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On Weighted Weak Statistical Convergence

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Abstract

The purpose of the present work is to introduce extended notion of weak statistical convergence on normed spaces. Furthermore, some certain properties of this mode of convergence are given.

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1. Introduction

Zygmund introduced the idea of statistical convergence in [1]. Fast and Steinhaus introduced statistical convergence to assign limit to sequences which are not convergent in the usual sense independently in the same year (see [2],[3]).

We begin by recalling the notion of asymptotic (or natural) density of a set $A \subset \mathbb{N}$ such that

$$\delta(A) = \lim_{n\to\infty} \frac{1}{n} \left|\{k \le n : k \in A\}\right|,$$

whenever the limit exists. $|\{.\}|$ indicates the cardinality of the enclosed set. A sequence (x_k) of numbers is called statistically convergent to a number x provided that for $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n} \left|\{k \le n : |x_k - x| \ge \varepsilon\}\right| = 0.$$

In this case, $S - \lim_{k\to\infty} x_k = x$. This notion is used an effective tool to resolve many problems in ergodic theory, fuzzy set theory, trigonometric series and Banach spaces. Also many researchers studied related topics with summability theory. (see [4]-[6]).

A sequence (x_k) in a normed space X is said to be weakly convergent to $x \in X$ provided that $\lim_{k\to\infty} \varphi(x_k - x) = 0$ for each $\varphi \in X^*$, the continuous dual of X. In this case, we write $W - \lim_{k\to\infty} x_k = x$.

Connor *et al.* introduced weak statistical convergence and used it to give description of Banach spaces with seperable duals in [7].

A sequence (x_k) in a normed space X is said to be weakly statistically convergent to $x \in X$ provided that, for each $\varepsilon > 0$, $\delta(\{k \le n : |\varphi(x_k - x)| \ge \varepsilon\}) = 0$ for each $\varphi \in X^*$. In this case, we write $WS - \lim_{k\to\infty} x_k = x$. The set of all weakly statistically convergent sequences is denoted by WS.

Bhardwaj *et al.* defined weak statistical Cauchy sequences in a normed space X and studied weak statistical convergence in l_p spaces in [8]. Meenakshi *et al.* studied weak λ-statistical convergence, weak λ-statistically Cauchy and weak (V,λ)-summability in a normed space X in [9].

Weighted statistical convergence introduced by Karakaya and Chisti in [10]. Also Küçükaslan studied this concept in [11]. Then the modified definition is given by Mursaleen *et al.* in [12] as follow:

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Let (p_k) be a positive sequence of nonnegative numbers such that $p_0 > 0$ and $P_n = \sum_{k=0}^{n} p_k \to \infty$ as $n \to \infty$. A sequence (x_k) is weighted statistically convergent (or $S_{\bar{N}}$-convergent) to x if for $\varepsilon > 0$, the set $\{k \in \mathbb{N} : p_k |x_k - x| \geq \varepsilon\}$ has weighted density zero, i.e.

$$\lim_{n\to\infty} \frac{1}{P_n} |\{k \leq P_n : p_k |x_k - x| \geq \varepsilon\}| = 0.$$

It is denoted by $S_{\bar{N}} - \lim_{k\to\infty} x_k = x$. $S_{\bar{N}}$ denote the set of these sequences. Ghosal studied the concept of weighted statistical convergence of order α in [13].

2. Main Results

In this section, we give the notion of weighted norm statistical convergence, weighted weak statistical convergence, weighted weak statistical Cauchy sequence and weighted weak $(\bar{N}, t_n)$-summability. Then we establish the relationship between these notions and give some important properties related to the modes of these covergences. Let $K \subseteq \mathbb{N}$, the set of positive integers, the weighted density of K is defined by

$$\delta_{\bar{N}}(K) = \lim_{n\to\infty} \frac{1}{P_n} |\{k \leq P_n : k \in K\}|.$$

In particular, if we choose $p_k = 1$, then it reduces to natural density. After that we use (t_k) instead of (p_k) in our results to avoid confusing and $T_n = \sum_{k=0}^{n} t_k \to \infty$ as $n \to \infty$. Throughout the paper, X denotes a normed linear space, X^* is its continuous dual.

Now, we begin with the following definitions.

Definition 2.1. A sequence (x_k) in X is called weighted norm statistically convergent (or $S(\bar{N})$-convergent) to $x \in X$ if for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{T_n} |\{k \leq T_n : t_k \|x_k - x\| \geq \varepsilon\}| = 0$$

In this case we write $S(\bar{N}) - \lim_{k\to\infty} x_k = x$. $S(\bar{N})$ denotes the set of these sequences in X.

Definition 2.2 A sequence (x_k) in X is called weighted weakly statistically convergent (or $WS_{\bar{N}}$-convegent) to $x \in X$ if for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{T_n} |\{k \leq T_n : t_k |\varphi(x_k - x)| \geq \varepsilon\}| = 0$$

for every $\varphi \in X^*$. In this case we write $WS_{\bar{N}} - \lim_{k\to\infty} x_k = x$. $WS_{\bar{N}}$ denotes the set of these sequences in X.

Definition 2.3. A sequence (x_k) in X is called weighted weakly statistically Cauchy sequence (or $WS_{\bar{N}}$-Cauchy) if for every $\varepsilon > 0$, there exists a number $n = n_0(\varepsilon)$ such that

$$\lim_{n\to\infty} \frac{1}{T_n} |\{k \leq T_n : t_k |\varphi(x_k - x_{n_0})| \geq \varepsilon\}| = 0$$

for every $\varphi \in X^*$.

Definition 2.4. A sequence (x_k) in X is called weighted weakly $(\bar{N}, t_n)$-summable (or $W(\bar{N}, t_n)$-summable) to $x \in X$ provided that

$$\lim_{n\to\infty} \frac{1}{T_n} \sum_{k=0}^{n} t_k |\varphi(x_k - x)| = 0$$

for each $\varphi \in X^*$. In this case we write $W(\bar{N}, t_n) - \lim_{k\to\infty} x_k = x$. $W(\bar{N}, t_n)$ denotes the set of these sequences in X.

For particular case $t_k = 1$, definition 2.1, definition 2.2 coincide with norm statistical convergence, weak statistical convergence which are defined in [7], definition 2.4 coincide with weak C_1-summability is given in [15] and definition 2.3 coincide with weak statistically Cauchy is given in [8].

Theorem 2.5 For any sequence (x_k) in X, if $(x_k) \in WS_{\bar{N}}$, then it has a unique limit value.

Proof. Suppose that there exists two limit value such as $x_1 \neq x_2$. Choose $\varepsilon = \frac{1}{2} |\varphi(x_1 - x_2)| > 0$ and $t_k > c > 0$. Then we have

$$\begin{aligned} 1 &\leq \frac{1}{T_n} |\{k \leq T_n : t_k |\varphi(x_1 - x_2)| \geq \varepsilon c\}| \\ &\leq \frac{1}{T_n} \left|\left\{k \leq T_n : t_k |\varphi(x - x_1)| \geq \frac{\varepsilon c}{2}\right\}\right| + \frac{1}{T_n} \left|\left\{k \leq T_n : t_k |\varphi(x - x_2)| \geq \frac{\varepsilon c}{2}\right\}\right|. \end{aligned}$$

The right hand side limit is equal to zero. Hence $x_1 \neq x_2$ is impossible.

Theorem 2.6 Let (x_k) and (y_k) be the sequences in X and c is a scalar. Then the following statements hold.
(i) If $WS_{\bar{N}}-\lim_{k\to\infty} x_k = x$, then $WS_{\bar{N}}-\lim_{k\to\infty} cx_k = cx$.
(ii) If $WS_{\bar{N}}-\lim_{k\to\infty} x_k = x$ and $WS_{\bar{N}}-\lim_{k\to\infty} y_k = y$, then $WS_{\bar{N}}-\lim_{k\to\infty}(x_k+y_k) = x+y$.

Theorem 2.7 A sequence (x_k) in X is $WS_{\bar{N}}$-convergent to x if and only if there exists a set $K=\{k_1<k_2<k_3<...\}\subset\mathbb{N}$ such that $\delta_{\bar{N}}(K)=1$ and $\lim_{k\in K} t_k|\varphi(x_k-x)|=0$ for each $\varphi\in X^*$.

Proof. Assume that the sequence (x_k) is weighted weakly statistically convergent to x. Let $\varepsilon>0$ and $\varphi\in X^*$ be arbitrary. Consider $M_r=\left\{k: t_k|\varphi(x_k-x)|\geq\frac{1}{r}\right\}$ and $K_r=\left\{k: t_k|\varphi(x_k-x)|<\frac{1}{r}\right\}$ for $r=1,2,...$. From the assumption $\delta_{\bar{N}}(M_r)=0$. Also

$$K_1\supset K_2\supset...\supset K_i\supset K_{i+1}\supset... \tag{2.1}$$

and

$$\delta_{\bar{N}}(K_r)=1,\ r=1,2,...\ . \tag{2.2}$$

Now to prove the desired result assume that $\lim_{k\in K_r} p_k|\varphi(x_k-x)|\neq 0$. Hence there exist $\varepsilon>0$ for which $t_k|\varphi(x_k-x)|\geq\varepsilon$ for infinitely terms. If we take $K_\varepsilon=\{k: t_k|\varphi(x_k-x)|<\varepsilon\}$ and $\varepsilon>\frac{1}{r}$ $(r=1,2,...)$, then $\delta_{\bar{N}}(K_\varepsilon)=0$ and by (2.1), $K_r\subset K_\varepsilon$. This means $\delta_{\bar{N}}(K_r)=0$ and it contradicts by (2.2). As a result we have $\lim_{k\in K_r} t_k|\varphi(x_k-x)|=0$ for each arbitrary $\varphi\in X^*$.

Conversely, suppose that there exists a set $K=\{k_1<k_2<k_3<...\}\subset\mathbb{N}$ with $\delta_{\bar{N}}(K)=1$ and $\lim_{k\in K} t_k|\varphi(x_k-x)|=0$ for every $\varphi\in X^*$. So we can find a positive integer n_0 such that $t_k|\varphi(x_k-x)|<\varepsilon$ for all $k\geq n_0$, $k\in K$ and $\varphi\in X^*$. $M_\varepsilon=\{k: t_k|\varphi(x_k-x)|\geq\varepsilon\}\subseteq\mathbb{N}-\left\{k_{n_0+1},k_{n_0+2},k_{n_0+3},...\right\}$ and therefore $\delta_{\bar{N}}(M_\varepsilon)=0$. This shows that (x_k) is $WS_{\bar{N}}$-convergent to x.

Theorem 2.8 For any sequence (x_k) in X, if $t_k<1$ and $W-\lim_{k\to\infty} x_k=x$, then $WS_{\bar{N}}-\lim_{k\to\infty} x_k=x$.

Proof. Assume that a sequence (x_k) in X is weakly convergent to x and $t_k<1$. Then for every $\varepsilon>0$ and each $\varphi\in X^*$, there exists a positive integer n_0 such that $|\varphi(x_k-x)|<\varepsilon$ for all $k\geq n_0$. Thus the set $M_\varepsilon=\{k\in\mathbb{N}:|\varphi(x_k-x)|\geq\varepsilon\}$ is finite and we have

$$\frac{1}{T_n}|\{k\leq T_n:|\varphi(x_k-x)|\geq\varepsilon\}|\geq\frac{1}{T_n}|\{k\leq T_n: t_k|\varphi(x_k-x)|\geq\varepsilon\}|.$$

The left hand side tends to zero. This means (x_k) is also weighted weakly statistically convergent to x.

The converse of this result is not true. It will be seen from following example.

Example 2.9 Let $t_k<1$ and $(x_k)\in\ell_p$ $(1<p<\infty)$ defined by

$$x_j^{(k)}=\begin{cases} m, & if\ j\leq k,\ k=m^2,\\ \frac{1}{k}, & if\ j\leq k,\ k\neq m^2,\\ 0, & otherwise.\end{cases}$$

For $k\neq m^2$ and arbitrary $\varphi\in\ell_p^*$, there is unique $y\in\ell_q$ such that

$$\begin{aligned}|\varphi(x_k)| &= \left|\sum_{j=1}^{\infty} x_j^{(k)} y_j\right|\\ &\leq \|x\|_p\|y\|_q\\ &\leq \left(\sum_{j=1}^{\infty}\left|x_j^{(k)}\right|^p\right)^{1/p}\left(\sum_{j=1}^{\infty}|y_j|^q\right)^{1/q}\\ &\leq \left(\sum_{j=1}^{k}\frac{1}{k^p}\right)^{1/p} H^{1/q}\\ &= \left(\frac{H}{k}\right)^{1/q}\to 0\end{aligned}$$

for some positive constant H as $k\to\infty$. Hence we have $WS_{\bar{N}}-\lim_{k\to\infty} x_k=0$ by Theorem 2.7. For $k=m^2$, consider the functional defined on ℓ_p by $\varphi_j(x)=x_j$, where $(x_k)\in\ell_p$. Clearly, $\varphi_j(x_k)=x_j^{(k)}=\sqrt{k}\to\infty$, as $k\to\infty$. Hence, (x_k) is not weakly convergent.

Theorem 2.10 $S(\bar{N})$-convergence implies $WS_{\bar{N}}$-convergence with same limit in X but the converse is not true.

Proof. Let (x_k) in X, be a sequence which is $S(\bar{N})$-convergent to x. Then for every $\varepsilon>0$,

$$\lim_{n\to\infty}\frac{1}{T_n}|\{k\leq T_n: t_k\|x_k-x\|\geq\varepsilon\}|=0.$$

Now for every $\varepsilon > 0$ and each $\varphi \in X^*$,

$$\begin{aligned}\frac{1}{T_n}\left|\left\{k \le T_n : t_k \left|\varphi\left(x_k - x\right)\right| \ge \varepsilon\right\}\right| &\le \frac{1}{T_n}\left|\left\{k \le T_n : t_k \left\|\varphi\right\| \left\|x_k - x\right\| \ge \varepsilon\right\}\right| \\ &= \frac{1}{T_n}\left|\left\{k \le T_n : t_k \left\|x_k - x\right\| \ge \frac{\varepsilon}{\left\|\varphi\right\|}\right\}\right|.\end{aligned}$$

This means x is also $WS_{\bar{N}}$-convergent to x.

We give an example to show that the converse of this result is not true.

Example 2.11 Let consider the space $L_p(-1,1)$ for $p > 1$. Then define $x_k : (-1,1) \to \mathbb{R}$ by

$$x_k(a) = \begin{cases} k^{1/p}, & if \;\; a \in [0, \frac{1}{n}] \\ 0, & otherwise. \end{cases}$$

and let choose $t_k = 1/2$. Then we have $W - \lim_{k\to\infty} x_k = 0$ in $L_p(-1,1)$ [14]. By Theorem 2.8 we have $WS_{\bar{N}} - \lim_{k\to\infty} x_k = 0$. Next we show that it is not $S(\bar{N})$-convergent to 0. Since $\|x_k\|_{L_p(-1,1)} = 1$ we have

$$\lim_{n\to\infty} \frac{2}{n}\left|\left\{k \le \frac{n}{2} : \frac{1}{2}\left\|x_k - 0\right\| \ge \varepsilon\right\}\right| \ne 0$$

for $0 < \varepsilon < \frac{1}{2}$. Hence (x_k) is not a $S(\bar{N})$-convergent sequence.

Theorem 2.12 If a sequence (x_k) in X is weighted weakly $(\bar{N}, t_n)$-summable to x, then it is weighted weakly statistically convergent to x.

Proof. Let (x_k) is weighted weakly $(\bar{N}, t_n)$-summable to x. Then for each $\varphi \in X^*$ and $\varepsilon > 0$, we have

$$\frac{1}{T_n}\sum_{k=0}^{\infty} t_k \left|\varphi\left(x_k - x\right)\right| \ge \frac{1}{T_n}\sum_{\substack{k=0 \\ k \in K_{t_n}(\varepsilon)}}^{n} t_k \left|\varphi\left(x_k - x\right)\right| \ge \frac{\varepsilon}{T_n}\left|\left\{k \le T_n : t_k \left|\varphi\left(x_k - x\right)\right| \ge \varepsilon\right\}\right|$$

where $K_{t_n}(\varepsilon) = \left|\left\{k \le T_n : t_k \left|\varphi\left(x_k - x\right)\right| \ge \varepsilon\right\}\right|$. Hence (x_k) is also weighted weakly statistically convergent to x.

Theorem 2.13 If $(t_k) \in \ell_\infty$ and $WS_{\bar{N}} - \lim_{k\to\infty} x_k = x$, then $W(\bar{N}, t_n) - \lim_{k\to\infty} x_k = x$ for a sequence (x_k) in X.

Proof. Suppose that $(t_k) \in \ell_\infty$ and (x_k) is weighted weakly statistically convergent to x. Since $\varphi \in X^*$, φ is bounded and we have $t_k \left|\varphi\left(x_k - x\right)\right| \le H$ for all k.

$$\begin{aligned}\frac{1}{T_n}\sum_{k=1}^{n} t_k \left|\varphi\left(x_k - x\right)\right| &= \frac{1}{T_n}\sum_{\substack{k=1 \\ k \in K_{t_n}(\varepsilon)}}^{n} t_k \left|\varphi\left(x_k - x\right)\right| + \frac{1}{T_n}\sum_{\substack{k=1 \\ k \in K_{t_n}^c(\varepsilon)}}^{n} t_k \left|\varphi\left(x_k - x\right)\right| \\ &\le \frac{H}{T_n}\left|\left\{k \le T_n : t_k \left|\varphi\left(x_k - x\right)\right| \ge \varepsilon\right\}\right| + \varepsilon.\end{aligned}$$

This means that (x_k) is weighted weakly $(\bar{N}, t_n)$-summable to x.

Theorem 2.14 Let

$$\liminf_{n\to\infty} \frac{T_n}{n} \ge 1 \text{ and } \limsup_{n\to\infty} \frac{T_n}{n} < \infty. \tag{2.3}$$

(i) If (2.3) and $t_k < 1$ hold, then $WS(X) \subset WS_{\bar{N}}(X)$.
(ii) If (2.3) and $t_k \ge 1$ hold, then $WS_{\bar{N}}(X) \subset WS(X)$.

Proof. (i) If we take a sequence (x_k) from the set $WS(X)$ and by (2.3), we have

$$\begin{aligned}\frac{1}{n}\left|\left\{k \le n : \left|\varphi\left(x_k - x\right)\right| \ge \varepsilon\right\}\right| &\ge \frac{1}{n}\left|\left\{k \le T_n : t_k \left|\varphi\left(x_k - x\right)\right| \ge \varepsilon\right\}\right| \\ &= \left(\frac{T_n}{n}\right)\frac{1}{T_n}\left|\left\{k \le T_n : t_k \left|\varphi\left(x_k - x\right)\right| \ge \varepsilon\right\}\right|\end{aligned}$$

for arbitrary $\varepsilon > 0$. Also taking the limit as $n \to \infty$ we have (x_k) belongs to $WS_{\bar{N}}(X)$.

(ii) If we take a sequence (x_k) from the set $WS_{\bar{N}}(X)$ and by (2.3), we have

$$\begin{aligned}\frac{1}{n}\left|\left\{k \le n : \left|\varphi\left(x_k - x\right)\right| \ge \varepsilon\right\}\right| &\le \frac{1}{n}\left|\left\{k \le T_n : t_k \left|\varphi\left(x_k - x\right)\right| \ge \varepsilon\right\}\right| \\ &= \left(\frac{T_n}{n}\right)\frac{1}{T_n}\left|\left\{k \le T_n : t_k \left|\varphi\left(x_k - x\right)\right| \ge \varepsilon\right\}\right|\end{aligned}$$

for arbitrary $\varepsilon > 0$.

Corollary 2.15 Let (2.3) holds and $t_k \geq 1$. Then the following statements hold.
(i) Weighted norm statistical convergence implies norm statistical convergence.
(ii) Weighted weak $(\bar{N}, t_n)$-summability implies weak C_1-summability.
(iii) Weighted weak statistical convergence implies weak statistical convergence.
(iv) Every weighted weak statistical Cauchy sequence is weak statistical Cauchy sequence, in a normed space X.

Theorem 2.16 For a complete normed space X, if a sequence (x_k) is $WS_{\bar{N}}$-Cauchy, then it is $WS_{\bar{N}}$-convergent.

Proof. Suppose that (x_k) is $WS_{\bar{N}}$-Cauchy sequence. For given $\varepsilon > 0$,

$$\begin{aligned}\frac{1}{T_n}\left|\{k \leq T_n : t_k\left|\varphi\left(x_k - x\right)\right| \geq \varepsilon\}\right| &\leq \frac{1}{T_n}\left|\left\{k \leq T_n : t_k\left|\varphi\left(x_k - x_{K(n)}\right)\right| \geq \frac{\varepsilon}{2}\right\}\right| \\ &+\frac{1}{T_n}\left|\left\{k \leq T_n : t_k\left|\varphi\left(x_{K(n)} - x_k\right)\right| \geq \frac{\varepsilon}{2}\right\}\right|,\end{aligned}$$

which gives that (x_k) is $WS_{\bar{N}}$-convergent. The converse implication is true while $t_k < 1$ such as if (x_k) is $WS_{\bar{N}}$-convergent then it is $WS_{\bar{N}}$-Cauchy sequence.

Theorem 2.17 If $\dim X < \infty$, then $S(\bar{N})$-convergence is equivalent to $WS_{\bar{N}}$-convergence.

Proof. We have from Theorem 2.10 that $S(\bar{N})$-convergence implies $WS_{\bar{N}}$-convergence. Hence we need to prove that $WS_{\bar{N}}$-convergence implies $S(\bar{N})$-convergence. Consider a basis $\{e_1, e_2, e_3, ..., e_n\}$ for X and the sequence (x_k) which is $WS_{\bar{N}}$-convergent to x, where

$$x_k = a_1^k e_1 + a_2^k e_2 + ... + a_n^k e_n \text{ for } k = 1, 2, \dots,$$

and

$$x = a_1 e_1 + a_2 e_2 + ... + a_n e_n.$$

Let define the linear functionals $\varphi_i \in X^*$ $(i = 1, 2, ..., n)$ as follows:

$$\varphi_i\left(e_j\right) = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j. \end{cases}$$

Since (x_k) is $WS_{\bar{N}}$-convergent to x, it follows that $\varphi_i(x_k)$ is $S(\bar{N})$-convergent to $\varphi_i(x)$. This implies $a_i^{(k)}$ is $S(\bar{N})$-convergent to a_i, as $\varphi_i(x_k) = a_i^{(k)}$ and $\varphi_i(x) = a_i$. For $\varepsilon > 0$,

$$\frac{1}{T_n}\left|\left\{k \leq T_n : t_k\left|a_i^{(k)} - a_i\right| \geq \varepsilon\right\}\right| = 0 \tag{2.4}$$

for $i = 1, 2, ..., n$. Now we have

$$t_k\left\|x_k - x\right\| = t_k\left\|\sum_{i=1}^n\left(a_i^{(k)} - a_i\right)e_i\right\| \leq t_k\sum_{i=1}^n\left|a_i^{(k)} - a_i\right|\left\|e_i\right\| \leq t_k H\sum_{i=1}^n\left|a_i^{(k)} - a_i\right|$$

where $H = \max\left\|e_i\right\|$. Hence we have for $\varepsilon > 0$,

$$\begin{aligned}\{k \leq T_n : t_k\left\|x_k - x\right\| \geq \varepsilon\} &\subseteq \left\{k \leq T_n : t_k\sum_{i=1}^n\left|a_i^{(k)} - a_i\right| \geq \frac{\varepsilon}{H}\right\} \\ &= \left\{k \leq T_n : t_k\left|a_1^{(k)} - a_1\right| \geq \frac{\varepsilon}{H}\right\} \cup ... \cup \left\{k \leq T_n : t_k\left|a_n^{(k)} - a_n\right| \geq \frac{\varepsilon}{H}\right\}.\end{aligned}$$

Consequently we have the desired result from (2.4).

3. Conclusion

The concept of statistical convergence has applications in different fields of mathematics. In this paper, the concepts of weighted weak statistical convergence, weighted norm statistically convergence, weighted weak statistical Cauchy sequence, weighted weak $(\bar{N}, t_n)$-summability are introduced. Some topological properties of these concepts are investigated. Introduced constructions and obtained results in this paper open new directions for further research.

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