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BEYOND EUCLIDEAN: A METRIC-OPTIMIZED TYPE-1 FUZZY SVR FUNCTIONS ARCHITECTURE FOR TIME SERIES FORECASTING[1]

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Abstract: This paper introduces a distance-aware forecasting framework based on type-1 fuzzy support vector regression functions, designed to examine the impact of distance metric selection on fuzzy time series prediction performance. The proposed approach extends fuzzification mechanisms by incorporating nine alternative distance functions into a fuzzy clustering-driven regression structure. These distance measures determine the formation of fuzzy partitions and directly influence the weighting scheme used in cluster-specific regression models. A unified optimization strategy is employed to jointly tune both fuzzification and regression components using a grid search procedure. This integrated optimization allows the framework to dynamically adapt to varying data distributions, temporal dependencies, and structural characteristics of time series data. The proposed method is validated through extensive experiments conducted on four real-world datasets from production and financial domains. The empirical findings reveal that distance metric choice significantly affects forecasting accuracy. In many experimental scenarios, non-Euclidean distance measures outperform classical Euclidean-based metrics. Distance functions emphasizing directional similarity, scale normalization, or maximum deviation frequently produce superior results. Chebyshev, standardized Euclidean, and Cosine distances achieve the lowest prediction errors across several datasets and evaluation periods. The results indicate that no single distance metric is universally optimal, underscoring the importance of metric flexibility and data-aware selection. The study demonstrates that integrating distance metric optimization into type-1 fuzzy support vector regression function-based architectures leads to consistent improvements in forecasting accuracy. The proposed architecture offers a generalizable solution for time series forecasting.

Keywords: Fuzzy Time Series Forecasting, Distance Metric Selection, Type-1 Fuzzy Regression Functions, Support Vector Regression, Fuzzy C-Means Clustering

JEL Code: C52, C53, C61, C63, C45

ÖKLİDİN ÖTESİNDE: ZAMAN SERİSİ ÖNGÖRÜSÜ İÇİN METRİK-OPTİMİZE TİP-1 BULANIK DVR FONKSİYONLARI MİMARİSİ

Özet: Bu çalışma, bulanık zaman serisi tahmin performansı üzerinde uzaklık metriği seçiminin etkisini incelemek amacıyla, tip-1 bulanık destek vektör regresyon fonksiyonlarına dayalı, uzaklık metriklerine duyarlı bir tahmin çerçevesi önermektedir. Önerilen yaklaşım, bulanıklaştırma mekanizmalarını genişleterek, dokuz farklı uzaklık fonksiyonunu bulanık kümeleme temelli bir regresyon yapısına entegre etmektedir. Bu uzaklık metrikleri, bulanık bölümlerin oluşumunu belirlemekte ve kümeye özgü regresyon modellerinde kullanılan ağırlıklandırma şemasını doğrudan etkilemektedir. Bulanıklaştırma ve regresyon bileşenlerinin birlikte eniyilenmesi amacıyla, ızgara araması temelli birleşik bir optimizasyon stratejisi uygulanmıştır. Bu bütünleşik optimizasyon süreci, önerilen yapının farklı veri dağılımlarına, zamansal bağımlılıklara ve zaman serisi verilerinin yapısal özelliklerine dinamik biçimde uyum sağlamasına olanak tanımaktadır. Önerilen yöntem, üretim ve finans alanlarını temsil eden dört farklı zamansal davranış özelliklerine sahip gerçek hayat veri setleri üzerinde gerçekleştirilerek doğrulanmıştır. Bulgular, uzaklık metriği seçiminin tahmin doğruluğu üzerinde belirleyici bir etkiye sahip olduğunu ortaya koymaktadır. Birçok senaryoda, Öklid dışındaki uzaklık metriklerinin, Öklid tabanlı metriklere kıyasla daha başarılı sonuçlar ürettiği gözlemlenmiştir. Yönlülüğe duyarlı, ölçek normalizasyonunu dikkate alan ve maksimum

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sapmayı vurgulayan uzaklık fonksiyonları daha düşük tahmin hataları sağlamaktadır. Çebişev, standartlaştırılmış Öklid ve Kosinüs uzaklıkları, farklı veri kümeleri ve değerlendirme dönemleri boyunca en düşük tahmin hatalarını elde etmiştir. Elde edilen sonuçlar, tek bir uzaklık metriğinin tüm durumlar için evrensel olarak en iyi seçenek olmadığını göstermekte ve metrik esnekliği ile veri odaklı metrik seçiminin önemini ortaya koymaktadır. Çalışma, uzaklık metriği optimizasyonunun tip-1 bulanık destek vektör regresyon fonksiyonları tabanlı mimarilere entegre edilmesinin tahmin doğruluğunda tutarlı iyileşmeler sağladığını göstermekte ve zaman serisi tahmini için genellenebilir bir çözüm sunmaktadır.

Anahtar Kelimeler: Bulanık Zaman Serisi Öngörüsü, Uzaklık Metriği Seçimi, Tip-1 Bulanık Regresyon Fonksiyonları, Destek Vektör Regresyonu, Bulanık C-Ortalamalar Kümeleme

JEL Kodu: C52, C53, C61, C63, C45

Introduction

Introduced by Zadeh (1965), fuzzy set theory offers a foundational framework for managing uncertainty in complex systems, including time series. While traditional forecasting models are analytically robust, they often depend on rigid statistical assumptions that fail in noisy or incomplete real-world scenarios. In contrast, fuzzy logic-based approaches -particularly Mamdani and Assilian (1975) and Takagi and Sugeno (1985) type fuzzy inference systems (FISs)- effectively model vagueness and ambiguity using membership functions. However, their reliance on manually constructed rules poses scalability and complexity limitations. To address these limitations, type-1 fuzzy functions (T1FFs), also referred to as type-1 fuzzy regression functions (T1FRFs) in some literature, have been proposed by Turksen (2008) as a flexible, rule-free alternative capable of capturing nonlinearities and uncertainty without relying on explicit rule definitions. These functions have been incorporated into hybrid frameworks, including autoregressive models, evolutionary algorithms, and more recently, support vector regression (SVR), to enhance forecasting accuracy and adaptability.

In T1FRFs-based frameworks, fuzzification -transforming numerical inputs into fuzzy representations- is commonly performed using fuzzy c-means (FCM) clustering. Here, the choice of distance metric becomes a key modeling decision, as it defines how similarity is measured in the input space. While classical metrics such as Euclidean and Mahalanobis remain prevalent, recent studies increasingly explore context-aware distances tailored to data dimensionality and structure. Despite recent methodological advancements, a significant research gap remains: few studies offer a systematic, comparative evaluation of distance metrics within unified fuzzy regression pipelines. This study addresses that gap by proposing a novel architecture -Distance Metric-based Type-1 Fuzzy Support Vector Regression Functions (DM-T1FSVRFs)- that evaluates the performance of nine distance metrics in a T1FRF-SVR forecasting context. A grid search procedure jointly optimizes both the clustering metric and SVR parameters to enhance prediction accuracy under uncertainty. As real-world time series data become increasingly complex and diverse, the integration of fuzzy logic with advanced clustering and regression techniques presents a promising, though underexplored, research avenue. The proposed DM-T1FSVRFs framework aims to fill this gap by examining how different distance metrics influence fuzzy-SVR forecasting accuracy. To that end, the model is empirically tested across four distinct datasets having varied dynamic behaviors, including trend shifts, volatility, and diverse temporal characteristics. The remainder of this paper is structured as follows: the *Proposed Method* section outlines the methodology, including the algorithmic design and hyperparameter optimization procedure. The *Results and Discussion* section presents the experimental setup and findings. Finally, the *Conclusion* section summarizes the study and suggests directions for future research.

To contextualize and further motivate our approach, we examine relevant literature, with particular focus on fuzzy time series forecasting models and the role of distance metrics in clustering-based frameworks.

Fuzzy time series (FTS) models were originally introduced by Song and Chissom (1993a, 1993b, 1994) as a mathematical framework for modeling time-dependent data that exhibit imprecision and linguistic vagueness -characteristics commonly encountered in real-world temporal datasets. Their pioneering work laid the foundation for a new paradigm distinct from traditional statistical time series models, particularly in its ability to accommodate qualitative information through fuzzy set theory. This innovation allowed FTS models to circumvent restrictive assumptions such as linearity or stationarity. Building upon this foundation, Chen (1996) proposed a more computationally efficient version of the original model by introducing arithmetic operations to define fuzzy logical relationships (FLRs), eliminating the need for complex matrix calculations. This simplified structure became the de facto standard and catalysed a wave of research focused on extending and refining the FTS methodology. Chen's later works (2002) expanded the use of linguistic variable definitions and partitioning techniques, influencing most subsequent model formulations.

Over the past three decades, a wide variety of FTS model variants have been developed to address various forecasting challenges. Structural advancements include high-order FTS models (Aladag et al. (2009); Cagcag Yolcu et al. (2016)), multivariate FTS (Chen & Tanuwijaya, 2011), and probabilistic or interval-based fuzzy models. These approaches aim to increase the expressiveness of the models, enhance pattern recognition, and accommodate multiple input variables or uncertainty in a more granular way. Notable contributions in these areas come from researchers such as Aladag, Egrioglu, Yolcu, and Uslu, who proposed high-order and adaptive FTS frameworks integrating neural networks, fuzzy clustering (e.g., fuzzy c-means, Gustafson–Kessel), and nonlinear mapping functions (Cagcag Yolcu et al. (2016); Egrioglu et al. (2013)). In parallel, a growing body of work has focused on the hybridization of FTS models with computational intelligence techniques. Adaptive Neuro-Fuzzy Inference Systems (ANFIS) (Egrioglu et al. (2014); Sharma et al. (2024); Wan & Si (2017)), support vector machines (SVM) (Baser & Demirhan (2017); Chen & Kao (2013); Luo et al. (2023)), evolutionary algorithms (Aladag et al. (2014); Aladag et al. (2012); Bas et al. (2014); Bas et al. (2020); Chen & Kao (2013); Egrioglu et al. (2019); Yolcu et al. (2014)), and more recently, deep learning structures such as convolutional neural networks (Sadaei et al., 2019) and long short-term memory (LSTM) networks (Kocak et al., 2021) have been combined with fuzzy frameworks to improve performance. These hybrid models address common challenges such as overfitting, data sparsity, and loss of interpretability, particularly in high-dimensional or non-stationary contexts. Castillo and Melin (2020), for example, integrated fuzzy logic with fractal dimension analysis to forecast COVID-19 time series, illustrating the practical relevance of such hybrids in epidemiological modeling.

Recent developments also point to an increasing interest in advanced fuzzy set structures. Interval type-2 fuzzy systems, intuitionistic fuzzy sets (Luo et al., 2019), and graph-theoretic modeling approaches such as visibility graphs (Zhang et al., 2017) have enabled richer modeling of uncertainty and interdependencies. In line with these trends, probabilistic extensions such as non-stationary fuzzy time series proposed by De Lima E Silva et al. (2020) are capable of adapting to shifts in data distributions over time. These models offer enhanced robustness and predictive power in volatile environments, such as financial markets and energy systems. Several researchers have also proposed algorithmic enhancements for key components of FTS modeling, such as universe of discourse partitioning, fuzzification, FLR generation, and defuzzification. For example, Singh and Dhiman (2018) employed granular computing and bio-inspired optimization to fine-tune fuzzy intervals, while Pattanayak et al. (2021) leveraged probabilistic intuitionistic fuzzy sets to better model uncertainty in high-order forecasting tasks. These strategies are often accompanied by extensive benchmark testing against traditional FTS models using datasets like TAIEX and University of Alabama enrolment series.

In a recent bibliometric review, Palomero et al. (2022) analyzed 118 peer-reviewed studies published between 2017 and 2021. Their findings highlight several notable trends: (*i*) the rapid hybridization of fuzzy systems with machine learning and optimization tools; (*ii*) the expansion of FTS applications into domains such as air quality monitoring, energy demand, and environmental modeling; and (*iii*) a shift toward interpretable and scalable forecasting systems. This review identified Guimaraes, Sadaei, Singh, and Egrioglu among the most influential contributors in the field. Scalability and real-time adaptability have become central concerns in modern FTS research. In this context, libraries such as pyFTS (Lucas et al., 2022) have emerged, offering modular and extensible Python implementations of classical and advanced FTS models, including support for probabilistic, multivariate, and interval forecasting. Similarly, systems such as multiple input multiple output fuzzy aggregation models with neural networks (Soto et al., 2019) provide robust architectures for multiple time series prediction problems.

Comprehensive reviews by Bose and Mali (2019), Panigrahi and Behera (2020), and Ojha et al. (2019) reinforce the multidimensional nature of contemporary FTS research, revealing a convergence of statistical modeling, fuzzy logic, artificial intelligence, and optimization. These contributions underscore the importance of model interpretability, accuracy, and domain-specific customization in real-world forecasting applications. In conclusion, the FTS research landscape has matured significantly, evolving from simple linguistic-based models to complex hybrid systems that offer both high predictive accuracy and interpretability. With ongoing advancements in computational intelligence and data analytics, fuzzy time series models continue to hold promise for a wide range of applications where data uncertainty, linguistic ambiguity, and nonlinear dynamics are prevalent.

Building upon early rule-based FISs, function-based modeling approaches such as T1FRFs, first introduced by Turksen (2008), have emerged as scalable and interpretable alternatives. These approaches bypass the need for expert-defined rules by directly modeling the relationship between input variables and outputs using least squares estimation on fuzzy membership values. Notably, these functions can include nonlinear transformations of membership degrees and input variables to capture complex dependencies. T1FRFs have been successfully applied to time series forecasting tasks in domains such as energy consumption, stock market prediction, and meteorological forecasting (Aladag et al. (2014); Dalar et al. (2015)). These studies showed that even basic implementations -using FCM (Bezdek et al., 1984) and linear regression- outperform conventional rule-based fuzzy systems. (Beyhan & Alci, 2010) applied T1FRF-based models to autoregressive exogenous forecasting structures, demonstrating their effectiveness in capturing external influences in time series. Zarandi et al. (2013) enhanced fuzzy function modeling by integrating Imperialist Competitive Algorithm optimization, improving convergence in complex forecasting scenarios. Bas et al. (2021) further demonstrated the forecasting potential of fuzzy function-based models by integrating modified clustering techniques tailored to diverse real-world time series datasets. Extensions using ridge regression (Bas et al., 2019) and elastic net regularization (Tak & Inan, 2022) have been proposed to address multicollinearity that arises from highly correlated nonlinear transformations of fuzzy inputs.

More recent work has led to a proliferation of structurally enriched T1FRF variants. Recurrent fuzzy functions (R-T1FFs), proposed by Tak (2020b), integrate autoregressive and moving average (ARMA) components into the regression model, using metaheuristic optimization algorithms such as particle swarm optimization and grey wolf optimizer. These models dynamically update residual terms and have demonstrated superior performance on financial datasets with regime changes. Additionally, Tak (2022) proposed ARMA-type Possibilistic Fuzzy Functions (ARMA-PFFs), integrating autoregressive structures with possibilistic clustering to better handle uncertainty in financial and environmental time series. A recent advancement proposed by Egrioglu et al. (2024) combines fuzzy regression functions with Gaussian Process Regression, achieving enhanced performance under high-uncertainty conditions. Ensemble-based and aggregated approaches have also emerged. Tak (2018) introduced Meta Fuzzy Functions (MFFs), which leverage fuzzy clustering to combine multiple T1FRF variants with varying parameterizations. This ensemble methodology has proven especially effective in volatile or noisy data conditions. In addition, Dalar and Egrioglu (2018) introduced a bootstrapped fuzzy regression function framework, aiming to enhance forecast reliability by generating multiple resampled fuzzy models and aggregating their outputs. Similarly, Dalar and Egrioglu (2025) presented hybrid models combining T1FRFs with statistical bootstrapping to produce both point forecasts and confidence intervals.

In addition to structural innovations, recent studies have tackled robustness under outlier contamination. Modified fuzzy regression functions with noise clusters (Chakravarty et al., 2022) and robust intuitionistic fuzzy regression functions using Welsch and Huber estimators (Egrioglu & Bas, 2023) show that fuzzy systems can maintain accuracy in the presence of extreme data. These methods are particularly beneficial in environmental and financial forecasting contexts where data irregularities are common. Further addressing robustness issues, Bas (2022) incorporated robust regression loss functions -such as Andrews, Bisquare, and Cauchy estimators- into the fuzzy function construction process, demonstrating improved resistance against outliers. Similarly, Khammar et al. (2020) proposed a novel distance-based fuzzy regression framework tailored for LR-type fuzzy numbers (i.e., fuzzy numbers defined by separate left and right shape functions), providing enhanced robustness under noisy environments. Moreover, the clustering component of T1FRFs has evolved beyond traditional fuzzy c-means. Gustafson–Kessel clustering has been used to accommodate elliptical data distributions (Bas & Egrioglu, 2022), and possibilistic clustering (Tak, 2020a) has been incorporated to improve model resilience against overlapping clusters and noise.

Finally, hybrid architectures that combine T1FRFs with neural networks or deep learning layers -such as the fuzzy regression network functions (Aktoprak & Cagcag Yolcu, 2025) and intuitionistic fuzzy regression functions based on elastic net regularization (IFRFs-bENR) with LSTM (Cagcag Yolcu & Yolcu, 2024)- have demonstrated that fuzzy logic can effectively complement modern learning paradigms, especially when applied to high-variance, nonlinear time series. Demirhan and Baser (2024) developed a hierarchical fuzzy regression, where the clustering structure dynamically evolves based on data density, offering improved flexibility in nonstationary time series. While significant strides have been made in function-based fuzzy forecasting models, the effectiveness of these models fundamentally depends on the quality of the underlying fuzzy partitions. This, in turn, is strongly influenced by the distance metrics employed during the fuzzification stage, prompting growing attention towards metric selection in recent research.

Fuzzy clustering -particularly via the FCM algorithm (Bezdek et al., 1984)- is central to converting raw data into fuzzy representations. The distance metric used in this process plays a critical role in defining membership degrees, thereby shaping the resulting model structure. While early studies primarily relied on Euclidean distance, more recent work (e.g., López-Oriona et al., (2023)) has explored ordinal-sensitive and time-aligned metrics. Despite these advancements, many forecasting models still rely on default distances without systematic comparison. While Euclidean distance has traditionally dominated fuzzy clustering applications due to its simplicity and analytical tractability, emerging studies increasingly emphasize the limitations of this metric in capturing anisotropic, skewed, or magnitude-variant data structures. As a result, alternative distances such as Cosine (direction-sensitive), Chebyshev (maximum-deviation-sensitive), and Mahalanobis (covariance-aware) have gained prominence for their ability to encode richer notions of similarity aligned with domain-specific requirements.

Beyond classical metrics, several studies have investigated the influence of distance functions on fuzzy clustering performance. Bezdek et al. (1984) provided early foundations for evaluating the role of different metrics in cluster shape and separability. Yang et al. (2005) emphasized that non-Euclidean metrics could enhance clustering outcomes, especially in high-dimensional datasets. Kumar et al. (2014) systematically compared Euclidean, Manhattan, and Minkowski distances within FCM algorithms, demonstrating that distance choice critically affects clustering accuracy. Among the comparative studies on distance metrics in FCM, Arora et al. (2019) provide a systematic analysis of Euclidean, Mahalanobis, Minkowski, and Chebyshev distances using synthetic and real datasets. Their findings validate that the choice of distance measure significantly alters clustering behavior, iteration counts, and misclassification rates. Building upon these insights, Lin et al. (2013) proposed a flexible hybrid metric combining multiple distance measures to better accommodate varying data structures. Similarly, Zhao et al. (2015) compared Euclidean and Mahalanobis metrics in fuzzy clustering and highlighted metric suitability depending on data distribution characteristics. In non-traditional domains, Velez-Falconi et al. (2020) assessed the effectiveness of multiple distance measures -including Canberra and Spearman distances- in image segmentation tasks, further underlining the need for context-aware distance selection. Finally, Thrun (2021) discussed intrinsic biases in clustering algorithms, emphasizing that metric choice plays a critical role that cannot be mitigated solely by global optimization strategies. These findings collectively support the argument that metric selection should be treated as a fundamental modeling decision, rather than a default assumption. Moreover, Bas and Egrioglu (2022) employed Gustafson–Kessel clustering within the T1FRF framework, enabling adaptive cluster shapes to better capture anisotropic data structures during the fuzzification stage. Given the foundational role of clustering accuracy in fuzzy modeling, the integration of fuzzy representations with advanced regression models such as SVR demands careful consideration of fuzzification strategies, particularly the choice of distance metrics.

Support vector regression (Drucker et al. (1996); Vapnik (2000)) offers robust capabilities for modeling nonlinear relationships using kernel functions. Early hybridizations of fuzzy systems and support vector regression, such as the work by Celikyilmaz and Turksen (2007), highlighted the synergy between FISs and kernel-based learning, paving the way for later fuzzy-SVR integrations. Recent studies have combined SVR with T1FRF-based frameworks to enhance performance in high-uncertainty environments. For instance, Baser and Demirhan (2017) demonstrated that fuzzy functions combined with SVR outperform rule-based fuzzy systems in modeling solar radiation. Their study also emphasized that kernel function selection critically influences model performance, particularly when combined with fuzzy membership-based input transformations. Nevertheless, most existing approaches focus predominantly on kernel selection and SVR hyperparameter tuning, often overlooking the critical influence of the fuzzification stage -especially the distance metric employed during clustering- on overall model performance. Despite advancements in both clustering and regression components, there remains a notable gap in systematically evaluating how different distance metrics affect the performance of unified fuzzy-SVR frameworks.

Although distance metrics are known to significantly influence both fuzzy clustering and regression performance, comparative studies in this area remain limited. Few works -such as Tak and Inan (2022)- have investigated the impact of distance-induced multicollinearity in fuzzy regressors. To date, no prior study has systematically compared multiple distance metrics within a unified fuzzy-SVR framework with integrated hyperparameter optimization. This gap motivates the current study, which aims to evaluate nine distance metrics across diverse datasets and temporal contexts. Building upon the methodological and conceptual gaps identified in the literature, this section outlines the proposed approach, termed Distance Metric-based Type-1 Fuzzy Support Vector Regression Functions. The method integrates a fuzzy clustering stage enhanced with multiple distance metrics and a support vector regression model optimized via grid search. The algorithmic structure, clustering mechanism, distance computations, and regression architecture are discussed in detail below.

Material and Methods

This study introduces a novel forecasting framework, termed Distance Metric-based Type-1 Fuzzy Support Vector Regression Functions. The core idea is to enhance type-1 fuzzy SVR models by incorporating nine distinct distance metrics into the fuzzification process. These include Euclidean, Squared Euclidean (Sqeuclidean), Standardized Euclidean (Seuclidean), City block (Manhattan), Minkowski (with *p*=3), Chebyshev, Cosine, Canberra, and Mahalanobis.

The methodology consists of three primary stages: (i) fuzzification of lagged input vectors using distance-augmented fuzzy c-means (FCM), (*ii*) training SVR models for each fuzzy cluster, and (*iii*) aggregating cluster-specific forecasts using fuzzy membership weights. Hyperparameters—including the distance metric and SVR configuration—are tuned through grid search based on validation performance.

FCM clustering with multiple distance metrics

FCM is employed to partition the lagged input vectors into C fuzzy clusters. Each data point $x_i \in \mathbb{R}^d$ is associated with a membership value u_{ik}, representing its degree of belonging to cluster k. These values are normalized such that $\sum_{k=1}^{C} u_{ik} = 1$. The FCM objective function minimized is:

$$J_m(U, C) = \sum_{i=1}^{N} \sum_{k=1}^{C} u_{ik}^m d_{ik}^2 \ , \quad U \in \mathbb{R}^{N \times C}, \tag{1}$$

where N is the total number of data points, C is the number of clusters, u_{ik} denotes the degree of membership of data point x_i in cluster k, $m > 1$ is the fuzzification coefficient, and d_{ik} represents the distance between x_i and cluster center c_k, computed using one of the selected metrics. Cluster centers are iteratively updated based on the weighted mean of inputs, and membership degrees are recalculated accordingly (Bezdek et al., 1984). Cluster centers are iteratively updated using the weighted mean of the inputs:

$$c_k = \frac{\sum_{i=1}^{N} u_{ik}^m x_i}{\sum_{i=1}^{N} u_{ik}^m}, \quad k = 1, 2, \dots, C. \tag{2}$$

Membership degrees are recalculated after each iteration:

$$u_{ik} = \frac{1}{\sum_{j=1}^{C} \left(\frac{d_{ik}}{d_{ij}}\right)^{\frac{2}{m-1}}}, \quad i = 1, 2, \dots, N; \ k = 1, 2, \dots, C. \tag{3}$$

An α-cut threshold α is applied to suppress low-confidence memberships. Any $u_{ik} < \alpha$ is set to zero to sharpen cluster boundaries. This promotes the emergence of more distinct and interpretable clusters.

$$u_{ik}^{new} = \begin{cases} u_{ik}, & if\ u_{ik} \geq \alpha \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

The practical implications of using diverse distance metrics in FCM clustering, as discussed by Arora et al. (2019), further support the inclusion of non-Euclidean functions. Their empirical benchmarks reveal that metrics like Chebyshev and Minkowski often outperform Euclidean distance under conditions of asymmetry or varying cluster volumes.

Each distance metric is formally defined through its respective function (see Equations 5–13), encompassing concepts such as angular similarity (Cosine), scale normalization (Standardized Euclidean), and statistical dispersion (Mahalanobis). The distance d_{ik} is computed using one of the following metrics:

$$d_{ik}^{(\text{euclidean})} = \sqrt{\sum(x_i - c_k)^2}, \tag{5}$$

$$d_{ik}^{(\text{minkowski})} = \left(\sum |x_i - c_k|^p\right)^{\frac{1}{p}}, \tag{6}$$

$$d_{ik}^{(\text{manhattan})} = \sum|x_i - c_k|, \tag{7}$$

$$d_{ik}^{(\text{mahalanobis})} = \sqrt{(x_i - c_k)^T S^{-1}(x_i - c_k)}, \text{ where } S \text{ is the covariance matrix,} \tag{8}$$

$$d_{ik}^{(\text{chebyshev})} = \max_j |x_{ij} - c_{kj}|, \tag{9}$$

$$d_{ik}^{(\text{cosine})} = 1 - \frac{x_i \cdot c_k}{\|x_i\|\|c_k\|}, \tag{10}$$

$$d_{ik}^{(\text{canberra})} = \sum \frac{|x_i - c_k|}{|x_i| + |c_k|}, \tag{11}$$

$$d_{ik}^{(sq\text{euclidean})} = \sum(x_i - c_k)^2, \tag{12}$$

$$d_{ik}^{(\text{seuclidean})} = \sqrt{\sum \frac{(x_i - c_k)^2}{v}}, \text{ where } v \text{ is the variable} - \text{wise variance} \tag{13}$$

This distance-flexible fuzzification approach makes it possible to assess how different spatial assumptions (e.g., angular similarity, scale-normalized dispersion, variance sensitivity) influence the clustering phase and, by extension, the forecasting model's downstream behavior.

The nine-distance metrics were selected to represent complementary notions of similarity commonly used in clustering applications while remaining fully compatible with the standard fuzzy c-means objective on lagged numerical vectors. Euclidean, squared Euclidean, Manhattan, Minkowski, and Chebyshev distances cover different norm-based behaviors; standardized Euclidean accounts for scale differences across variables; Cosine captures directional similarity; Canberra emphasizes relative component-wise deviations; and Mahalanobis incorporates covariance structure. More specialized sequence-alignment or rank-based distances were not included because their use would require a modified clustering framework or additional modeling assumptions beyond the scope of the present comparative study.

Support vector regression

The SVR is a supervised learning technique that extends SVM to regression problems (Drucker et al., 1996; Vapnik, 2000). It aims to find a function that approximates the target values within a specified error tolerance ϵ, while minimizing model complexity.

In the proposed DM-T1FSVRFs architecture, a separate SVR model is trained for each fuzzy cluster identified during the FCM stage. The regression function takes the following form:

$$f(X) = w^T \emptyset(X) + b, \tag{14}$$

where X is the input feature vector, $\emptyset(X)$ is a nonlinear transformation function (kernel), w is the weight vector, and b is the bias term. The training objective is to minimize the regularized risk function:

$$\min_{w,b,\xi,\xi^*} \frac{1}{2}\|w\|^2 + C_{\text{SVR}} \sum_{i=1}^{N} (\xi_i + \xi_i^*) \tag{15}$$

$$\text{subject to the constraints: } \begin{cases} y_i - f(x_i) \leq \epsilon + \xi_i\,, \\ f(x_i) - y_i \leq \epsilon + \xi_i^*, \\ \quad \xi_i, \xi_i^* \geq 0, \end{cases} \tag{16}$$

where y_i is the observed value, ξ_i , ξ_i^* are slack variables allowing margin violations, ϵ defines the permissible error threshold, and C_{SVR} is the regularization parameter that balances the trade-off between model complexity and prediction error. To capture nonlinearity in the data, the following kernel functions are considered:

- Linear kernel:

$$K(X_i, X_j) = X_i^T X_j. \tag{17}$$

- Radial Basis Function (RBF) kernel:

$$K(X_i, X_j) = \exp(-\gamma \|X_i - X_j\|^2). \tag{18}$$

- Polynomial kernel:

$$K(X_i, X_j) = (\gamma X_i^T X_j + r)^d. \tag{19}$$

- Sigmoid kernel:

$$K(X_i, X_j) = \tanh(\gamma X_i^T X_j + r). \tag{20}$$

The kernel and its relevant hyperparameters—γ (when applicable) and ϵ—were selected through a grid search process using validation data. This ensures that each SVR model is optimally tuned for the corresponding cluster's data structure. This cluster-specific SVR approach allows the overall model to account for heterogeneity in the data and tailor the regression functions to the localized patterns revealed through fuzzy clustering.

The Proposed method

The proposed DM-T1FSVRFs framework consists of seven sequential stages designed to transform raw time series data into accurate and adaptable forecasts. The entire pipeline integrates fuzzification, regression, and aggregation processes while optimizing key hyperparameters via validation feedback. An algorithm for the proposed DM-T1FSVRFs can be given step-by-step as follows.

Step 1. *Data partitioning*: Each time series dataset is divided into three subsets: training (x_{train}), validation (x_{val}), and test (x_{test}) sets. The last n_{test} observations are allocated to the test set, the preceding n_{test} to validation, and the remaining to training ($x_{train} = x[: n - (2 \times n_{test})]$, $x_{val} = x[n - (2 \times n_{test}) + 1 : n - n_{test}]$, $x_{test} = x[n - n_{test} + 1 :]$). This chronological split ensures temporal consistency and prevents information leakage.

Step 2. *Data standardization*: All data subsets are normalized using the mean and standard deviation of the training set:

$$x' = \frac{x - \mu_{train}}{\sigma_{train}}. \tag{21}$$

This ensures that information from the validation and test sets is not used in the normalization process, thereby preventing data leakage.

Step 3: *Generation of lagged features*: To capture the temporal dependencies in the data, n_l lagged versions of the time series are created. For validation and test sets, historical values from the training and validation sets respectively are used to maintain continuity.

Step 4: *Fuzzy clustering*: FCM clustering is applied to the lagged training data using a pre-defined number of clusters C and a selected distance metric. An α-cut threshold is employed to prune low-membership values, enhancing cluster interpretability. The resulting membership degrees are used to fuzzify the validation and test sets with respect to the same cluster centers.

Step 5: *SVR training per cluster*: For each cluster k, an individual SVR model is trained using the following feature matrix:

$$X = [1,\ u_{ik},\ \sqrt{u_{ik}},\ u_{ik}^2,\ e^{u_{ik}},\ \text{lagged features}].$$

A separate SVR function $f_k(X)$ is learned for each cluster, capturing the local data dynamics within that cluster.

Step 6: *Prediction aggregation*: The final prediction $\hat{y}_i$ for each data point is obtained by weighting the outputs of cluster-specific SVR models by their respective membership degrees:

$$\hat{y}_i = \frac{\sum_{k=1}^{C} u_{ik} f_k(X)}{\sum_{k=1}^{C} u_{ik}}. \tag{22}$$

This aggregation ensures that the prediction reflects the degree of association of the data point to each fuzzy region.

Step 7: *Performance evaluation*: Predictions are transformed back to the original scale ($\hat{y}_i^{original} = \hat{y}_i \times \sigma_{train} + \mu_{train}$).

Performance is evaluated using root mean squared error (RMSE) and mean absolute percentage error (MAPE):

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (y_i - \hat{y}_i)^2}\ ; \qquad \text{MAPE} = \frac{1}{N}\sum_{i=1}^{N} \left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100. \tag{23}$$

Validation set results are used for hyperparameter tuning, while test set results reflect the model's generalization capability on unseen data. To provide an overview of the proposed DM-T1FSVRFs framework, a workflow diagram is presented in Appendix Figure 1 (Figure A1). This flowchart illustrates the seven sequential steps involved in the model construction, including data preprocessing, fuzzy clustering with metric selection, SVR training, and prediction aggregation. While the *Results and Discussion* section details each step algorithmically, the diagram facilitates a more intuitive understanding of the full modeling pipeline.

As shown in Figure A1, the proposed framework integrates both metric-flexible fuzzification and localized regression modeling in a coherent and reproducible pipeline. In this section, we present the results of extensive experiments conducted across four real-world time series datasets: ABP, DJI, TAIEX, and VNI30. The performance of the model is analyzed in terms of forecasting accuracy, and the influence of distance metric selection is critically examined. The outcomes are discussed in comparison with benchmark models to evaluate the generalization capacity and practical effectiveness of the proposed framework.

Results and Discussion

Experimental setup summary

To evaluate the forecasting performance of the proposed DM-T1FSVRFs framework, experiments were conducted on four real-world time series datasets, each exhibiting distinct temporal and domain-specific characteristics:

(i) Australian Beer Production (ABP) – representing seasonal production patterns – the dataset was retrieved from the GitHub repository accompanying the book Practical Time Series Analysis (Packt Publishing, 2022);
(ii) Dow Jones Industrial Average (DJI) – reflecting financial market volatility – historical data were obtained from Investing.com (2024a);
(iii) Vietnam Ho Chi Minh Index (VNI30) – characterizing frontier market dynamics – the dataset was collected from Investing.com (2024b); and
(iv) Taiwan Stock Exchange Index (TAIEX) – typifying emerging market behavior – datasets were obtained from the official Taiwan Stock Exchange (TWSE, 2024).

The empirical evaluation covers four datasets with distinct temporal spans: ABP (March 1956 to June 1994), DJI (2010-2014), TAIEX (1999-2004), and VNI30 (2010-2013). Each dataset was chronologically split into training, validation, and test sets to maintain temporal consistency and avoid information leakage. A grid search procedure was employed to jointly optimize SVR parameters (C_{SVR}, ϵ, kernel type, γ) and fuzzification parameters (distance type, lag size, number of clusters, and α_{cut}). The best-performing configurations were compared with benchmark models reported in the literature. The explored SVR search space comprised C_{SVR} in {0.1000, 0.2154, 0.4641, 1.0000, 2.1544, 4.6416}, epsilon in {0.0020, 0.0126, 0.0796, 0.5024}, gamma in {0.0159, 0.0634, 0.2524}, and kernel type in {linear, radial basis function, polynomial, sigmoid}. The fuzzification search space covered the nine distance metrics, dataset-specific lag ranges, the number of clusters, and α_{cut} values, as summarized in Appendix Table 1 (Table A1). This addition supports full reproducibility while maintaining the narrative flow of the main text.

All implementations were conducted using Python version 3.10.6. The support vector regression component of the proposed framework was executed via the SVR class provided by the scikit-learn (Pedregosa et al., 2011) library. No additional machine learning libraries were used beyond standard Python packages.

Australian beer production

The Australian Beer Production dataset, which measures quarterly beer production in Australia in megalitres from March 1956 to June 1994, exhibits seasonal production behavior with moderate trend shifts, making it a suitable candidate for examining the sensitivity of fuzzy-SVR models to clustering distance metrics.

The forecasting results obtained for the ABP dataset are summarized in Table 1. Before presenting these results, it is important to clarify the benchmark models and their abbreviations. The model L&NL-ANN refers to the linear and nonlinear artificial neural network proposed by Yolcu et al. (2013), while T1FRFs represents the original type-1 fuzzy regression functions introduced by Turksen (2008). T1FFRR, developed by Bas et al. (2019), extends the original formulation by incorporating ridge regression to address multicollinearity. The T1PFFFs model is a possibilistic extension introduced by Tak (2020a) to enhance robustness in noisy environments, and T1GKFRFs, proposed by Bas and Egrioglu (2022), integrates Gustafson–Kessel clustering to better capture elliptical data distributions.

Table 1. Test-set RMSE values of DM-T1FSVRFs and benchmark models on the ABP dataset.

	Methods	**ABP**
Benchmarks	L&NL-ANN	18.7888
	T1FRFs	17.3926
	T1FFRR	17.0845
	T1PFFFs	15.1914
	T1GKFRFs	14.9780
The Proposed Method	Canberra distance	13.0652
	Chebyshev distance	13.1427
	Cosine distance	13.3134
	Euclidean distance	13.4257
	Mahalanobis distance	13.4291
	Manhattan distance	13.2670
	Minkowski (p=3) distance	**13.0063**
	Seuclidean distance	13.2336
	Sqeuclidean distance	13.2012

Table 1 summarizes the RMSE results obtained on the ABP dataset. Among the nine distance metrics, Minkowski (with p=3) achieved the best performance (RMSE = 13.0063), followed by Canberra and Chebyshev. These metrics, which emphasize proportional or maximum absolute differences, outperformed classical Euclidean-based measures. The corresponding SVR models predominantly used polynomial and sigmoid kernels, with lag sizes between 1 and 5 and cluster counts between 2 and 5—highlighting the importance of tuning distance-kernel interactions in seasonal datasets. Compared to benchmark models such as T1GKFRFs (14.9780), T1PFFFs (15.1914), and T1FRFs (17.3926), the proposed approach demonstrated significant improvements in prediction accuracy.

Dow Jones industrial average

The proposed DM-T1FSVRFs model was applied to the DJI dataset spanning five years (2010–2014). The forecasting performance of the proposed model on the DJI is detailed in Table 2. Before examining the numerical results, we briefly clarify the benchmark models included in the comparison. ANFIS-GP and ANFIS-SC represent grid partitioning and subtractive clustering variants of the ANFIS, originally proposed by Jang (1993) and Yager and Filev (1994), respectively. However, the comparative performance results used in this study were adopted from the evaluation conducted by Bas and Egrioglu (2022) and Egrioglu et al. (2014). MANFIS, introduced by Egrioglu et al. (2014), refers to a modified ANFIS model that integrates memory-based structures for time series.

Table 2. Test-set RMSE values of DM-T1FSVRFs and benchmark models on the DJI dataset.

	Methods	2010	2011	2012	2013	2014	Mean
Benchmarks	ANFIS-GP	39.7287	132.3766	93.4754	197.8908	177.7626	128.2468
	ANFIS-SC	30.1699	130.1214	102.3491	176.4183	202.5687	128.3255
	MANFIS	21.7519	123.5594	100.5147	95.2613	160.9774	100.4129
	T1FRFs	20.5995	123.9386	97.7848	92.0896	178.1782	102.5181
	T1PFFFs	20.0783	108.2084	95.3382	89.1043	167.1738	95.9806
	T1GKFRFs	19.7272	113.5598	96.3100	88.2134	157.8565	95.1334
The Proposed Method	Canberra	**18.1655**	**96.8242**	84.6957	79.5921	141.3813	84.1318
	Chebyshev	18.9680	97.2580	**84.0685**	79.0528	140.2762	83.9247
	Cosine	18.9751	96.8976	84.9845	**79.0306**	142.0796	84.3935
	Euclidean	18.2343	97.3266	84.9491	79.1169	140.1063	83.9466
	Mahalanobis	18.8307	97.5023	84.9141	79.4019	142.4574	84.6213
	Manhattan	18.8333	97.2301	84.7532	79.3650	142.1161	84.4595
	Minkowski (*p*=3)	18.3630	97.7854	84.8738	79.7806	141.5936	84.4793
	Seuclidean	18.3292	97.3256	84.0773	79.3254	**140.0016**	**83.8118**
	Sqeuclidean	18.2345	97.6995	84.2883	79.3884	141.3328	84.1887

Table 2 shows the annual RMSE values for the DJI. The best overall performance was achieved with Standardized Euclidean distance, which attained the lowest mean RMSE of 83.8118 and slightly outperformed Chebyshev (83.9247) and Euclidean (83.9466) distances. When considering yearly performance, Standardized Euclidean distance produced the lowest RMSE in 2014, whereas Canberra was the most effective for 2010. The Chebyshev distance achieved the best results in 2012, followed closely by Standardized Euclidean and Squared Euclidean distances. In 2011 and 2013, Canberra and Cosine distances, respectively, were also highly competitive, demonstrating the importance of aligning distance metrics with temporal patterns and yearly dynamics.

These outcomes emphasize that no single distance metric dominated across all years, reinforcing the significance of metric flexibility in the proposed fuzzy clustering framework. Moreover, the diverse optimal metrics across years suggest that financial time series forecasting benefits from tailored distance metric selection based on annual variability and market volatility. Top-performing SVRs used polynomial and sigmoid kernels with moderate lag sizes (2–5) and cluster counts (3–7). When benchmarked against other models, the proposed DM-T1FSVRFs method demonstrated clear superiority. Notably, it reduced the mean RMSE by more than 15 points compared to the best ANN-based alternative (MANFIS: 100.4129), underscoring its effectiveness in capturing complex patterns inherent in financial datasets.

To visually reinforce the numerical findings for the DJI, Figure 1 presents the mean RMSE performance of the proposed DM-T1FSVRFs framework alongside benchmark models. The figure highlights the pronounced performance gap between traditional fuzzy or neural approaches and distance metric-optimized configurations. To enable a more detailed comparison among the distance-based variants of the proposed method, a zoomed-in inset is included, illustrating the relatively narrow RMSE differences across competing distance metrics. Within these configurations, Chebyshev and Standardized Euclidean distances achieve the lowest mean RMSE values, demonstrating the effectiveness of non-Euclidean distance formulations in capturing the dynamics of financial time series.

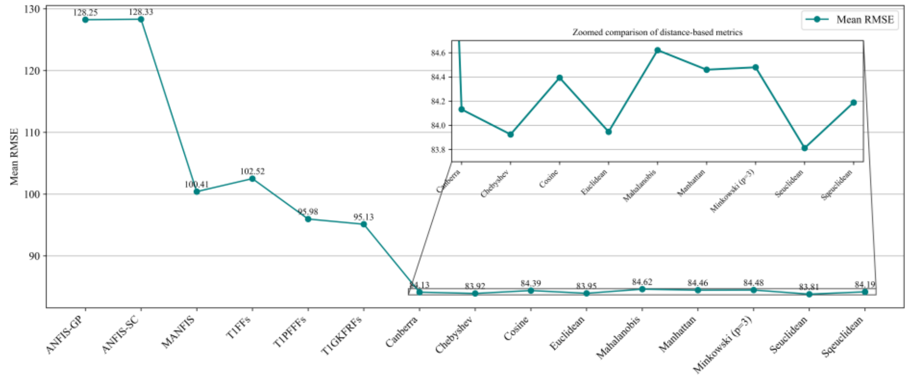

Figure 1. Mean RMSE comparison of the proposed DM-T1FSVRFs framework and benchmark models on the DJI.

Taiwan stock exchange index

The TAIEX dataset consists of yearly evaluation segments spanning the 1999-2004 period and represents an emerging-market financial series with substantial structural variation. Table 3 summarizes the RMSE values obtained from the proposed DM-T1FSVRFs framework using nine distance metrics across six consecutive years of the TAIEX. The values reported for Peng et al. (2015), Ye et al. (2016), and Chen and Jian (2017) correspond to fuzzy time series models incorporating rule-based structures with varying defuzzification techniques. Chen and Phuong (2017) propose a neural network-based hybrid framework integrating fuzzy time series. Tak et al. (2018) and Tak (2020b) focus on recurrent and intuitionistic fuzzy regression extensions to handle uncertainty and noise.

Table 3. Test-set RMSE values of DM-T1FSVRFs and benchmark models on the TAIEX data.

	Methods	1999	2000	2001	2002	2003	2004	Mean
Benchmarks	Peng et al. (2015)	92.1900	123.3300	116.7300	63.6600	50.9000	53.6300	83.4067
	Ye et al. (2016)	101.2900	125.4200	113.2200	63.9900	52.9900	52.4000	84.8850
	Chen and Jian (2017)	101.8200	128.9500	110.6600	60.4100	50.6500	52.8600	84.2250
	Chen and Phuong (2017)	99.9700	126.5900	110.1700	61.6200	53.0100	53.2800	84.1067
	Tak et al. (2018)	98.3300	128.1800	106.4800	65.1400	52.3800	53.7800	84.0483
	Bas et al. (2019)	99.1200	119.7300	113.1700	62.5500	48.7300	51.6600	82.4933
	Tak (2020b)	97.8100	122.2300	106.8100	64.2400	51.5000	52.7900	82.5633
	Tak (2020a)	-	-	113.3400	66.3900	53.1300	54.3000	71.7900
	Tak and Inan (2022)	96.0110	118.1430	103.6050	63.9280	54.3000	51.5950	81.2637
	Bas and Egrioglu (2022)	-	-	106.3700	65.8200	53.1500	51.1500	69.1225
The Proposed Method	Canberra	88.2782	103.1293	93.6903	52.2293	43.4711	45.9669	71.1275
	Chebyshev	88.2196	102.4604	94.9947	51.0465	43.2309	45.0408	70.8322
	Cosine	85.6952	103.1445	**92.0579**	51.4865	43.2471	**45.0227**	**70.1090**
	Euclidean	87.6922	101.1821	94.4805	50.7382	43.2562	45.7516	70.5168
	Mahalanobis	86.6240	103.8337	94.7571	51.8271	43.1585	45.6991	70.9833
	Manhattan	88.2596	102.3673	94.0212	50.8428	43.2215	45.7463	70.7431
	Minkowski (*p*=3)	**85.2122**	104.0742	94.9360	51.1132	43.2299	45.2587	70.6374
	Seuclidean	87.7129	101.5645	94.3957	**50.4266**	43.8332	45.7510	70.6140
	Sqeuclidean	86.5583	**100.0116**	92.2383	53.9739	**43.1298**	45.2894	70.2002

Table 3 indicates that Cosine distance yielded the best overall performance with a mean RMSE of 70.1090, outperforming other metrics such as Squared Euclidean (70.2002), Euclidean (70.5168), and Chebyshev (70.8322). Notably, Cosine distance also achieved the lowest RMSE for the years 2001 and 2004. Minkowski (p=3) demonstrated superior performance in 1999, while Squared Euclidean and Standardized Euclidean were optimal for

2000 and 2002, respectively. These outcomes underscore the importance of aligning the distance metric to the temporal and structural characteristics of each year within the dataset. The diversity of optimal metrics across years reveals that no single metric consistently dominates, further justifying the use of a metric-flexible fuzzy clustering framework. When compared to benchmark methods, including those proposed by Peng et al. (2015), Ye et al. (2016), Chen and Jian (2017), and Tak and Inan (2022), the proposed model demonstrates a clear advantage. The best-performing configuration achieves a mean RMSE improvement exceeding 10 points over the strongest literature benchmark (Tak and Inan (2022): 81.2637).

This substantial reduction in forecasting error validates the effectiveness of incorporating distance metric diversity in the fuzzification stage. It also confirms that the adaptability of the proposed DM-T1FSVRFs architecture—achieved through joint optimization of clustering and regression components—enables more precise modeling of complex, non-stationary financial series like TAIEX.

Vietnam Ho Chi Minh index

The VNI30 dataset covers the 2010-2013 period and provides a frontier-market financial series characterized by nonlinear and nonstationary behavior. Table 4 presents the RMSE values obtained from both benchmark models and the proposed DM-T1FSVRFs framework over four years of the VNI30 Index, along with their mean performance. ML-FF-ANN refers to a standard multilayer feedforward artificial neural network (Svozil et al., 1997). PS-ANN-ABC represents a hybrid model proposed by Egrioglu et al. (2019), combining a pi-sigma artificial neural network with training based on the artificial bee colony algorithm. The FTS-N model was introduced by Bas et al. (2015), incorporating fuzzy time series and neural network integration. PSO-SMN-ANN denotes a single multiplicative neural network trained using particle swarm optimization. Chen's model (1996) is a classical fuzzy time series forecasting method (C02) widely used for comparison.

Table 4. Test-set RMSE values of DM-T1FSVRFs and benchmark models on the VNI30 data.

	Methods	2010	2011	2012	2013	Mean
Benchmarks	ML-FF-ANN	4.4109	12.7627	3.4072	2.5316	5.7781
	PS-ANN-ABC	4.8138	34.9308	4.0435	2.4531	11.5603
	T1FRFs	4.0447	3.5254	3.6163	3.5883	3.6937
	C02	5.7849	6.0450	5.0223	4.7204	5.3932
	FTS-N	4.1072	3.5386	3.3799	2.8301	3.4640
	PSO-SMN-ANN	4.8178	21.5609	3.6322	3.0972	8.2770
	T1GKFRFs	3.3065	3.3911	3.2900	3.3768	3.3411
The Proposed Method	Canberra	2.8231	3.0713	2.8939	2.2153	2.7509
	Chebyshev	2.6435	2.9749	**2.8256**	2.1840	**2.6570**
	Cosine	**2.6149**	3.0249	2.9103	2.1805	2.6826
	Euclidean	2.6648	2.9646	2.9099	**2.1711**	2.6776
	Mahalanobis	2.9224	3.0654	2.8824	2.1898	2.7650
	Manhattan	2.7416	2.9781	2.8583	2.1931	2.6928
	Minkowski (p=3)	2.7160	2.9686	2.9575	2.1724	2.7036
	Seuclidean	2.6639	3.0087	2.9517	2.1907	2.7037
	Sqeuclidean	3.0592	**2.9595**	2.9762	2.1724	2.7918

As observed in Table 4, the proposed method consistently outperformed all benchmark models across all years. The lowest mean RMSE among the proposed methods was achieved using the Chebyshev distance, with a mean of 2.6570. It was closely followed by Euclidean (2.6776), Cosine (2.6826), and Manhattan (2.6928) metrics. Notably, Chebyshev yielded the best annual RMSE in 2012 (2.8256), and Squared Euclidean was optimal in 2011 (2.9595), although it ranked among the less favorable in mean performance. Across the four years, the top-performing metrics (Chebyshev, Euclidean, Cosine) were remarkably consistent in maintaining RMSE below 2.7, highlighting the effectiveness of these spatial assumptions in handling VNI30's nonlinear, nonstationary nature.

Compared to benchmark models such as ML-FF-ANN (5.7781), PS-ANN-ABC (11.5603), and even advanced fuzzy methods like T1FRFs (3.6937) and T1GKFRFs (3.3411), the proposed DM-T1FSVRFs framework led to substantial improvements, reducing the mean RMSE by more than 1 point.

These findings confirm that when optimized for the right distance metric, fuzzy-SVR systems can effectively model emerging and frontier markets, often outperforming more complex or hybrid alternatives. For the VNI30 dataset, the proposed DM-T1FSVRFs framework consistently outperforms benchmark models in terms of mean RMSE. Among the distance-based configurations, Chebyshev and Euclidean distance measures achieve the lowest mean RMSE values, with scores of 2.6570 and 2.6776, respectively, indicating stable and effective forecasting performance across evaluation periods. A detailed visual comparison of all benchmark and distance-based configurations for the VNI30 dataset is provided in Figure A2.

To further explore how each benchmark and proposed method performed across different years within the VNI30 dataset, Figure A3 presents year-wise RMSE distributions using radar plots. Each subplot corresponds to a specific year from 2010 to 2013, visually highlighting method-specific consistency and comparative ranking. Note that the RMSE values are presented on their original scales without normalization, allowing for a straightforward interpretation of absolute forecasting errors across methods. As illustrated in Figure A3, the proposed distance-metric-based configurations (especially those using Chebyshev, Euclidean, and Cosine metrics) consistently occupy the inner regions of the radar plots, indicating superior accuracy. Benchmark models, particularly ANN-based approaches, show wider and more irregular patterns, emphasizing their instability across years. These visual insights confirm the advantage of incorporating adaptive distance metrics in fuzzy-SVR frameworks, especially in structurally dynamic environments like frontier markets.

To better understand the configurations that led to the best forecasting performance, Table A2 summarizes the optimal hyperparameter combinations identified for each dataset and year. These include the selected distance metric, number of lagged inputs, cluster settings, and SVR kernel parameters. This consolidation provides a comprehensive view of how different distance-based clusterings interact with SVR tuning, and it lays the groundwork for the comparative analysis that follows. As shown in Table A2, the optimal configurations vary considerably across datasets and time periods, reaffirming the importance of context-aware model design. Notably, even simple distance functions such as Cosine or Chebyshev emerged as top performers in multiple settings, while more complex metrics like Mahalanobis appeared less frequently. These patterns are further examined in the following section, which provides a comparative analysis of the distance metrics used throughout the study.

To complement the quantitative error evaluations, we visualized the observed and forecasted values for one representative time window from each dataset in Figure 2. As shown in Figure 2, the proposed method closely tracks observed values across various patterns and scales, demonstrating reliable temporal generalization. The alignment between forecasted and actual series confirms the model's adaptability, particularly in settings with trend shifts and short-term volatility. To ensure transparency and completeness, visualizations of all 16 forecast scenarios—spanning each time series dataset and evaluation year—are provided in the Appendix Figures A4–A19.

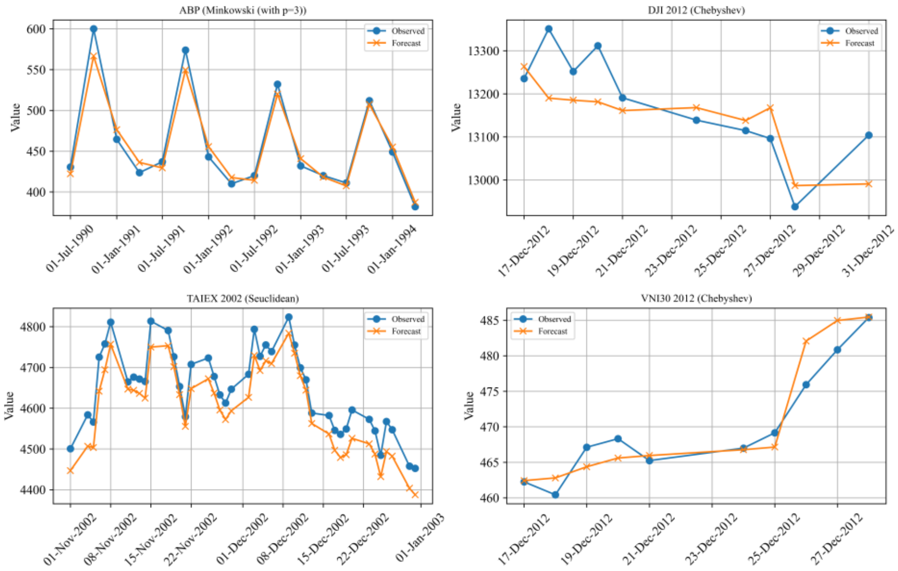

Figure 2. Forecast vs. observed values for the test period of four benchmark datasets.

Metric comparison and observations

A comparative analysis of the nine distance metrics integrated within the proposed DM-T1FSVRFs framework reveals insightful trends regarding their suitability across diverse time series structures. While no single metric emerged as universally optimal, consistent patterns were observed across datasets.

As presented in Appendix Table A2, Chebyshev, Euclidean, and Cosine distances frequently appeared among the top-performing configurations. Notably, Cosine distance, as shown in Table 3, achieved the best mean RMSE in the TAIEX dataset and delivered strong results across both financial and production series. This suggests that directional similarity—captured effectively by Cosine—may be particularly useful when modeling temporal dynamics where relative changes matter more than absolute values. Euclidean and Chebyshev distances, although simpler in formulation, consistently yielded stable performance. Chebyshev's sensitivity to the largest deviations made it especially effective in high-volatility datasets like VNI30. Similarly, Euclidean distance showed stable performance in datasets with moderate variance and trend continuity.

While Canberra distance showed promising results in TAIEX, it was less competitive in other datasets, indicating its potential in handling dimensional disparities. In contrast, Mahalanobis distance, despite accounting for covariance, rarely ranked among the best-performing metrics—likely due to the low dimensionality of input vectors, which limits the benefit of covariance-based scaling. The performance variability of metrics such as Squared Euclidean and Standardized Euclidean highlights the importance of dataset-specific metric selection. These metrics excelled in isolated years (e.g., DJI 2014, TAIEX 2000) but lacked consistency across all conditions. These findings underscore the necessity of adaptive and data-aware metric selection in fuzzy modeling.

Comparative evaluation with benchmarks

To further assess the predictive capabilities of the proposed DM-T1FSVRFs framework, we compared its results against a variety of benchmark models from the literature, including feedforward neural networks (ML-FF-ANN), evolutionary and hybrid ANN architectures (PS-ANN-ABC, PSO-SMN-ANN), and established fuzzy regression models (T1FRFs, T1GKFRFs, FTS-N). These models were previously validated across the same datasets, offering a reliable basis for performance comparison. The comparative results, detailed in Tables 1 through 4, demonstrate that the proposed method outperformed all benchmark approaches across all datasets in terms of mean RMSE. Specifically:

- On the ABP dataset, the best DM-T1FSVRFs configuration (Minkowski, RMSE = 13.0063) showed a clear advantage over T1GKFRFs (14.9780) and ANN-based models like L&NL-ANN (18.7888).
- For DJI, the lowest mean RMSE (83.8118) was achieved using Standardized Euclidean distance, significantly lower than T1FRFs (102.5181) and ANFIS variants (Mean RMSE≥128.2468).
- In the TAIEX dataset, Cosine distance yielded the best result (70.1090), markedly surpassing results from Peng et al. (2015) (83.4067), Ye et al. (2016) (84.8850), and Tak and Inan (2022) (81.2637).
- For VNI30, Chebyshev distance achieved the lowest mean RMSE (2.6570), outperforming not only other distances but also advanced benchmarks such as T1GKFRFs (3.3411) and FTS-N (3.4640).

These improvements are attributable to two main factors: (*i*) the integration of multiple distance metrics in the fuzzification phase, which allows for more context-appropriate similarity assessments, and (*ii*) the dynamic tuning of SVR parameters per fuzzy cluster, enabling better local adaptation to nonstationary data patterns.

While ANN-based models often require large training datasets and suffer from overfitting in smaller samples, the fuzzy-SVR functions framework employed here capitalizes on the structural interpretability of fuzzy sets and the generalization ability of kernel-based regression.

Hence, the proposed DM-T1FSVRFs model not only improves accuracy but also introduces a flexible methodological alternative that is particularly well-suited to time series domains characterized by structural variation and limited data availability.

Figure 3 illustrates the mean performance rank of each distance metric across all datasets. Ranks are calculated individually for each dataset based on RMSE, with lower RMSE corresponding to better ranks. These ranks are then averaged to assess overall effectiveness across heterogeneous time series. This ranking approach abstracts away from scale differences and highlights the comparative utility of metrics in a normalized fashion.

As seen in Figure 3, Chebyshev distance achieved the best overall rank (3.25), followed closely by Euclidean and Standardized Euclidean metrics. Cosine distance also maintained a competitive mean rank. These findings align with the heatmap analysis, reaffirming that metrics which emphasize magnitude extremes (e.g., Chebyshev) or directional patterns (e.g., Cosine) can offer consistently strong performance across diverse datasets. Mahalanobis distance, despite its theoretical advantage in handling correlated features, ranked lowest overall. Overall, this ranking visualization underscores the importance of adaptive metric selection and confirms the contextual nature of distance metric effectiveness in fuzzy-SVR frameworks.

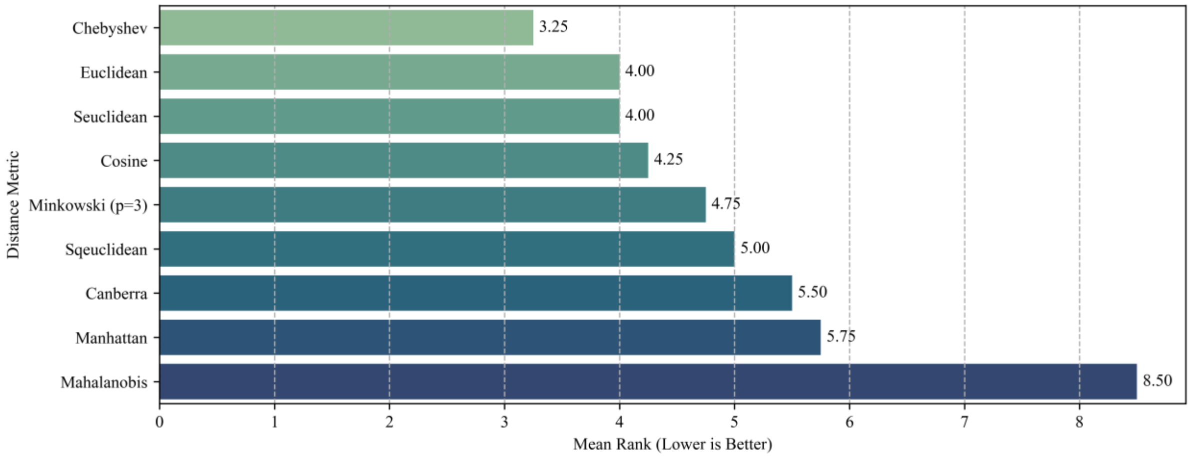

Figure 3. Mean ranking of distance metrics across four datasets based on test-set RMSE performance (lower ranks denote better mean performance).

Conclusion

The experimental results across four real-world datasets provide several key insights into the performance of the proposed DM-T1FSVRFs framework for time series forecasting.

First, incorporating distance metric selection into the fuzzification stage significantly influenced predictive accuracy. The absence of a universally optimal metric highlights the dataset-dependent nature of similarity assessments. These results support the idea that different geometric structures—such as directionality, variance, or scale—require tailored distance formulations for optimal clustering performance. In addition to the selected visualizations presented in Figure 2, comprehensive plots comparing forecasted and observed values for all experimental scenarios are provided as Appendix Figures A4–A19. These extended results allow for a more detailed inspection of temporal forecasting patterns across different datasets and years.

Second, allowing SVR parameters to be optimized per fuzzy cluster enabled the model to better capture local patterns, including regime shifts, volatility clusters, and seasonality. This adaptive configuration proved superior to global models with static parameters, particularly in dynamic financial datasets such as DJI and TAIEX.

Third, the findings suggest that even mathematically simple metrics like Euclidean or Chebyshev can be highly effective when paired with appropriate SVR kernels and tuned hyperparameters. In particular, Chebyshev achieved the top mean rank across all datasets, while Euclidean and Cosine also performed consistently well. This challenges the common assumption that more complex metrics, such as Mahalanobis, always yield better results—especially in low-dimensional time series.

Additionally, the results emphasize the value of jointly considering clustering and regression stages in fuzzy modeling. By treating fuzzification as a tuneable component rather than a preprocessing step, the framework enhances both interpretability and adaptability.

Building on the experimental findings, this study presented DM-T1FSVRFs, a novel fuzzy time series forecasting framework that integrates a diverse set of distance metrics into the fuzzification process of type-1 fuzzy support vector regression models. The framework enables systematic exploration of metric-cluster interactions and supports dynamic hyperparameter optimization at both the clustering and regression stages. The proposed method consistently outperformed benchmark models in terms of RMSE, validating its capacity for adaptive, high accuracy forecasting in complex time series settings. In addition to empirical gains, the framework offers an interpretable and flexible alternative to traditional fuzzy or neural models. By embedding fuzzification within the optimization loop, it allows context-aware modeling of data structure and temporal patterns.

Despite the encouraging empirical results, several limitations of the present study should be acknowledged. First, although the forecasting performances of the distance-metric-based configurations were compared across multiple datasets and evaluation periods, no formal statistical significance test was conducted on the forecast error sequences. Therefore, the reported differences among distance metrics should be interpreted as comparative empirical evidence rather than definitive proof of statistically significant superiority. Second, the joint grid-search procedure over distance metrics, lag structure, cluster number, alpha-cut level, and SVR hyperparameters increases computational cost, which may become substantial for longer or higher-dimensional time series. Future research may address these limitations by incorporating formal significance analyses and more efficient optimization strategies, such as adaptive search procedures or learned distance selection mechanisms.

Building on these limitations, future research may extend this work in several directions, including:

- *(i)* dynamic or learned distance metric selection,
- *(ii)* application to high-dimensional multivariate time series,
- *(iii)* integration with type-2 or probabilistic fuzzy systems for improved uncertainty handling, and
- *(iv)* automated feature selection in the fuzzification pipeline.

Overall, the proposed DM-T1FSVRFs provides a promising step toward interpretable and adaptable forecasting solutions in data-rich but structurally complex environments.

Author Contributions

İbrahim Cimşit: Conceptualization, Methodology, Software, Formal analysis, Writing – original draft, Writing – review & editing. ***Ali Zafer Dalar***: Conceptualization, Methodology, Software, Formal analysis, Supervision, Resources, Investigation, Data curation, Validation, Writing – original draft, Writing – review & editing. ***Ufuk Yolcu***: Conceptualization, Methodology, Software, Formal analysis, Supervision, Investigation, Validation, Writing – original draft, Writing – review & editing.

Ethics Committee Approval

There is no need for ethics committee approval for the study.

Declaration of Competing Interest

The authors declare that they have no competing interests.

Declaration of Generative AI Use

During the preparation of this manuscript, the authors made limited use of ChatGPT (developed by OpenAI) for translation assistance and language refinement. All scientific content was thoroughly reviewed and revised by the authors, who assume full responsibility for the final version of the paper.

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Appendix

Table A1. Summary of dataset characteristics and hyperparameter search spaces used in the proposed DM-T1FSVRFs framework.

<table>
<tr><th>Dataset</th><th>Obs. (N)</th><th>n_{test}</th><th>n_l</th><th>C</th><th>α-cut</th><th>Kernel Type</th><th>C_{SVR}</th><th>ϵ</th><th>γ</th></tr>
<tr><td>ABP</td><td>154</td><td>16</td><td>1:5</td><td>2:5</td><td rowspan="16">0.0000, 0.0500, 0.2321</td><td rowspan="16">Linear, Radial Basis Function, Polynomial, Sigmoid</td><td rowspan="16">0.1000, 0.2154, 0.4641, 1.0000, 2.1544, 4.6416</td><td rowspan="16">0.0020, 0.0126, 0.0796, 0.5024</td><td rowspan="16">0.0159, 0.0634, 0.2524</td></tr>
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<tr><td>TAIEX 1999</td><td>241</td><td>45</td></tr>
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<tr><td>VNI30 2013</td><td>250</td><td>10</td></tr>
</table>

Table A2. Optimal hyperparameter configurations for the best-performing models across all datasets and years.

Series		Distance Type	n_l	C	α-cut	C_{SVR}	ϵ	Kernel Type	γ
ABP		Minkowski (p=3)	2	4	0.2321	0.2154	0.002	Sigmoid	0.0634
DJI	2010	Canberra	4	7	0.2321	1.0000	0.0796	Linear	0.0634
	2011	Canberra	3	5	0.2321	4.6416	0.0126	Polynomial	0.0159
	2012	Chebyshev	3	5	0.0500	0.4642	0.5024	Polynomial	0.0634
	2013	Cosine	5	7	0.2321	0.2154	0.0020	Polynomial	0.0634
	2014	Seuclidean	3	3	0.2321	1.0000	0.0796	Polynomial	0.2524
TAIEX	1999	Minkowski (*p*=3)	2	6	0.2321	0.2154	0.0126	Sigmoid	0.0159
	2000	Sqeuclidean	1	5	0.0500	2.1544	0.0020	Sigmoid	0.0159
	2001	Cosine	4	2	0.0500	1.0000	0.0126	Sigmoid	0.0159
	2002	Seuclidean	4	6	0.0500	4.6416	0.0020	Sigmoid	0.0159
	2003	Sqeuclidean	2	2	0.2321	1.0000	0.0126	Polynomial	0.0634
	2004	Cosine	1	7	0.0500	1.0000	0.5024	RBF	0.0159
VNI30	2010	Cosine	3	2	0.0500	4.6416	0.0796	Polynomial	0.2524
	2011	Sqeuclidean	4	5	0.2321	0.4642	0.0020	Sigmoid	0.0159
	2012	Chebyshev	3	5	0.2321	2.1544	0.0126	Polynomial	0.0159
	2013	Euclidean	5	3	0.2321	0.1000	0.0796	RBF	0.0159

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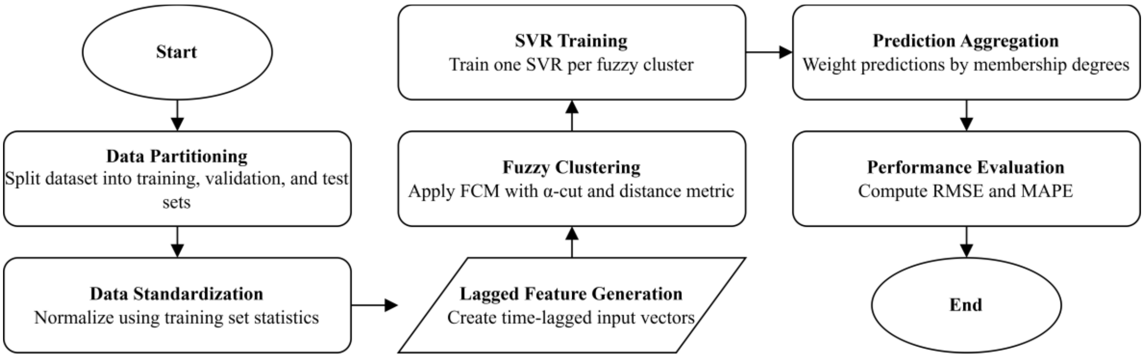

Figure A1. Flowchart of the proposed DM-T1FSVRFs framework

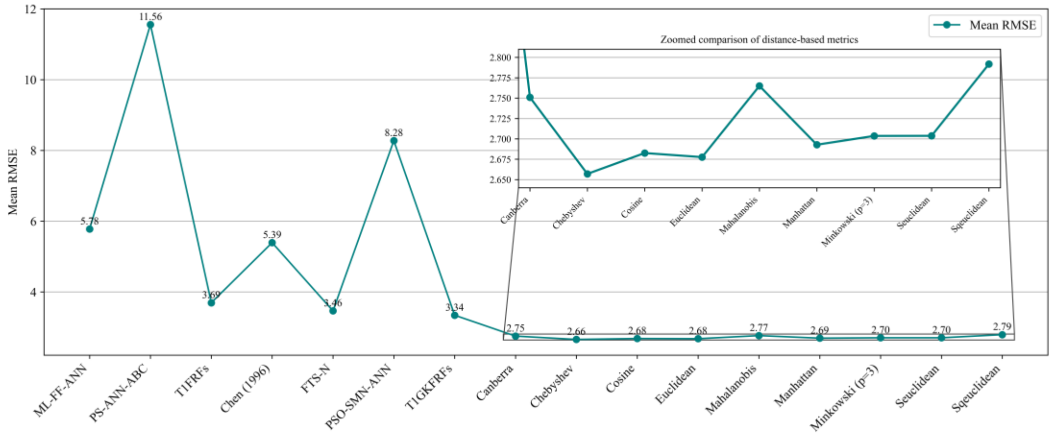

Figure A2. Mean RMSE comparison for VNI30 dataset on the test set

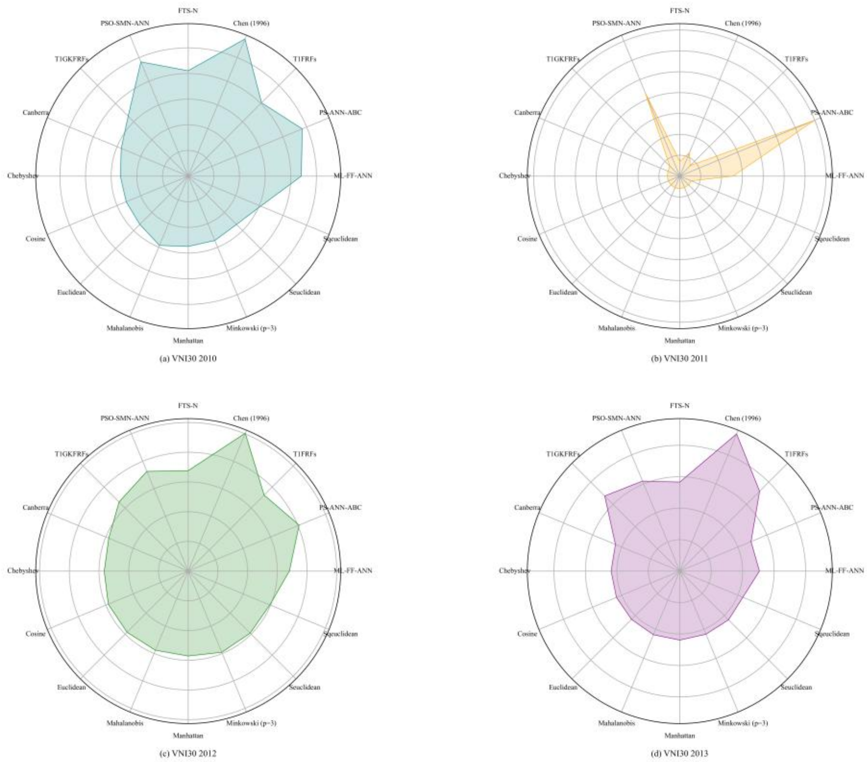

Figure A3. Radar chart of test-set RMSE performance for VNI30 dataset

DOI: 10.37880/cumuiibf.1885704

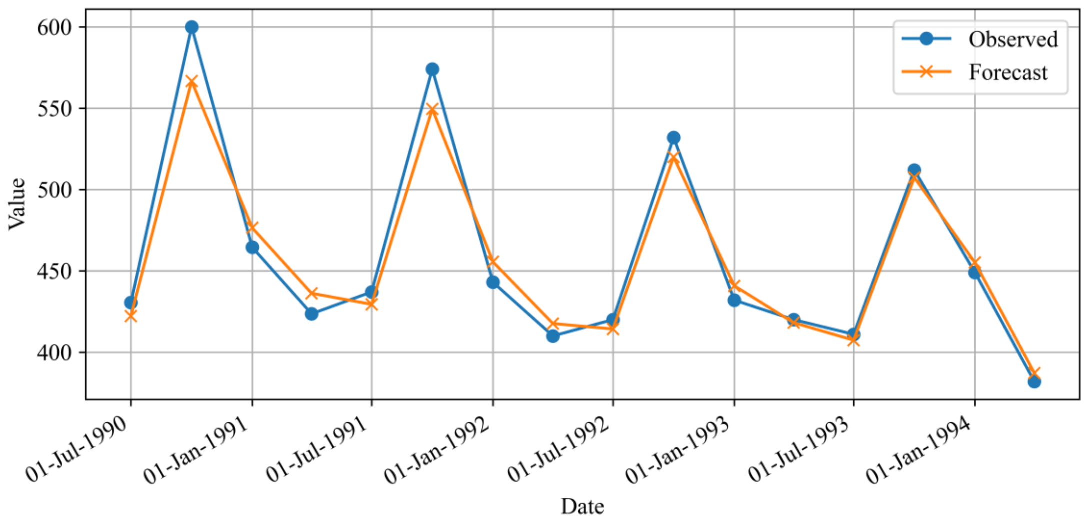

Figure A4. Forecast vs. observed values for ABP dataset on the test set

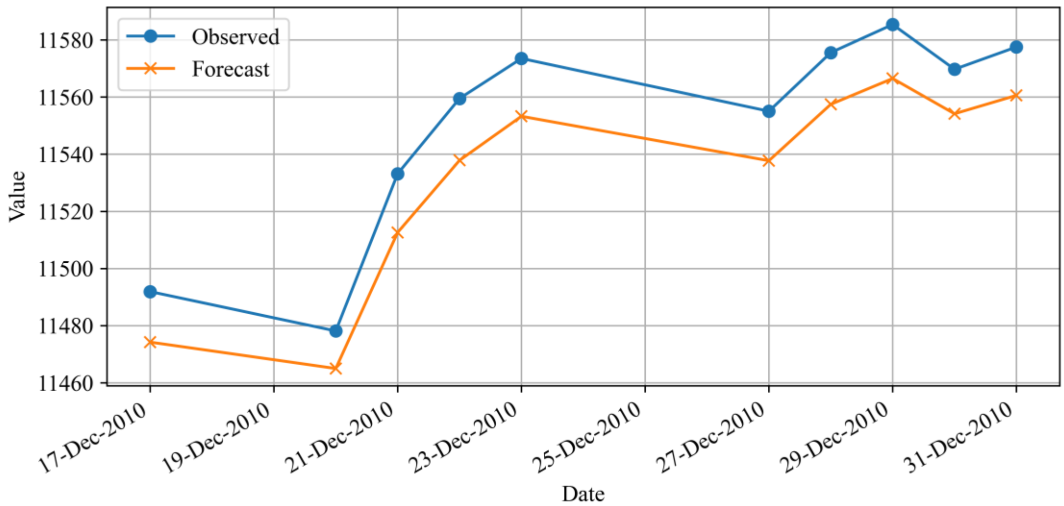

Figure A5. Forecast vs. observed values for DJI 2010 on the test set

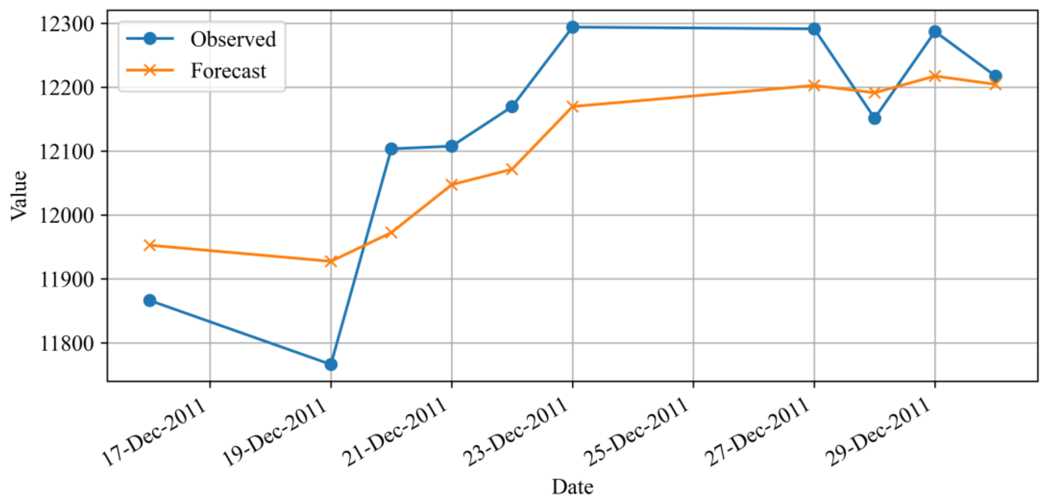

Figure A6. Forecast vs. observed values for DJI 2011 on the test set

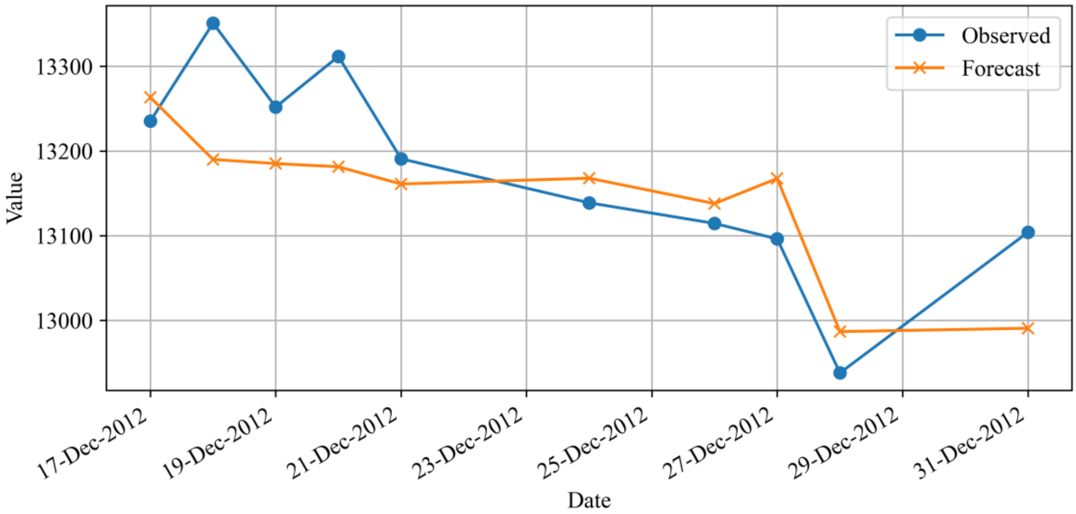

Figure A7. Forecast vs. observed values for DJI 2012 on the test set

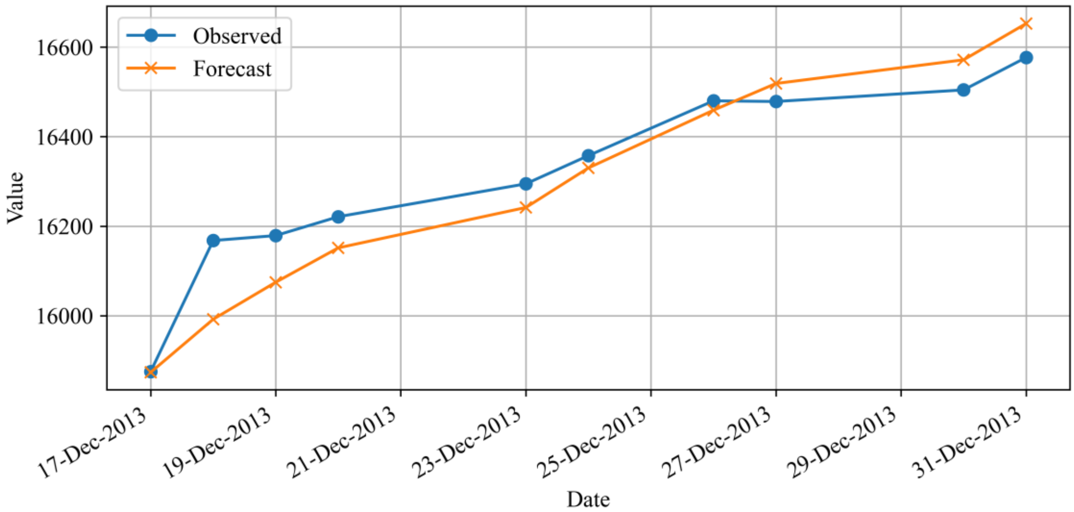

Figure A8. Forecast vs. observed values for DJI 2013 on the test set

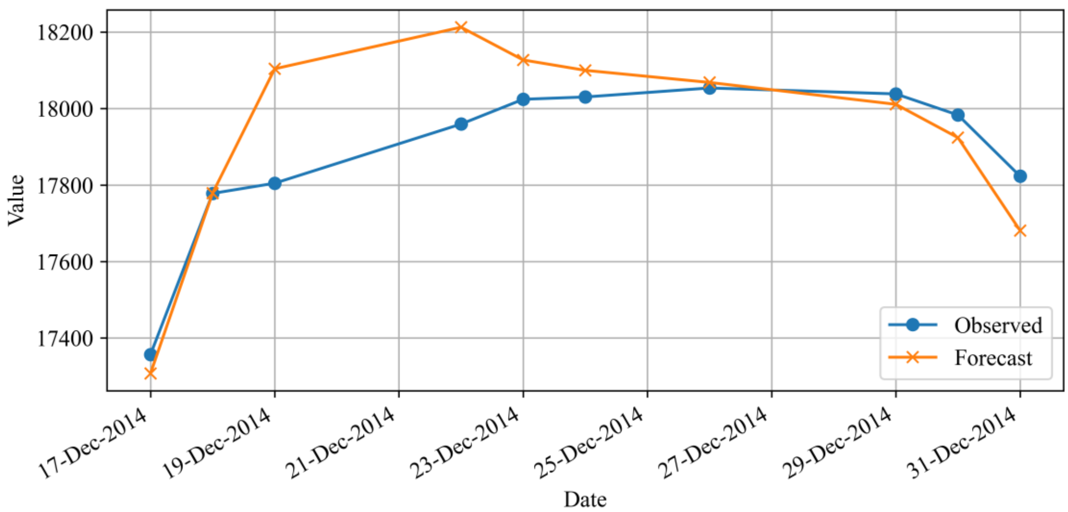

Figure A9. Forecast vs. observed values for DJI 2014 on the test set

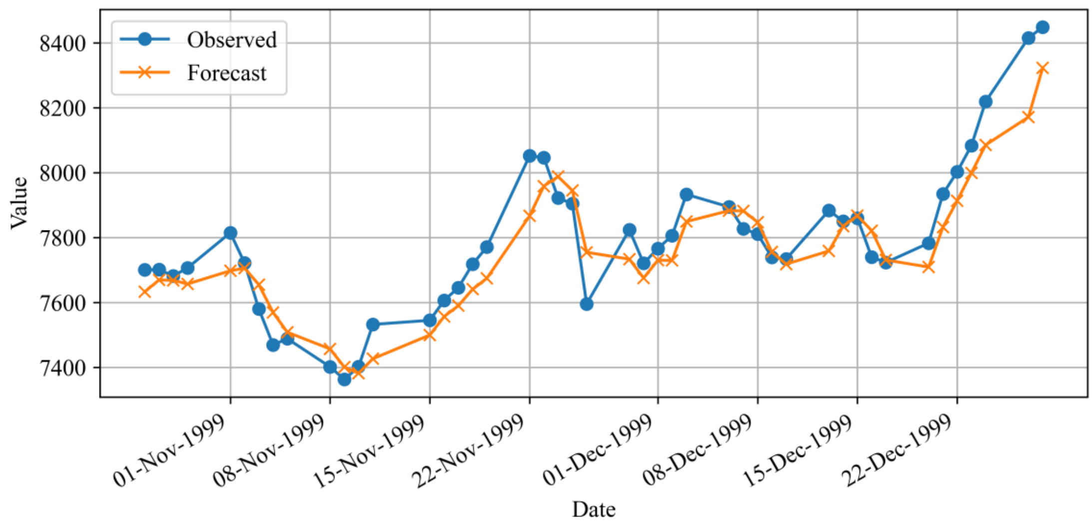

Figure A10. Forecast vs. observed values for TAIEX 1999 on the test set

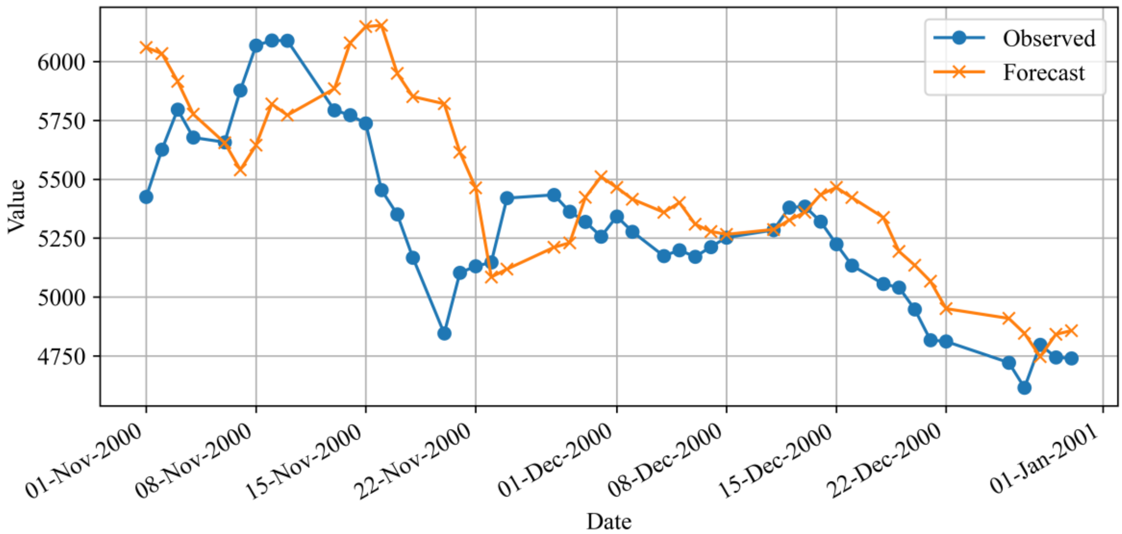

Figure A11. Forecast vs. observed values for TAIEX 2000 on the test set

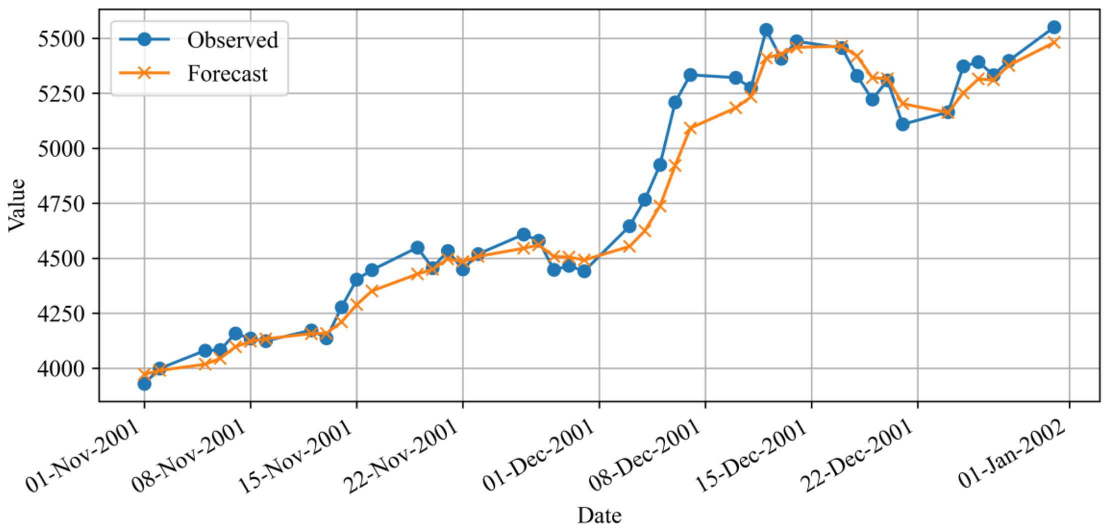

Figure A12. Forecast vs. observed values for TAIEX 2001 on the test set

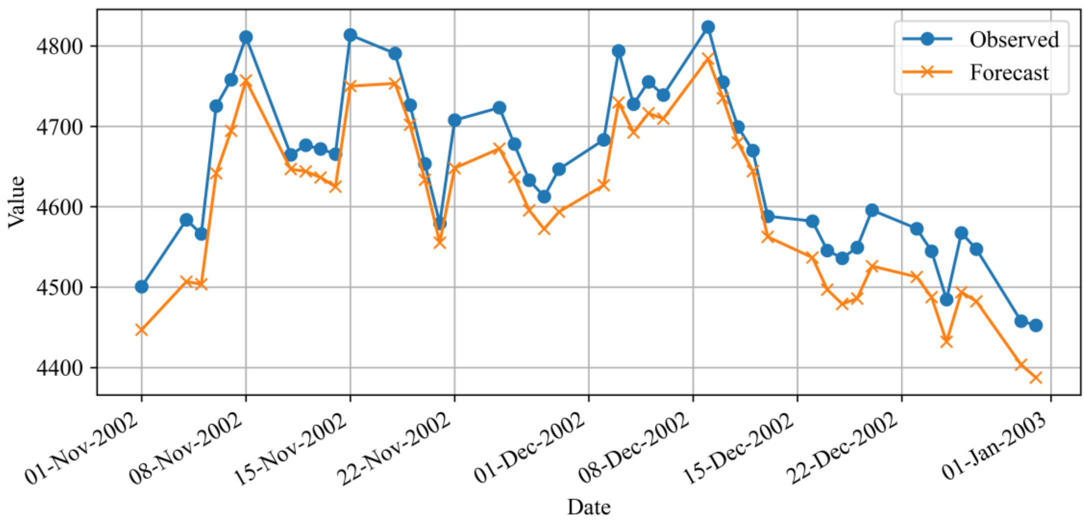

Figure A13. Forecast vs. observed values for TAIEX 2002 on the test set

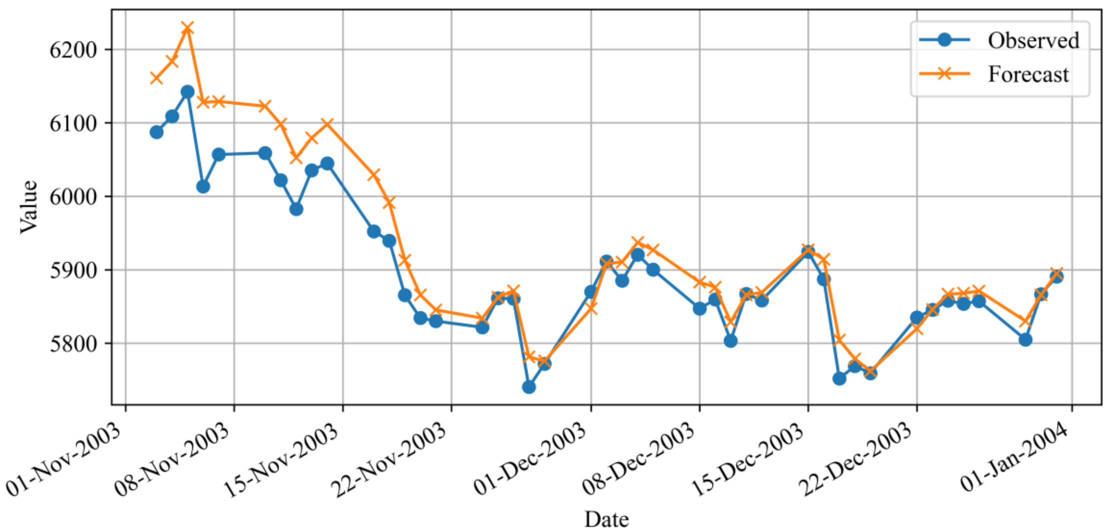

Figure A14. Forecast vs. observed values for TAIEX 2003 on the test set

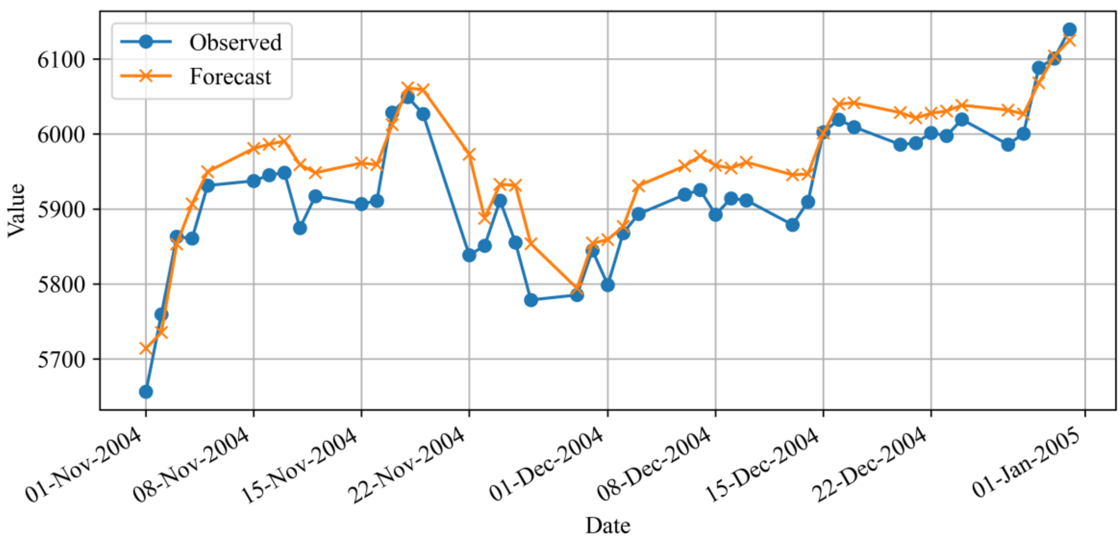

Figure A15. Forecast vs. observed values for TAIEX 2004 on the test set

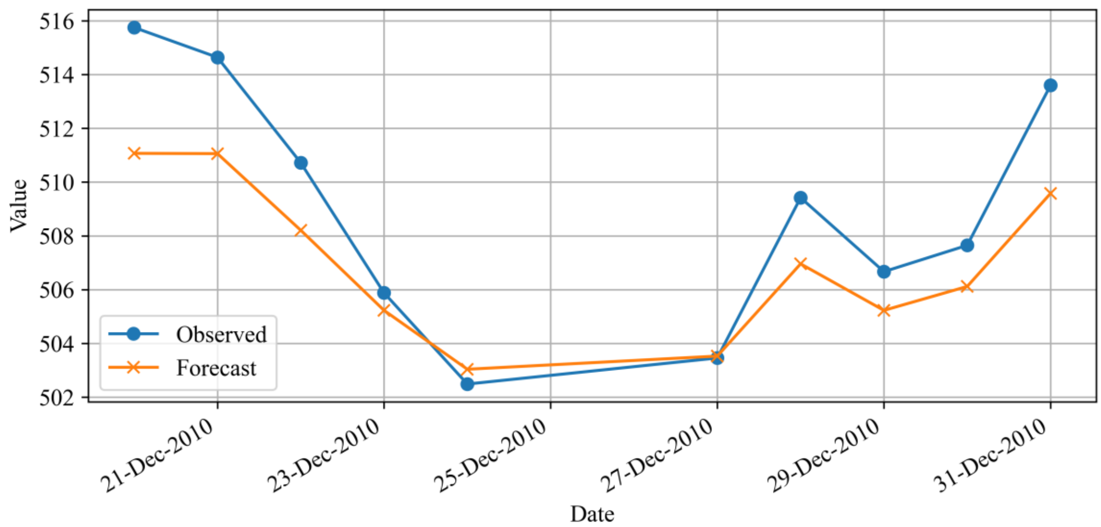

Figure A16. Forecast vs. observed values for VNI30 2010 on the test set

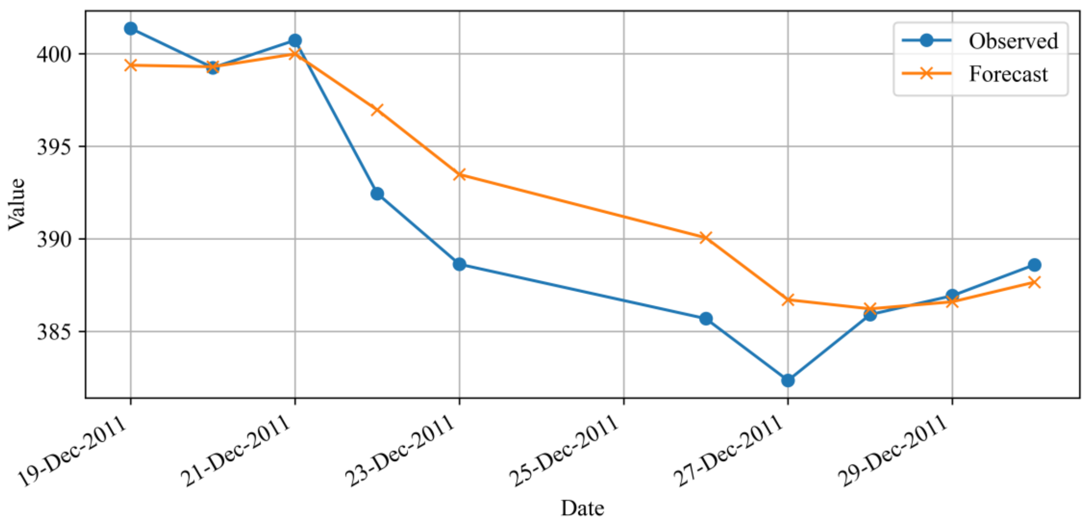

Figure A17. Forecast vs. observed values for VNI30 2011 on the test set

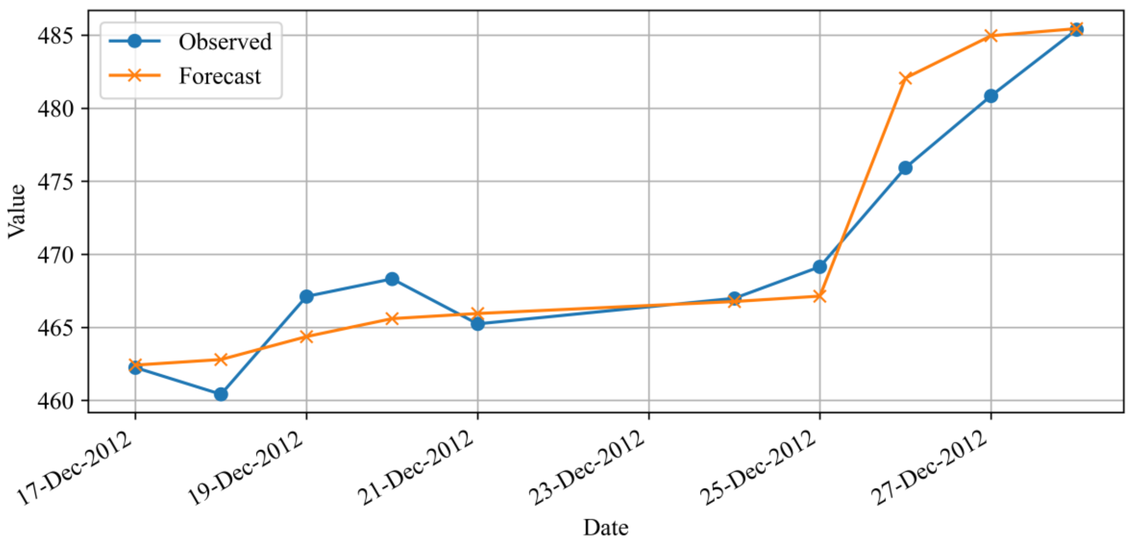

Figure A18. Forecast vs. observed values for VNI30 2012 on the test set

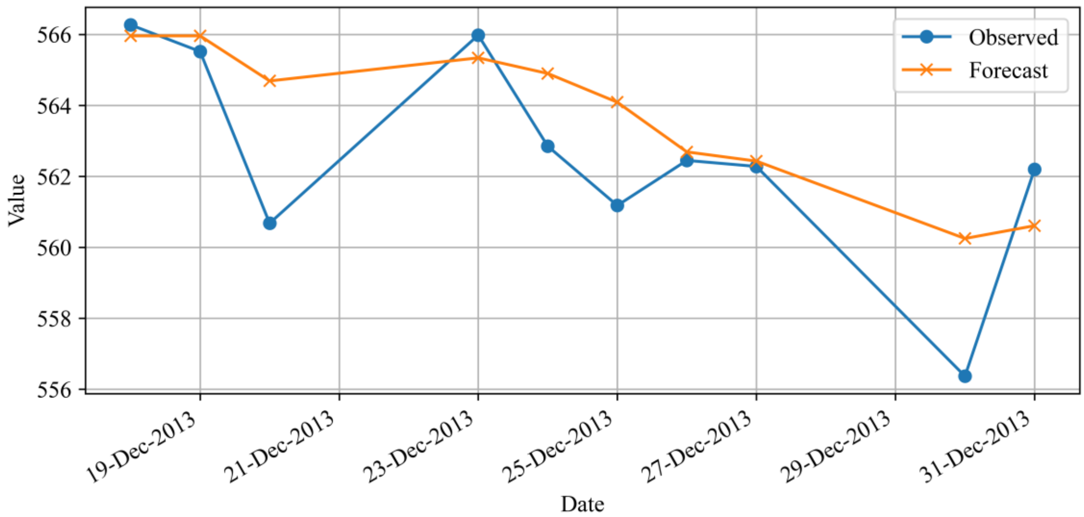

Figure A19. Forecast vs. observed values for VNI30 2013 on the test set