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Some Curves on α-para Kenmotsu manifolds with semisymmetric metric connections

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Abstract

The manuscript should contain an abstract. The abstract should be self-contained and citation-free and should not exceed 200 words. The abstract should state the purpose, approach, results and conclusions of the work. The author should assume that the reader has some knowledge of the subject but has not read the paper. Thus, the abstract should be intelligible and complete in it-self (no numerical references); it should not cite figures, tables, or sections of the paper. The abstract should be written using third person instead of first person.

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1. Introduction

Now a days the studies of different curves on almost contact or contact manifolds as variational problems have become popular topic of research. Legendre curve is a curve on contact manifolds like a helix in the Euclidean space R^3. C. Baikoussis and D.E. Blair first studied Legendre curves on Sasakian manifolds in the paper [1]. Recently we have studied Legendre curves on three-dimensional quasi-Sasakian manifolds with semisymmetric metric connections[27]. In [29], we have studied curves on α-para Kenmotsu manifolds with Levi-Civita connections. We also have studied curves on some classes of Kenmotsu manifolds [28]. Certain curves on some classes of three-dimensional almost contact manifolds have been studied by the first author [26]. The almost contact analogue of Legendre curves are generally known as almost contact curves [33], [15].

Biharmonic curves are studied as problems in variational calculus [5]. S. Montaldo and C. Oniciuc studied biharmonic maps between Riemannian manifolds [20]. In the paper [12] D. Fetcu studied Biharmonic Legendre curves in Sasakian space forms. Several authors have studied Legendre curves or almost contact curves [21], [24], [33]. Slant curves in 3-dimensional normal almost paracontact metric manifolds have been studied by J. Welyczko in the paper [34]. For further details we refer [6], [9], [10], [16].

α-para Kenmotsu manifolds have been studied by several authors [19], [32]. The theory of connections is an important topic in differential geometry. Generally geometric properties are studied with the help of Levi-Civita connections. On the other hand semisymmetric metric connections are frequently used by differential geometers to analyse different geometric properties. The notion of semisymmetric metric connections was introduced by K. Yano [35]. Later several authors have studied it. The present paper is organized as follows:

After the introduction, we give some preliminaries in Section 2. In Section 3, we study biharmonic almost contact curves on α-para Kenmotsu manifolds with respect to semisymmetric metric connections. In Section 4, we study slant curves with respect to semisymmetric metric connections. Section 5 deals with locally ϕ-symmetric almost contact curves on α-para Kenmotsu manifolds with respect to semisymmetric metric connections. Section 6 contains example. The last section gives the summary of the obtained results.

2. Preliminaries

Let M be a $2n+1$ dimensional differentiable manifold. Then (ϕ,ξ,η) is called an almost para contact structure on M if

$$\phi^2X = X-\eta(X)\xi, \qquad \eta(\xi)=1, \qquad \phi\xi=0, \qquad \eta(\phi)=0, \tag{2.1}$$

where ϕ be an 1-1 tensor field, ξ a vector field and η a 1-form on M.

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The tensor field ϕ induces an almost paracomplex structure on the distribution $\mathscr{D}=\ker\eta$, that is, the eigen distributions $\mathscr{D}^{+}$, $\mathscr{D}^{-}$ corresponding to the eigen values 1, -1 of ϕ, respectively, have equal dimension n.
M is said to be almost paracontact manifold if it is endowed with an almost paracontact structure [7], [11], [18], [36].
Let M be an almost paracontact manifold. M will be called an almost paracontact metric manifold if it is additionally endowed with a pseudo-Riemannian metric g of signature $(n+1,n)$ and such that

$$g(\phi X,\phi Y)=-g(X,Y)+\eta(X)\eta(Y), \tag{2.2}$$

$$g(X,\xi)=\eta(X) \tag{2.3}$$

for all $X,Y\in\chi(M)$.
We can define the fundamental form corresponding to the structure as a skew-symmetric 2-form Φ by $\Phi(X,Y)=g(X,\phi Y)$. It is to be observed that $\eta\wedge\Phi^{n}$ is up to a constant factor the Riemannian volume element of M.
On an almost paracontact manifold, the (2,1)- tensor field $N^{(1)}$ is defined by

$$N^{(1)}(X,Y)=[\phi,\phi](X,Y)-2d\eta(X,Y)\xi,$$

where $[\phi,\phi]$ is the Nijenhuis torsion of ϕ given by

$$[\phi,\phi](X,Y)=\phi^{2}[X,Y]+[\phi X,\phi Y]-\phi[\phi X,Y]-\phi[X,\phi Y].$$

If $N^{(1)}$ vanishes identically, then the almost paracontact manifold(structure) is said to be normal [7], [18], [36]. The normality condition means that the almost paracomplex structure J defined on $M\times R$ by

$$J(X,\lambda\frac{d}{dt})=(\phi X+\lambda\xi,\eta(X)\frac{d}{dt})$$

is integrable (paracomplex).
In this paper, we are interested in dimension 3.
In a 3-dimensional α-para Kenmotsu manifold the following results hold [32].

$$\begin{aligned} R(X,Y)Z &= (\frac{r}{2}+2\alpha^{2})[g(Y,Z)X-g(X,Z)Y] \\ &- (\frac{r}{2}+3\alpha^{2})[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)]\xi \\ &+ (\frac{r}{2}+3\alpha^{2})[\eta(X)Y-\eta(Y)X]\eta(Z), \end{aligned} \tag{2.4}$$

where R is the curvature tensor of the manifold.

$$(\nabla_{X}\eta)Y=\alpha[g(X,Y)-\eta(X)\eta(Y)], \tag{2.5}$$

$$(\nabla_{X}\phi)Y=\alpha[g(\phi X,Y)\xi-\eta(Y)\phi X], \tag{2.6}$$

$$\nabla_{X}\xi=\alpha(X-\eta(X)\xi) \tag{2.7}$$

for all vector fields $X,Y,Z\in\chi(M)$.
A curve γ on M is called Frenet curve with respect to Levi-Civita connection on M if

$$\nabla_{T}T=kN, \tag{2.8}$$

$$\nabla_{T}N=-kT+\tau B, \tag{2.9}$$

$$\nabla_{T}B=-\tau N, \tag{2.10}$$

where k, τ are the curvature and torsion of the curve with respect to Levi-Civita connection and $\{$T,N,B$\}$ is an orthonormal Frenet frame and $T=\dot{\gamma}$.
A linear connection on a Riemannian manifold M is called symmetric [31], [35] if torsion tensor of the connection is zero on the manifold M. The connection is called semisymmetric if the torsion $\tilde{T}$ is given by

$$\tilde{T}(X,Y)=\eta(Y)X-\eta(X)Y.$$

The semi symmetric metric connections $\tilde{\nabla}$ and the Levi-Civita connections ∇ on an almost contact metric manifold are related by

$$\tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X,Y)\xi \tag{2.11}$$

for all vector fields X,Y on M.
A curve γ on M is called Frenet curve with respect to semi-symmetric metric connections if it satisfies

$$\tilde{\nabla}_T T = \tilde{k}N, \tag{2.12}$$

$$\tilde{\nabla}_T N = -\tilde{k}T + \tilde{\tau}B, \tag{2.13}$$

$$\tilde{\nabla}_T B = -\tilde{\tau}N, \tag{2.14}$$

where $\tilde{k}, \tilde{\tau}$ are the curvature and torsion of the curve with respect to semi symmetric metric connections, {T, N, B} is an orthonormal frame with $\dot{\gamma} = T$.
The relation between the curvature tensor R of the Levi-Civita connections and the curvature tensor $\tilde{R}$ of the semisymmetric metric connections is given by [2]

$$\begin{aligned} \tilde{R}(X,Y)Z &= R(X,Y)Z + \delta(X,Z)Y - \delta(Y,Z)X \\ &+ g(X,Z)QY - g(Y,Z)QX, \end{aligned} \tag{2.15}$$

where δ is a tensor field of type (0,2) and a tensor field Q of type (1,1) is given by

$$\delta(Y,Z) = g(QY,Z) = (\nabla_Y \eta)(Z) - \eta(Y)\eta(Z) + \frac{1}{2}\eta(\xi)g(Y,Z).$$

Now $\eta(\xi) = 1$, so from above we get,

$$\delta(Y,Z) = g(QY,Z) = (\nabla_Y \eta)(Z) - \eta(Y)\eta(Z) + \frac{1}{2}g(Y,Z). \tag{2.16}$$

A frenet curve γ in an almost contact metric manifold is said to be almost contact curve if it is an integral curve of the distribution $\mathscr{D} = \ker\eta$. Formally, it is also said that a Frenet curve γ in an almost contact metric manifold is an almost contact curve if and only if $\eta(\dot{\gamma}) = 0$ and $g(\dot{\gamma},\dot{\gamma}) = 1$. For details we refer [1], [15], [33]. It is to be noted that in the paper [15], curves satisfying the above properties on almost contact manifolds have been named as almost contact curve, while Welyczko [33] has named such curves on almost contact manifolds as Legendre curves. By Legendre curves on almost contact manifolds we shall mean almost contact curves.
A Frenet curve γ is called a slant curve if it makes a constant angle with the Reeb vector field ξ [10]. If a unit speed curve γ on an almost contact metric manifold is slant curve, then $\eta(\dot{\gamma}) = \cos\theta$, where θ is a constant and is called slant angle. In particular, if the angle is $\frac{\pi}{2}$, the curve becomes almost contact curve. A slant curve γ is called proper if it is neither parallel nor perpendicular to the Reeb vector field ξ.

3. Biharmonic almost contact curves on three-dimensional α-para Kenmotsu manifolds with respect to semisymmetric metric connections

Definition 3.1. An almost contact curve γ on a three-dimensional α-para Kenmotsu manifold is called biharmonic with respect to semi-symmetric metric connection $\tilde{\nabla}$ if it satisfies [25]

$$\tilde{\nabla}_T^3 T + \tilde{\nabla}_T \tilde{T}(\tilde{\nabla}_T T, T)T + \tilde{R}(\tilde{\nabla}_T T, T)T = 0, \tag{3.1}$$

where $\dot{\gamma}$=T, $\tilde{T}$ is the torsion of semi symmetric connection and $\tilde{R}$ is the curvature of the semisymmetric metric connection.
In this section we obtain the following:
Theorem 3.1. The necessary condition for a non-geodesic almost contact curve on a three-dimensional α-para Kenmotsu manifold to be biharmonic with respect to semisymmetric metric connection is $\tilde{k}$=a non-zero constant and $\tilde{k}^2 + \tilde{\tau}^2 = \frac{r}{2} + 2\alpha^2 - 2\alpha - 1$.
Proof. Let us consider a biharmonic almost contact curve.
For semisymmetric metric connection, we have
$\tilde{\nabla}_T \tilde{T}(\tilde{\nabla}_T T, T)T = 0$.
Using Serret-Frenet formula in (3.1) we get

$$\tilde{\nabla}_T^3 T + \tilde{k}\tilde{R}(N,T)T = 0. \tag{3.2}$$

Using (2.5) in (2.16) we get,

$$QY = (\alpha + \frac{1}{2})Y - (\alpha + 1)\eta(Y)\xi. \tag{3.3}$$

Again we have,

$$\delta(Y,Z) = (\alpha+\frac{1}{2})g(Y,Z) - (\alpha+1)\eta(Y)\eta(Z). \tag{3.4}$$

Now putting (3.3), (3.4) in (2.15) we get,

$$\begin{aligned}\tilde{R}(X,Y)Z &= R(X,Y)Z + 2(\alpha+\frac{1}{2})g(X,Z)Y - 2(\alpha+\frac{1}{2})g(Y,Z)X \\ &+ (\alpha+1)[\eta(Y)X - \eta(X)Y]\eta(Z) \\ &+ (\alpha+1)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\xi. \end{aligned} \tag{3.5}$$

Since we have considered Frenet frame as $\{T, \phi T, \xi\}$ where $\phi T = -N$ and so for an almost contact curve we get $\eta(T) = 0, \quad \eta(N) = 0$. Using these facts and putting $X = N$, $Y = T$, $Z = T$ in (3.5) and also using (2.4) we get,

$$\tilde{R}(N,T)T = (\frac{r}{2} + 2\alpha^2 - 2\alpha - 1)N. \tag{3.6}$$

Again by Serret-Frenet formula we get,

$$\tilde{\nabla}^3_T T = -3\tilde{k}\tilde{k'}T + (\tilde{k''} - \tilde{k^3} - \tilde{k}\tilde{\tau^2})N + (2\tilde{\tau}\tilde{k'} + \tilde{k}\tilde{\tau'})B. \tag{3.7}$$

From (3.2), (3.6) and (3.7) we get,
$-3\tilde{k}\tilde{k'}T + (\tilde{k''} - \tilde{k^3} - \tilde{k}\tilde{\tau^2} + \tilde{k}\frac{r}{2} + 2\tilde{k}\alpha^2 - 2\alpha\tilde{k} - \tilde{k})N + (2\tilde{\tau}\tilde{k'} + \tilde{k}\tilde{\tau'})B = 0.$
So we have,

$$-3\tilde{k}\tilde{k'} = 0. \tag{3.8}$$

From above, we get $\tilde{k}$=a non-zero constant, provided $\tilde{k} \neq 0$
Again,

$$\tilde{k''} - \tilde{k^3} - \tilde{k}\tilde{\tau^2} + \tilde{k}\frac{r}{2} + 2\tilde{k}\alpha^2 - 2\alpha\tilde{k} - \tilde{k} = 0. \tag{3.9}$$

$$2\tilde{\tau}\tilde{k'} + \tilde{k}\tilde{\tau'} = 0. \tag{3.10}$$

For $\tilde{k}$=constant, from (3.9) we have
$\tilde{k}(\tilde{k^2} + \tilde{\tau^2} - \frac{r}{2} - 2\alpha^2 + 2\alpha + 1) = 0.$
Since $\tilde{k} \neq 0$, we have

$$\tilde{k}^2 + \tilde{\tau}^2 = \frac{r}{2} + 2\alpha^2 - 2\alpha - 1.$$

This completes the proof.

4. Slant curves on three-dimensional α-para Kenmotsu manifolds with respect to semisymmetric metric connections

In this section, we study slant curves on α-para Kenmotsu manifolds with respect to semisymmetric metric connection and prove the following:
Theorem 4.1. A proper slant curve γ on α-para Kenmotsu manifolds with respect to semisymmetric metric connection is a geodesic if and only if $\alpha = -1$.
Proof. Let us consider a proper slant curve γ on a α-para Kenmotsu manifold with respect to semisymmetric metric connection.
Here $\dot{\gamma}(s) = T(s)$ is given by

$$\cos\theta(s) = g(T(s), \xi), \tag{4.1}$$

where θ is the constant slant angle.
By covariant differentiation with respect to $\tilde{\nabla}$ we get from (4.1)

$$-\sin\theta.\theta^{'} = -g(\tilde{\nabla}_T T, \xi) - g(T, \tilde{\nabla}_T \xi). \tag{4.2}$$

From (2.7) and (2.11) we get

$$\tilde{\nabla}_T \xi = T(\alpha+1) - \eta(T)(\alpha+1)\xi. \tag{4.3}$$

Now using (2.12), (4.2) and (4.3) we get,

$$\begin{aligned} -\sin\theta.\theta^{'} &= -g(\tilde{k}N,\xi)-g(T,T(\alpha+1)-\eta(T)(\alpha+1)\xi) \\ &= -\tilde{k}\eta(N)-(\alpha+1)+(\eta(T))^2(\alpha+1) \\ &= -\tilde{k}\eta(N)-(\alpha+1)+\cos^2\theta.(\alpha+1) \\ &= -\tilde{k}\eta(N)-\sin^2\theta.(\alpha+1). \end{aligned} \tag{4.4}$$

If θ= a non-zero constant, then from (4.4) we get, $\tilde{k}\eta(N)=-(\alpha+1)\sin^2\theta$.
Hence $\tilde{k}=0$ if and only if $\alpha=-1$.
This completes the proof.

5. Locally ϕ-symmetric almost contact curves on three-dimensional α-para Kenmotsu manifolds with respect to semisymmetric metric connections

Definition 5.1. With respect to semi-symmetric metric connection an almost contact curve γ on a three-dimensional α-para Kenmotsu manifold is called locally ϕ-symmetric if it satisfies [24]

$$\phi^2(\tilde{\nabla}_T\tilde{R})(\tilde{\nabla}_T T,T)T=0, \tag{5.1}$$

where $T=\dot{\gamma}$.
Here we obtain the following:
Theorem 5.1. A locally ϕ-symmetric curve defined on a three-dimensional α-para Kenmotsu manifold with non-zero constant structure function α and constant scalar curvature r is a geodesic with respect to semisymmetric metric connection.
Proof. Putting $X=\tilde{\nabla}_T T,\quad Y=Z=T$ in (3.5) and using (2.4) and then using Serret-Frenet formula, we get,

$$\tilde{R}(\tilde{\nabla}_T T,T)T=\tilde{k}(\frac{r}{2}+2\alpha^2-2\alpha-1)N. \tag{5.2}$$

Again putting $X=B, Y=Z=T$ in (3.5) and (2.4) we get,

$$\tilde{R}(B,T)T=(\frac{r}{2}+2\alpha^2-2\alpha-1)B. \tag{5.3}$$

By definition of covariant differentiation of $\tilde{R}$ with respect to semisymmetric metric connection and using Serret-Frenet formula, we get,

$$\begin{aligned} (\tilde{\nabla}_T\tilde{R})(\tilde{\nabla}_T T,T)T &= \tilde{\nabla}_T\tilde{R}(\tilde{\nabla}_T T,T)T-\tilde{k}\tilde{\tau}\tilde{R}(B,T)T \\ &- \tilde{k}'\tilde{R}(N,T)T-\tilde{k}^2\tilde{R}(N,T)N. \end{aligned} \tag{5.4}$$

Now from (5.4) and using Serret-Frenet formula we get,

$$\begin{aligned} \tilde{\nabla}_T\tilde{R}(\tilde{\nabla}_T T,T)T &= -[\frac{r}{2}\tilde{k^2}+2\alpha^2\tilde{k^2}-2\alpha\tilde{k^2}-\tilde{k^2}]T+[2\alpha^2\tilde{k}'+\frac{r}{2}\tilde{k}'+\frac{r'}{2}\tilde{k} \\ &+ 4\tilde{k}'\alpha\alpha'-2\tilde{k}\alpha-2\alpha\tilde{k}'-\tilde{k}']N \\ &+ [\frac{r}{2}\tilde{k}\tilde{\tau}+2\tilde{k}\tilde{\tau}\alpha^2-2\tilde{k}\tilde{\tau}\alpha-\tilde{k}\tilde{\tau}]B. \end{aligned} \tag{5.5}$$

Again putting $X=N, Y=T, Z=N$ in (3.5) and (2.4) we get,

$$\tilde{R}(N,T)N=(2\alpha+1-\frac{r}{2}-2\alpha^2)T. \tag{5.6}$$

Now from (3.6), (5.3), (5.4), (5.5) and (5.6) we get,

$$(\tilde{\nabla}_T\tilde{R})(\tilde{\nabla}_T T,T)T=(\frac{r'}{2}\tilde{k}+4\tilde{k}'\alpha\alpha'-2\tilde{k}\alpha)N. \tag{5.7}$$

Applying ϕ^2 on bothsides of (5.7) we get,

$$\phi^2(\tilde{\nabla}_T\tilde{R})(\tilde{\nabla}_T T,T)T=(\frac{r'}{2}\tilde{k}+4\tilde{k}'\alpha\alpha'-2\tilde{k}\alpha)N. \tag{5.8}$$

If the curves are locally ϕ-symmetric, then we have

$$\tilde{k}(\frac{r'}{2}+4\alpha\alpha'-2\alpha)=0.$$

If we consider the structure function α is a non-zero constant and scalar curvature r is constant, then we get the theorem.
This completes the proof.

6. Example

In this section we give example of an almost contact curve and slant curve on α-para Kenmotsu manifolds. To give the examples, we have followed the paper [34]
Let $M=\mathbb{R}^2\times\mathbb{R}$. Then a normal almost para contact structure (ϕ,ξ,η) can be defined as follows: $\phi(e_1)=e_2,\quad \phi(e_2)=e_1,\quad \phi(e_3)=0,\quad \xi=e_3,\quad \eta=dz.$
Let the vector fields be

$$e_1=\frac{\partial}{\partial x},\quad e_2=\frac{\partial}{\partial y},\quad e_3=\frac{\partial}{\partial z}$$

are linearly independent at each point of M. Let g be the pseudo-Riemannian metric defined by

$$g(e_1,e_3)=g(e_2,e_3)=g(e_1,e_2)=0,\qquad g(e_1,e_1)=-2z,\qquad g(e_2,e_2)=2z,\qquad g(e_3,e_3)=1.$$

The quadraruple (ϕ,ξ,η,g) becomes a normal almost para contact metric structure on M.
For the Levi-Civita connection, By Koszul formula, we have

$$\nabla_{e_1}e_3=\frac{1}{2z}e_1,\qquad \nabla_{e_1}e_2=0,\qquad \nabla_{e_1}e_1=e_3,$$

$$\nabla_{e_2}e_3=\frac{1}{2z}e_2,\qquad \nabla_{e_2}e_2=-e_3,\qquad \nabla_{e_2}e_1=0,$$

$$\nabla_{e_3}e_3=0,\qquad \nabla_{e_3}e_2=\frac{1}{2z}e_2,\qquad \nabla_{e_3}e_1=\frac{1}{2z}e_1.$$

Using the above and (2.7), we get $\alpha=\frac{1}{2z}$. Hence the manifold is a α-para Kenmotsu manifold.
Let us now calculate $\tilde{\nabla}_{e_i}e_j$ by

$$\tilde{\nabla}_{e_1}e_3=\frac{1}{2z}e_1+e_1,\qquad \tilde{\nabla}_{e_1}e_2=0,\qquad \tilde{\nabla}_{e_1}e_1=0,$$

$$\tilde{\nabla}_{e_2}e_3=\frac{1}{2z}e_2+e_2,\qquad \tilde{\nabla}_{e_2}e_2=-2e_3,\qquad \tilde{\nabla}_{e_2}e_1=0,$$

$$\tilde{\nabla}_{e_3}e_3=0,\qquad \tilde{\nabla}_{e_3}e_2=\frac{1}{2z}e_2,\qquad \tilde{\nabla}_{e_3}e_1=\frac{1}{2z}e_1.$$

Using the above and (2.7) we get $\alpha=\frac{1}{2z}+1$. Hence the manifold is a α-para Kenmotsu manifold with semisymmetric metric connections.
Consider a curve $\gamma: I\longrightarrow M$ defined by $\gamma(s)=(\sqrt{\frac{5}{7}}s,\sqrt{\frac{2}{7}}s,1)$.
This curve is an almost contact curve.
Again Consider a curve $\gamma: I\longrightarrow M$ defined by $\gamma(s)=(\sqrt{\frac{5}{7}}s,\sqrt{\frac{2}{7}}s,s)$.
This curve is slant curve.

7. summary of the new results and their significance

Semisymmetric metric connections have a beautiful interpretation in Riemannian geometry. If a person moves on earth's surface always facing a definite point, then his displacement path is semisymmetric and metric [30]. Linear connections defined along such paths are semisymmetric. So study of semisymmetric metric connections is geometrically meaningful.
Here we have obtained a necessary and sufficient condition for an almost contact curve to be bi-harmonic with respect to semisymmetric metric connection. We also obtained a necessary and sufficient condition for a slant curve to be geodesic with respect to semisymmetric metric connection. Again, we have given a characterization of locally ϕ-symmetric curves. **The new fact is that the results regarding curvatures and torsions with respect to semisymmetric connections are not necessarily same with those with respect to Levi-Civita Connections. But for locally ϕ symmetric curve it is so. In these respects our results are new. Again a new problem can be studied with respect to quarter symmetric metric connections in this line**.
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