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Mean–Variance–Skewness–Kurtosis Approach to Portfolio Optimization: An Application in İstanbul Stock Exchange

Portföy Optimizasyonunda Ortalama-Varyans-Çarpıklık-BasıklıkYaklaşımı: İMKB Uygulaması

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ABSTRACT

Portfolio optimization, the construction of the best combination of investment instruments that will meet the investors' basic expectations under certain limitations, has an important place in the finance world. In the portfolio optimization, the Mean Variance model of Markowitz (1952) that expresses a tradeoff between return and risk for a set of portfolios, has played a critical role and affected other studies in this area.

In the Mean Variance model, only the covariances between securities are considered in determining the risk of portfolios. The model is based on the assumptions that investors have a quadratic utility function and the return of the securities is distributed normally. Various studies that investigate the validity of these assumptions find evidence against them. Asset returns have significant skewness and kurtosis. In the light of these findings, it is seen that in recent years researchers use higher order of moments in the portfolio selection (Konno et al, 1993; Chunhachinda et al, 1997; Liu et al, 2003; Harvey et al, 2004; Jondeau and Rockinger, 2006; Lai et al, 2006; Jana et al, 2007; Maringer and Parpas, 2009; Briec et al, 2007; Taylan and Tatlıdil, 2010).

In this study, in the mean- variance- skewness- kurtosis framework, multiple conflicting and competing portfolio objectives such as maximizing expected return and skewness and minimizing risk and kurtosis simultaneously, will be addressed by construction of a polynomial goal programming (PGP) model. The PGP model will be tested on Istanbul Stock Exchange (ISE) 30 stocks. Previous empirical results indicate that for all investor preferences and stock indices, the PGP approach is highly effective in order to solve the multi conflicting portfolio goals in the mean – variance - skewness – kurtosis framework. In this study, portfolios will be formed in accordance with the investor preferences over incorporation of higher moments. The effects of preferences both on the combination of stocks in the portfolios and descriptive statistics of portfolio returns will be analyzed. Another aim of this study is to investigate the impacts of the incorporation of skewness and kurtosis of asset returns into the portfolio optimization on portfolios' returns descriptive statistics.

Keywords: Portfolio optimization, mean-variance-skewness-kurtosis approach, Istanbul stock exchange (ISE) 30.

ÖZET

Belli kısıtlar altında yatırımcıların temel beklentilerini karşılayacak en iyi yatırım araçları karmasının oluşturulması olan portföy optimizasyonu. finans dünyasında önemli bir yere sahiptir. Portföy optimizasyonunda, oluşturulan portföyler için getiri ve risk arasında bir dengelemeyi ifade eden Markowitz'in (1952) Ortalama Varyans modeli, bu alanda kritik bir role sahiptir ve yapılan diğer çalışmaları da etkilemiştir.

Markowitz'in Ortalama-Varyans modelinde, portföyün riski belirlenirken sadece menkul kıymet getirilerinin kovaryans değerleri dikkate alınmaktadır. Bu model, yatırımcıların kuadratik fayda fonksiyonuna sahip olduğu ve hisse senedi getirilerin normal dağıldığı varsayımlarına dayandırılmıştır. Bu varsayımların geçerliliğini inceleyen çok sayıda çalışmada karşıt bulgulara ulaşılmıştır. Varlık getirilerinin anlamlı derecede çarpıklık ve basıklık özelliği gösterdiği saptanmıştır. Bu bulgular ışığında, son yıllarda araştırmacıların portföy seçiminde yüksek dereceden momentleri kullandıkları görülmektedir (Konno et al, 1993; Chunhachinda et al, 1997; Liu et al, 2003; Harvey et al, 2004; Jondeau and Rockinger, 2006; Lai et al, 2006; Jana et al, 2007; Maringer and Parpas, 2009; Briec et al, 2007; Taylan and Tatlıdil, 2010).

Bu çalışmada, ortalama-varyans-çarpıklık ve basıklık modeli çerçevesinde, beklenen getiri ve çarpıklığın maksimize edilmesi, varyans ve basıklığın minimize edilmesi gibi birbiri ile çelişen ve aynı anda karşılanması gereken portföy amaçları, oluşturulacak polinomal hedef programlama yöntemi ile ele alınacaktır. Oluşturulacak PGP modeli, İstanbul Menkul Kıymetler Borsası (İMKB) 30 hisse senetleri üzerinde test edilecektir. Daha önce yapılmış olan çeşitli ampirik çalışma sonuçları, tüm yatırımcı tercihleri ve hisse senedi endeksleri için, ortalama-varyans-çarpıklık-basıklık çerçevesinde çoklu çelişen portföy amaçlarının çözümünde PGP yaklaşımının etkili bir yol olduğunu işaret etmektedir. Bu çalışmada, yatırımcıların yüksek dereceden momentler ile ilgili tercihlerine göre portföyler oluşturulacaktır. Bu tercihlerin hem portföy içindeki hisse senedi dağılımına, hem de portföylerin getirilerinin tanımlayıcı istatistiklerine etkileri incelenecektir. Bu çalışmanın bir diğer amacı da, portföy optimizasyonunda hisse senetlerinin getirilerinin çarpıklık ve basıklığının göz önünde bulundurulmasının portföy getirilerinin tanımlayıcı istatistikleri üzerinde yarattığı etkilerin de incelenmesidir.

Anahtar Kelimeler: Portföy optimizasyonu, ortalama–varyans–çarpıklık–basıklık yaklaşımı, İstanbul menkul kıymetler borsası (İMKB) 30

1. INTRODUCTION

Investors want to maximize their returns by allocating their capitals among a set of potential investments. The aim in this allocation process is to achieve a desired tradeoff between their risk and return preferences. In other words, investors aim to optimize their portfolios in accordance with their preferences.

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For long years, portfolio selection and optimization problem is an attractive topic for investors. Following the seminal work of Markowitz, returns of financial assets are typically described their mean, while risk is described by variance (Maringer and Parpas, 2009: 219). Subsequently, an abundant literature emerged, questioning the adequacy of the mean-variance criterion proposed by Markowitz (1952) for allocating wealth (Xu et. al., 2007: 2488). This literature finds evidence against the model and shows that asset returns are characterized by significant skewness and kurtosis. As a result of these findings, more recently researchers tended to concern for higher moments in the portfolio optimization problem and lots of techniques have been developed to solve this problem (Konno et al., 1993; Chunhachinda et al., 1997; Liu et al., 2003; Harvey et al., 2004; Jondeau and Rockinger, 2006; Lai et al., 2006; Jana et al, 2007; Maringer and Parpas, 2009; Briec et al., 2007; Taylan and Tatlıdil, 2010).

Lai(1991), Chunhachinda, et al. (1997), Prakash et al. (2003) and Sun and Yan (2003) applied the polynomial goal programming approach to the portfolio selection with skewness. Later, kurtosis is incorporated into the portfolio selection by Jondeau and Rockinger (2004).

In this study, in the mean- variance- skewness-kurtosis framework, portfolio optimization problem will be addressed. In the presence of higher order moments, portfolio selection contains multiple conflicting and competing portfolio objectives such as maximizing expected return and skewness and minimizing risk and kurtosis simultaneously. In this framework, portfolio allocation depends on investor preferences for these moments. This multi objective problem will be solved by using a polynomial goal programming (PGP) model.

The existing literature about portfolio optimization indicates that the PGP approach is highly effective in order to solve the multi conflicting portfolio goals in the mean – variance - skewness – kurtosis framework for all investor preferences and stock indices. In this study, the PGP model will be tested on a small sample of stocks in ISE and the existence of an optimal solution will be investigated under different investor preferences. The effects of preferences both on the combination of stocks in the portfolios and descriptive statistics of portfolios' returns will be analyzed.

In this context, the concepts, portfolio and portfolio optimization are reviewed in section 2. The approach of PGP and existing literature about this approach are discussed in section 3. Section 4 represents our empirical analysis of the PGP approach. And section 5 concludes the paper.

2. PORTFOLIO OPTIMIZATION IN THE MEAN–VARIANCE–SKEWNESS-KURTOSIS FRAMEWORK

In financial terms, a portfolio is an appropriate mix or collection of investments held by an institution or private individuals. The portfolio optimization problem is a well-known difficult problem occurring in the finance world. The problem consists of choosing an optimal set of assets in order to minimize the risk and maximize the profit of the investment. The investor's objective is to get the maximum possible return on an investment with the minimum possible risk. This objective is achieved through asset diversification (Singh et al., 2010: 75).

The mean-variance framework for portfolio selection, developed by Markowitz (1952), continues to be the most popular method for portfolio construction (Kale, 2009: 439). Since Markowitz's pioneering work was published, the mean-variance model has revolutionized the way people think about portfolio of assets, and numerous studies on portfolio selection have been made based on only the first two moments of return distributions (Lai et al, 2006: 1) Most serious investors use mean-variance optimization to form portfolios, in part, because it requires knowledge only of a portfolio's expected return and variance. Yet this convenience comes at some expense, because the legitimacy of mean-variance optimization depends on questionable assumptions. Either investors have quadratic utility or portfolio returns are normally distributed. Neither of these assumptions is literally true (Cremers et al., 2003:2; Harvey et al., 2004: 4; Lai et al., 2006:1). Strong empirical evidence suggests that returns are driven by asymmetric and/or fat-tailed distributions (Jondeau and Rockinger, 2006: 29). The mean – variance model by Markowitz is important in portfolio optimization but this model should be expanded.

The classical Markowitz (1952, 1959) model for portfolio selection has been studied in the past by simplifying it or reformulating it into different models. Several practitioners pointed out to the computational difficulty of Markowitz model which is associated with solving a large-scale quadratic programming (Simimou and Thulasiram, 2010: 481). Several alternative approaches have been developed in the financial literature to incorporate the individual preferences for higher-order moments into optimal asset allocation problems (Jurczenko et al., 2005: 2; Taylan and Tatlıdil, 2010: 349). Samuelson (1970) also showed that the higher moment is relevant to investors' decision-making in portfolio selection and, furthermore, almost all investors would prefer a portfolio with a larger third moment if the first and second moments are the same (Liu et al, 2003: 255). In this framework, portfolio selection with skewness is determined. But the fourth moment, kurtosis, which is neglected by most researchers, is also important for portfolio selection if return distribution is non-normal, or utility functions are higher than quadratic, or higher moments are relevant to the investor's decision (Lai et al, 2006: 1). In the light of these findings, it is seen that in recent years researchers use higher order of moments in the portfolio selection.

In this study, following Lai (2006) the PGP will be used in order to find solutions to portfolio optimization problem that contains multiple conflicting and competing portfolio objectives that are maximizing expected return and skewness and minimizing risk and kurtosis simultaneously.

As told by Lai et al (2006)', PGP was first introduced by Tayi and Leonard to facilitate bank balance sheet management with competing and conflicting objectives (Lai et al. 2006: 2). Along with, Lai (1991), Chunhachinda et al. (1997), and Prakash et al. (2003) applied the PGP approach to the portfolio selection with skewness. All these studies provided evidence that incorporating skewness into the portfolio decision causes major changes in the optimal portfolio (Jondeau and Rockinger, 2006: 30; Lai et al. 2006: 2). In the study of Taylan and Tatlıdil (2010), it is seen that the portfolio optimization is achieved by shortage function and higher order moments. By construction of a PGP, they tried to analyze multiple competing portfolio allocation objectives such as maximizing expected portfolio return and skewness, minimizing risk and kurtosis simultaneously and investor's preferences over incorporation of higher moments (Taylan and Tatlıdil, 2010: 348).

To sum up, more recently in local and foreign literature, higher order moments -especially mean- variance- skewness- kurtosis- based portfolio optimization has attracted a great deal of attention. In this study, to achieve portfolio optimization in the framework of four moments, the PGP is used. In the following sections of the study a brief review of the PGP will be given and it will be followed by the research section of the study.

3. POLYNOMIAL GOAL PROGRAMMING

Goal programming (GP) is an important category in linear programming. In this idea, instead of trying to optimize each objective function, the decision maker is asked to specify a goal or target value that realistically is the most desirable value for that function (Hashemi et al. ,2006: 507). The overall purpose of goal programming is to minimize the deviations between the achievement of goals and their aspiration levels (Chang, 2002: 62 – 63).

In this study, we deal with PGP. The PGP is a multi-objective goal programming technique that allows us to incorporate higher order moments in portfolio selection. The PGP model accommodates both intra-level and inter-level preference trade-offs via the specification of the objective function as a polynomial expression (Deckro and Hebert, 2002: 149).

There are numerous studies in the literature indicating that portfolio returns are not normally distributed. As a result of the evidence against the normality assumption of the Markowitz's model, higher order moments are started to be considered in the portfolio selection problem.

Starting from this point, Lai (1991) proposed a multiobjective portfolio selection model to incorporate the skewness of return distributions. The optimal solution of this model is to select a portfolio component such that its multiple objectives are optimized. That is to maximize the expected rate of return and skewness, while minimizing the variance (Chen and Shia, 2007: 133). Like Lai, Harvey et al (2004), Jurczenko et al (2006), Lai et al.(2006), Chen, and Shia (2007) and Taylan and Tatlıdil (2010) applied the PGP method to portfolio optimization with fourth moment.

As Chunhachinda et al. (1997) mentioned, the important features of polynomial goal programming include (Chen and Shia, 2007: 131):

- The existence of an optimal solution,

- The flexibility in incorporating investor preferences, and

- The relative simplicity of the computational requirements (Chen and Shia, 2007: 131).

The advantage of the PGP framework is that it is general enough to accommodate investor desires for higher moments: skewness and kurtosis through preference parameters. It solves the trade-off among the competing objectives for the return distribution properties (Proelss and Schweizer 2009: 1).

In the application of the PGP model, we compute the first four moments of asset returns (see Lai et al., 2006):

$$Mean = R(x) = X^T \bar{R} = \sum_{i=1}^{n} x_i R_i \quad (1)$$

$$Variance = V(x) = X^T VX = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \quad i \neq j \quad (2)$$

$$Skewness = S(x) = E(X^T (R - \bar{R}))^3 = \sum_{i=1}^{n} x_i^3 s_i^3 + 3\sum_{i=1}^{n} (\sum_{j=1}^{n} x_i^2 x_j s_{iij} + \sum_{j=1}^{n} x_i x_j^2 s_{ijj}) \ (i \neq j) \quad (3)$$

$$Kurtosis = K(x) = E(X^T (R - \bar{R}))^4 = \sum_{i=1}^{n} x_i^4 s_i^4 + 4\sum_{i=1}^{n} (\sum_{j=1}^{n} x_i^3 x_j k_{iiij} + \sum_{j=1}^{n} x_i x_j^3 k_{ijjj}) + 6\sum_{i=1}^{n} \sum_{j=1}^{n} x_i^2 x_j^2 k_{iijj} \quad (i \neq j) \quad (4)$$

where R is the distribution of returns and $\bar{R}$ is mean of the return, $X_T=(x_1,x_2,\ldots,x_n)$ is the transpose of the weight vector used to combine the portfolio, xi is the percentage of wealth invested in the ith risky asset. V, S, and K is the variance - covariance, skewness-coskewness, and kurtosis – cokurtosis matrices of R, respectively.

To combine the multiple objectives such that maximization of the expected return and skewness of return while minimization of the variance and kurtosis of return, we use the same multiobjective programming technique with Lai et.al. (2006). The formula

$$\left.\begin{array}{ll} \text{Maximize} & R(x) = X^T \bar{R} \\ \text{Minimize} & V(x) = X^T VX \\ \text{Maximize} & S(x) = E(X^T (R-\bar{R}))^3 \\ \text{Minimize} & K(x) = E(X^T (R-\bar{R}))^4 \\ \text{subject to} & X^T I = 1 \\ & X \geq 0 \end{array}\right\} P1 \quad (5)$$

To combine these objectives into a single objective function, we use a PGP approach. Let d1, d2, d3 and d4 be the goal variables which account for the deviations of expected return, variance, skewness and kurtosis from the optimal scores of, R*, V*, S* and K*, respectively. To obtain the optimal scores, the given model, P1, is divided into four subproblems and solved them individually (see Lai et. al.(2006)).

After calculating the optimal scores of each moment, we use the PGP model that was proposed by Lai et. al. (2006) to find portfolio allocations for different investors' preferences. The PGP model (P2) is

$$\left.\begin{array}{ll} \text{Minimize} & Z = |\frac{d_1}{R^*}|^{\lambda 1} + |\frac{d_2}{V^*}|^{\lambda 2} + |\frac{d_3}{S^*}|^{\lambda 3} + |\frac{d4}{K^*}|^{\lambda 4} \\ \text{subject to} & X^T \bar{R} + d_1 = R^* \\ & X^T VX - d_2 = V^* \\ & E(X^T (R-\bar{R}))^3 + d_3 = S^* \\ & E(X^T (R-\bar{R}))^4 - d_4 = K^* \\ & X^T I = 1 \\ & X \geq 0 \\ & d_i \geq 0 \ \ i = 1,\ldots,4 \end{array}\right\} P2 \quad (6)$$

The PGP problem solution involves a two- step procedure. First the optimal scores of *R**, *V**, *S** and *K**. Then the optimal scores are substituted into P2, and the minimum value of Z can be found for a given set of investor preferences (Lai et. al.,2006:3).

4. EXPERIMENTAL ANALYSIS

4.1. Data Set

In this study, Istanbul Stock Exchange (ISE) 30 stocks are examined. Our data set contains daily prices of permanently traded stocks in ISE- 30 index during the last five years. Among the permanent stocks in ISE-30 Index, we choose the ones with positive average daily returns for the period January 4, 2010 to December 31,2010. As a result, we obtain 8 stocks for implementation. We use logarithmic returns in our analysis.

4.2. Experiment Results

In this study our main objective is to show the effects of investors' preferences both on the combination of stocks in the portfolios and descriptive statistics of portfolios' returns in the four moment framework. In this part, we present all process followed in performing the PGP approach. The distribution properties of the analysed stocks are given in the table below.

In addition to individual distribution properties of asset returns, covariance, coskewness and cokurtosis of asset returns are calculated. Tables 2- 4 show these statistics.

Table 1: The Distribution Properties Of Assets Return

	DOHOL	EREGL	GARAN	KCHOL	SAHOL	SISE	TUPRS	YKBNK
Mean	0,000335	0,000501	0,000833	0,002286	0,0009	0,001499	0,001042	0,001573
Variance	0,00045	0,000298	0,000536	0,000447	0,0004	0,000363	0,000437	0,000443
Skewness	0,649758	-0,15737	-0,050961	0,207003	-0,222917	-0,087963	-0,468247	-0,001743
Kurtosis	7,242042	3,847797	3,311221	5,025932	3,763371	4,021011	4,367271	4,002585

Table 2: The Variance- Covariance Of Asset Returns

	DOHOL	EREGL	GARAN	KCHOL	SAHOL	SISE	TUPRS	YKBNK
DOHOL	0,000448	0,000134	0,00015	0,000153	0,000112	0,000137	0,000103	0,000159
EREGL	0,000134	0,000296	0,000236	0,000173	0,000193	0,000182	0,000149	0,000234
GARAN	0,00015	0,000236	0,000534	0,000237	0,000226	0,000209	0,000211	0,000329
KCHOL	0,000153	0,000173	0,000237	0,000446	0,000196	0,000169	0,000164	0,000228
SAHOL	0,000112	0,000193	0,000226	0,000196	0,000398	0,000174	0,000146	0,000224
SISE	0,000137	0,000182	0,000209	0,000169	0,000174	0,000362	0,000187	0,000194
TUPRS	0,000103	0,000149	0,000211	0,000164	0,000146	0,000187	0,000435	0,000184
YKBNK	0,000159	0,000234	0,000329	0,000228	0,000224	0,000194	0,000184	0,000442

Table 3: The Skewness-Coskewness Of Asset Returns

		DOHOL	EREGL	GARAN	KCHOL	SAHOL	SISE	TUPRS	YKBNK
		sii1	sii2	sii3	sii4	sii5	sii6	sii7	sii8
DOHOL	s11j	0,653644	-0,087190	-0,079630	-0,003210	0,003417	0,127786	-0,056170	-0,081270
EREGL	s22j	-0,217280	-0,158310	-0,038770	-0,190140	-0,186370	-0,146260	-0,200020	-0,144050
GARAN	s33j	-0,084350	0,007886	-0,051270	0,065867	0,035003	0,065449	-0,009390	0,054833
KCHOL	s44j	-0,021440	0,054581	0,212902	0,208264	0,101706	0,103053	0,085062	0,187865
SAHOL	s55j	-0,147340	-0,179060	-0,088440	-0,003780	-0,224260	-0,019990	-0,121190	-0,072150
SISE	s66j	-0,011250	-0,131400	-0,025820	0,057503	0,009054	-0,088490	-0,085330	-0,076770
TUPRS	s77j	-0,197620	-0,310660	-0,187350	-0,146890	-0,089230	-0,294030	-0,471110	-0,240170
YKBNK	s88j	-0,231090	-0,096540	0,091403	0,000069	-0,027300	-0,106740	-0,066230	-0,001750

		DOHOL	EREGL	GARAN	KCHOL	SAHOL	SISE	TUPRS	YKBNK
		si11	si22	si33	si44	si55	si66	si77	si88
DOHOL	s1jj	0,653644	-0,217280	-0,084350	-0,021440	-0,147340	-0,011250	-0,197620	-0,231090
EREGL	s2jj	-0,087190	-0,158310	0,007886	0,054581	-0,179060	-0,131400	-0,310660	-0,096540
GARAN	s3jj	-0,079630	-0,038770	-0,051270	0,212902	-0,088440	-0,025820	-0,187350	0,091403
KCHOL	s4jj	-0,003210	-0,190140	0,065867	0,208264	-0,003780	0,057503	-0,146890	0,000069
SAHOL	s5jj	0,003417	-0,186370	0,035003	0,101706	-0,224260	0,009054	-0,089230	-0,027300
SISE	s6jj	0,127786	-0,146260	0,065449	0,103053	-0,019990	-0,088490	-0,294030	-0,106740
TUPRS	s7jj	-0,056170	-0,200020	-0,009390	0,085062	-0,121190	-0,085330	-0,471110	-0,066230
YKBNK	s8jj	-0,081270	-0,144050	0,054833	0,187865	-0,072150	-0,076770	-0,240170	-0,001750

Table 4: The Kurtosis-Cokurtosis Of Asset Returns

		DOHOL	EREGL	GARAN	KCHOL	SAHOL	SISE	TUPRS	YKBNK
		kiii1	kiii2	kiii3	kiii4	kiii5	kiii6	kiii7	kiii8
DOHOL	k111j	7,212449	1,610523	1,097499	1,672190	1,112366	3,000571	0,878768	1,205393
EREGL	k222j	1,412179	3,832007	2,308827	2,212096	2,353864	2,090209	1,881659	2,637629
GARAN	k333j	1,222239	2,161866	3,297904	2,224837	1,859786	1,786888	1,495437	2,517438
KCHOL	k444j	1,648645	2,564858	2,730197	5,006121	2,706373	2,748459	2,149946	2,907206
SAHOL	k555j	1,035759	2,434517	2,125608	2,189090	3,748129	1,857535	1,354983	2,261866
SISE	k666j	1,754716	2,415424	1,885555	1,886981	1,805583	4,004761	1,941591	2,034175
TUPRS	k777j	1,201212	2,053733	1,621370	1,935859	1,198417	2,282303	4,350196	1,711587
YKBNK	k888j	1,424798	2,658861	2,751696	2,516464	2,227594	2,163343	1,756077	3,986494

		DOHOL	EREGL	GARAN	KCHOL	SAHOL	SISE	TUPRS	YKBNK
		ki111	ki222	ki333	ki444	ki555	ki666	ki777	ki88
DOHOL	k1jjj	7,212450	1,412180	1,222240	1,648650	1,035760	1,754720	1,201210	1,424800
EREGL	k2jjj	1,610520	3,832010	2,161870	2,564860	2,434520	2,415420	2,053730	2,658860
GARAN	k3jjj	1,097500	2,308830	3,297900	2,730200	2,125610	1,885560	1,621370	2,751700
KCHOL	k4jjj	1,672190	2,212100	2,224840	5,006120	2,189090	1,886980	1,935860	2,516460
SAHOL	k5jjj	1,112370	2,353860	1,859790	2,706370	3,748130	1,805580	1,198420	2,227590
SISE	k6jjj	3,000570	2,090210	1,786890	2,748460	1,857540	4,004760	2,282300	2,163340
TUPRS	k7jjj	0,878770	1,881660	1,495440	2,149950	1,354980	1,941590	4,350200	1,756080
YKBNK	k8jjj	1,205390	2,637630	2,517440	2,907210	2,261870	2,034180	1,711590	3,986490

		DOHOL	EREGL	GARAN	KCHOL	SAHOL	SISE	TUPRS	YKBNK
		kii11	kii22	kii33	kii44	kii55	kii66	kii77	kii88
DOHOL	k11jj	7,212450	1,281070	1,334490	1,442300	1,100570	2,272350	1,348470	1,399090
EREGL	k22jj	1,281070	3,832010	2,200300	2,378300	2,275320	2,336750	2,144040	2,508450
GARAN	k33jj	1,334490	2,200300	3,297900	2,679980	1,997520	1,860230	1,562260	2,565240
KCHOL	k44jj	1,442300	2,378300	2,679980	5,006120	2,394260	2,377050	2,106750	3,031240
SAHOL	k55jj	1,100570	2,275320	1,997520	2,394260	3,748130	1,757020	1,616530	2,159900
SISE	k66jj	2,272350	2,336750	1,860230	2,377050	1,757020	4,004760	2,138240	2,096560
TUPRS	k77jj	1,348470	2,144040	1,562260	2,106750	1,616530	2,138240	4,350200	1,738650
YKBNK	k88jj	1,399090	2,508450	2,565240	3,031240	2,159900	2,096560	1,738650	3,986490

Table 5: Optimal Solution Set Of PGP Portfolio Optimization Score

Optimal solution set of the PGP portfolio optimization scores				
Objectives	Mean*	Variance*	Skewness*	Kurtosis*
Optimal Scores	0.002286	0.000195042	0.6497580	0,377116

By dividing the P1 model into four subproblems and solved them individually, the optimal scores of four moments are obtained.

With the optimal solution of individual objective, we solve the P2 with the PGP approach. In the Tables 6-7, the first four moment and asset allocations for optimal portfolio with different investors' preferences are given.

Table 6: First Four Moments For Optimal Portfolio With Different Investors' Preferences

Portfolio	λ	Mean	Variance	Skewness	Kurtosis
1	(1,0,0,0)	0,002286	0,000450	0,207003	5,025932
2	(0,1,0,0)	0,000913	0,000200	-0,102202	0,727841
3	(0,0,1,0)	0,000335	0,000450	0,649758	7,242042
4	(0,0,0,1)	0,001069	0,000210	-0,021681	0,377116
5	(1,1,0,0)	0,001456	0,000220	0,005468	1,010926
6	(1,3,0,0)	0,001859	0,000270	0,106986	2,871934
7	(1,1,1,1)	0,001105	0,000200	-0,036900	0,385332
8	(1,1,3,0)	0,001970	0,000300	0,166534	3,838471
9	(1,3,0,1)	0,001103	0,000210	-0,040328	0,382693
10	(3,1,1,0)	0,001418	0,000220	0,038676	1,270270
11	(3,1,3,1)	0,001126	0,000206	-0,025766	0,397462
12	(1,3,1,3)	0,001255	0,000215	-0,005834	0,484610

Table 7: Asset Allocations For Optimal Portfolio With Different Preferences

Portfolio	1	2	3	4	5	6	7	8	9	10	11	12
λ	1,0,0,0	0,1,0,0	0,0,1,0	0,0,0,1	1,1,0,0	1,3,0,0	1,1,1,1	1,1,3,0	1,3,0,1	3,1,1,0	3,1,3,1	1,3,1,3
DOHOL	0	0,213744	1	0,15462	0,110922	0	0,154833	0	0,144324	0,194909	0,160173	0,117268
EREGL	0	0,244591	0	0,114306	0,027188	0	0,106314	0	0,107392	0,014361	0,095224	0,067110
GARAN	0	0	0	0,126631	0	0	0,114072	0,017252	0,122201	0	0,124502	0,128510
KCHOL	1	0,091472	0	0,105096	0,287503	0,482804	0,125544	0,603369	0,117687	0,317925	0,141342	0,192949
SAHOL	0	0,143965	0	0,131615	0,092039	0	0,129972	0	0,128376	0,10721	0,126833	0,115750
SISE	0	0,133024	0	0,11481	0,255637	0,319474	0,12607	0,275081	0,121814	0,270997	0,129842	0,147138
TUPRS	0	0,173205	0	0,139439	0,140621	0,065807	0,132204	0	0,139281	0,062073	0,113835	0,101850
YKBNK	0	0,000000	0	0,113483	0,08609	0,131915	0,110991	0,104299	0,118924	0,032526	0,108248	0,129425

The portfolios formed in accordance with the investor's preferences over incorporation of higher moments are given above. In order to analyze the effects of preferences both on the combination of stocks in the portfolios and descriptive statistics of portfolios' returns, different levels of preferences are investigated. Investors' preferences of (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1), (1,1,0,0), (1,3,0,0), (1,1,1,1), (1,1,3,0), (1,3,0,1), (3,1,1,0), (3,1,3,1) and (1,3,1,3) are included in our experiment.

In the first four portfolio, the first, the second, the third and the fourth moment are optimized. The portfolio 5, (1,1,0,0) is the Markowitz mean-variance portfolio. Investors higher preference for variance in portfolio 6, resulting in an increase in each moment investigated. When we consider changing the preference parameters from (1,3,0,0) to (1,1,1,1), it is seen that each moment investigated decreases. The decrease in the preference for variance by holding expected return is constant and considering the third and fourth moment in addition to the first two moments in portfolio formation, leads to lower expected return, variance, skewness and kurtosis. Portfolio 8-12 represent different combinations of investors' preferences for expected returns, variance, skewness and kurtosis.

5. CONCLUSION

Investors aim to allocate their capitals among a set of potential investments to achieve a desired tradeoff between their risk and return preferences. One of the most important preferred investment instrument is the securities. The important questions that have to be answered here is how the portfolio will be formed and what the best combination of investment instruments in the portfolio will be.

In this study, we try to answer these questions in the mean- variance- skewness- kurtosis framework by using a PGP model. In this model, multiple conflicting and competing portfolio objectives such as maximizing expected return and skewness and minimizing risk and kurtosis simultaneously are considered in accordance with different investors' preferences. Our results reveal that the investors' preferences affect both asset allocations of portfolio and descriptive statistics of descriptive statistics of asset returns.

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