http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 19.04.2018 *Year*: **2018**, *Number*: **25**, *Pages*: **50-58**
Published: 28.10.2018 *Original Article*

Bδg-Homeomorphisms

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Abstaract — In this paper we introduce two new classes of mappings called Bδg-homeomorphism and Bδgc-homeomorphism which are defined using Bδg-closed sets and study their basic properties. We also investigate its group structure of their subgroups. We also investigate its relationship with other types of mappings.

Keywords — *Semi-generalized set, semi-homeomorphism mapping, Bδg-homeomorphism mapping, Bδg-closed set.*

1 Introduction

Maki et al. [13] introduced the notions of generalized homeomorphism (briefly g-homeomorphism). Devi et al. [2] introduced two classes of mappings called generalized semi-homeomorphism (briefly gs-homeomorphism) and semi- generalized homeomorphism (briefly sg-homeomorphisms). In this present paper we introduce new class of generalization of homeomorphisms called Bδg-homeomorphisms using Bδg-closed sets. We further introduce generalization of homeomorphisms called Bδgc-homeomorphism. Basic properties of these two mappings are studied and the relation between these types and other existing ones are established.

2 Preliminary

Throughout this paper, a space (X,τ) (or simply X) represents a topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and

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the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. (i) semi-open set [11] if A $\subseteq$ cl(int(A)).
(ii) preopen set [15] if A $\subseteq$ int(cl(A)).
(iii) α-open set [19] if A $\subseteq$ int(cl(int(A))).
(iv) regular open set [23] if A $\doteq$ int(cl(A)).

The complement of a semi-open (resp. preopen, -open) set is called semi-closed (resp. preclosed, -closed).

The α-closure (resp. semi-closure, preclosure) of A $\subseteq$ X is the smallest -closed (resp. semi-closed, preclosed) set containing A. cl(A) (resp. scl(A), pcl(A)) is called the -closure (resp. semi-closure, preclosure) of A.

Definition 2.2. The -interior [27] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by int_g (A). The subset A is called δ-open [27] if A = int_g (A), i.e. a set is δ -open if it is the finite union of regular open sets. The complement of a δ-open set is called δ-closed. Alternatively, a set A $\subseteq$ (X, τ)is called δ-closed[27] if A = $cl_\delta(A)$, where

$$cl_\delta(A) = \{x \in X : int(cl(U) \bigcap A \neq \phi, U \in \tau \text{ and } x \in U\}$$

Definition 2.3. A subset A of a space (X, τ) is called a
(i) generalized closed (briefly g-closed) set [12] if cl(A)$\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
(ii) generalized semi-closed (briefly gs-closed) set [1] if scl(A)$\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).
(iii) α-generalized closed (briefly αg-closed) set [13] if αcl(A)$\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).
(iv) δ-generalized closed (briefly δg-closed) set [3] if $cl_\delta(A)$ $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).
(v) $\widehat{g}$-closed set [26] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).
(vi) $\widehat{g}$-closed (briefly δ $\widehat{g}$-closed) set [8] if $cl_\delta(A)$ $\subseteq$ U whenever A $\subseteq$ U and U is $\widehat{g}$-open in (X, τ).

The complement of a g-closed (resp. gs-closed, αg-closed, δg-closed, $\widehat{g}$-closed and δ $\widehat{g}$ -closed)set is called g-open (resp. gs-open, αg-open, δg-open,$\widehat{g}$-open and δ $\widehat{g}$-open)

Definition 2.4. A subset A of a space (X, τ) is called a
(i) t-set infinite(A) = int(cl(A))
(ii) B-set if A = G $\cap$ F where G is open and F is a t-set in X.

Definition 2.5. A space (X, τ) is called a
(i) $T_{\frac{1}{2}}$-space [12] if every g-closed set in it is closed.
(ii) $T_{\frac{3}{4}}$ [3] if every δg-closed set in it is δ-closed.
(iii) $T_{\frac{3}{4}}$-space [8] if every δ $\widehat{g}$-closed set in it is δ - closed.

Definition 2.6. A map f:(X,τ) → (Y,σ)is called

(i) g-continuous [2] if f^{-1}(V) is g-closed in (X,τ) for every closed set V of (Y,σ).

(ii) gs-continuous [2] if f^{-1}(V) is gs-closed in (X,τ) for every closed set V of (Y,σ).

(iii) δ $\widehat{g}$-continuous [2] if f^{-1}(V) is δ $\widehat{g}$-closed in (X,τ) for every closed set V of (Y,σ).

Definition 2.7. A map f:(X,τ) → (Y,σ) is called

(i) generalized closed (briefly g-closed) (resp. g-open) [15] if the image of every closed (resp. open) set in (X,τ) is g-closed (resp. g-open) in (Y,σ).

(ii) generalized semi-closed (briefly gs-closed) (resp. gs-open) [2] if the image of every closed (resp. open) set in (X,τ) is gs-closed (resp. gs-open) in (Y,σ).

(iii) δ-closed [19] if f(V) is δ-closed in (Y,σ)for every δ-closed set V of (X,τ)

(iv) δ $\widehat{g}$-closed [8] if the image of every closed set in (X,τ) is δ $\widehat{g}$-closed in (Y,σ).

Definition 2.8. A map f:(X,τ) → (Y,σ)is called

(i) g-homeomorphism [13] if f is bijection, g-open and g-continuous.

(ii) gs-homeomorphism [2] if f is bijection, gs-open and gs-continuous.

(iii) δ-closed [19] if f(V) is δ-closed in (Y,σ)for every δ-closed set V of (X,τ)

(iv) δ $\widehat{g}$-homeomorphism [8] if f is bijection, δ $\widehat{g}$ - open and δ $\widehat{g}$ - continuous.

Proposition 2.9 (8)**.** Every δ -closed set is δ $\widehat{g}$ -closed set.

3 Properties Of Bδg-homeomorphisms

Definition 3.1. A subset A of (X,τ) is called Bδg- closed if cl_δ(A) ⊆ U whenever A ⊆ U and U is B-set

The Complement of Bδg - closed set is Bδg -open.

Definition 3.2. A bijection map f:(X,τ) → (Y,σ) is called Bδg - continuous if f is both Bδg -continuous and Bδg -open.

Definition 3.3. A bijection map f:(X,τ) → (Y,σ) is called Bδg - continuous if f^{-1}(V) is Bδg -closed in (X,τ) for every closed set V of (Y,σ)

Definition 3.4. A bijection map f:(X,τ) → (Y,σ) is called Bδg - irresolute if f^{-1}(V) is Bδg -closed in (X,τ) for every closed set V of (Y,σ)

Definition 3.5. A bijection map f:(X,τ) → (Y,σ) is called Bδg - closed if the image of closed set in (X,τ) is Bδg - closed in (Y,σ)

Definition 3.6. A bijection map f:(X,τ) → (Y,σ) is called Bδg - homeomorphism if f is both Bδg - continuous and Bδg - open.

Definition 3.7. A space X is called a $_BT_{\delta g}$- space if every Bδg ? closed set in it is δ closed.

Theorem 3.8. Every Bg-homeomorphism is gs-homeomorphism..

Proof. Let f:(X,τ) $\rightarrow$ (Y,σ) be Bδg-homeomorphism. Then f is bijective, Bδg-continuous and Bδg-open map. Let V be an closed set in (Y,σ). Then $f^{-1}(V)$ is Bδg-closed in (X,τ). Every Bδg-closed set is gs-closed and hence $f^{-1}(V)$ is gs-closed in (X,τ). This implies that f is gs-continuous. Let U be an open set in (X,τ). Then f(U) is Bδg-open in (Y,σ). This implies f is gs-open map. Hence f is gs-homeomorphism. □

Remark 3.9. The following example shows that the converse of the above theorem need not be true.

Example 3.10. Let X = $\{a, b, c\}$ and Y = $\{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{p\}, \{q\}, \{p, q\}, Y\}$. Define f: (X,τ) $\rightarrow$ (Y,σ) by $f(a) = p$; $f(b) = q$ and $f(c) = r$. Clearly f is gs-homeomorphism but f is not Bδg-homeomorphism because f($\{b\}$) = f$\{q\}$ is not a Bδg-open in (Y,σ) where $\{b\}$ is open in (X,τ)

Theorem 3.11. Every Bδg-homeomorphism is g-homeomorphism.

Proof. Follows from the fact that every Bδg- continuous map is g- continuous map and every Bδg-open map is g-open map. □

Remark 3.12. The converse of the above theorem need not be true as it can be seen from the following example.

Example 3.13. Let X = $\{a, b, c\}$ and Y = $\{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{p\}, \{p, r\}, Y\}$. Define f:(X,τ) $\rightarrow$ (Y,σ) by $f(a) = p$; $f(b) = r$ and $f(c) = q$. Clearly f is g-homeomorphism but f is not Bδg-homeomorphism because f($\{a, b\}$) = f$\{p, r\}$ is not a Bδg-open in (Y,σ) where (a,b) is open in (X,τ)

Remark 3.14. Homeomorphisms and Bδg-homeomorphisms are independent of each other as shown in the following examples.

Example 3.15. Let X = $\{a, b, c\}$ and Y = $\{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{p\}, \{r\}, \{p, r\}, \{q, r\}, Y\}$. Define a map f:(X,τ) $\rightarrow$ (Y,σ) by $f(a) = p$; $f(b) = q$ and $f(c) = r$. Then f is Bδg - open and Bδg - continuous. Hence f is a Bδg homeomorphism. However $f^{-1}(\{p, q\}) = \{a, b\}$ is not closed in (X,τ) where $\{p, q\}$ is closed in (Y,σ) and hence f is not continuous. Therefore, f is not homeomorphism.

Example 3.16. Let X = $\{a, b, c\}$ and Y = $\{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{q\}, \{p, q\}, Y\}$. Define a function f:(X,τ) $\rightarrow$ (Y,σ) by $f(a) = q$; $f(b) = p$ and $f(c) = r$. Then f is a homeomorphism. The set $\{a, b\}$ is open in (X,τ) but $f(\{a, b\}) = \{p, q\}$ is not Bδg - open in (Y,σ). This implies that f is not Bδg - open map. Hence f is not a Bδg - homeomorphism.

Remark 3.17. The concepts of Bδg-homeomorphism and $\delta\ \widehat{g}$-homeomorphism are independent of each other as shown in the following examples.

Example 3.18. Let X = $\{a, b, c\}$ and Y = $\{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{q\}, Y\}$. Define a map f:(X,τ) $\rightarrow$ (Y,σ) by $f(a) = q$; $f(b) = p$ and $f(c) = r$. Then f is a $\delta\ \widehat{g}$-homeomorphism. Here the set $\{a\}$ is open in (X,τ) but $f(\{a\}) = \{q\}$ is not Bδg - open in (Y,σ). This implies that f is not Bδg - open map. Hence f is not a Bδg - homeomorphism.

Example 3.19. Let X = $\{a, b, c\}$ and Y = $\{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{r\}, \{p, q\}, Y\}$. Define a map f:(X,τ) $\rightarrow$ (Y,σ) by $f(a) = q$; $f(b) = p$ and $f(c) = r$. Then f is a Bδg-homeomorphism but f is not $\delta\ \widehat{g}$-homeomorphism because $f(\{b, c\}) = \{p, r\}$ is not $\delta\ \widehat{g}$- open in (Y,σ) where $\{b, c\}$ is open in (X,τ).

Proposition 3.20. For any bijective map f:(X,τ) $\rightarrow$ (Y,σ)the following statements are equivalent.
(i) f^{-1}:(Y,σ)$\rightarrow$(X,τ)is Bδg-continuous map.
(ii) f is a Bδg-open map.
(iii) f is a Bδg-closed map.

Proof. (i) $\Rightarrow$ (ii) Let U be an open set in (X,τ). Since, f^{-1} is Bδg continuous, $(f^{-1})^{-1}$(U) is Bδg- open in (Y,σ). Hence f is Bδg open map. (ii) $\Rightarrow$(iii). Let F be a closed set in (X,τ). Then F^c is open in (X,τ).Since f is Bδg open map, $f(F^c)$ is Bδg open set in (Y,σ). But $f(F^c) = f(F^c)$ is Bδg open set in (Y,σ). This implies that $f(F^c)$ is Bδg open set in (Y,σ). Hence f is Bδg closed map.(iii)?(i).Let V be a closed set of(X,τ).Since f is Bδg closed map, f(V) is Bδg closed in (Y,σ). That is $(f^{-1})^{-1}$(V) is Bδg closed set in (Y,σ). Hence f^{-1} is Bδg continuous. □

Theorem 3.21. Let f:(X,τ) $\rightarrow$ (Y,σ) be a bijective and Bδg - continuous map. Then the following statements are equivalent.
(i) f is a Bδg-open map.
(ii) f is a Bδg-homeomorphism.
(iii) f is an Bδg-closed map.

Proof. (i) $\Rightarrow$ (ii). Let f be Bδg-open map. By hypothesis, f is bijective and Bδg-continuous. Hence f is Bδg-homeomorphism. (ii) $\Rightarrow$ (iii). Let f be Bδg- homeomorphism. Then f is Bδg-open. By Proposition 3.19 f is Bδg-closed map.(iii) $\Rightarrow$ (i). It is obtained from Proposition 3.19. □

Remark 3.22. The composition of two Bδg-homeomorphisms need not be Bδg-homeomorphism as the following example shows.

Example 3.23. Let X = $\{a, b, c\}$ = Y = Z with the topologies $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{c\}, \{a, b\}, Y\}$ and $\eta = \{\phi, \{a\}, \{b\}, \{a, b\}, Z\}$. Let f:(X,τ) $\rightarrow$ (Y,σ) and g:(Y,σ) $\rightarrow$ (Z,η) be two identity maps. Then both f and g are Bδg-homeomorphism. The set $\{b, c\}$ is open in (X,τ) but $g \circ f$ ($\{b, c\}$) = $\{b, c\}$ is not Bδg - open in (Z,η). This implies that $g \circ f$ is not Bδg - open and hence $g \circ f$ is not Bδg - homeomorphism.

We introduce the following definition.

Definition 3.24. A bijection map f:(X,τ) $\rightarrow$ (Y,σ) is said to be Bδgc - homeomorphism if f is Bδg - irresolute and its inverse f^{-1} is Bδg - irresolute.

Remark 3.25. Bδgc - homeomorphisms and Bδg- homeomorphisms are independent notions as shown in the following examples.

Example 3.26. Let X $= \{a, b, c\}$ and Y $= \{p, q, r\}$ with the topologies $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{q\}, Y\}$. Define a map f:(X,τ) $\rightarrow$ (Y,σ) by $f(a) = p$; $f(b) = q$ and $f(c) = r$. Then f is a Bδg - homeomorphism. The set $\{q, r\}$ is Bδg - closed in (Y,σ) but $f^{-1}(\{q, r\}) = \{b, c\}$ is not Bδg - closed in (X,τ). Therfore f is not Bδg - irresolute and hence f is not a Bδgc - homeomorphism.

Example 3.27. Let X $= \{a, b, c\}$ and Y $= \{p, q, r\}$ with the topologies $\tau = \{\phi, \{b\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{r\}, \{p, r\}, Y\}$. Define a map f:(X,τ) $\rightarrow$ (Y,σ) by $f(a) = r$; $f(b) = q$ and $f(c) = p$. Then f is Bδgc - homeomorphism. But f is not Bδg - homeomorphism because $f(\{b, c\})$ is not Bδg - open in (Y,σ) where $\{b, c\}$ is open in (X,τ).

Remark 3.28. From the above discussion we get the following diagram. A $\rightarrow$ B represents A implies B. A $\nrightarrow$ B represents A does not implies B.

2

6 3

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4

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Figure 1: 1. Bδg-Homeomorphism 2. gs-Homeomorphism 3. g-Homeomorphism 4. $\delta\widehat{g}$-Homeomorphism 5. Bδgc-Homeomorphism 6.Homeomorphism

Theorem 3.29. The composition of two Bδgc-homeomorphisms is Bδgc-homeomorphism.

Proof. Let f:(X,τ) $\rightarrow$ (Y,σ) and g:(Y,σ) $\rightarrow$ (Z,η) be two Bδgc-homeomorphisms. Let F be a Bδg-closed set in (Z,η). Since g is Bδg-irresolute map, $g^{-1}(F)$ is Bδg-closed in (Y,σ). Since f is Bδg-irresolute, $f^{-1}(g^{-1}(F))$ is Bδg-closed in (X,τ). That is $(g \bigcirc f)^{-1}(F)$ is Bδg-closed in (X,τ). This implies $g \bigcirc f$ is Bδg-irresolute. Let G be Bδg-closed in (X,τ). Since f^{-1} is Bδg-irresolute, $(f^{-1})^{-1}(G) = f(G)$ is Bδg-closed in (Y,σ). Since g^{-1} is Bδg-irresolute, $(g^{-1})^{-1}(f(G))$ is Bδg-closed in (Z,η). That is $g(f(G))$ is Bδg-closed in (Z,η). Therefore $(g \bigcirc f)(G)$ is Bδg-closed in (Z;?)(Z,η). This implies that$((g \bigcirc f)^{-1})^{-1}(G)$ is Bδg-closed in (Z,η). This shows that $(g \bigcirc f)^{-1}$ is Bδg-irresolute. Hence $g \bigcirc f$ is Bδgc-homeomorphism. $\square$

4 Application

Theorem 4.1. Every Bδg-homeomorphism from a $_BT_{\delta g}$-space into another $_BT_{\delta g}$-space is a homeomorphism.

Proof. Let f:(X,τ) $\to$ (Y,σ) be Bδg-homeomorphism. Then f is bijective, Bδg-open and Bδg-continuous maps. Let U be an open in (X,τ). Since f is Bδg-open and since (Y,σ) is $_BT_{\delta g}$ - space, f(U) is open set in (Y,σ). This implies f is open map. Let V be a closed set in (Y,σ). Since f is Bδg-continuous and since (X,τ) is $_BT_{\delta g}$-space, f^{-1}(V) is closed in (X,τ). Therefore f is continuous. Hence f is homeomorphism. □

Theorem 4.2. Let (Y,σ) be $_BT_{\delta g}$-space. If f:(X,τ) $\to$ (Y,σ) and g:(Y,σ) $\to$ (Z,η) are Bδg-homeomorphism then $g \bigcirc f$ is Bδg- homeomorphism.

Proof. Let f:(X,τ) $\to$ (Y,σ) and g:(Y,σ) $\to$ (Z,η) be two Bδg - homeomorphism. Let U be an open set in (X,τ). Since f is Bδg-open map, f(U) is Bδg-open in (Y,σ). Since (Y,σ) is $_BT_{\delta g}$-space, f(U) is open in (Y,σ). Also since g is Bδg-open map, $g(f$(U)) is Bδg-open in (Z,η). Hence $g \bigcirc f$ is Bδg-open map. Let V be a closed set in (Z,η). Since g is Bδg-continuous and since (Y,σ) is $_BT_{\delta g}$-space, g^{-1}(V) is closed in (Y,σ). Since f is Bδg-continuous, $f^{-1}(g^{-1}(\text{V})) = (g \bigcirc f)^{-1}(\text{V})$ is Bδg-closed set in (X,τ). That is $g \bigcirc f$ is Bδg-continuous. Hence $g \bigcirc f$ is Bδg-homeomorphism. □

Theorem 4.3. Every Bδg-homeomorphism from a $_BT_{\delta g}$-space into another $_BT_{\delta g}$-space is $\delta\ \widehat{g}$-homeomorphism.

Proof. Let f:(X,τ) $\to$ (Y,σ) be Bδg-homeomorphism. Then f is bijective, Bδg-open and Bδg-continuous maps. Let U be an open set (X,τ). Since f is Bδg-open, and since (Y,σ) is $_BT_{\delta g}$-space, f(U) is δ-closed. By Proposition 2.8 every δ-closed set is $\delta\ \widehat{g}$-closed. Hence f(U) is $\delta\ \widehat{g}$-closed in (Y,σ). This implies f is $\delta\ \widehat{g}$-open. Let V be a closed set in (Y,σ). Since f is Bδg-continuous and since (X,τ) $_BT_{\delta g}$-space, f^{-1}(V) is $\delta\ \widehat{g}$-closed in (X,τ). Therefore f is $\delta\ \widehat{g}$-continuous. Thus f is $\delta\ \widehat{g}$-homeomorphism. □

Theorem 4.4. Every Bδg-homeomorphism from a $_BT_{\delta g}$-space into another $_BT_{\delta g}$-space is Bδgc-homeomorphism.

Proof. Let f :(X,τ) $\to$ (Y,σ) be Bδg-homeomorphism. Let U be Bδg-closed in (Y,σ). Since (Y,σ) is $_BT_{\delta g}$-space, U is closed in (Y,σ). Also Since f is Bδg-continuous, f^{-1}(U) is Bδg-closed in (X,τ). Hence f is Bδg-irresolute map. Let V be Bδg-open in (X,τ). Since (X,τ) is $_BT_{\delta g}$-space, V is open in (X,τ). Also since f is Bδg-open, f(V) is Bδg-open set in (Y,σ). That is $(f^{-1})^{-1}$(V) is Bδg-open in (Y,σ) and hence f^{-1} is Bδg-irresolute. Thus f is Bδgc- homeomorphism. □

We shall introduce the group structure of the set of all Bδgc- homeomorphisms from a topological space (X,τ) onto itself by Bδgc-h(X,τ).

Theorem 4.5. The set Bδgc-h(X,τ) is a group under composition of mappings.

Proof. By Theorem 3.28 $g \bigcirc f \in$ Bδgc-h(X,τ) for all f, $g \in$ Bδgc-h(X,τ). We know that the composition of mappings is associative. The identity map belonging to Bδgc-h(X,τ) acts as the identity element. If $f \in$ Bδgc-h(X,τ) then $f^{-1} \in$ Bδgc-h(X,τ). such that $f^{-1} \bigcirc f = f \bigcirc f^{-1} = I$ and so inverse exists for each element of Bδgc-h(X,τ). Hence Bδgc-h(X,τ) is a group under the composition of mappings. □

Theorem 4.6. Let f : Bδgc-h(X,τ)$\rightarrow$ Bδgc-h(Y,σ) be Bδgc-homeomorphism. Then f induces an isomorphism from the group Bδgc-h(X,τ) onto the group Bδgc-h(Y,σ).

Proof. We define a map f : Bδgc-h(X,τ)$\rightarrow$ Bδgc-h(Y,σ) by $f*(k) = f \bigcirc k \bigcirc f^{-1}$ for every $k \in$ Bδgc-h(X,τ).Then f is a bijection and also for all $k_1, k_2 \in$ Bδgc-h(X,τ), $f*(k_1 \bigcirc k_2) = f \bigcirc (k_1 \bigcirc k_2) \bigcirc f^{-1} = (f \bigcirc k_1 \bigcirc f^{-1}) \bigcirc (f \bigcirc k_2 \bigcirc f^{-1}) = f*(k_1) \bigcirc f*(k_2)$. Hence $f*$ is homeomorphism and so it is an isomorphism induced by f. □

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