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A NEW APPROACH TO STATISTICALLY QUASI CAUCHY SEQUENCES

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Abstract. A sequence (α_k) of points in $\mathbb{R}$, the set of real numbers, is called *ρ-statistically p quasi Cauchy* if

$$\lim_{n\to\infty} \frac{1}{\rho_n} |\{k \leq n : |\Delta_p \alpha_k| \geq \varepsilon\}| = 0$$

for each $\varepsilon > 0$, where $\boldsymbol{\rho} = (\rho_n)$ is a non-decreasing sequence of positive real numbers tending to ∞ such that $\limsup_n \frac{\rho_n}{n} < \infty$, $\Delta \rho_n = O(1)$, and $\Delta_p \alpha_{k+p} = \alpha_{k+p} - \alpha_k$ for each positive integer k. A real-valued function defined on a subset of $\mathbb{R}$ is called ρ-statistically p-ward continuous if it preserves ρ-statistical p-quasi Cauchy sequences. ρ-statistical p-ward compactness is also introduced and investigated. We obtain results related to ρ-statistical p-ward continuity, ρ-statistical p-ward compactness, p-ward continuity, continuity, and uniform continuity.

1. Introduction

The concept of continuity and any concept involving continuity play a very important role not only in pure mathematics but also in other branches of sciences involving mathematics especially in computer science, information theory, biological science.

The idea of statistical convergence was formerly given under the name "almost convergence" by Zygmund in the first edition of his celebrated monograph published in Warsaw in 1935 in [39]. The concept was formally introduced by Fast [26] and later was reintroduced by Schoenberg [34], and also independently by Buck [2]. Although statistical convergence was introduced over nearly the last eighty years, it has become an active area of research for thirty years with the contributions by several authors, Salat ([33]), Fridy [27], Caserta and Kocinac [24], Maio and Kocinac ([28]), Caserta, Maio and Kocinac ([25]), Patterson and Savas ([32],Mursaleen ([29]), Cakalli and Khan ([17]), Yildiz ([37], and [38]).

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A sequence (α_k) of points in $\mathbb{R}$ is called *ρ-statistically convergent* to an element ℓ of $\mathbb{R}$ if

$$\lim_{n\to\infty}\frac{1}{\rho_n}|\{k\leq n: |\alpha_k-\ell|\geq\varepsilon\}|=0$$

for each $\varepsilon>0$, where $\boldsymbol{\rho}=(\rho_n)$ is a non-decreasing sequence of positive real numbers tending to ∞ such that $\limsup_n \frac{\rho_n}{n}<\infty$, and $(\Delta\rho_n)$ is a bounded sequence ([14]). This is denoted by $st_\rho-\lim_{k\to\infty}\alpha_k=\ell$. We note that such sequences were introduced without the assumption of boundedness of the downward difference sequence of $\boldsymbol{\rho}$ in [30], and was called quasi statistical convergence. The sequential method $st_\rho-\lim$ is a regular sequential method since any convergent sequence is ρ-statistically convergent ([30, page 13].

A sequence (α_k) of points in $\mathbb{R}$, the set of real numbers, is called *ρ-statistically quasi Cauchy* if

$$\lim_{n\to\infty}\frac{1}{\rho_n}|\{k\leq n: |\Delta\alpha_k|\geq\varepsilon\}|=0$$

for each $\varepsilon>0$, where $\Delta\alpha_k=\alpha_{k+1}-\alpha_k$ for each positive integer k ([14]).

Using the idea of continuity of a real function in terms of sequences in the sense that a function preserves a certain kind of sequences, many kinds of continuities were introduced and investigated, not all but some of them we recall in the following: slowly oscillating continuity ([7]), quasi-slowly oscillating continuity ([23]), ward continuity ([12]), δ-ward continuity ([8]), statistical ward continuity ([10]), and N_θ-ward continuity ([4]) which enabled some authors to obtain conditions on the domain of a function for some characterizations of uniform continuity (see [36, Theorem 6],[3, Theorem 1 and Theorem 2],[23, Theorem 2.3], [3, Theorem 1], and [21, Theorem 5].

The purpose of this paper is to introduce and investigate the concept of ρ-statistical p-ward continuity of a real function, and prove interesting theorems.

2. Results

Now we introduce the concept of ρ-statistically p quasi Cauchyness.

Definition 2.1. *A sequence (α_k) of points in $\mathbb{R}$, the set of real numbers, is called ρ-statistically p quasi Cauchy if*

$$\lim_{n\to\infty}\frac{1}{\rho_n}|\{k\leq n: |\Delta_p\alpha_k|\geq\varepsilon\}|=0$$

for each $\varepsilon>0$, where $\Delta_p\alpha_{k+p}=\alpha_{k+p}-\alpha_k$ for each positive integer k, p is a fixed positive integer.

Any quasi-Cauchy sequence is ρ-statistically p-quasi-Cauchy, but the converse is not always true. Any ρ-statistically convergent sequence is ρ-statistically p-quasi-Cauchy. There are ρ-statistically p-quasi-Cauchy sequences which are not ρ-statistically convergent. The sum of two ρ-statistical p-quasi-Cauchy sequences is ρ-statistically p-quasi-Cauchy, but the product of two ρ-statistical p-quasi-Cauchy sequences need not be ρ-statistically p-quasi-Cauchy.

Now we give the definition of ρ-statistical p-ward compactness.

Definition 2.2. *A subset E of $\mathbb{R}$ is called ρ-statistically p-ward compact if any sequence of points in E has a ρ-statistical p-quasi-Cauchy subsequence.*

First, we note that any finite subset of $\mathbb{R}$ is ρ-statistically p-ward compact, the union of two ρ-statistically p-ward compact subsets of $\mathbb{R}$ is ρ-statistically p-ward compact and the intersection of any family of ρ-statistically p-ward compact subsets of $\mathbb{R}$ is ρ-statistically p-ward compact. Any G-sequentially compact subset of $\mathbb{R}$ is ρ-statistically p-ward compact for a regular subsequential method G (see [6], and [9]). Furthermore any subset of a ρ-statistically p-ward compact set is ρ-statistically p-ward compact, any bounded subset of $\mathbb{R}$ is ρ-statistically p-ward compact, any slowly oscillating compact subset of $\mathbb{R}$ is ρ-statistically p-ward compact (see [7] for the definition of slowly oscillating compactness). These observations suggest to us the following.

Theorem 2.1. *A subset E of $\mathbb{R}$ is bounded if and only if it is ρ-statistically p-ward compact.*

Proof. If E is a bounded subset of $\mathbb{R}$, then any sequence of points in E has a convergent subsequence which is also ρ-statistically p-quasi-Cauchy. Conversely, suppose that E is not bounded. If it is not bounded below, then pick an element α_1 of E less than 0. Then we can choose an element α_2 of E such that $\alpha_2 < -p-\rho_1+\alpha_1$. Similarly we can choose an element α_3 of E such that $\alpha_3 < -p-\rho_2+\alpha_2$. We can inductively choose α_k satisfying $\alpha_{k+1} < -p-\rho_k+\alpha_k$ for each $k \in \mathbb{N}$. Hence $\alpha_k - \alpha_{k+p} > p+\rho_k$ for each $k \in \mathbb{N}$. Thus $|\alpha_{k+p}-\alpha_k| > p+\rho_k$ for each $k \in \mathbb{N}$. Then the sequence (α_k) does not have any ρ-statistically p-quasi Cauchy subsequence. If E is unbounded above, then we can find a β_1 greater than 0. Then we can pick a β_2 such that $\beta_2 > \rho_1 + p + \beta_1$. We can successively find for each $k \in \mathbb{N}$ a β_{k+1} such that $\beta_{k+1} > \rho_k + p + \beta_k$. Then $\beta_{k+p} - \beta_k > p + \rho_k$ for each $k \in \mathbb{N}$. Thus $|\beta_{k+p} - \beta_k| > p + \rho_k$ for each $k \in \mathbb{N}$. Then the sequence (β_k) does not have any ρ-statistical p-quasi Cauchy subsequence. Thus E is not ρ-statistically p-ward compact. This completes the proof. □

Corollary 2.2. *A subset E of $\mathbb{R}$ is ρ-statistically p-ward compact if and only if it is both upward and downward statistically compact.*

Proof. The proof follows from the preceding theorem and [13, Theorem 3.3 and Theorem 3.6]. □

Theorem 2.3. *If a function f is uniformly continuous on a subset E of $\mathbb{R}$, then $(f(\alpha_k))$ is ρ-statistically p-quasi Cauchy whenever (α_k) is a quasi-Cauchy sequence of points in E.*

Proof. Take any p-quasi-Cauchy sequence (α_k) of points in E, and let ε be any positive real number. By uniform continuity of f, there exists a $\delta > 0$ such that $|f(\alpha)-f(\beta)| < \varepsilon$ whenever $|\alpha-\beta| < \delta$ and $\alpha, \beta \in E$. Since (α_k) is a p-quasi-Cauchy sequence, there exists a positive integer k_0 such that $|\alpha_{k+p} - \alpha_k| < \delta$ for $k \geq k_0$. Thus

$$|\{k \leq n : |f(\alpha_{k+p}) - f(\alpha_k)| \geq \varepsilon\}| \leq k_0.$$

Hence

$$\lim_{n\to\infty} \frac{1}{\rho_n} |\{k \leq n : |f(\alpha_{k+p}) - f(\alpha_k)| \geq \varepsilon\}| = 0$$

Thus $(f(\alpha_k))$ is a ρ-statistically p-quasi Cauchy sequence. This completes the proof of the theorem. □

Definition 2.3. *A function defined on a subset E of $\mathbb{R}$ is called ρ-statistically p-ward continuous if it preserves ρ-statistically p-quasi-Cauchy sequences, i.e. $(f(\alpha_n))$ is a ρ-statistically p-quasi-Cauchy sequence whenever (α_n) is.*

We see that the sum of two ρ-statistically p-ward continuous functions is ρ-statistically p-ward continuous, and cf is ρ-statistically p-ward continuous whenever c is a constant real number and f is a ρ-statistically p-ward continuous function.

Theorem 2.4. *If f is ρ-statistically p-ward continuous on a subset E of $\mathbb{R}$, then it is ρ-statistically continuous on E.*

Proof. Assume that f is a ρ-statistically p-ward continuous function on E. Let (α_n) be any ρ-statistically convergent sequence with $st_\rho - \lim_{k\to\infty} \alpha_k = \ell$. Then the sequence

$$(\alpha_1, \alpha_1, ..., \alpha_1, \ell, \ell, ..., \ell, \alpha_2, \alpha_2, ..., \ell, \ell, ..., \alpha_n, \alpha_n, ..., \ell, \ell, ...)$$

is ρ-statistically convergent to ℓ, where the same terms repeat p times. Hence it is ρ-statistically quasi-Cauchy, so is ρ-statistically p-quasi-Cauchy. As f is ρ-statistically p-ward continuous, the sequence

$$(f(\alpha_1), f(\alpha_1), ..., f(\alpha_1), f(\ell), f(\ell), ..., f(\ell), f(\alpha_2), f(\alpha_2), ..., f(\ell), f(\ell), ..., f(\alpha_n), f(\alpha_n), ..., f(\ell), f(\ell), ...)$$

is ρ-statistically p-quasi-Cauchy. Hence it follows that the sequence $(f(\alpha_n))$ is ρ-statistically converges to $f(\ell)$. This completes the proof of the theorem. □

Related to G-continuity we have the following result.

Corollary 2.5. *If f is ρ-statistically p-ward continuous, then it is G-continuous for any regular subsequential method G.*

The preceding corollary ensures that ρ-statistically p-ward continuity implies either of the following continuities; ordinary continuity, statistical continuity, lacunary statistical continuity ([5]), strongly lacunary continuity ([4]), λ-statistical continuity, I-sequential continuity for any non trivial admissible ideal I of $\mathbb{N}$ ([16]).

It is well known that any continuous function on a compact subset E of $\mathbb{R}$ is uniformly continuous on E. For ρ-statistically p-ward continuous functions we have the following.

Theorem 2.6. *Let E be a ρ-statistically p-ward compact subset E of $\mathbb{R}$ and let $f : A \longrightarrow \mathbb{R}$ be a ρ-statistically p-ward continuous function on E. Then f is uniformly continuous on E.*

Proof. Suppose that f is not uniformly continuous on E so that there exists an $\varepsilon_0 > 0$ such that for any $\delta > 0$ $x, y \in E$ with $|x - y| < \delta$ but $|f(x) - f(y)| \geq \varepsilon_0$. For each positive integer n, there are α_n and β_n such that $|\alpha_n - \beta_n| < \frac{1}{n}$, and $|f(\alpha_n) - f(\beta_n)| \geq \varepsilon_0$. Since E is ρ-statistically p-ward compact, there exists a ρ-statistical p-quasi-Cauchy subsequence (α_{n_k}) of the sequence (α_n). It is clear that the corresponding subsequence (β_{n_k}) of the sequence (β_n) is also ρ-statistically p-quasi-Cauchy, since $(\beta_{n_{k+p}} - \beta_{n_k})$ is a sum of three ρ-statistical null sequences, i.e.

$$\beta_{n_{k+p}} - \beta_{n_k} = (\beta_{n_{k+p}} - \alpha_{n_{k+p}}) + (\alpha_{n_{k+p}} - \alpha_{n_k}) + (\alpha_{n_k} - \beta_{n_k}).$$

Then the sequence

$$(a_{n_1}, a_{n_1}, ..., a_{n_1}, \beta_{n_1}, \beta_{n_1}, ..., \beta_{n_1}, \alpha_{n_2}, \alpha_{n_2}, ..., \alpha_{n_2}, \beta_{n_2}, \beta_{n_2}, ..., \beta_{n_2}, , ..., \alpha_{n_k}, \alpha_{n_k}, ..., \alpha_{n_k}, \beta_{n_k}, \beta_{n_k}, ..., \beta_{n_k}, ...)$$

is ρ-statistical p-quasi-Cauchy since the sequence $(\alpha_{n_k} - \beta_{n_k})$ is ρ-statistically convergent to 0. But the transformed sequence is not ρ-statistically p-quasi-Cauchy. Thus f does not preserve ρ-statistical p-quasi-Cauchy sequences. This contradiction completes the proof of the theorem. □

Corollary 2.7. *If a function f is ρ-statistically p-ward continuous on a bounded subset E of $\mathbb{R}$, then it is uniformly continuous on E.*

Proof. The proof follows from Theorem 2.6 and Theorem 2.1. □

Theorem 2.8. *ρ-statistical p-ward continuous image of any ρ-statistically p-ward compact subset of $\mathbb{R}$ is ρ-statistically p-ward compact.*

Proof. Assume that f is a ρ-statistically p-ward continuous function on a subset E of $\mathbb{R}$, and A is a ρ-statistically p-ward compact subset of E. Let (β_n) be any sequence of points in $f(A)$. Write $\beta_n = f(\alpha_n)$ where $\alpha_n \in A$ for each positive integer n. ρ-statistically ward compactness of A implies that there is a subsequence $(\gamma_k) = (\alpha_{n_k})$ of (α_n) with $st_\rho - \lim_{k\to\infty} \Delta\gamma_k = 0$. Write $(t_k) = (f(\gamma_k))$. As f is ρ-statistically p-ward continuous, $(f(\gamma_k))$ is ρ-statistically p-quasi-Cauchy. Thus we have obtained a subsequence (t_k) of the sequence $(f(\alpha_n))$ with $st_\rho - \lim_{k\to\infty} \Delta^p t_k = 0$. Thus $f(A)$ is ρ-statistically p-ward compact. This completes the proof of the theorem. □

Corollary 2.9. *ρ-statistically p-ward continuous image of any compact subset of $\mathbb{R}$ is ρ-statistically ward compact.*

The proof follows from the preceding theorem.

Corollary 2.10. *ρ-statistically p-ward continuous image of any bounded subset of $\mathbb{R}$ is bounded.*

The proof follows from Theorem 2.1 and Theorem 2.8.

Corollary 2.11. *ρ-statistical p-ward continuous image of a G-sequentially compact subset of $\mathbb{R}$ is ρ-statistically p-ward compact for any subsequential regular method G.*

It is a well known result that uniform limit of a sequence of continuous functions is continuous. This is also true in case of ρ-statistical p-ward continuity, i.e. uniform limit of a sequence of ρ-statistical p-ward continuous functions is ρ-statistically p-ward continuous.

Theorem 2.12. *If (f_n) is a sequence of ρ-statistically p-ward continuous functions on a subset E of $\mathbb{R}$ and (f_n) is uniformly convergent to a function f, then f is ρ-statistically p-ward continuous on E.*

Proof. Let ε be a positive real number and (α_k) be any ρ-statistical p-quasi-Cauchy sequence of points in E. By the tuniform convergence of (f_n) there exists a positive integer N such that $|f_n(x) - f(x)| < \frac{\varepsilon}{3}$ for all $x \in E$ whenever $n \geq N$. As f_N is ρ-statistically p-ward continuous on E, we have

$$\lim_{n\to\infty} \frac{1}{\rho_n} |\{k \leq n : f_N(\alpha_{k+p}) - f_N(\alpha_k)| \geq \frac{\varepsilon}{3}\}| = 0.$$

On the other hand we have
$\{k \leq n : |f(\alpha_{k+p}) - f(\alpha_k)| \geq \varepsilon\} \subset \{k \leq n : |f(\alpha_{k+p}) - f_N(\alpha_{k+p})| \geq \frac{\varepsilon}{3}\}$
$\cup\{k \leq n : |f_N(\alpha_{k+p}) - f_N(\alpha_k)| \geq \frac{\varepsilon}{3}\} \cup \{k \leq n : |f_N(\alpha_k) - f(\alpha_k)| \geq \frac{\varepsilon}{3}\}$

Now it follows from this inclusion that
$\lim_{n\to\infty} \frac{1}{\rho_n}|\{k \leq n : |f(\alpha_{k+p}) - f(\alpha_k)| \geq \varepsilon\}|$

$$\leq \lim_{n\to\infty} \frac{1}{\rho_n}|\{k \leq n : |f(\alpha_{k+p}) - f_N(\alpha_{k+p})| \geq \frac{\varepsilon}{3}\}| + \lim_{n\to\infty} \frac{1}{\rho_n}|\{k \leq n : |f_N(\alpha_{k+p}) - f_N(\alpha_k)| \geq \frac{\varepsilon}{3}\}|$$

$+ \lim_{n\to\infty} \frac{1}{\rho_n}|\{k \leq n : |f_N(\alpha_k) - f(\alpha_k)| \geq \frac{\varepsilon}{3}\}| = 0 + 0 + 0 = 0.$

This completes the proof of the theorem. □

Theorem 2.13. *The set of all ρ-statistically p-ward continuous functions on a subset E of $\mathbb{R}$ is a closed subset of the set of all continuous functions on E, i.e. $\overline{\Delta\rho^p SWC(E)} = \Delta\rho^p SWC(E)$ where $\Delta\rho^p SWC(E)$ is the set of all ρ-statistically p-ward continuous functions on E, $\overline{\Delta\rho^p SWC(E)}$ denotes the set of all cluster points of $\Delta\rho^p SWC(E)$.*

Proof. f be any element in $\overline{\Delta\rho^p SWC(E)}$. Then there exists a sequence of points in $\Delta\rho^p SWC(E)$ such that $\lim_{k\to\infty} f_k = f$. To show that f is ρ-statistically p-ward continuous, take any ρ-statistical p-quasi-Cauchy sequence (α_k) of points in E. Let $\varepsilon > 0$. Since (f_k) converges to f, there exists an N such that for all $x \in E$ and for all $n \geq N$, $|f(x) - f_n(x)| < \frac{\varepsilon}{3}$. As f_N is ρ-statistically p-ward continuous, we have $\lim_{n\to\infty} \frac{1}{\rho_n}|\{k \leq n : f_N(\alpha_{k+p}) - f_N(\alpha_k)| \geq \frac{\varepsilon}{3}\}| = 0$. On the other hand,
$\{k \leq n : |f(\alpha_{k+p}) - f(\alpha_k)| \geq \varepsilon\} \subset \{k \leq n : |f(\alpha_{k+p}) - f_N(\alpha_{k+p})| \geq \frac{\varepsilon}{3}\}$
$\cup\{k \leq n : |f_N(\alpha_{k+p}) - f_N(\alpha_k)| \geq \frac{\varepsilon}{3}\} \cup \{k \leq n : |f_N(\alpha_k) - f(\alpha_k)| \geq \frac{\varepsilon}{3}\}$
Now it follows from this inclusion that
$\lim_{n\to\infty} \frac{1}{\rho_n}|\{k \leq n : |f(\alpha_{k+p}) - f(\alpha_k)| \geq \varepsilon\}|$

$$\leq \lim_{n\to\infty} \frac{1}{\rho_n}|\{k \leq n : |f(\alpha_{k+p}) - f_N(\alpha_{k+p})| \geq \frac{\varepsilon}{3}\}| + \lim_{n\to\infty} \frac{1}{\rho_n}|\{k \leq n : |f_N(\alpha_{k+p}) - f_N(\alpha_k)| \geq \frac{\varepsilon}{3}\}|$$

$+ \lim_{n\to\infty} \frac{1}{\rho_n}|\{k \leq n : |f_N(\alpha_k) - f(\alpha_k)| \geq \frac{\varepsilon}{3}\}| = 0 + 0 + 0 = 0.$

This completes the proof of the theorem. □

Corollary 2.14. *The set of all ρ-statistically p-ward continuous functions on a subset E of $\mathbb{R}$ is a complete subspace of the space of all continuous functions on E.*

Proof. The proof follows straightforward from the preceding theorem.

□

3. Conclusion

The results in this paper not only generalize results studied in [10], and [11] as a special case, i.e. $\rho_n = n$ for each $n \in \mathbb{N}$, but also includes results which are also new for the special case. It turns out that the set of uniformly continuous functions includes the set of ρ-statistical ward continuous functions on bounded sets. We suggest to investigate ρ-statistically p-quasi-Cauchy sequences of fuzzy points or soft points (see [19], for the definitions and related concepts in fuzzy setting, and see [1] related concepts in soft setting). We also suggest to investigate ρ-statistically p-quasi-Cauchy double sequences (see for example [18] for the definitions and related concepts in the double sequences case). For another further study, we suggest to investigate ρ-statistically p-quasi-Cauchy sequences in abstract metric spaces (see [22], [31], [20], and [35]).

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