Cilt: 11 • Özel Sayı • 2011
ss. 125-134

Fuzzy ELECTRE I Method for Evaluating Catering Firm Alternatives

Yemek Firması Alternatiflerini Değerlendirmede Bulanık ELECTRE I Yöntemi

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ABSTRACT

Decision making in an uncertain environment is a complex task for decision makers. Fuzzy multi criteria decision making methods are proposed in the literature where the subjective criteria and the weights of all criteria are assessed in linguistic variables since conventional methods cannot take into consideration the subjectivity and uncertainty in the decision process. Fuzzy ELECTRE I (Elimination Et Choix Traduisant la Realité) is one of the fuzzy multi criteria decision making methods for resolving the ambiguity of concepts that are associated with decision makers' judgments. In this paper fuzzy ELECTRE I method is applied to catering firm selection problem of a textile company. By this way uncertainty in the problem has been considered and the most appropriate catering firm has been selected among the alternatives.

Keywords: Fuzzy set, ELECTRE I, fuzzy ELECTRE I, catering firm selection

ÖZET

Belirsiz bir ortamda karar verme, karar vericiler için karmaşık bir iştir. Geleneksel yöntemler, karar verme sürecinde subjektifliği ve belirsizliği dikkate alamadığı için literatürde subjektif kriterler ve bütün kriterlerin ağırlıklarının dilsel değişkenlerle değerlendirildiği çok kriterli karar verme yöntemleri önerilir. Bulanık ELECTRE I, karar vericilerin kararları ile ilgili kavramların belirsizliğini çözmek için kullanılan bulanık çok kriterli karar verme yöntemlerinden biridir. Bu çalışmada Bulanık ELECTRE I yöntemi, bir tekstil işletmesinin yemek firması seçim problemine uygulanmıştır. Bu şekilde, problemdeki belirsizlik dikkate alınmış ve alternatifler arasından en uygun yemek firması seçilmiştir.

Anahtar Kelimeler: Bulanık küme, ELECTRE I, bulanık ELECTRE I, yemek firması seçimi

1. INTRODUCTION

Catering is the act of providing foods and services or it may be defined as preparing or providing food for someone else to serve; or preparing, delivering and serving food at the premises of another person or event (Kahraman et al., 2004). Catering firm selection problem is strategically important for companies. While selecting prospective catering firms, the company judges each catering firm's ability to meet consistently and cost-effectively its needs using selection criteria and appropriate measures. So catering firm selection decision inherently is a multi criteria decision making (MCDM) problem. In real world, needs are uncertain and they are often expressed as qualitative concepts. So the nature of this decision usually is complex and unstructured (Kahraman et al., 2003).

In the literature MCDM including several methods, allows rating a range of criteria and then ranking them with the opinions of decision makers. The MCDM method has high potential to reduce the cost and time and increases the accuracy of decisions and can be an appropriate framework for solving the problems (Asghari et al., 2010). ELECTRE I is one of the MCDM methods which is based on the study of outranking relations and it uses concordance and discordance indexes to analyze the outranking relations among the alternatives. Concordance and discordance indexes can be viewed as measurements of satisfaction and dissatisfaction that a decision maker chooses one alternative over the other. However in this method the ratings and the weights of the selection criteria are known precisely and thus are inadequate for dealing with the imprecise or vague nature of linguistic assessment (Sevkli, 2010). So in the literature ELECTRE I has been combined with fuzzy set

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theory. Fuzzy set theory resembles human reasoning in its use of approximate information and uncertainty to generate decisions (Kahraman et al., 2004). Since the conventional ELECTRE I cannot reflect the human thinking style, fuzzy ELECTRE I is used to solve a catering firm selection problem in this study.

The organization of this paper is as follows. In the second section the ELECTRE I method is explained. In the third section fuzzy sets, fuzzy numbers and their fuzzy algebraic operations are introduced. In the fourth section concept of fuzzy ELECTRE I method is introduced briefly then the literature review regarding fuzzy ELECTRE and the formulation of fuzzy ELECTRE I are given. In the fifth section a comparison among five catering firms is made by using fuzzy ELECTRE I method. In the last section conclusions and findings are interpreted.

2. ELECTRE I

ELECTRE (ELimination Et Choix Traduisant la REalite´) is one of the MCDM methods. This method allows decision makers to select the best choice (action) with maximum advantage and minimum conflict in the function of various criteria (Asghari et al., 2010). The ELECTRE method for choosing the best action(s) from a given set of actions was devised in 1965, and was later referred to as ELECTRE I (Sevkli, 2010). Different versions of ELECTRE have been developed including ELECTRE I, II, III, IV and TRI. All methods are based on the same fundamental concepts but differ both operationally and according to the type of the decision problem. Specifically, ELECTRE I is designed for selection problems, ELECTRE TRI for assignment problems and ELECTRE II, III and IV for ranking problems (Marzouk, 2010).

The main idea is the proper utilization of "outranking relations" (Vahdani and Hadipour, 2011). ELECTRE creates the possibility to model a decision process by using coordination indices. These indices are concordance and discordance matrices (Asghari et al., 2010). The decision maker uses concordance and discordance indices to analyze outranking relations among different alternatives and to choose the best alternative using the crisp data (Wu and Chen, 2011). These outranking relations in ELECTRE I are shown as S, whose meaning is "at least as good as." Considering two alternatives A_f and A_g, four situations may arise (Hatami-Marbini and Tavana, 2011);

- A_f S A_g and not A_g S A_f (A_f is strictly preferred to A_g),
- A_g S A_f and not A_f S A_g (A_g is strictly preferred to A_f),
- A_f S A_g and A_g S A_f (A_f is indifferent to A_g)
- Not A_f S A_g and not A_g S A_f (A_f is incomparable to A_g).

The steps of ELECTRE method can be summarized as follows (Sevkli, 2010):

Step 1: For starting the method it's supposed that the problem has m alternatives or actions ($A_1, A_2, \ldots, A_m$) and n decision criteria/attributes ($C_1, C_2, \ldots, C_n$). Each alternative is evaluated with respect to the n criteria/attributes. All the values/ratings assigned to the alternatives with respect to each criterion form a decision matrix denoted by $X = (x_{ij})_{mxn}$. Let $W = (w_1, w_2, \ldots, w_n)$ be the relative weight vector about the criteria, satisfying $\sum_{j=1}^{n} w_j = 1$.

Step 2: The decision matrix $X = \left(x_{ij}\right)_{mxn}$ is normalized by calculating r_{ij} which represents the normalized criteria/attribute value/rating.

For the minimization objective;

$$r_{ij} = \frac{1/x_{ij}}{\sqrt{\sum_{i=1}^{m} 1/x_{ij}^2}}, \ i=1,2,\ldots,m \text{ and } j=1,2,\ldots,n \quad (1a)$$

For the maximization objective;

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \ i=1,2,\ldots,m \text{ and } j=1,2,\ldots,n \quad (1b)$$

Step 3: The weighted normalized decision matrix $V = \left(v_{ij}\right)_{mxn}$ is calculated.

$$v_{ij} = r_{ij}.w_j \text{ and } i=1,2,\ldots,m \text{ and } j=1,2,\ldots,n \quad (2)$$

w_j is the relative weight of the jth criterion or attribute and $\sum_{i=1}^{n} w_j = 1$

Step 4: The concordance set is determined. If alternative A_f is preferred to alternative A_g for all criteria, the concordance set is composed. This can be written as

$$C(f,g) = \left\{ j \middle| v_{fj} \geq v_{gj} \right\} \quad (3)$$

In the formula v_{fj} is the weighted normalized rating of alternative A_f with respect to the jth criterion and $C(f, g)$ is the collection of attributes where A_f is better than or equal to A_g. Then concordance indexes are calculated. The concordance index of $C(f, g)$ is defined as;

$$C_{fg} = \sum_{j^*} w_{j^*} \quad (4)$$

j* are attributes/criteria contained in the concordance set $C(f, g)$.

Step 5: The discordance set is determined. It contains all criteria for which A_f is worse than A_g. This can be written as;

$$D(f,g) = \left\{ j \middle| v_{fj} < v_{gj} \right\} \quad (5)$$

Then discordance indexes are calculated. The discordance index of $D(f, g)$ is defined as;

$$D_{fg} = \frac{\sum_{j^+}\left|v_{fj^+} - v_{gj^+}\right|}{\sum_{j}\left|v_{fj} - v_{gj}\right|} \tag{6}$$

j^+ are attributes/criteria contained in the discordance set D(f, g).

Step 6: Outranking relations between alternatives are determined. A_f outranks A_g when $C_{fg} \geq \bar{C}$ and $D_{fg} \leq \bar{D}$ where $\bar{C}$ and $\bar{D}$ are the averages of C_{fg} and D_{fg} respectively.

3. FUZZY SETS AND FUZZY NUMBERS

In order to deal with imprecision of human thought, Zadeh (1965) first introduced the fuzzy set theory. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one (Zadeh, 1965). A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or non-membership at all, whereas fuzzy sets allow partial membership. In other words, an element may partially belong to a fuzzy set.

Fuzzy sets theory providing a more widely frame than classic sets theory, has been contributing to capability of reflecting real world. Modeling using fuzzy sets has proven to be an effective way for formulating decision problems where the information available is subjective and imprecise (Zimmermann, 1992).

A fuzzy number $\tilde{M}$ is a convex normalized fuzzy set $\tilde{M}$ of the real line R such that:

- It exists such that one $x_0 \in R$ with $\mu_{\tilde{M}}(x_0) = 1$ (x_0 is called mean value of $\tilde{M}$)

- $\mu_{\tilde{M}}(x)$ is piecewise continuous (Zimmermann, 1992).

Triangular fuzzy numbers can be defined as a triplet (a,b,c). The parameters a, b and c respectively, indicate the smallest possible value, the most promising value and the largest possible value that describe a fuzzy event (Kahraman et al., 2003). The membership function of a fuzzy triangular number can be described as;

$$\mu(x/\tilde{M}) = \begin{cases} 0, & x < a, \\ (x-a)/(b-a), & a \leq x \leq b, \\ (c-x)/(c-b), & b \leq x \leq c, \\ 0, & x > c \end{cases} \tag{7}$$

In this study Hamming distance is used while finding distance between two fuzzy numbers. For any fuzzy numbers $\tilde{A}$ and $\tilde{B}$, the Hamming distance $\left(\tilde{A},\tilde{B}\right)$ can be found as (Hatami-Marbini and Tavana, 2011);

$$d\left(\tilde{A},\tilde{B}\right) = \int_R \left|\mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)\right| dx \tag{8}$$

4. FUZZY ELECTRE I

In traditional ELECTRE methods, the weights of the criteria and the ratings of alternatives on each criterion are known precisely and crisp values are used in the evaluation process. However under many conditions, exact or crisp data are inadequate to model real-life situations. Therefore, these data may have some structures such as fuzzy data, bounded data, ordinal data and interval data (Vahdani et al., 2010). In fuzzy ELECTRE, linguistic preferences can easily be converted to fuzzy numbers (Kaya and Kahraman, 2011). In other words decision makers utilize fuzzy numbers instead of single values in the evaluation process of the ELECTRE (Wu and Chen, 2011). A fuzzy outranking relation, $k\ S\ l$, can be characterized by a membership function (k, l) which indicates the degree of outranking associated with each pair of alternatives (A_k, A_l) in fuzzy ELECTRE (Kaya and Kahraman, 2011).

There are many studies in the literature that combine all types of ELECTRE methods with fuzzy sets. Recent studies in the literature about ELECTRE I and fuzzy sets are given in this section. Sevkli (2010) used fuzzy ELECTRE approach to the supplier selection problem. Wu and Chen (2011) developed a new method, the intuitionistic fuzzy ELECTRE method, for solving multi-criteria decision making problems. Vahdani and Hadipour (2011) presented the interval-valued fuzzy ELECTRE method for solving MCDM problems in which the weights of criteria were unequal, using interval-valued fuzzy set concepts and applied the methodology to maintenance strategy selection problem. Hatami-Marbini and Tavana (2011) proposed a new methodology for fuzzy ELECTRE based on Hamming distance, applied the new methodology to the application of Chen et al. (2006) and compared the obtained results with Chen et al.'s results obtained by TOPSIS. Kaya and Kahraman (2011) used fuzzy ELECTRE with AHP to propose an environmental impact assessment methodology for urban industrial planning.

In this paper, fuzzy ELECTRE I method is considered which was proposed by Hatami-Marbini and Tavana (2011). The algorithm of this method can be described as follows;

Step 1: First of all a committee of decision makers is formed. In a decision committee that has K decision makers; fuzzy rating of each decision maker $DM_k\left(k = 1,2,\ldots K\right)$ can be represented as triangular fuzzy number $\tilde{R}_k\left(k = 1,2,\ldots K\right)$ with membership function $\mu_{\tilde{R}_k}(x)$.

Step 2: Then evaluation criteria are determined.

Step 3: After that, appropriate linguistic variables are chosen for evaluating criteria and attributes.

Step 4: If the fuzzy ratings of all decision makers are described as triangular fuzzy numbers $\tilde{R}_k = (a_k, b_k, c_k)$, $k = 1, 2, \ldots, K$, then the aggregated fuzzy rating can be determined as $\tilde{R} = (a, b, c)$, $k = 1, 2, \ldots, K$. Here;

$$a = \frac{1}{K}\sum_{k=1}^{K} a_k, \quad b = \frac{1}{K}\sum_{k=1}^{K} b_k, \quad c = \frac{1}{K}\sum_{k=1}^{K} c_k \tag{9}$$

If the fuzzy rating and importance weight of the kth decision maker are $\tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk})$ and $\tilde{w}_{jk} = (w_{jk}^L, w_{jk}^M, w_{jk}^U)$, $i = 1, 2, \ldots m$, $j = 1, 2, \ldots n$ respectively, then the aggregated fuzzy ratings $(\tilde{x}_{ij})$ of alternatives with respect to each criterion can be found as;

$$(\tilde{x}_{ij}) = (a_{ij}, b_{ij}, c_{ij}) \tag{10}$$

$$a_{ij} = \frac{1}{K}\sum_{k=1}^{K} a_{ijk}, \quad b_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ijk}, \quad c_{ij} = \frac{1}{K}\sum_{k=1}^{K} c_{ijk} \tag{11}$$

Then the aggregated fuzzy weights $(\tilde{w}_{ij})$ of each criterion are calculated as;

$$(\tilde{w}_j) = (w_j^L, w_j^M, w_j^U) \tag{12}$$

$$w_j^L = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^L, \quad w_j^M = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^M, \quad w_j^U = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^U \tag{13}$$

Step 5: Then the fuzzy decision matrix is constructed as;

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}, \tilde{W} = [\tilde{w}_1, \tilde{w}_2, \cdots \tilde{w}_n] \tag{14}$$

where $\tilde{x}_{ij} = (a_{ji}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_j^L, w_j^M, w_j^U)$; $i = 1, 2, \ldots m$, $j = 1, 2, \ldots n$ can be approximated by positive triangular fuzzy numbers.

Step 6: After constructing the fuzzy decision matrix, it is normalized. Instead of using complicated normalization formula, the linear scale transformation can be used to transform the various criteria scales into a comparable scale. Therefore, the normalized fuzzy decision matrix can be obtained as $\tilde{R}$.

$$\tilde{R} = [\tilde{r}_{ij}]_{mxn} \quad i = 1, 2, \ldots, m; \; j = 1, 2, \ldots n \tag{15}$$

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right), \; c_j^* = \max_i c_{ij} \tag{16}$$

Step 7: Considering the different weight of each criterion, the weighted normalized decision matrix is computed by multiplying the importance weights of evaluation criteria and the values in the normalized fuzzy decision matrix. The weighted normalized fuzzy decision matrix $\tilde{V}$ is defined as;

$$\tilde{V} = \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \cdots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \cdots & \tilde{v}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \cdots & \tilde{v}_{mn} \end{bmatrix} \tag{17}$$

where $\tilde{v}_{ij} = \tilde{r}_{ij}(.)\tilde{w}_j \quad i = 1, 2, \ldots, m \quad j = 1, 2, \ldots, n$

and here $\tilde{w}_j$ represents the importance weight of criterion C_j.

Step 8: Concordance and discordance matrices are calculated using the weighted normalized fuzzy decision matrix ($\tilde{V}$) and the pair wise comparison among the alternatives. Considering two alternatives and the concordance set can be defined as;

$$J_C = \{ j \mid \tilde{v}_{gj} \geq \tilde{v}_{fj} \} \tag{18}$$

here J_C is the index of all criteria belonging to the concordance coalition with the outranking relation $A_g \, S \, A_f$.

In this paper Hamming distance shown in Eq. (8) is used for comparing any two alternatives g and f on each criterion. Firstly their least upper bound, $\max(\tilde{v}_{gj}, \tilde{v}_{fj})$, is determined. Then Hamming distances $d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{gj})$ and $d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj})$ are calculated. $\tilde{v}_{gj} \geq \tilde{v}_{fj}$ if and only if $d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj}) \geq d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{gj})$.

The discordance set can be defined as;

$$J_D = \{ j \mid \tilde{v}_{gj} < \tilde{v}_{fj} \} \tag{19}$$

here J_D is the index of all criteria belonging to the discordance coalition and it is against the assertion "A_g is at least as good as A_f." Similarly for comparing each criterion of alternatives g and f, Hamming distance is used which assumes that $\tilde{v}_{gj} < \tilde{v}_{fj}$ if and only if $d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj}) < d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{gj})$.

Step 9: The concordance matrix for each pair wise comparison of the alternatives can be defined as;

$$\tilde{C} = \begin{bmatrix} - & \cdots & \tilde{c}_{1f} & \cdots & \tilde{c}_{1m} \\ \vdots & \ddots & \vdots & \cdots & \vdots \\ \tilde{c}_{g1} & \cdots & \tilde{c}_{gf} & \cdots & \tilde{c}_{gm} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \tilde{c}_{m1} & \cdots & \tilde{c}_{mf} & \cdots & - \end{bmatrix} \tag{20}$$

The elements of the concordance matrix are determined as fuzzy summation of the fuzzy weights of all criteria in the concordance set.

$$\tilde{c}_{gf} = \left(c^L_{gf}, c^M_{gf}, c^U_{gf}\right) = \sum_{j\in J_C} \tilde{W}_j = \left(\sum_{j\in J} w^L_j, \sum_{j\in J} w^M_j, \sum_{j\in J} w^U_j\right) \quad (21)$$

Step 10: The discordance matrix can be defined as;

$$D = \begin{bmatrix} - & \cdots & d_{1f} & \cdots & d_{1m} \\ \vdots & \ddots & \vdots & \cdots & \vdots \\ d_{g1} & \cdots & d_{gf} & \cdots & d_{gm} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ d_{m1} & \cdots & d_{mf} & \cdots & - \end{bmatrix} \quad (22)$$

$$d_{gf} = \frac{\max_{j\in J_D} \left|\tilde{v}_{gj} - \tilde{v}_{fj}\right|}{\max_{j} \left|\tilde{v}_{gj} - \tilde{v}_{fj}\right|} = \frac{\max_{j\in J_D} \left|d\left(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj}\right)\right|}{\max_{j} \left|d\left(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj}\right)\right|} \quad (23)$$

Step 11: According to the concordance level, the value of the concordance matrix elements are evaluated. The concordance level $\tilde{\bar{C}} = \left(c^L, c^M, c^U\right)$ can be defined as the average of the elements in the concordance matrix represented by;

$$c^L = \sum_{f=1}^{m}\sum_{g=1}^{m} c^L_{gf} \Big/ m(m-1) \quad , \quad c^M = \sum_{f=1}^{m}\sum_{g=1}^{m} c^M_{gf} \Big/ m(m-1) \quad ,$$
$$c^U = \sum_{f=1}^{m}\sum_{g=1}^{m} c^U_{gf} \Big/ m(m-1) \quad (24)$$

Step 12: Boolean matrix B is formed according to the minimum concordance level $\tilde{\bar{C}}$ as;

$$B = \begin{bmatrix} - & \cdots & b_{1f} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots & \cdots & \vdots \\ b_{g1} & \cdots & b_{gf} & \cdots & b_{gm} \\ \vdots & \ddots & \vdots & \vdots & \\ b_{m1} & \cdots & b_{mf} & \cdots & - \end{bmatrix} \quad (25)$$

$$\begin{cases} \tilde{c}_{gf} \geq \tilde{\bar{C}} \Leftrightarrow b_{gf} = 1 \\ \tilde{c}_{gf} < \tilde{\bar{C}} \Leftrightarrow b_{gf} = 0 \end{cases} \quad (26)$$

Hamming distance is used for comparing $\tilde{c}_{gf}$ and $\tilde{\bar{C}}$. In the matrix B if b_{gf} =1 it is said to be alternative g dominates f.

Step 13: The elements of the discordance matrix are measured by a discordance level. The discordance level $\bar{D}$ can be defined as the average of the elements in the discordance matrix.

$$\bar{D} = \sum_{f=1}^{m}\sum_{g=1}^{m} d_{gf} \Big/ m(m-1) \quad (27)$$

Step 14: Boolean matrix H is measured by a minimum discordance level

$$H = \begin{bmatrix} - & \cdots & h_{1f} & \cdots & h_{1m} \\ \vdots & \ddots & \vdots & \cdots & \vdots \\ h_{g1} & \cdots & h_{gf} & \cdots & h_{gm} \\ \vdots & \ddots & \vdots & \vdots & \\ h_{m1} & \cdots & h_{mf} & \cdots & - \end{bmatrix} \quad (28)$$

$$\begin{cases} d_{gf} < \bar{D} \Leftrightarrow h_{gf} = 1 \\ d_{gf} \geq \bar{D} \Leftrightarrow h_{gf} = 0 \end{cases} \quad (29)$$

The elements of this matrix measure the power of the discordant coalition, in other words if its element value surpasses a given level, D, the assertion is no longer valid. Discordant coalition exerts no power whenever $d_{gf} < \bar{D}$. In other words, the elements of matrix H with the value of 1 show the dominance relations among the alternatives.

Step 15: The global matrix Z is calculated by peer to peer multiplication of the elements of the matrices B and H as follows;

$$Z = B \otimes H \quad (30a)$$

here each element (z_{gf}) of matrix Z is obtained as;

$$z_{gf} = b_{gf} . h_{gf} \quad (30b)$$

Step 16: The final step of this procedure consists of exploitation of the above outranking relation (matrix Z) in order to identify as small as possible a subset of alternatives, from which the best compromise alternative could be selected. Consequently, it is extremely useful to build a simple graph $G = (V,J)$, where V is the set of vertices and J is the set of arcs. For each alternative, we associate a vertex and for each pair of alternatives A_g and A_f, an arc exists between them if either A_g is preferred to A_f or A_g is indifferent to A_f. An alternative A_g outranks A_f if an arc exists between A_g and A_f and the arrow points from A_g to A_f (for this case, z_{gf} = 1). A_g and A_f are incomparable if no arc exists between A_g and A_f (for this case, z_{gf} = 0). A_g and A_f are indifferent if an arc exists between A_g and A_f and an arrow exists in both directions (for this case, z_{gf} = 1 and z_{fg} =1) (Hatami-Marbini and Tavana, 2011).

5. APPLICATION

Catering services can be considered as a new and rapidly growing sector in Turkish industry. The companies of catering service sector in Turkey have to be very competitor. Their customers change frequently their supplier of catering contractor, because it is easy to replace them when a complaint or nonconformity happens and there are too many companies in the sector (Kahraman et al., 2004). In this part the mentioned methodology is applied to a real industrial case, which refers to one of the textile company operated in Denizli in Turkey. The catering selection problem of this textile company is solved through fuzzy ELECTRE I.

Firstly a committee includes three decision makers (DM_1, DM_2, DM_3) from the textile company is formed. Then decision makers determine five catering firm alternatives (A_1, A_2, A_3, A_4, A_5) and define criteria while they are evaluating the catering firm alternatives. These criteria are hygiene (C_1), references (C_2), taste and variety of meals (C_3), service quality (C_4), price (C_5) and adequacy of structure (C_6).

Linguistic terms for determining the weights of criteria and performance ratings for catering firm alternatives are determined and given in Table 1 and Table 2 respectively.

Table 1: The Linguistic Variables for the Importance Weights of Criteria

Linguistic Variable	Very Low (VL)	Low (L)	Medium (M)	High (H)	Very High (VH)
Fuzzy Number	(0 , 0 , 0.2)	(0 , 0.2 , 0.4)	(0.2 , 0.4 , 0.6)	(0.4 , 0.6 , 0.8)	(0.6 , 0.8 , 1)

Table 2: The Linguistic Variables for Performance Ratings

Linguistic Variable	Very Poor (VP)	Poor (P)	Medium (M)	Good (G)	Very High (VG)
Fuzzy Number	(0 , 0 , 2)	(0 , 2 , 4)	(2 , 4 , 6)	(4 , 6 , 8)	(6 , 8 , 10)

Then decision makers reach a consensus to evaluate the importance weights of criteria using the linguistic terms defined in Table 3. The linguistic assessments of the five alternatives on each criterion provided by the three decision makers are presented in Table 4.

Table 3: The Importance Weights of the Six Criteria by Three DMs

	DM_1	**DM_2**	**DM_3**
C_1	H	VH	VH
C_2	H	H	H
C_3	H	VH	VH
C_4	H	H	VH
C_5	VH	H	M
C_6	M	H	H

Table 4: Linguistic Assessments of Three DMs on Five Alternatives

Criteria	**Alternatives**	**DMs**		
		DM_1	**DM_2**	**DM_3**
C_1	***A_1***	VG	G	G
	A_2	G	M	M
	A_3	M	P	P
	A_4	M	M	G
	A_5	VG	G	M
C_2	***A_1***	VG	G	G
	A_2	G	G	M
	A_3	G	M	M
	A_4	G	G	M
	A_5	G	VG	G
C_3	***A_1***	VG	VG	G
	A_2	VG	G	VG
	A_3	G	VG	M
	A_4	M	M	P
	A_5	G	M	M
C_4	***A_1***	VG	G	G
	A_2	VG	G	VG
	A_3	G	M	M
	A_4	G	VG	G
	A_5	G	M	M
C_5	***A_1***	P	M	M
	A_2	M	M	M
	A_3	VG	G	G
	A_4	M	M	M
	A_5	G	M	M
C_6	***A_1***	G	G	M
	A_2	G	M	M
	A_3	G	M	P
	A_4	G	M	M
	A_5	G	G	G

Then the fuzzy weights of criteria and fuzzy ratings of alternatives for each criterion are aggregated through Eq. (11) and Eq. (13). Aggregated fuzzy weights of criteria are shown in Table 5. Aggregated fuzzy ratings of alternatives for each criterion with fuzzy decision matrix are shown in Table 6. And also normalized fuzzy decision matrix is shown in Table 7. Finally weighted normalized fuzzy decision matrix as shown in Table 8 is formed by using Table 5 and Table 7.

Table 9 obtained by using Table 8, shows distance between two alternatives g and f with respect to each criterion. Hamming distance method shown in Eq. (8) is used to calculate distances. Table 10 shows the concordance matrix obtained by using Eq. (21). Also Table 11 shows the discordance matrix obtained by using Eq. (23). Minimum concordance and discordance levels are shown in the last rows of these tables.

Table 5: Aggregated Fuzzy Weights of Six Criteria

Criteria	***Fuzzy weights***
C_1	(0.533 , 0.733 , 0.933)
C_2	(0.400 , 0.600 , 0.800)
C_3	(0.533 , 0.733 , 0.933)
C_4	(0.467 , 0.667 , 0.867)
C_5	(0.400 , 0.600 , 0.800)
C_6	(0.333 , 0.533 , 0.733)

Table 6: The Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(4.667 , 6.667 , 8.667)	(4.667 , 6.667 , 8.667)	(5.333 , 7.333 , 9.333)	(4.667 , 6.667 , 8.667)	(1.333 , 3.333 , 5.333)	(3.333 , 5.333 ,7.333)
A_2	(2.667 , 4.667 , 6.667)	(3.333 , 5.333 , 7.333)	(5.333 , 7.333 , 9.333)	(5.333 , 7.333 , 9.333)	(2.000 , 4.000 , 6.000)	(2.667 , 4.667, 6.667)
A_3	(0.667 , 2.667 , 4.667)	(2.667 , 4.667 , 6.667)	(4.000 , 6.000 , 8.000)	(2.667 , 4.667 , 6.667)	(4.667 , 6.667 , 8.667)	(2.000 , 4.000 , 6.000)
A_4	(2.667 , 4.667 , 6.667)	(3.333 , 5.333 , 7.333)	(1.333 , 3.333 , 5.333)	(4.667 , 6.667 , 8.667)	(2.000 , 4.000 , 6.000)	(2.667 , 4.667 , 6.667)
A_5	(4.000 , 6.000 , 8.000)	(4.667 , 6.667 , 8.667)	(2.667 , 4.667 , 6.667)	(2.667 , 4.667 , 6.667)	(2.667 , 4.667 , 6.667)	(4.000 , 6.000 , 8.000)

Table 7: The Normalized Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.538 , 0.769 , 1.000)	(0.538 , 0.769 , 1.000)	(0.571 , 0.786 , 1.000)	(0.500 , 0.714 , 0.929)	(0.154 , 0.385 , 0.615)	(0.417 , 0.667 , 0,917)
A_2	(0.308 , 0.538 , 0.769)	(0.385 , 0.615 , 0.846)	(0.571 , 0.786 , 1.000)	(0.571 , 0.786 , 1.000)	(0.231 , 0.462, 0.692)	(0.333 , 0.583 , 0,833)
A_3	(0.077 , 0.308 , 0.538)	(0.308 , 0.538 , 0.769)	(0.429 , 0.643 , 0.857)	(0.286 , 0.500 , 0.714)	(0.538 , 0.769 , 1.000)	(0.250 , 0.500 , 0,750)
A_4	(0.308 , 0.538 , 0.769)	(0.385 , 0.615 , 0.846)	(0.143 , 0.357 , 0.571)	(0.500 , 0.714 , 0.929)	(0.231, 0.462 , 0.692)	(0.333 , 0.583 , 0,833)
A_5	(0.462 , 0.692 , 0.923)	(0.538 , 0.769 , 1.000)	(0.286 , 0.500 , 0.714)	(0.286 , 0.500 , 0.714)	(0.308 , 0.538 , 0.769)	(0.500 , 0.750 , 1,000)

Table 8: The Weighted Normalized Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.287 , 0.564 , 0.933)	(0.215 , 0.462 , 0.800)	(0.305 , 0.576 , 0.933)	(0.233 , 0.476, 0.805)	(0.062 , 0.231 , 0.492)	(0.139 , 0.356 , 0.672)
A_2	(0.164 , 0.395 , 0.718)	(0.154 , 0.369 , 0.677)	(0.305 , 0.576 , 0.933)	(0.267 , 0.524 , 0.867)	(0.092 , 0.277 , 0.554)	(0.111 , 0.311 , 0.611)
A_3	(0.041 , 0.226 , 0.503)	(0.123 , 0.323 , 0.615)	(0.229 , 0.471 , 0.800)	(0.133 , 0.333 , 0.619)	(0.215 , 0.462 , 0.800)	(0.083 , 0.267 , 0.550)
A_4	(0.164 , 0.395 , 0.718)	(0.154 , 0.369 , 0.677)	(0.076 , 0.262 , 0.533)	(0.233 , 0.476 , 0.805)	(0.092 , 0.277 , 0.554)	(0.111 , 0.311 , 0.611)
A_5	(0.246 , 0.508 , 0.862)	(0.215 , 0.462 , 0.800)	(0.152 , 0.367 , 0.667)	(0.133 , 0.333 , 0.619)	(0.123 , 0.323 , 0.615)	(0.167 , 0.400 , 0.733)

Table 9: The Distance Between Two Alternatives *g* and *f* with respect to Each Criterion

	X_{11}	X_{21}	X_{31}	X_{41}	X_{51}
X_{11}	-	(0 , 0.05)	(0 , 0.09)	(0 , 0.05)	(0 , 0.02)
X_{21}	-	-	(0 , 0.05)	(0 , 0)	(0.03 , 0)
X_{31}	-	-	-	(0.05 , 0)	(0.08 , 0)
X_{41}	-	-	-	-	(0.03 , 0)
X_{51}	-	-	-	-	-
	X_{12}	X_{22}	X_{32}	X_{42}	X_{52}
X_{12}	-	(0 , 0.03)	(0 , 0.05)	(0 , 0.03)	(0 , 0)
X_{22}	-	-	(0 , 0.02)	(0 , 0)	(0.03 , 0)
X_{32}	-	-	-	(0.02 , 0)	(0.05 , 0)
X_{42}	-	-	-	-	(0.03 , 0)
X_{52}	-	-	-	-	-
	X_{13}	X_{23}	X_{33}	X_{43}	X_{53}
X_{13}	-	(0 , 0)	(0 , 0.03)	(0 , 0.09)	(0 , 0.06)
X_{23}	-	-	(0 , 0.03)	(0 , 0.09)	(0 , 0.06)
X_{33}	-	-	-	(0 , 0.06)	(0 , 0.03)
X_{43}	-	-	-	-	(0.03 , 0)
X_{53}	-	-	-	-	-
	X_{14}	X_{24}	X_{34}	X_{44}	X_{54}
X_{14}	-	(0.01 , 0)	(0 , 0.04)	(0 , 0)	(0 , 0.04)
X_{24}	-	-	(0 , 0.06)	(0 , 0.01)	(0 , 0.06)
X_{34}	-	-	-	(0.04 , 0)	(0 , 0)
X_{44}	-	-	-	-	(0 , 0.04)
X_{54}	-	-	-	-	-
	X_{15}	X_{25}	X_{35}	X_{45}	X_{55}
X_{15}	-	(0.02 , 0)	(0.08 , 0)	(0.02 , 0)	(0.03 , 0)
X_{25}	-	-	(0.06 , 0)	(0 , 0)	(0.02 , 0)
X_{35}	-	-	-	(0 , 0.06)	(0 , 0.05)
X_{45}	-	-	-	-	(0.02 , 0)
X_{55}	-	-	-	-	-
	X_{16}	X_{26}	X_{36}	X_{46}	X_{56}
X_{16}	-	(0 , 0.02)	(0 , 0.03)	(0 , 0.02)	(0.02 , 0)
X_{26}	-	-	(0 , 0.02)	(0 , 0)	(0.03 , 0)
X_{36}	-	-	-	(0.02 , 0)	(0.05 , 0)
X_{46}	-	-	-	-	(0.03 , 0)
X_{56}	-	-	-	-	-

Table 10: The Concordance Matrix

	A_1	A_2	A_3	A_4	A_5
A_1	-	(1.800 , 2.600 , 3.400)	(2.267 , 3.267 , 4.267)	(2.267 , 3.267 , 4.267)	(1.933, 2.733, 3.533)
A_2	(1.400, 2.000, 2.600)	-	(2.267 , 3.267 , 4.267)	(2.667 , 3.867 , 5.067)	(1.000 , 1.400 , 1.800)
A_3	(0.400 , 0.600 , 0.800)	(0.400 , 0.600 , 0.800)	-	(0.933 , 1.333 , 1.733)	(1.400 , 2.000 , 2.600)
A_4	(0.867 , 1.267 , 1.667)	(1.667 , 2.467 , 3.267)	(1.733 , 2.533 , 3.333)	-	(0.467 , 0.667 , 0.867)
A_5	(1.133 , 1.733 , 2.333)	(1.667 , 2.467 , 3.267)	(1.733 , 2.533 , 3.333)	(2.200 , 3.200 , 4.200)	-
$\overline{C} = (1.509 , 2.189 , 2.869)$					

Table 11: The Discordance Matrix

	A_1	A_2	A_3	A_4	A_5
A_1	-	0.4	0.88	0.22	0.5
A_2	1	-	1	0	0.5
A_3	1	1	-	0.83	1
A_4	1	1	1	-	0.75
A_5	1	1	0.625	1	-
$\overline{D} = 0{,}79$					

Table 12: Boolean Matrix *B* based on the Minimum Concordance Level

	A_1	A_2	A_3	A_4	A_5
A_1	-	1	1	1	1
A_2	0	-	1	1	0
A_3	0	0	-	0	0
A_4	0	1	1	-	0
A_5	0	1	1	1	-

Table 13: Boolean Matrix *H* based on the Minimum Discordance Level

	A_1	A_2	A_3	A_4	A_5
A_1	-	1	0	1	1
A_2	0	-	0	1	1
A_3	0	0	-	0	0
A_4	0	0	0	-	1
A_5	0	0	1	0	-

Table 14: The Global Matrix *Z*

	A_1	A_2	A_3	A_4	A_5
A_1	-	1	0	1	1
A_2	0	-	0	1	0
A_3	0	0	-	0	0
A_4	0	0	0	-	0
A_5	0	0	1	0	-

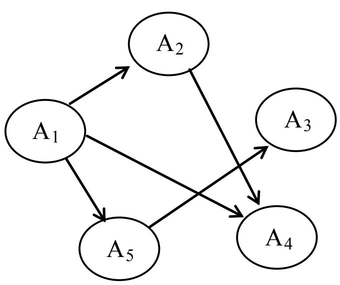

Figure 1: The Decision Graph for the Application

Boolean matrices B and H are shown in Table 12 and Table 13. The global matrix (Z) shown in Table 14 is obtained by using Table 12 and Table 13.

Finally the decision graph is formed and shown in Figure 1. According to Figure 1, A_1 is selected as the best catering firm among five catering firm alternatives for the textile company.

6. CONCLUSION

In real world decision making problems take place in a complex environment where conflicting systems of logic, uncertain and imprecise knowledge have to be considered. To face such complexity, multi criteria decision making methods are used (Montazer et al., 2009). One of these methods is fuzzy ELECTRE I. In this paper, fuzzy ELECTRE I proposed by Hatami-Marbini and Tavani (2011) is applied to catering firm selection problem. Because determining the best catering firm that fits with the organizational requirements is an extremely difficult and critical decision. An unsuitable selection can significantly affect not only the success of the implementation but also performance of the company (Cebeci, 2005). Five catering firms in Denizli are compared with this method and the best catering firm is selected providing the most satisfaction for the textile company operated in Denizli.

The difficulty of dealing with ambiguous and imprecise nature of the linguistic assessment in traditional ELECTRE I is overcome with fuzzy ELECTRE I. It also integrates experts' judgment, experience and expertise in more flexible and realistic manner using membership functions and linguistic variables.

For future studies other multi criteria decision making methods can be used while comparing catering firms and compared the results of them. Different weights and criteria can be used according to firm's structure and needs. Distance measured in the process of concordance and discordance index calculation in this paper can be replaced different kinds of distance methods. And also different linguistic variables and fuzzy numbers can be used according to the problem. Finally the method can be applied to other selection problems.

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