

The Effect of Axial Selection on Static Solution Results in Heat Affected Non-Prismatic Elementary Frames

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Abstract: In this study, the influence of the bar axis selection on the static solution results was investigated in-plane carrier systems consisting of non-prismatic bar elements (height variable bar elements, also known as hunched, along the axis) in terms of equal or different heat exchange effect. On the sample considered, the non-prismatic elements are considered as straight-axis bars. The results obtained from the classical analysis (the section change is taken into account only at the bending stiffness in this method) and the solution results, which are proposed for non-prismatic elements, obtained by considering the weight axis of the element as a bar axis are compared and the relative differences was detected with differences between two results.

Keywords: *non-prismatic, hunched, different heat, stiffness, bars, equal heat*

Introduction

In the industrial structures with the large span, cross sections are increased in areas near nodes in order to increase the load carrying capacities of the elements. In this way, the heights of such elements, and thus the cross-sectional areas and moments of inertia become variable throughout the element. The most common situation is that the bar section along the element changes linear, parabolically or remains stable zone by zone (Figure 1). In some of the industrial structures, heat effect is the subject by force of the purpose of using the structure. (Cross *et al.*, 1958) and (Portland Cement Association 1958) gave the fixed end moments, stiffness and transfer factors of the variable section elements, Finite Element Models (FEM) for variable sections were given in (Resende *et al.*, 1981). In (Eisenberg 1985), stiffness formulation was written by moving from the flexibility of the variable section elements, in (Mezaini *et al.* 1991) the linear elastic behaviour of variable sectioned frames was investigated using ISO-parametric plane stress finite elements and it had been found that there are big differences between the obtain results and fixing moments, stiffness and transport factors in the literature for the variable cross-section elements and the models of the classical frame analysis were proposed by investigating the ranks and sources of the errors. In (Topçu, 1992), the basic stiffness coefficients of the variable section elements were given and the calculation of fixing moments for various methods were given using the analytical and numerical integration method. Karaduman (1993) investigated the effect of heat on carrier systems consisting of variable bar sections. Elasticity moduli were regarded as variable in (Fertis *et al.* 1990). Behaviour of Non-prismatic beam vibration was investigated in (Ruta, 1999). This study (Yüksel, 2012) aimed to investigate the behaviour of non-prismatic beams with symmetrical parabolic haunches. In this study (Archundia-Aranda *et al.* 2013), Behaviour of reinforced concrete hunched beams subjected to cyclic shear loading was investigated. Investigation of performance of a minimum weight restressed concrete beam adopting a non-prismatic section is aimed in (Raju *et al.* 2014). Shear behaviour of non-prismatic steel reinforced concrete beams was investigated in (Orr *et al.* 2014). Influence of the cross section shape on the behaviour of SRG-confined prismatic unreinforced concrete specimens was investigated in (Thermou *et al.* 2015).

The purpose of this work is to investigate the effect of the rod axis selection on the static solution results and determine the differences in the case of equal or different heat effect in the frames consisting of non-prismatic elements.

General Behaviour of Non-Prismatic Elements

The behaviour of the non-prismatic elements is different from the prismatic elements due to the discontinuity of the neutral axis and the variation of the cross-section along the element. In this section, the effects of the geometric axis obtained from the examination for external loads, the change

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of section and the results of stress distribution in non-prismatic elements in (Mezaini et al., 1991) will be summarized.

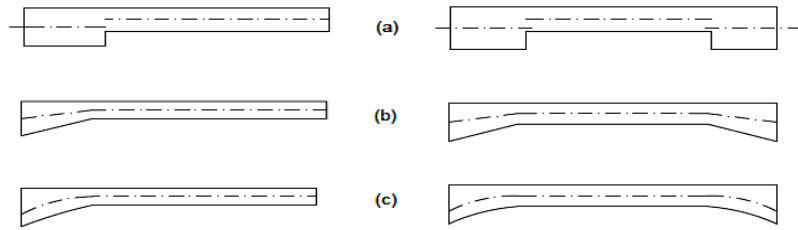


Figure 1. Type of non-prismatic elements a) Fix haunched elements b) Linear haunched elements c) Parabolic haunched elements

The centre of gravity axis discontinuity

When viewed mathematically, the middle axis is non-continuous in stepped or hunched elements. The given differential equations for beam bending are not valid in discontinuous points (Mezaini et al., 1991). Two types of variable-section elements and their properties are shown in Figure 2. a & b to investigate the effects of neutral axis discontinuity.

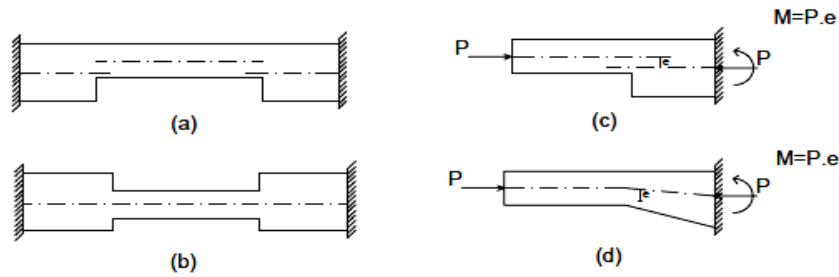


Figure 2. Discontinuous axes

Here, in the case of equal temperature change, only the axial force is generated in the element shown in Figure 2.b, and the bending moment is also generated in the element shown in Figure 2.a. Furthermore, if two separate elements are subjected to vertical load or end rotations, axial force is generated in element Figure 2.a, whereas no axial force is generated in element Figure 2.b. The completely different behaviour of these two elements is due to the difference in geometry in the axes. If the change in height is only one edge as shown in Figure 2.c&d, similar behaviours are existed.

Stress distribution on cross-section

In non-prismatic elements under the same internal forces, the stress distribution on the cross section is different from the stress distribution of a prismatic element of the same dimension. The results obtained from finite element analysis of such stress distributions are given in Figure 3 (Mezaini et al., 1991). In Figure 3, it is clearly seen that the stress distribution of each element depends on shape of section changing. In this case, the neutral axis is in gusset region and region near gusset above the centre of gravity axis. This is because the dissymmetrical shear stress around the centre of gravity axis (Mezaini et al., 1991).

Stress flow in non-prismatic elements

The reason for the confusion of the stress flow at the cut-off point where the sudden section changing in the elements is geometry. This area, which is exposed to an insignificant stress due to sudden section changing, is called invalid area. The presence of these invalid areas causes stiffness reduction (Figure 4.a; Mezaini et al.1991).

In the haunched element where the continuous section changing such as in figure 4.b, the fold are occurred on part of changed gradient in the centre of gravity. General behaviour of such elements resembles behaviour of elements of sudden section changing. The only difference is that stiffness reduction due to the stress flow in the end elements is less important (Mezaini et al., 1991).

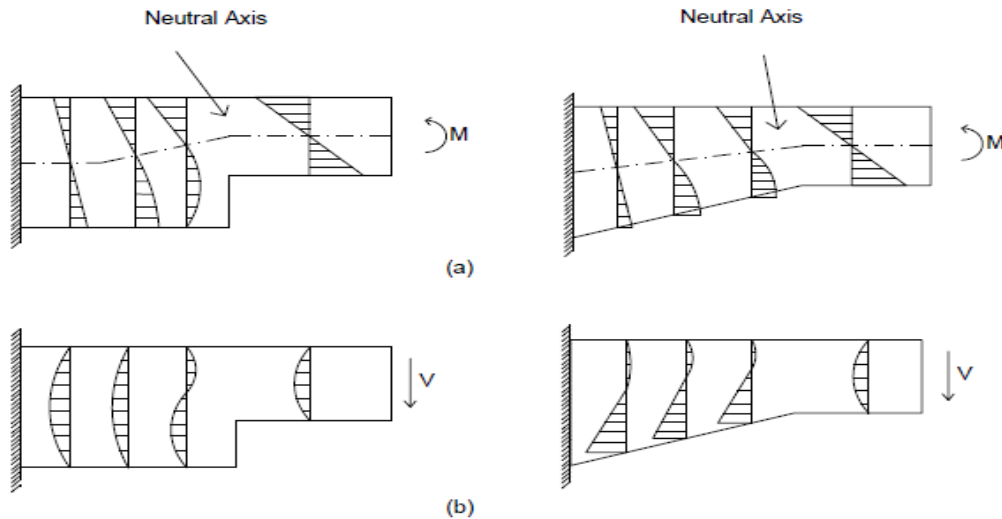


Figure 3. Stress distribution on non-prismatic elements a) Normal stress due to only bending moment
b) Shear stress due to constant shear force

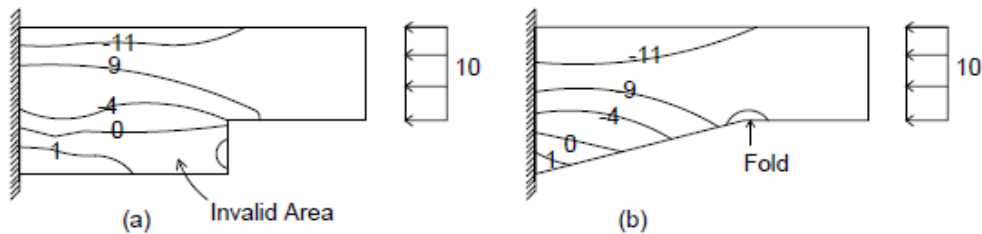


Figure 4. Main stresses on exterior surface

Non-prismatic elements modelling

As a result of studies on non-prismatic elements, models which give the closest values to the results obtained from finite element analysis for such elements are suggested by developing by (Mezaini et al, 1991). In this study, these models, which are suggested for solving of plane load-bearing system consisting of non-prismatic elements under effect of equal or different heat exchange, are used.

Linearly Haunched Elements

By basing on the results obtained finite element analysis, it is indicated that %75 of gusset length is effective by cutting out the invalid areas in fold region and the model shown in figure 5 gives most suitable results for linearly haunched elements (Mezaini et al, 1991).



Figure 5. Linearly haunched elements modelling

Parabolic Haunched Element

The effects of the parabolic haunch length were investigated by following the same procedure as for the linear haunch and the best results were obtained in the model shown in figure 6 (Mezaini et al, 1991).

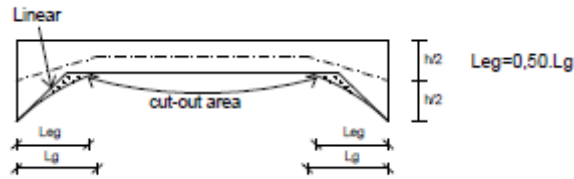


Figure 6. Parabolic haunched elements modelling

Gradual Elements

The three models shown in Figure 7.a.b.c for the gradual element were analysed and it was seen that in the three models, the model shown in Figure 7.b showed the best adaptation with the finite element analysis. In this model, the corner is reduced by lifting it at an angle of 45° and the average area and moment of inertia are used for transition from one part to another.

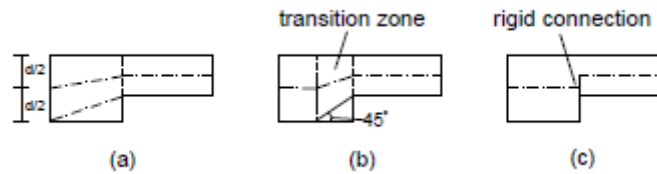


Figure 7. Gradual elements modelling

Recommended Model Using

The recommended model for linear and parabolic haunched elements applies only to fixed support and continuous terminals as shown in figure 8.a.b. If the element is not continuous or is connected to the column as shown in figure 8.c, the best results are obtained by adding the entire original center of gravity centered gusset region to the calculator (Mezaini et al, 1991).

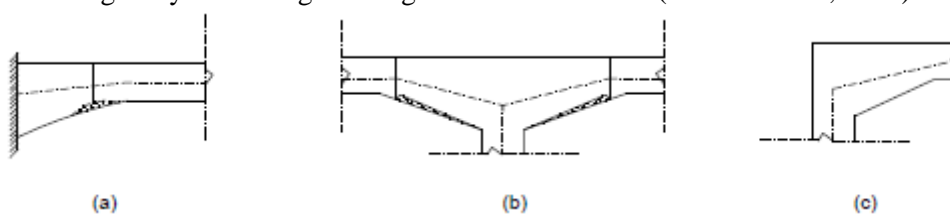


Figure 8. Connection states of non-prismatic elements

The Importance of Axis Selection in The Effect of Heat Exchange

In this part of the study, a sample of a variable cross-section frame affected by equal or different heat exchange was solved by a computer program that was prepared in BASIC programming language given in reference 7. While sample of the frame solves, firstly, the non-prismatic elements presumed as in-line bar are analysed with classical solution by taking into consideration only the bending stiffness of the section change and the internal forces for equal and different heat exchanges are obtained. After that, in the same sample of the frame, the real analysis mentioned in part 2 is carried out by considering the bar axis as the element weight axis and the internal forces for equal and different heat exchanges are obtained (Tables 1 - 4). The relative differences between the internal forces obtained from the two solution methods are summarized in table 5 and 6. When the tables are examined, it is understood that the results obtained from the classical solution are significantly different from the real solution results. As a result, it is understood that the fact that the real bar axis is taken into consideration in the selection of the bar axis in the frames consisting of the variable cross-sectional elements under equal and different heat exchanges effect.

Numerical application

The sample of frame given in figure 9 is solved with classical solution and real solution under equal heat exchange and different heat exchange ($\Delta T_1 = -5^{\circ}C$ and $\Delta T_2 = 25^{\circ}C$). Differences between the internal forces are obtained. In solution, $b = 0.40$ m; $I = 0.0114$ m⁴; $E = 2.10$ t/m² are regarded.

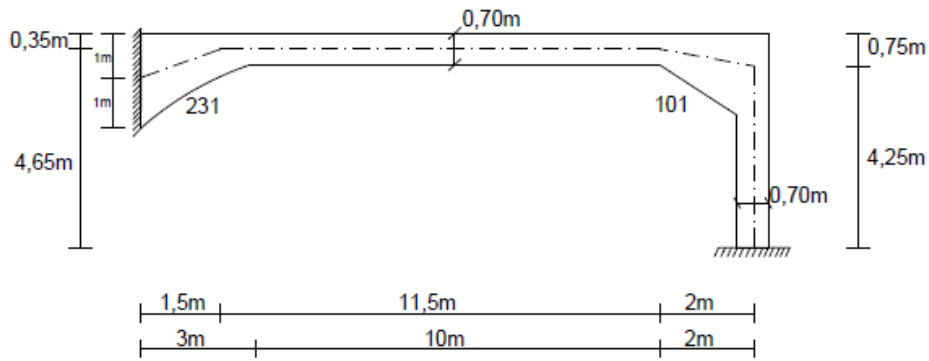


Figure 9. Sample of variable cross-sectional frame

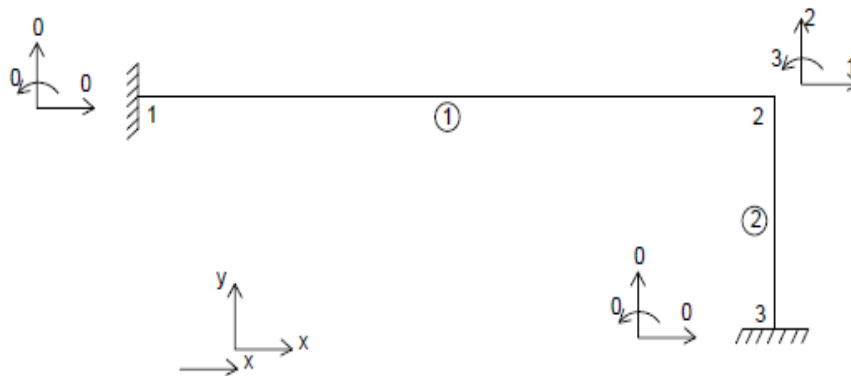


Figure 10. Coding state of the classical model

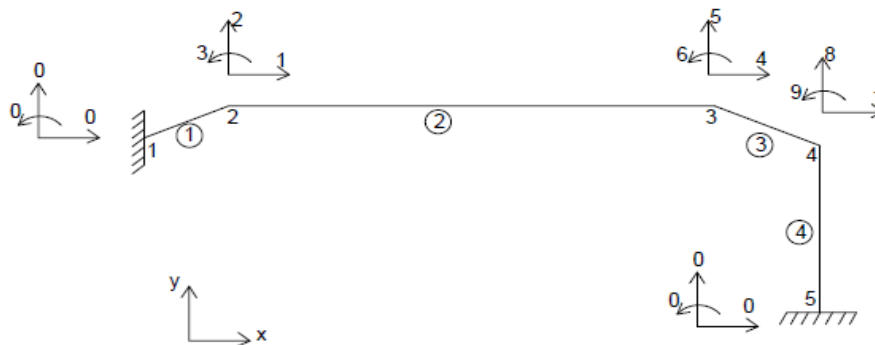


Figure 11. Coding state of the suggested model

Table 1. Static analysis results of classical model under $\Delta T=30\text{ }^{\circ}\text{C}$ equal heat exchange

Internal forces of elements								
No	i	j	$N_i(t)$	$N_j(t)$	$V_i(t)$	$V_j(t)$	$M_i(t.m)$	$M_j(t.m)$
1	1	2	-8.026	8.026	1.537	-1.537	9.795	13.266
2	2	3	-1.537	1.537	-8.026	8.026	-13.266	-24.055

Table 2. Static analysis results of suggested model under $\Delta T=30\text{ }^{\circ}\text{C}$ equal heat exchange

Internal Forces Of Elements								
No	i	j	$N_i(t)$	$N_j(t)$	$V_i(t)$	$V_j(t)$	$M_i(t.m)$	$M_j(t.m)$
1	1	2	-7.700	7.700	3.623	-3.623	11.230	-6.156
2	2	3	-8.091	8.091	1.465	-1.465	6.156	10.694
3	3	4	-8.383	8.383	-0.153	0.153	-10.694	10.400
4	4	5	-1.465	1.465	-8.091	8.091	-10.400	-23.984

Table 3. Static analysis results of classical model under $\Delta T_1=-5\text{ }^\circ\text{C}$, $\Delta T_2=25\text{ }^\circ\text{C}$ difference heat exchange

Internal forces of elements								
No	i	j	$N_i(t)$	$N_j(t)$	$V_i(t)$	$V_j(t)$	$M_i(t.m)$	$M_j(t.m)$
1	1	2	-1.701	1.701	0.702	-0.702	2.736	7.799
2	2	3	-0.702	0.702	-1.701	1.701	-7.799	-0.109

Table 4. Static analysis results of suggested model under $\Delta T_1=-5\text{ }^\circ\text{C}$, $\Delta T_2=25\text{ }^\circ\text{C}$ difference heat exchange

Internal forces of elements								
No	I	J	$N_i(t)$	$N_j(t)$	$V_i(t)$	$V_j(t)$	$M_i(t.m)$	$M_j(t.m)$
1	1	2	-1.824	1.824	1.294	-1.294	3.263	-1.450
2	2	3	-2.025	2.025	0.754	-0.754	1.450	7.225
3	3	4	-2.176	2.176	0.349	-0.349	-7.225	7.896
4	4	5	-0.754	0.754	-2.025	2.025	-7.896	-0.712

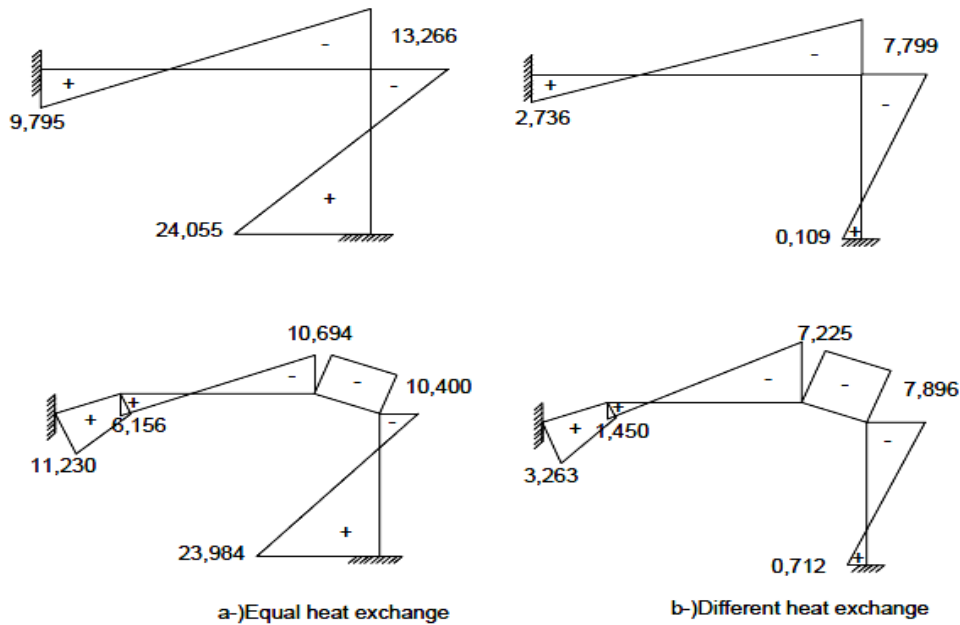


Figure 12. Moment diagrams for results

Table 5. The relative differences between two solution methods under $\Delta T=30\text{ }^\circ\text{C}$ equal heat exchange

	Bending Moment (t.m)	Shear Force (t)	Normal Force (t)
Left end of beam	$(11.230-9.795)/11.230=\%+12.78$	$(3.623-1.537)/3.623=\%+57.57$	$(7.700-8.026)/7.700=\%-4.23$
Right end of beam	$(10.400-13.266)/10.400=\%-27.17$	$(0.153-(-1.537))/0.153=\%+1104$	$(8.383-8.026)/8.383=\%+4.25$
Top end of column	$(10.400-13.266)/10.400=\%-27.17$	$(8.091-8.026)/8.091=\%+0.80$	$(1.465-1.537)/1.465=\%-4.91$
Bottom end of Column	$(23.984-24.055)/23.984=\%-0.30$	$(8.091-8.026)/8.091=\%+0.80$	$(1.465-1.537)/1.465=\%-4.91$

Table 6. The relative differences between two solution methods under $\Delta T_1=-5\text{ }^\circ\text{C}$, $\Delta T_2=25\text{ }^\circ\text{C}$ difference heat exchange

	Bending Moment (t.m)	Shear Force (t)	Normal Force (t)
Left end of beam	$(3.263-2.736)/3.263=\%+16.15$	$(1.294-0.702)/1.294=\%+45.75$	$(1.824-1.701)/1.824=\%+6.74$
Right end of beam	$(7.896-7.799)/7.896=\%+1.23$	$(0.349-0.752)/0.349=\%-115.47$	$(2.716-1.701)/2.716=\%+37.37$
Top end of column	$(7.896-7.799)/7.896=\%+1.23$	$(2.025-1.701)/2.025=\%+16$	$(0.754-0.702)/0.754=\%+6.90$
Bottom end of Column	$(0.712-0.109)/0.712=\%+84.69$	$(2.025-1.701)/2.025=\%+16$	$(0.754-0.702)/0.754=\%+6.90$

Results

In this study, plane carrier systems consisting of non-prismatic bar elements under effect of equal or different heat exchange is investigated. The non-prismatic elements presumed as a in-line bar are analysed with classical solution by taking into consideration only the bending stiffness of the section change and the internal forces for equal and different heat exchanges are obtained. The real analysis mentioned in part 2 is carried out by regarding as the element weight axis the bar axis and the internal forces for equal and different heat exchanges are obtained. The relative differences between the internal forces obtained from the two solution methods are compared. As a result, it is important to take into consideration the cross-sectional variation in the solution of the carrier systems consisting of non-prismatic elements. It is also necessary to consider the bar axis as the bar weight axis in terms of the accuracy of the solution.

References

- Cross H, Morgan N, (1958) *Continuous Frames Of Reinforced Concrete*, 14th Ed., John Wiley and Sons Inc., New York, pp.126-155.
- Portland Cement Association, (1958) *Handbook of frame constants*, Beam Factors and moment coefficients for members of variable section, Portland Cement Association, Chicago,III.
- Resende and Doyle (1981) NONPRI-An effective nonprismatic three dimensional beam finite element. *Comp. & Struc.*, **14**, 71-77.
- Eisenberg,M.,(1985) Explicit Stiffness Matrices Nonprismatic Members, *Comp & Struc.*, **20**, 715-720.
- Mezaini N, Balkaya, C, Çıtıptıoğlu E, (1991) Analysis of Frames with Nonprismatic Members, *ASCE*, **117**, 1573-1591.
- Topçu A, (1992) Değişken Kesitli Düzlem çerçeve Elemanların Temel Rijitlik Katsayılarının ve Ankastrelik Kuvvetlerinin Analitik ve Romberg integrasyon Yöntemi ile Hesabı, *İnsaat Müh. Bilgisayar Kullanımı III. Semp.*, İ.T.U., İstanbul.
- Karaduman A, (1993) Değişken Kesitli Düzlem Taşıyıcı Sistemlerin Matris Deplasman Yöntemi ile Statik Çözümü, Selçuk Üniversitesi, Fen Bilimleri Enstitüsü, İnşaat Mühendisliği Anabilim Dalı, Konya, 132 s.
- Fertis DG, Keene ME, (1990), Elastic and Inelastic Analysis of Non-Prismatic Members, *J Struct. Eng.*, **116**, 475-489.
- Ruta, F, (1999), Application of Chebychev series to solution of non-prismatic beam vibration problems, *J. Sound & Vibration* **227**, 449-467.
- Yuksel SB, (2012), Assessment of non-prismatic beams having symmetrical parabolic haunches with constant haunch length ratio of 0.5, *Struct. Engi. & Mech.*, **22**, 849-866.
- Archundia-Aranda H, Grande-Vega A, Tena-Colunga A, (2013) Behaviour of reinforced concrete haunched beams subjected to cyclic shear loading, *J. Struct. Eng.*, **49**, 27-42.
- Raju P.M, Rajsekhar K, Sandeep T.R,(2014), Performance of non-prismatic simply supported prestressed concrete beams, *Struct. Eng. & Mech.*, **52**, 723-738.
- Orr JJ, Ibell TJ, Darby AP, (2014) Shear behavior of non-prismatic steel reinforced concrete beams, *Eng. Struct.*, **71**, 48-59.
- Thermou GE, Katakalos K., Manos G.(2015), Influence of the cross section shape on the behaviour of SRG-confined prismatic concrete specimens, *Materials and Struct./Materiaux et Constructions*, 19p
- Topçu A, (1992) Değişken Kesitli Düzlem çerçeve Elemanların Temel Rijitlik Katsayılarının ve Ankastrelik Kuvvetlerinin Analitik ve Romberg integrasyon Yöntemi ile Hesabı, *İnsaat Mühendisliğinde Bilgisayar Kullanımı III. Sempozyumu*, İ.T.U., İstanbul.
- Çakıroğlu A, Çetmeli E, (1976) *Yapı Statiği*, Cilt II, İ.T.Ü.Matbaası, İstanbul.