MANAS Journal of Engineering MJEN 2 (2) 2014 9-22

Solutions Of The System Of Maximum Difference Equations

Dağıstan Şimşek
Department of Applied Mathematics and Informatics, Faculty of Science,Cal Campüs,Kyrgyzstan-Turkey Manas University, Bishkek, Kyrgzstan, dagistan.simsek@manas.edu.kg
Ahmet Doğan
Department of Mathematics,Faculty of Science,Cal Campüs,Kyrgyzstan-Turkey Manas University,Bishkek,Kyrgzstan
ahmet.dogan@manas.edu.kg

Received: *24.06.2014* *Reviewed:* *10.08.2014* *Accepted:* *10.08.2014*

Abstract The behaviour of the solutions of the following system of difference equations is examined.

$$x_{n+1} = \max\left\{\frac{1}{x_{n-3}}, \frac{y_{n-3}}{x_{n-3}}\right\}; y_{n+1} = \max\left\{\frac{1}{y_{n-3}}, \frac{x_{n-3}}{y_{n-3}}\right\} \quad (1)$$

Where the initial conditions are positive real numbers.

Keywords *Difference Equation, Maximum Operations, Semicycle*

Maksimumlu Fark Denklem Sisteminin Çözümleri

Özet Aşağıdaki fark denklem sisteminin çözümlerinin davranışları incelenmiştir.

$$x_{n+1} = \max\left\{\frac{1}{x_{n-3}}, \frac{y_{n-3}}{x_{n-3}}\right\}; y_{n+1} = \max\left\{\frac{1}{y_{n-3}}, \frac{x_{n-3}}{y_{n-3}}\right\} \quad (1)$$

Başlangıç şartları pozitif reel sayılardır.

Anahtar sözcükler *Fark Denklemi, Maksimum Operatörü, Yarı Dönmeler*

1. INTRODUCTION

Recently, there has been a great interest in studying the periodic nature of nonlinear difference equations. Although difference equations are relatively simple in form, it is, unfortunately, extremely diffucult to understand thorougly the periodic behavior of their solutions. The periodic natüre of nonlinear difference equations of the max type has been investigated by many authors. See for example [1-17].

Definition 1: Let I be an interval of real numbers and let $f : I^{s+1} \to I$ be a continuously differentiable function where s is a non-negative integer. Consider the difference equation

$$x_{n+1} = f(x_n, x_{n-1}, \ldots, x_{n-s}) \text{ for } n = 0,1,2,\ldots \tag{2}$$

with the initial values $x_{-s}, \ldots, x_0 \in I$. A point $\bar{x}$ called an equilibrium point of Eq.(2) if

$$\bar{x} = f(\bar{x}, \ldots, \bar{x}).$$

Definition 2: A positive semicycle of a solutions $\{x_n\}_{n=-s}^{\infty}$ of Eq.(2) consist of a string of terms $\{x_l, x_{l+1}, \ldots, x_m\}$ all greater than or equal to equilibrium $\bar{x}$ with $l \geq -s$ and $m \leq \infty$ such that either $l = -s$ or $l > -s$ ve $x_{l-1} < \bar{x}$ and either $m = \infty$ or $m < \infty$ and $x_{m+1} < \bar{x}$.

Definition 3: A negative semicycle of a solutions $\{x_n\}_{n=-s}^{\infty}$ of Eq.(2) consist of a string of terms $\{x_l, x_{l+1}, \ldots, x_m\}$ all less than or equal to equilibrium $\bar{x}$ with $l \geq -s$ and $m \leq \infty$ such that either $l = -s$ or $l > -s$ and $x_{l-1} \geq \bar{x}$ and either $m = \infty$ or $m \leq \infty$ and $x_{m+1} \geq \bar{x}$.

Definition 4 : Fibonacci sequence is $f_1 = 1, f_2 = 1$ and for $n \geq 3$, $f_n = f_{n-1} + f_{n-2}$.

2. MAIN RESULTS

Let $\bar{x}$ and $\bar{y}$ be the unique positive equilibrium of Eq.(1), then clearly

$$\bar{x} = \max\left\{\frac{1}{\bar{x}}, \frac{\bar{y}}{\bar{x}}\right\}; \bar{y} = \max\left\{\frac{1}{\bar{y}}, \frac{\bar{x}}{\bar{y}}\right\}$$

$$\bar{x} = \frac{1}{\bar{x}} \Rightarrow \bar{x}^{2} = 1 \Rightarrow \bar{x} = 1$$

$$\bar{y} = \frac{1}{\bar{y}} \Rightarrow \bar{y}^{2} = 1 \Rightarrow \bar{y} = 1$$

We can obtain $\bar{x} = 1$ and $\bar{y} = 1$.

Lemma 1: Assume that,

$1 < x_{-3} < y_{-3}$, $1 < x_{-2} < y_{-2}$, $1 < x_{-1} < y_{-1}$ and $1 < x_0 < y_0$

Then the following statements are true for the solutions of Eq.(1) :

- **a)** Every positive semicycle consist of four terms,
- **b)** Every negative semicycle consist of four terms,
- **c)** Every positive semicycle of length four is followed by a negative semicycle of length four,
- **d)** Every negative semicycle of length four is followed by a positive semicycle of length four.

Proof:

a) and b)

$N \geq 0$ and $1 < x_{-3} < y_{-3}$, the solution x_n, y_n can be obtained as follows :

If $x_n < y_n$, $x_n > \bar{x}$ and $y_n > \bar{y}$ then,

$1 < x_{-3} < y_{-3}$, $1 < x_{-2} < y_{-2}$, $1 < x_{-1} < y_{-1}$ ve $1 < x_0 < y_0$ with the initial conditions,

$$x_{N+1} = \max\left\{\frac{1}{x_{N-3}}, \frac{y_{N-3}}{x_{N-3}}\right\} = \frac{y_{N-3}}{x_{N-3}} > \bar{x}$$

$$y_{N+1} = \max\left\{\frac{1}{y_{N-3}}, \frac{x_{N-3}}{y_{N-3}}\right\} = \frac{x_{N-3}}{y_{N-3}} < \bar{y}$$

$$x_{N+2} = \max\left\{\frac{1}{x_{N-2}}, \frac{y_{N-2}}{x_{N-2}}\right\} = \frac{y_{N-2}}{x_{N-2}} > \bar{x}$$

$$y_{N+2} = \max\left\{\frac{1}{y_{N-2}}, \frac{x_{N-2}}{y_{N-2}}\right\} = \frac{x_{N-2}}{y_{N-2}} < \bar{y}$$

$$x_{N+3} = \max\left\{\frac{1}{x_{N-1}}, \frac{y_{N-1}}{x_{N-1}}\right\} = \frac{y_{N-1}}{x_{N-1}} > \bar{x}$$

$$y_{N+3} = \max\left\{\frac{1}{y_{N-1}}, \frac{x_{N-1}}{y_{N-1}}\right\} = \frac{x_{N-1}}{y_{N-1}} < \bar{y}$$

$$x_{N+4} = \max\left\{\frac{1}{x_N}, \frac{y_N}{x_N}\right\} = \frac{y_N}{x_N} > \bar{x}$$

$$y_{N+4} = \max\left\{\frac{1}{y_N}, \frac{x_N}{y_N}\right\} = \frac{x_N}{y_N} < \bar{y}$$

$$x_{N+5} = \max\left\{\frac{1}{x_{N+1}}, \frac{y_{N+1}}{x_{N+1}}\right\} = \max\left\{\frac{x_{N-3}}{y_{N-3}}, \frac{x_{N-3}^2}{y_{N-3}^2}\right\} = \frac{x_{N-3}}{y_{N-3}} < \bar{x}$$

$$y_{N+5} = \max\left\{\frac{1}{y_{N+1}}, \frac{x_{N+1}}{y_{N+1}}\right\} = \max\left\{\frac{y_{N-3}}{x_{N-3}}, \frac{y_{N-3}^2}{x_{N-3}^2}\right\} = \frac{y_{N-3}^2}{x_{N-3}^2} > \bar{y}$$

$$x_{N+6} = \max\left\{\frac{1}{x_{N+2}}, \frac{y_{N+2}}{x_{N+2}}\right\} = \max\left\{\frac{x_{N-2}}{y_{N-2}}, \frac{x_{N-2}^2}{y_{N-2}^2}\right\} = \frac{x_{N-2}}{y_{N-2}} < \bar{x}$$

$$y_{N+6} = \max\left\{\frac{1}{y_{N+2}}, \frac{x_{N+2}}{y_{N+2}}\right\} = \max\left\{\frac{y_{N-2}}{x_{N-2}}, \frac{y_{N-2}^2}{x_{N-2}^2}\right\} = \frac{y_{N-2}^2}{x_{N-2}^2} > \bar{y}$$

$$x_{N+7} = \max\left\{\frac{1}{x_{N+3}}, \frac{y_{N+3}}{x_{N+3}}\right\} = \max\left\{\frac{x_{N-1}}{y_{N-1}}, \frac{x_{N-1}^2}{y_{N-1}^2}\right\} = \frac{x_{N-1}}{y_{N-1}} < \bar{x}$$

$$y_{N+7} = \max\left\{\frac{1}{y_{N+3}}, \frac{x_{N+3}}{y_{N+3}}\right\} = \max\left\{\frac{y_{N-1}}{x_{N-1}}, \frac{y_{N-1}^2}{x_{N-1}^2}\right\} = \frac{y_{N-1}^2}{x_{N-1}^2} > \bar{y}$$

$$x_{N+8}=\max\left\{\frac{1}{x_{N+4}},\frac{y_{N+4}}{x_{N+4}}\right\}=\max\left\{\frac{x_N}{y_N},\frac{x_N^2}{y_N^2}\right\}=\frac{x_N}{y_N}<\bar{x}$$

$$y_{N+8}=\max\left\{\frac{1}{y_{N+4}},\frac{x_{N+4}}{y_{N+4}}\right\}=\max\left\{\frac{y_N}{x_N},\frac{y_N^2}{x_N^2}\right\}=\frac{y_N^2}{x_N^2}>\bar{y}$$

.

.

.

Hence we obtained,

$x_{N+1}>\bar{x}$, $x_{N+2}>\bar{x}$, $x_{N+3}>\bar{x}$, $x_{N+4}>\bar{x}$, $x_{N+5}<\bar{x}$, $x_{N+6}<\bar{x}$, $x_{N+7}<\bar{x}$, $x_{N+8}<\bar{x}$, …

$y_{N+1}<\bar{y}$, $y_{N+2}<\bar{y}$, $y_{N+3}<\bar{y}$, $y_{N+4}<\bar{y}$, $y_{N+5}>\bar{y}$, $y_{N+6}>\bar{y}$, $y_{N+7}>\bar{y}$, $y_{N+8}>\bar{y}$, … .

Hence Every positive semicycle consist of four terms, Every negative semicycle consist of four terms.

c) Using the proof **a) and b)**
If for $N\geq 0$, then

$x_{N+1}>\bar{x}$, $x_{N+2}>\bar{x}$, $x_{N+3}>\bar{x}$, $x_{N+4}>\bar{x}$, $x_{N+5}<\bar{x}$, $x_{N+6}<\bar{x}$, $x_{N+7}<\bar{x}$, $x_{N+8}<\bar{x}$, …

Therefore every positive semicycle of length four is followed by a negative semicycle of length four.

d) Using the proof **a) and b)**
If for $N\geq 0$, then

$y_{N+1}<\bar{y}$, $y_{N+2}<\bar{y}$, $y_{N+3}<\bar{y}$, $y_{N+4}<\bar{y}$, $y_{N+5}>\bar{y}$, $y_{N+6}>\bar{y}$, $y_{N+7}>\bar{y}$, $y_{N+8}>\bar{y}$, …

Therefore every negative semicycle of length four is followed by a positive semicycle of length four.

Teorem 1 : Let $(x_n;y_n)$ be a solution of Eq.(1) for $1<x_{-3}<y_{-3}$, $1<x_{-2}<y_{-2}$, $1<x_{-1}<y_{-1}$ and $1<x_0<y_0$. Then, for n = 0,1,2,… we have

$$x_{8n+1}=\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2n+2)};\ x_{8n+2}=\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2n+2)};\ x_{8n+3}=\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2n+2)};\ x_{8n+4}=\left(\frac{y_0}{x_0}\right)^{f(2n+2)};$$

$$x_{8n+5}=\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2n+2)};\ x_{8n+6}=\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2n+2)};\ x_{8n+7}=\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2n+2)};\ x_{8n+8}=\left(\frac{x_0}{y_0}\right)^{f(2n+2)}$$

$$y_{8n+1}=\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2n+1)};\ y_{8n+2}=\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2n+1)};\ y_{8n+3}=\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2n+1)};\ y_{8n+4}=\left(\frac{x_{0}}{y_{0}}\right)^{f(2n+1)};$$

$$y_{8n+5}=\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2n+3)};\ y_{8n+6}=\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2n+3)};\ y_{8n+7}=\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2n+3)};\ y_{8n+8}=\left(\frac{y_{0}}{x_{0}}\right)^{f(2n+3)}$$

İspat: We obtain

$$x_1=\max\left\{\frac{1}{x_{-3}},\frac{y_{-3}}{x_{-3}}\right\}=\frac{y_{-3}}{x_{-3}}>\bar{x}$$

$$y_1=\max\left\{\frac{1}{y_{-3}},\frac{x_{-3}}{y_{-3}}\right\}=\frac{x_{-3}}{y_{-3}}<\bar{y}$$

$$x_2=\max\left\{\frac{1}{x_{-2}},\frac{y_{-2}}{x_{-2}}\right\}=\frac{y_{-2}}{x_{-2}}>\bar{x}$$

$$y_2=\max\left\{\frac{1}{y_{-2}},\frac{x_{-2}}{y_{-2}}\right\}=\frac{x_{-2}}{y_{-2}}<\bar{y}$$

$$x_3=\max\left\{\frac{1}{x_{-1}},\frac{y_{-1}}{x_{-1}}\right\}=\frac{y_{-1}}{x_{-1}}>\bar{x}$$

$$y_3=\max\left\{\frac{1}{y_{-1}},\frac{x_{-1}}{y_{-1}}\right\}=\frac{x_{-1}}{y_{-1}}<\bar{y}$$

$$x_4=\max\left\{\frac{1}{x_{0}},\frac{y_{0}}{x_{0}}\right\}=\frac{y_{0}}{x_{0}}>\bar{x}$$

$$y_4=\max\left\{\frac{1}{y_{0}},\frac{x_{0}}{y_{0}}\right\}=\frac{x_{0}}{y_{0}}<\bar{y}$$

$$x_5=\max\left\{\frac{1}{x_1},\frac{y_1}{x_1}\right\}=\max\left\{\frac{x_{-3}}{y_{-3}},\frac{x_{-3}^2}{y_{-3}^2}\right\}=\frac{x_{-3}}{y_{-3}}<\bar{x}$$

$$y_5=\max\left\{\frac{1}{y_1},\frac{x_1}{y_1}\right\}=\max\left\{\frac{y_{-3}}{x_{-3}},\frac{y_{-3}^2}{x_{-3}^2}\right\}=\frac{y_{-3}^2}{x_{-3}^2}>\bar{y}$$

$$x_6=\max\left\{\frac{1}{x_2},\frac{y_2}{x_2}\right\}=\max\left\{\frac{x_{-2}}{y_{-2}},\frac{x_{-2}^2}{y_{-2}^2}\right\}=\frac{x_{-2}}{y_{-2}}<\bar{x}$$

$$y_6=\max\left\{\frac{1}{y_2},\frac{x_2}{y_2}\right\}=\max\left\{\frac{y_{-2}}{x_{-2}},\frac{y_{-2}^2}{x_{-2}^2}\right\}=\frac{y_{-2}^2}{x_{-2}^2}>\bar{y}$$

$$x_7=\max\left\{\frac{1}{x_3},\frac{y_3}{x_3}\right\}=\max\left\{\frac{x_{-1}}{y_{-1}},\frac{x_{-1}^2}{y_{-1}^2}\right\}=\frac{x_{-1}}{y_{-1}}<\bar{x}$$

$$y_7=\max\left\{\frac{1}{y_3},\frac{x_3}{y_3}\right\}=\max\left\{\frac{y_{-1}}{x_{-1}},\frac{y_{-1}^2}{x_{-1}^2}\right\}=\frac{y_{-1}^2}{x_{-1}^2}>\bar{y}$$

$$x_8 = \max\left\{\frac{1}{x_4}, \frac{y_4}{x_4}\right\} = \max\left\{\frac{x_0}{y_0}, \frac{x_0^2}{y_0^2}\right\} = \frac{x_0}{y_0} < \bar{x}$$

$$y_8 = \max\left\{\frac{1}{y_4}, \frac{x_4}{y_4}\right\} = \max\left\{\frac{y_0}{x_0}, \frac{y_0^2}{x_0^2}\right\} = \frac{y_0^2}{x_0^2} > \bar{y}$$

.

.

.

for n = 0, from a) and b) equations. That is our assumption is true for n = 0.
Assume that our assumption is true for n = k. Then

$$x_{8k+1} = \left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)}; x_{8k+2} = \left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)}; x_{8k+3} = \left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)}; x_{8k+4} = \left(\frac{y_0}{x_0}\right)^{f(2k+2)};$$

$$x_{8k+5} = \left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)}; x_{8k+6} = \left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}; x_{8k+7} = \left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)}; x_{8k+8} = \left(\frac{x_0}{y_0}\right)^{f(2k+2)}$$

$$y_{8k+1} = \left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+1)}; y_{8k+2} = \left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+1)}; y_{8k+3} = \left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+1)}; y_{8k+4} = \left(\frac{x_0}{y_0}\right)^{f(2k+1)};$$

$$y_{8k+5} = \left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)}; y_{8k+6} = \left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)}; y_{8k+7} = \left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)}; y_{8k+8} = \left(\frac{y_0}{x_0}\right)^{f(2k+3)}$$

Lets Show that a) and b) equations is true for n = k+1. We have

$$x_{8k+9} = \max\left\{\frac{1}{x_{8k+5}}, \frac{y_{8k+5}}{x_{8k+5}}\right\} = \max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)}, \frac{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)}}{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)}}\right\}$$

$$= \max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)}, \left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)}\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)}\right\}$$

$$= \max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)}, \left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+4)}\right\}$$

$$= \left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+4)}$$

$$y_{8k+9}=\max\left\{\frac{1}{y_{8k+5}},\frac{x_{8k+5}}{y_{8k+5}}\right\}=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)},\frac{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)}}{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)},\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)}\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)},\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)}$$

$$x_{8k+10}=\max\left\{\frac{1}{x_{8k+6}},\frac{y_{8k+6}}{x_{8k+6}}\right\}=\max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)},\frac{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)}}{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)},\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)}\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)},\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+4)}$$

$$y_{8k+10}=\max\left\{\frac{1}{y_{8k+6}},\frac{x_{8k+6}}{y_{8k+6}}\right\}=\max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)},\frac{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}}{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-32}}{y_{-2}}\right)^{f(2k+3)},\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)},\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)}$$

$$x_{8k+11}=\max\left\{\frac{1}{x_{8k+7}},\frac{y_{8k+7}}{x_{8k+7}}\right\}=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)},\frac{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)}}{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)},\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)}\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)},\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+4)}$$

$$y_{8k+11}=\max\left\{\frac{1}{y_{8k+7}},\frac{x_{8k+7}}{y_{8k+7}}\right\}=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)},\frac{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)}}{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)},\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)}\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)},\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)}$$

$$x_{8k+12}=\max\left\{\frac{1}{x_{8k+8}},\frac{y_{8k+8}}{x_{8k+8}}\right\}=\max\left\{\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+2)},\frac{\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+3)}}{\left(\frac{x_{0}}{y_{0}}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+2)},\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+3)}\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+2)},\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+4)}$$

$$y_{8k+12}=\max\left\{\frac{1}{y_{8k+8}},\frac{x_{8k+8}}{y_{8k+8}}\right\}=\max\left\{\left(\frac{x_0}{y_0}\right)^{f(2k+3)},\frac{\left(\frac{x_0}{y_0}\right)^{f(2k+2)}}{\left(\frac{y_0}{x_0}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{x_0}{y_0}\right)^{f(2k+3)},\left(\frac{x_0}{y_0}\right)^{f(2k+2)}\left(\frac{x_0}{y_0}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{x_0}{y_0}\right)^{f(2k+3)},\left(\frac{x_0}{y_0}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{x_0}{y_0}\right)^{f(2k+3)}$$

$$x_{8k+13}=\max\left\{\frac{1}{x_{8k+9}},\frac{y_{8k+9}}{x_{8k+9}}\right\}=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+4)},\frac{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)}}{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+4)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+4)},\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)}\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+4)}\right\}$$

$$=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+4)},\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+5)}\right\}$$

$$=\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+4)}$$

$$y_{8k+13}=\max\left\{\frac{1}{y_{8k+9}},\frac{x_{8k+9}}{y_{8k+9}}\right\}=\max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)},\frac{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+4)}}{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)},\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+4)}\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)},\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+5)}\right\}$$

$$=\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+5)}$$

$$x_{8k+14}=\max\left\{\frac{1}{x_{8k+10}},\frac{y_{8k+10}}{x_{8k+10}}\right\}=\max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+4)},\frac{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)}}{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+4)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+4)},\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)}\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+4)}\right\}$$

$$=\max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+4)},\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+5)}\right\}$$

$$=\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+4)}$$

$$y_{8k+14}=\max\left\{\frac{1}{y_{8k+10}},\frac{x_{8k+10}}{y_{8k+10}}\right\}=\max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)},\frac{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+4)}}{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)},\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+4)}\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)},\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+5)}\right\}$$

$$=\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+5)}$$

$$x_{8k+15}=\max\left\{\frac{1}{x_{8k+11}},\frac{y_{8k+11}}{x_{8k+11}}\right\}=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+4)},\frac{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)}}{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+4)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+4)},\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)}\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+4)}\right\}$$

$$=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+4)},\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+5)}\right\}$$

$$=\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+4)}$$

$$y_{8k+15}=\max\left\{\frac{1}{y_{8k+11}},\frac{x_{8k+11}}{y_{8k+11}}\right\}=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)},\frac{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+4)}}{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)},\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+4)}\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)},\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+5)}\right\}$$

$$=\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+5)}$$

$$x_{8k+16}=\max\left\{\frac{1}{x_{8k+12}},\frac{y_{8k+12}}{x_{8k+12}}\right\}=\max\left\{\left(\frac{x_{0}}{y_{0}}\right)^{f(2k+4)},\frac{\left(\frac{x_{0}}{y_{0}}\right)^{f(2k+3)}}{\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+4)}}\right\}$$

$$=\max\left\{\left(\frac{x_{0}}{y_{0}}\right)^{f(2k+4)},\left(\frac{x_{0}}{y_{0}}\right)^{f(2k+3)}\left(\frac{x_{0}}{y_{0}}\right)^{f(2k+4)}\right\}$$

$$=\max\left\{\left(\frac{x_{0}}{y_{0}}\right)^{f(2k+4)},\left(\frac{x_{0}}{y_{0}}\right)^{f(2k+5)}\right\}$$

$$=\left(\frac{x_{0}}{y_{0}}\right)^{f(2k+4)}$$

$$y_{8k+16}=\max\left\{\frac{1}{y_{8k+12}},\frac{x_{8k+12}}{y_{8k+12}}\right\}=\max\left\{\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+3)},\frac{\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+4)}}{\left(\frac{x_{0}}{y_{0}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+3)},\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+4)}\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+3)},\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+5)}\right\}$$

$$=\left(\frac{y_{0}}{x_{0}}\right)^{f(2k+5)}$$

The proof is complete.

Teorem 3 : Let $(x_n; y_n)$ be a solution of Eq.(1) for $1 < x_{-3} < y_{-3}$, $1 < x_{-2} < y_{-2}$, $1 < x_{-1} < y_{-1}$ and $1 < x_0 < y_0$. Then, for n = 0,1,2,… we have

a) $\lim_{n\to\infty} x_{8n+1} = \infty;\ \lim_{n\to\infty} x_{8n+2} = \infty;\ \lim_{n\to\infty} x_{8n+3} = \infty;\ \lim_{n\to\infty} x_{8n+4} = \infty;$

$\lim_{n\to\infty} x_{8n+5} = 0;\ \lim_{n\to\infty} x_{8n+6} = 0;\ \lim_{n\to\infty} x_{8n+7} = 0;\ \lim_{n\to\infty} x_{8n+8} = 0$

b) $\lim_{n\to\infty} y_{8n+1} = 0;\ \lim_{n\to\infty} y_{8n+2} = 0;\ \lim_{n\to\infty} y_{8n+3} = 0;\ \lim_{n\to\infty} y_{8n+4} = 0;$

$\lim_{n\to\infty} y_{8n+5} = \infty;\ \lim_{n\to\infty} y_{8n+6} = \infty;\ \lim_{n\to\infty} y_{8n+7} = \infty;\ \lim_{n\to\infty} y_{8n+8} = \infty$

İspat: a) We obtain,

for $x_{-3} < y_{-3}$ $$\lim_{n\to\infty} x_{8n+1} = \lim_{n\to\infty}\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2n+2)} = \left(\frac{y_{-3}}{x_{-3}}\right)^{f(\infty)} = \left(\frac{y_{-3}}{x_{-3}}\right)^{\infty} = \infty \ ,$$

for $x_{-2} < y_{-2}$ $$\lim_{n\to\infty} x_{8n+2} = \lim_{n\to\infty}\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2n+2)} = \left(\frac{y_{-2}}{x_{-2}}\right)^{f(\infty)} = \left(\frac{y_{-2}}{x_{-2}}\right)^{\infty} = \infty \ ,$$

for $x_{-1} < y_{-1}$ $$\lim_{n\to\infty} x_{8n+3} = \lim_{n\to\infty}\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2n+2)} = \left(\frac{y_{-1}}{x_{-1}}\right)^{f(\infty)} = \left(\frac{y_{-1}}{x_{-1}}\right)^{\infty} = \infty \ ,$$

for $x_0 < y_0$ $$\lim_{n\to\infty} x_{8n+4} = \lim_{n\to\infty}\left(\frac{y_0}{x_0}\right)^{f(2n+2)} = \left(\frac{y_0}{x_0}\right)^{f(\infty)} = \left(\frac{y_0}{x_0}\right)^{\infty} = \infty \ ,$$

for $x_{-3} < y_{-3}$ $$\lim_{n\to\infty} x_{8n+5} = \lim_{n\to\infty}\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2n+2)} = \left(\frac{x_{-3}}{y_{-3}}\right)^{f(\infty)} = \left(\frac{x_{-3}}{y_{-3}}\right)^{\infty} = 0 \ ,$$

for $x_{-2} < y_{-2}$ $$\lim_{n\to\infty} x_{8n+6} = \lim_{n\to\infty}\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2n+2)} = \left(\frac{x_{-2}}{y_{-2}}\right)^{f(\infty)} = \left(\frac{x_{-2}}{y_{-2}}\right)^{\infty} = 0 \ ,$$

for $x_{-1} < y_{-1}$ $$\lim_{n\to\infty} x_{8n+7} = \lim_{n\to\infty}\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2n+2)} = \left(\frac{x_{-1}}{y_{-1}}\right)^{f(\infty)} = \left(\frac{x_{-1}}{y_{-1}}\right)^{\infty} = 0 \ ,$$

for $x_0 < y_0$ $$\lim_{n\to\infty} x_{8n+8} = \lim_{n\to\infty}\left(\frac{x_0}{y_0}\right)^{f(2n+2)} = \left(\frac{x_0}{y_0}\right)^{f(\infty)} = \left(\frac{x_0}{y_0}\right)^{\infty} = 0 \ ,$$

b) We obtain,

for $x_{-3} < y_{-3}$ $$\lim_{n\to\infty} y_{8n+1} = \lim_{n\to\infty}\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2n+1)} = \left(\frac{x_{-3}}{y_{-3}}\right)^{f(\infty)} = \left(\frac{x_{-3}}{y_{-3}}\right)^{\infty} = 0 \ ,$$

for $x_{-2} < y_{-2}$ $$\lim_{n\to\infty} y_{8n+2} = \lim_{n\to\infty}\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2n+1)} = \left(\frac{x_{-2}}{y_{-2}}\right)^{f(\infty)} = \left(\frac{x_{-2}}{y_{-2}}\right)^{\infty} = 0 \ ,$$

for $x_{-1} < y_{-1}$ $$\lim_{n\to\infty} y_{8n+3} = \lim_{n\to\infty}\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2n+1)} = \left(\frac{x_{-1}}{y_{-1}}\right)^{f(\infty)} = \left(\frac{x_{-1}}{y_{-1}}\right)^{\infty} = 0 \ ,$$

for $x_0 < y_0$ $$\lim_{n\to\infty} y_{8n+4} = \lim_{n\to\infty}\left(\frac{x_0}{y_0}\right)^{f(2n+1)} = \left(\frac{x_0}{y_0}\right)^{f(\infty)} = \left(\frac{x_0}{y_0}\right)^{\infty} = 0,$$

for $x_{-3} < y_{-3}$ $$\lim_{n\to\infty} y_{8n+5} = \lim_{n\to\infty}\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2n+3)} = \left(\frac{y_{-3}}{x_{-3}}\right)^{f(\infty)} = \left(\frac{y_{-3}}{x_{-3}}\right)^{\infty} = \infty ,$$

for $x_{-2} < y_{-2}$ $$\lim_{n\to\infty} y_{8n+6} = \lim_{n\to\infty}\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2n+3)} = \left(\frac{y_{-2}}{x_{-2}}\right)^{f(\infty)} = \left(\frac{y_{-2}}{x_{-2}}\right)^{\infty} = \infty ,$$

for $x_{-1} < y_{-1}$ $$\lim_{n\to\infty} y_{8n+7} = \lim_{n\to\infty}\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2n+3)} = \left(\frac{y_{-1}}{x_{-1}}\right)^{f(\infty)} = \left(\frac{y_{-1}}{x_{-1}}\right)^{\infty} = \infty ,$$

for $x_0 < y_0$ $$\lim_{n\to\infty} y_{8n+8} = \lim_{n\to\infty}\left(\frac{y_0}{x_0}\right)^{f(2n+3)} = \left(\frac{y_0}{x_0}\right)^{f(\infty)} = \left(\frac{y_0}{x_0}\right)^{\infty} = \infty .$$

This completes the proof.

3. DISCUSSIONS and CONCLUSION

We do this work in only one of the boundary conditions for (1) behavior of the system of equations has been studied. Researchers at the start of the conditions of the new studies for the different states.

REFERENCES

[1] Amleh A. M., Hoag J. and Ladas G. (1998). A Difference Equation With Eventualy Periodic Solutions, Comput. Math. Appl., 36(10-12), 401-404.

[2] Janowski, E. J., Kocic, V. L., Ladas, G. and Tzanetopoulos, G., (1998). Global behaviour of solutions of $x_{n+1} = \dfrac{\max\{x_n^k, A\}}{x_{n-1}}$, Journal of Difference Equations and Applications, 3, 297-310.

[3] Valicenti, S.,(1999). Periodicity and Global Attractivity of Some Difference Equations, University of Rhode Island, (PhD Thesis).

[4] Teixeria, C. T., (2000). Existence Stability Boundedness and Periodicity of Some Difference Equations, University of Rhode Island, (PhD Thesis).

[5] Papaschinopoulos, G. and Hatzifilippidis, V., (2001). On a max difference equation, Journal of Mathematical Analysis and Applications, 258, 258-268.

[6] Mishev D. P. and Patula W. T.,(2002). A Reciprocal Difference Equation With Maximum, Comput. Math. Appl., 43, 1021-1026.

[7] Voulov, H. D.,(2002). On the periodic character of some difference equations, Journal of Difference Equations and Applications, 8, 799-810.

[8] Voulov, H. D.,(2002). Periodic solutions to a difference equation with maximum, Proceedings of the American Mathematical Society, 131, 2155-2160.

[9] Feuer, J.,(2003). Periodic solutions of the Lyness max equation, Journal of Mathematical Analysis and Applications, 288, 147-160.

[10] Papaschinopoulos, G., Schinas, J. and Hatzifilippidis, V.,(2003). Global behaviour of the solutions of a max-equation and of a system of two max-equation, Journal of Computational Analysis and Applications, 5, 2, 237-247.

[11] Patula, W. T. and Voulov, H. D.,(2004). On a max type recursive relation with periodic coefficients, Journal of Difference Equations and Applications, 10, 3, 329-338.

[12] Cinar C., Stevic S. and Yalcinkaya I. (2005). On The Positive Solutions Of A Reciprocal Difference Equation With Minumum, J. Appl. Math. Computing, 17, 307-314.

[13] Simsek, D., Cinar, C. and Yalcinkaya, I.,(2006). On the solution of the difference equation $x_{n+1} = \max\left\{ {}^{1}\!/\!{}_{x_{n-1}}, x_{n-1} \right\}$, Int. J. Math. Sci., 1, 10, 481-487.

[14] Yan, X., Liao, X. and Li, C.,(2006). On a difference equation with maximum, Applied Mathematics and Computation, 181, 1-5.

[15] Simsek, D., Demir B. and Kurbanlı A.S.,(2009). x{n+1}=max{1/(x{n},y{n}/x{n}}; y{n+1}=max{1/y{n},x{n}/y{n}} Denklem Sistemlerinin Çözümleri Üzerine, Ahmet Keleşoğlu Eğitim Fakültesi Dergisi, 28, 91-104.

[16] Simsek D., Demir B. and Cinar C.,(2009). On the Solutions of the System of Difference Equations x{n+1}=max{A/x{n},y{n}/x{n}}, y{n+1}=max{A/y{n},x{n}/y{n}}, Discrete Dynamics in Nature and Society, Volume 2009, Article ID 325296, 11 pages.

[17] Simsek D., Kurbanlı A. S., Erdoğan M. E. , (2010). x(n+1) = max{1 \ x(n-1) ; y(n-1) \ x(n-1)} ; y(n+1) = max{1 \ y(n-1) ; x(n-1) \ y(n-1)} Fark Denklem Sisteminin Çözümleri, XXIII. Ulusal Matematik Sempozyumu, 153 pp, .04-07 Ağustos 2010, Erciyes Üniversitesi.