Holt	9.0	10.4	12.8	14.5	15.1	15.8	13.9	14.8	18.8	20.2	11.67	12.93	13.11	13.42	13.95	14.60
B-J automatic	9.2	10.4	12.2	13.9	14.0	14.6	13.0	14.1	17.8	19.3	11.42	12.39	12.52	12.78	13.33	13.99
ETS	8.8	9.8	12.0	13.5	13.9	14.7	13.0	14.1	17.6	18.9	11.04	12.13	12.32	12.66	13.14	13.77
$MHES(p^*,0)$	8.9	10.0	12.1	13.7	13.9	14.7	12.8	13.9	17.3	18.9	11.16	12.21	12.34	12.64	13.13	13.77
$MHES(p,1)$	8.4	9.7	11.5	12.9	13.6	14.2	12.9	15.4	18.9	20.9	10.64	11.72	11.94	12.66	13.32	14.09
$MHES-select$	8.7	9.6	11.5	12.9	13.1	13.7	12.1	13.7	17.3	18.7	10.69	11.58	11.69	12.09	12.64	13.28
$MHES(p^*,q)$	8.6	9.7	11.7	13.5	13.7	14.5	12.5	13.6	17.3	18.7	10.89	11.96	12.08	12.37	12.87	13.50

Table 2. Average symmetric MAPE across different forecast horizons: 862 seasonal series

	Forecasting horizons										Averages					
Method	1	2	3	4	5	6	8	12	15	18	1-4	1-6	1-8	1-12	1-15	1-18
Naive2	8.0	8.1	9.5	9.5	9.9	11.5	12.1	11.0	14.0	15.5	8.77	9.41	10.12	10.54	10.91	11.40
Single	7.1	7.4	8.8	8.7	9.3	10.9	11.3	10.7	13.1	14.6	8.02	8.71	9.42	9.78	10.13	10.62
Holt	6.5	6.9	8.2	8.4	9.4	10.6	11.2	11.5	13.2	15.3	7.50	8.33	9.15	9.66	10.09	10.67
B-J automatic	7.1	7.4	8.0	8.8	9.2	10.3	10.5	10.5	13.3	14.5	7.82	8.46	9.03	9.31	9.79	10.37
ETS	6.2	6.4	7.7	8.2	8.9	10.2	10.6	10.1	12.0	14.0	7.12	7.93	8.67	9.01	9.35	9.87
$MHES(p^*,0)$	6.5	6.9	8.0	8.3	9.2	10.9	11.1	10.7	12.3	14.2	7.45	8.30	9.09	9.44	9.75	10.22
$MHES(p,1)$	6.2	7.0	7.7	8.0	9.1	10.2	10.2	10.2	12.3	14.0	7.23	8.03	8.69	9.20	9.59	10.09
$MHES-select$	6.5	6.9	7.8	8.0	8.7	10.2	10.0	10.3	11.7	13.5	7.28	8.01	8.64	9.15	9.51	10.02
$MHES(p^*,q)$	6.4	6.8	7.8	8.1	8.9	10.6	10.6	10.4	11.8	14.0	7.26	8.10	8.85	9.20	9.49	9.96

Table 3. Average symmetric MAPE across different forecast horizons: 2141 nonseasonal series

	Forecasting horizons										Averages					
Method	1	2	3	4	5	6	8	12	15	18	1-4	1-6	1-8	1-12	1-15	1-18
Naive2	11.5	12.6	15.3	17.3	17.1	17.5	15.9	19.2	22.8	24.1	14.17	15.22	15.32	15.97	16.73	17.54
Single	10.4	11.9	14.3	16.3	16.3	16.5	14.5	17.0	21.8	22.5	13.22	14.28	14.30	14.70	15.40	16.21
Holt	10.0	11.9	14.8	17.4	18.1	18.5	16.2	17.8	23.5	24.8	13.52	15.10	15.23	15.69	16.43	17.26
B-J automatic	10.0	11.6	13.9	15.9	16.0	16.4	14.4	16.4	20.7	22.4	12.87	13.97	14.04	14.43	15.09	15.85
ETS	9.9	11.2	13.7	15.6	15.9	16.6	14.4	16.7	21.3	22.2	12.61	13.83	13.91	14.39	15.03	15.77
$MHES(p^*, 0)$	9.8	11.3	13.7	15.8	15.8	16.3	13.8	16.0	20.6	21.9	12.66	13.79	13.76	14.16	14.81	15.58
$MHES(p, 1)$	9.3	10.8	13.1	14.9	15.4	15.8	14.4	18.4	23.2	25.4	12.02	13.21	13.36	14.30	15.17	16.15
$MHES - select$	9.6	10.7	13.0	14.9	14.8	15.0	13.3	15.9	21.0	22.2	12.06	13.01	13.02	13.54	14.28	15.09
$MHES(p^*, q)$	9.5	10.9	13.3	15.7	15.7	16.1	13.6	15.7	20.8	21.8	12.34	13.52	13.49	13.87	14.54	15.31

Table 4. Average symmetric MAPE across different forecast horizons: 645 annual series

	Forecasting horizons						Averages	
Method	1	2	3	4	5	6	1-4	1-6
Naive2	8.5	13.2	17.8	19.9	23.0	24.9	14.85	17.88
Single	8.5	13.3	17.6	19.8	22.8	24.8	14.82	17.82
Holt	8.3	13.7	19.0	22.0	25.2	27.3	15.77	19.27
B-J automatic	8.6	13.0	17.5	20.0	22.8	24.5	14.78	17.73
ETS	9.3	13.6	18.3	20.8	23.4	25.8	15.48	18.53
$MHES(p^*, 0)$	9.1	13.5	17.6	19.9	22.8	25.1	15.04	18.00
$MHES(p, 1)$	8.3	12.2	16.8	18.6	21.5	23.3	13.95	16.78
$MHES - select$	8.3	11.5	15.6	17.7	20.5	22.0	13.28	15.94
$MHES(p^*, q)$	8.3	12.4	17.0	20.0	23.1	25.1	14.40	17.62

Table 5. Average symmetric MAPE across different forecast horizons: 756 quarterly series.

	Forecasting horizons							Averages		
Method	1	2	3	4	5	6	8	1-4	1-6	1-8
Naive2	5.4	7.4	8.1	9.2	10.4	12.4	13.7	7.55	8.82	9.95
Single	5.3	7.2	7.8	9.2	10.2	12.0	13.4	7.38	8.63	9.72
Holt	5.0	6.9	8.3	10.4	11.5	13.1	15.6	7.67	9.21	10.67
B-J automatic	5.5	7.4	8.4	9.9	10.9	12.5	14.2	7.79	9.10	10.26
ETS	5.0	6.6	7.9	9.7	10.9	12.1	14.2	7.32	8.71	9.94
$MHES(p^*, 0)$	5.2	7.1	7.8	9.7	10.1	11.8	13.5	7.45	8.62	9.71
$MHES(p, 1)$	5.3	6.8	7.6	9.1	9.9	11.0	12.4	7.19	8.28	9.24
$MHES - select$	5.2	7.0	7.7	9.2	9.6	11.1	12.3	7.28	8.30	9.21
$MHES(p^*, q)$	5.0	6.9	7.6	9.5	10.2	11.9	13.7	7.23	8.51	9.69

Table 6. Average symmetric MAPE across different forecast horizons: 1428 monthly series.

	Forecasting horizons										Averages					
Method	1	2	3	4	5	6	8	12	15	18	1-4	1-6	1-8	1-12	1-15	1-18
Naive2	15.0	13.5	15.7	17.0	14.9	14.4	15.6	16.0	19.3	20.7	15.30	15.08	15.26	15.55	16.16	16.89
Single	13.0	12.1	14.0	15.1	13.5	13.1	13.8	14.5	18.3	19.4	13.53	13.44	13.60	13.83	14.51	15.32
Holt	12.2	11.6	13.4	14.6	13.6	13.3	13.7	14.8	18.8	20.2	12.95	13.11	13.33	13.77	14.51	15.36
B-J automatic	12.3	11.7	12.8	14.3	12.7	12.3	13.0	14.1	17.8	19.3	12.78	12.70	12.86	13.19	13.95	14.80
ETS	11.5	10.6	12.3	13.4	12.3	12.3	13.2	14.1	17.6	18.9	11.93	12.05	12.43	12.96	13.64	14.45
$MHES(p^*, 0)$	11.5	10.8	12.6	13.8	12.6	12.5	12.9	13.9	17.3	18.9	12.20	12.33	12.78	12.98	13.67	14.49
$MHES(p, 1)$	11.0	10.9	12.2	13.4	12.8	12.8	13.8	15.4	18.9	20.9	11.86	12.16	13.19	14.07	15.01	15.33
$MHES - select$	11.6	11.0	12.6	13.8	12.4	12.3	12.7	13.7	17.3	18.7	12.22	12.26	12.65	12.84	13.53	14.31
$MHES(p^*, q)$	11.6	10.9	12.5	13.8	12.4	12.3	12.7	13.6	17.3	18.7	12.18	12.23	12.42	12.79	13.47	14.27

Table 7. Average symmetric MAPE across different forecast horizons: 174 other series.

	Forecasting horizons							Averages		
Method	1	2	3	4	5	6	8	1-4	1-6	1-8
Naive2	2.2	3.6	5.4	6.3	7.8	7.6	9.2	4.38	5.49	6.30
Single	2.1	3.6	5.4	6.3	7.8	7.6	9.2	4.36	5.48	6.29
Holt	1.9	2.9	3.9	4.7	5.8	5.6	7.2	3.32	4.13	4.81
B-J automatic	1.8	3.0	4.5	4.9	6.1	6.1	7.5	3.52	4.38	5.06
ETS	2.0	3.0	4.0	4.4	5.4	5.1	6.3	3.37	3.99	4.51
$MHES(p^*, 0)$	2.1	3.5	5.4	6.3	7.8	7.5	9.1	4.34	5.45	6.26
$MHES(p, 1)$	1.9	2.9	4.1	4.8	6.0	5.7	7.1	3.46	4.26	4.87
$MHES - select$	1.8	2.8	4.0	4.6	5.7	5.3	6.6	3.30	4.03	4.60
$MHES(p^*, q)$	1.7	2.6	3.7	4.3	5.4	4.9	6.2	3.09	3.77	4.30

4. Conclusion

The modification to Holt's linear trend method proposed in this paper is a simple, fast, computationally inexpensive and accurate alternative extrapolative forecasting method. Four versions of the proposed model (two special parameterizations, a model selection strategy out of these two special cases and a fully optimized version) were applied to the M3-competition data sets and it was shown that the proposed modification can perform as well as and in most cases much better than the models that are based on the two major forecasting approaches: Box Jenkins and exponential smoothing. An Excel file that contains the optimum parameter values for the proposed methods for each M3-competition data set along with the forecasts and errors is provided as supplementary material and can be accessed at the journal's website.

The models success can be attributed to the facts that it is less dependent on the initial values and is more flexible. When the smoothing parameters are equal to 1, i.e. $p = 1$ and $q = 1$, the method assigns equal weights to all past observations when estimating the level and trend which is a very intuitive starting point assuming the future is represented by the average of the past. From the tables in Section 3 it can be clearly seen how important it is to be able to assign equal weights to past observations which can not be achieved by other exponential smoothing models.

Note that we have not done any preprocessing of the data, identification of outliers or level shifts, or used any other strategy designed to improve the forecasts. These results are based on simple applications of the algorithms to the data. It should be expected that the results from the modification could be improved further if some sophisticated data preprocessing techniques are used as done by some of the competitors in the M3 competition. It is clear that the data sets include different types of trend (multiplicative, damped, multiplicative damped) and seasonal components (additive, multiplicative) in addition to different types of errors (additive and multiplicative errors) and when these components are incorporated in the proposed method then the forecasting performance of proposed method will improve further. In this study, the main focus was obtaining point forecasts only. Further study will be computation of prediction intervals and expanding this concept for other ES models.

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