



Three–Dimensional Stability Loss Problems of Local Near-Surface Buckling of a System Consisting of an Elastic Bond Layer and an Elastic Covering Layer

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Özet Bu çalışmada üç boyutlu lineerleştirilmiş teory kullanılarak (TLTS), bir boyutlu homojen sistem modeli çerçevesinde yerel yüzeye yakın tek katmanlı ve yarı-uzay elastic malzemeler ile kaplanmış bir sisteme bakılmıştır. Perturbasyo tekniği uygulayarak viskoelastik teorisinin üç boyutlu denklemleri ve sınır koşulları yardımı ile TLTS denklemleri elde edilmiştir. Laplace ve Fourier Yöntemlerinden yararlanarak, problem çözmek için yeni bir yöntem geliştirilmiştir.

Anahtar sözcükler *Burkulma kararsızlığı, kavisli dalga, kritik zaman ,yüzeeye yakın burkulma, kararlılık, viskoelastik yerel tabaka*

Üç Boyutlu Teory Çerçevesinde Yerel Yüzeye Yakın Tabakalarda Elastik ve Elastik Bağlı Sistemler İçin Kayıp Problemleri

Abstract Within the framework of a piecewise homogenous body model and by the use of a three-dimensional linearized theory of stability (TLTS), the local near-surface buckling of a material system consisting of a half-space which is covered by the single layer and half-space materials is elastic. The equations of TLTS are obtained from the three-dimensional geometrically nonlinear equations of the theory of viscoelasticity by using the boundary form perturbations technique. By employing the Laplace and Fourier transform, a method for solving the problem is developed. Numerical results on the critical compressive forces and the critical times are presented.

Keywords *Buckling instability, curved-layer, critical time, local near-surface buckling, stability, viscoelastic layer.*



1. INTRODUCTION

The results of the theoretical study of the growth of the initial imperfections one of which can be taken the insignificant curving of the reinforcing layers or fibers in the structure of the composite materials can be used for estimation of the loading carried capacity of these materials. In recent years it was established that the Carbon-Nanotubes or –Nanofibers have a curving in the structure of the Polymer-Nanocomposites [1]. This statement increases also the significance of the study for the influence of the curving of the reinforcing elements in the structure of the composite materials to the mechanical behavior of those. According to Refs. [2-4] and others, the curving of the reinforcing elements may be due to the design features (as in a woven composites), or to technological processes resulting from the action of various factors (as in a Polymer-Nanocomposites). Moreover the aforementioned curving can be taken [3, 5, 6, 26] as a geometrical model for the structure of the composite materials for the investigation of the various type of fracture (internal or near-surface stability loss) problems for the unidirectional composites under compression along the reinforcing elements. Owing to such modeling employing “boundary form perturbation” technique in the papers [5–11] the Three-dimensional Linearized Theory of Stability (TLTS) [12, 13] was developed for the internal and near surface stability loss problems for viscoelastic composite materials by employing the initial imperfection criterion [14]. In this case the development of these imperfections with the time flow is investigated within the scope of the piecewise homogeneous body model by the use of the three-dimensional geometrically non-linear field equations of the theory of the viscoelasticity. Using the series representation of the sought values in small parameter characterizing the degree of the initial insignificant imperfections of the reinforcing elements the solution of the non-linear boundary value problems is reduced to the solution of the series linear boundary-value problems. By direct verification it is proven that the linear equations and relations which are attained in these linear boundary value problems coincide with the corresponding ones of the TLTS. Just aforementioned statements allows the authors of the papers [5–11] to take into account the initial imperfection in the relations of the TLTS and employ the TLTS to investigate the stability loss problems of the time dependent materials within the framework of the initial imperfection criterion. Moreover, in the paper [5] it was proved that for the investigation of the stability loss problems and the determination of the values of the critical forces or critical time results obtained within the framework of only the zeros and first approximations are enough.

Now we consider some details of the results obtained in the papers [5-11] and start with the paper [5] in which it was assumed that the mode of the initial imperfection of the reinforcing layers is the co-phase periodical plane curving (the plane-strain state was considered). In this case by employing the aforementioned approach the values of the critical forces for elastic composites and the values of the critical time for the viscoelastic composites were determined and it was established that in the particular cases the values of the critical forces coincide with the corresponding results listed in [15] which were attained by employing the Euler approach. In the paper [6] the approach [5] was developed for the unidirectional fibrous viscoelastic composite materials. The near-surface stability loss problems for layered half-plane and half-space are studied in the paper [7] and [8], respectively.

As applied to various structural elements, the conditions of existence of internal as well as surface instability can be represented as $\min |p_{cr.}| < |p_{cr.}^{st}|$, $\ell_{cr.} \ll L$, where $\min |p_{cr.}|$ and $|p_{cr.}^{st}|$ are the critical loads corresponding to the internal or surface instability and buckling of the whole structural element in question, $\ell_{cr.}$ is the half-wavelength of the mode of internal or surface instability and L is the characteristic (minimum) dimension of the structural element. Thus, the phenomenon of internal or surface instability exists in the case where the dependence $|p_{cr.}| = |p_{cr.}(\chi)|$ has a well-defined minimum under $\chi \neq 0$ and the critical value of external load is determined as $\min |p_{cr.}|$ (where $\chi = \pi h / \ell$ for layered or $\chi = \pi R / \ell$ for fibrous materials; h is thickness of the reinforcing layer, R is a radius of fibre cross-section, ℓ is the half-wavelength of the initial imperfection mode). The value of the wave-generation parameter χ corresponding to $\min |p_{cr.}|$ is taken as critical value of that and denoted as $\chi_{cr.} = \pi h / \ell_{cr.}$.

Note that for the viscoelastic composites the foregoing procedures are made for the time $t=0$ and $t=\infty$ (where t is a time) separately and $p_{cr,0}$ (for $t=0$) and $p_{cr,\infty}$ (for $t=\infty$) are defined. For these cases where $p_{cr,\infty} < p < p_{cr,0}$ the values of the critical time (denoted by $t_{cr.}$) are defined.

In the paper [9] the composite consisting of the alternating layers of two materials is also considered and as initial imperfection in the structure of the material the local curving of the filler layers is taken. As in [5-8], it was assumed that this composite is compressed at infinity along the layers and the development of the aforementioned initial local imperfection with compressive force for elastic composite as well as with time for viscoelastic composite is investigated. As a result of these investigations which were made for the plane strain state, it was established that the critical values of the compressive force for the elastic composite as well as the values of the critical time for the viscoelastic composite does not depend on the initial local imperfection mode and on the values of the wave-generation type (as χ) parameter. By direct verification it is proved that the critical values of the compressive force coincide with the corresponding values of the Theoretical Strength Limit in Compression (TSLC) which are determined within the framework of the continual approach [2, 13]. Consequently, the values obtained for the critical time were related the TSLC of the corresponding viscoelastic composite material. Thus, in the paper [9] the approach proposed in [5] was developed for the determination of the TSLC of the unidirectional elastic and viscoelastic composite materials within the framework of the piecewise homogeneous body model.

However, up to now there were no studies on the near-surface local stability loss problems based on the development of the initial insignificant near-surface local curving (imperfection) of the reinforcing layer with external compressive force for the viscoelastic composite materials within the framework of the approach [5-11]. In the present paper the first attempt is made in this field and the approach [5-11] is developed and applied for the investigation on the near-surface local stability loss of the system consisting of the elastic (viscoelastic) substrate, viscoelastic (elastic) bond layer and elastic (viscoelastic) covering layer. As in [5-11], the equations and relations of the TLTS are attained from the geometrically non-linear exact equations of the theory of the viscoelasticity by employing the boundary-form perturbation technique.

2. PROBLEM FORMULATION

Let us consider a semi-infinite half-space joined with the stack consisting of the finite number of the filler (reinforcing) and binder layers. We are assuming here that the filler layers in this system have an insignificant initial imperfection in the curving form. To ease the evaluation one may suppose that the stack consists of three layers (see Figure1).

Values corresponding to the binder layers and half-space will be denoted by upper indices (2); values related with the filler layers by upper indices (1). Furthermore, the values related

with each selected layer and half-space will be represented by the additional index, indicating its sequence in the considered body. If we associate the corresponding Lagrangian coordinates

$O_m^{(k)} x_{1m}^{(k)} x_{2m}^{(k)} x_{3m}^{(k)}$ ($k=1,2,3$) which in their natural state coincide with Cartesian coordinates, then we obtain the new coordinate system from $Ox_1x_2x_3$ by parallel transfer along the Ox_2 -axis, with the middle surface of each layer of the filler and binder. Figure1 shows the cross section of the considered system at $x_{1m}^{(k)} = const(x_{3m}^{(k)} = const$).

In the evaluation the thickness of every filler layer will be assumed constant. It will also be proposed that the binder, filler and half-space materials are homogeneous, anisotropic and non-aging (hereditary) linearly viscoelastic. Now we investigate the stress deformation state in the above body under compression at "infinity" by uniformly distributed normal forces of intensity $p_1(p_3)$ in the direction of the $Ox_1(Ox_3)$ -axis. For each layer and for half-space we write the equilibrium equations, constitutive and geometrical relations as follows:

$$\frac{\partial}{\partial x_{jm}^{(k)}} \left[\sigma_{jn}^{(k)m} \left(\delta_i^n + \frac{\partial u_i^{(k)m}}{\partial x_{nm}^{(k)}} \right) \right] = 0,$$

$$\sigma_{ij}^{(k)m} = C_{ijrso}^{(k)m} \varepsilon_{rs}^{(k)m}(t) + \int_0^t C_{ijrso}^{(k)m}(t-\tau) \varepsilon_{rs}^{(k)m}(\tau) d\tau,$$

$$2\varepsilon_{ij}^{(k)m} = \frac{\partial u_i^{(k)m}}{\partial x_{jm}^{(k)}} + \frac{\partial u_j^{(k)m}}{\partial x_{im}^{(k)}} + \frac{\partial u_n^{(k)m}}{\partial x_{im}^{(k)}} \frac{\partial u_n^{(k)m}}{\partial x_{jm}^{(k)}}$$

(1)

$i, j, n, r, s = 1, 2, 3,$

$k, m = 1, 2$

We assume that between the components of the considered system there is a complete cohesion

$$\left[\sigma_{jn}^{(1)1} \left(\delta_i^n + \frac{\partial u_i^{(1)1}}{\partial x_{n1}^{(1)}} \right) \right]_{S_1^-} n_j^{1-} = \left[\sigma_{jn}^{(2)1} \left(\delta_i^n + \frac{\partial u_i^{(2)1}}{\partial x_{n1}^{(2)}} \right) \right]_{S_1^-} n_j^{1-},$$

$$u_i^{(1)1} \Big|_{S_1^-} = u_i^{(2)1} \Big|_{S_1^-},$$

$$\left[\sigma_{jn}^{(2)1} \left(\delta_i^n + \frac{\partial u_i^{(2)1}}{\partial x_{n1}^{(2)}} \right) \right]_{S_1^-} n_j^{2+} = \left[\sigma_{jn}^{(1)2} \left(\delta_i^n + \frac{\partial u_i^{(1)2}}{\partial x_{n2}^{(1)}} \right) \right]_{S_1^-} n_j^{2-},$$

(2)

$$u_i^{(2)1} \Big|_{S_2^+} = u_i^{(1)2} \Big|_{S_2^+},$$

$$\left[\sigma_{jn}^{(1)2} \left(\delta_i^n + \frac{\partial u_i^{(1)2}}{\partial x_{n2}^{(1)}} \right) \right]_{S_2^-} n_j^{2-} = \left[\sigma_{jn}^{(2)2} \left(\delta_i^n + \frac{\partial u_i^{(2)2}}{\partial x_{n2}^{(2)}} \right) \right]_{S_2^-} n_j^{2-},$$

$$u_i^{(1)2} \Big|_{S_2^-} = u_i^{(2)2} \Big|_{S_2^-},$$

In-Equ. (2) the values indicated by the upper index (2) 2 regard the half-space and these values satisfy also the decay conditions.

$$\sigma_{11}^{(2)2} \rightarrow p_1, \sigma_{33}^{(2)2} \rightarrow p_3, \sigma_{ij}^{(2)2} \rightarrow 0,$$

(3)

for $ij \neq 11; 33$ as $x_{22}^{(1)} \rightarrow -\infty.$

Furthermore, on the upper surface of the 1⁽¹⁾ st layer the conditions

$$\left[\sigma_{jn}^{(1)1} \left(\delta_j^n + \frac{\partial u_i^{(1)1}}{\partial x_n^{(1)}} \right) \right]_{S_1^+} n_j^{1+} = 0$$

(4)

met.

In -Equ.(2) and -Equ. (4), the components of the unit normal vector to the surfaces S_m^\pm re denoted $n_j^{m\pm}$. The other notation used in Eqs. (1), (2), (3) and (4) is well-known by the

specialist. The initial insignificant imperfection of the filler layers is expressed by the equations of the middle surface of those as

$$x_{2m}^{(1)} = F_m(x_{1m}^{(1)}, x_{3m}^{(1)}) = \varepsilon f_m(x_{1m}^{(1)}, x_{3m}^{(1)}), \quad (5)$$

where ε is a dimensionless small parameter ($0 \leq \varepsilon \leq 1$).

In addition, we suppose that the functions $F_m(x_{1m}^{(1)}, x_{3m}^{(1)})$ and their first-order derivatives are continuous and satisfy the following conditions

$$\left(\frac{\partial F_m}{\partial x_{1m}^{(1)}} \right)^2 + \left(\frac{\partial F_m}{\partial x_{3m}^{(1)}} \right)^2 \ll 1. \quad (6)$$

3. SOLUTION METHOD

The general concepts of the solution procedure, which is used in the present investigation, can be considered according [5]. If we take into account the condition of constant thickness $2h$ of the elastic layer and Eq. (6), we can derive the equations for the surfaces S^\pm (Fig.1), which can be presented as $x_i^{(k)\pm} = x_i^{(k)\pm}(t_1, h, \varepsilon, f(t_1, t_3))$ ($k; i=1, 2, 3$), where t_1, t_3 is a parameter ($t_1, t_3 \in (-\infty, +\infty)$) and h is the half-thickness of the elastic-reinforcing layer. Using these equations, expressions for the components n_i^\pm are also obtained. From the first condition in (9), expressions for $x_i^{(k)\pm}$ and $n_i^{(k)\pm}$ are expanded to the power series in the small parameter ε . From the discussions above, the quantities characterizing the stress-strain state of arbitrary components of the systems considered are indicated as series in the parameter ε as follows:

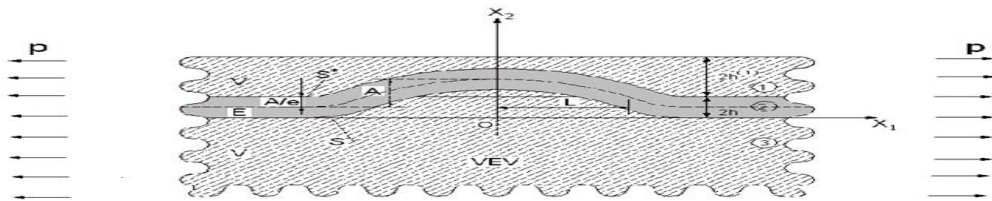


Fig.1: Stability Loss Problems of Local Near-Surface Buckling

$$\{\sigma_{ij}^{(k)}; \varepsilon_{ij}^{(k)}; u_i^{(k)}\} = \sum_{q=0}^{\infty} \varepsilon^q \{\sigma_{ij}^{(k),q}; \varepsilon_{ij}^{(k),q}; u_i^{(k),q}\} \quad (7)$$

If we substitute Eq. (7) into Eqs. (1)-(4) and compare identical powers of ε , we reach the corresponding closed system of equations for contact and boundary conditions. From the linearity of the mechanical relations, these relations can be fulfilled for each approximation (7) separately. Furthermore, the fourth condition in the Egu. (2) and the conditions attained from boundary conditions (2-4) are also satisfied for each approximation. The remaining relations and conditions attained from Eqs. (1), (2), and (4) for each qth approximation contain the values of all approximations made. In this case, for the quantities of zeros approximation, Eq. (1), contact and boundary conditions (2) and (4) are fulfilled at $x_1 = t_1; x_2 = \pm h$ (instead of surfaces S^\pm). For the quantities of the first and subsequent approximations, we attain linear equations and relations by direct verification, coinciding with the corresponding ones of the TLTS [13]. The equations of TLTS are also obtained from the corresponding non-linear equations by

employing the “boundary-form perturbation” technique. However, in [13] the equations of TLTS were attained from the non-linear equations, employing a linearization procedure. Exactly this distinction of the approach [3] from the approach [13] allows one to take into account the initial imperfection in the relations of TLTS and to employ the TLTS to investigate the stability loss problems of a time-dependent material within the framework of the initial imperfection criterion. Moreover, in [5-11], it was proved that, for investigating instability problems and for determining the critical forces or critical times, we need only the results obtained within the framework of the zeroth and first approximations.

Now we attempt to determine the quantities of zeros and first approximations. To do this, we assume that the materials of the half-plane and the layers are moderately rigid and quantities regarding the zeroth approximation can be found from the corresponding linear equations. In addition to that, we presume that $\left| \frac{\partial u_i^{(k),0}}{\partial x_j^{(k)}} \right| \ll 1$ and these quantities can be omitted in the equations of the first approximation.

Now we would like to apply the Laplace transform $\bar{\varphi}(s) = \int_0^{\infty} \varphi(t) \exp(-st) dt$ with $s > 0$ to

all equations and relations corresponding to the zeroth approximation. From the structural arrangement of the problem for this approximation and the principle of correspondence the Laplace transforms of quantities of this approximation are determined as follows

$$\bar{\sigma}_{11}^{(k)m,0} = \frac{\bar{E}^{*(k)m} (\bar{p}_1 + \bar{v}^{*(k)m} \bar{p}_3)}{\bar{E}^{*(2)2} (1 - (\bar{v}^{*(k)m})^2)} - \frac{\bar{E}^{*(k)m} (\bar{p}_3 + \bar{v}^{*(k)m} \bar{p}_1)}{\bar{E}^{*(2)2} (1 - (\bar{v}^{*(k)m})^2)}; \quad (8)$$

$$\bar{\sigma}_{11}^{(k)m,0} = \frac{\bar{E}^{*(k)m} (\bar{p}_1 + \bar{v}^{*(k)m} \bar{p}_3)}{\bar{E}^{*(2)2} (1 - (\bar{v}^{*(k)m})^2)} - \frac{\bar{E}^{*(k)m} (\bar{p}_3 + \bar{v}^{*(k)m} \bar{p}_1)}{\bar{E}^{*(2)2} (1 - (\bar{v}^{*(k)m})^2)}$$

$$\bar{\sigma}_{ij}^{(k)m,0} = 0 \quad \text{for } ij \neq 11, 33$$

where $\bar{E}^{*(k)m}$ and $\bar{v}^{*(k)m}$ are Laplace transforms of the operators

$$E^{*(k)m} = E_0^{(k)m} + \int_0^t E^{(k)m}(t-\tau) d\tau, \quad (9)$$

$$v^{*(k)m} = v_0^{(k)m} + \int_0^t v^{(k)m}(t-\tau) d\tau.$$

Here $E_0^{(k)m}$ and $v_0^{(k)m}$ are the instantaneous values of modulus of elasticity and Poisson's ratio of the (k) mth material. We determine the original of the sought values by the use of the Schapery's method [16].

Let us determine the values of the first approximation for which the following equations and relations are obtained. The governing field equations:

$$\frac{\partial \sigma_{ji}^{(k)m,1}}{\partial x_{jm}^{(k)}} + \sigma_{11}^{(k)m,0} \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2} + \sigma_{33}^{(k)m,0} \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} = 0. \quad (10)$$

The mechanical and geometrical relations:

$$\sigma_{ji}^{(k)m,1} = \lambda^{*(k)m} \theta^{(k)m,1} \delta_i^j + 2\nu^{*(k)m} \varepsilon_{ji}^{(k)m,1}, \theta^{(k)m,1} = \varepsilon_{11}^{(k)m,1} + \varepsilon_{22}^{(k)m,1} + \varepsilon_{33}^{(k)m,1},$$

$$\varepsilon_{ji}^{(k)m,1} = \frac{1}{2} \left(\frac{\partial u_i^{(k)m,1}}{\partial x_{jm}^{(k)}} + \frac{\partial u_j^{(k)m,1}}{\partial x_{im}^{(k)}} \right), \nu^{*(k)m} = \frac{1}{2} \left(\frac{E^{*(k)m} \nu^{*(k)m}}{(1 + \nu^{*(k)m})(1 - 2\nu^{*(k)m})} \right), \quad (11)$$

$$\mu^{*(k)m} = \frac{E^{*(k)m}}{2(1 + \nu^{*(k)m})}$$

The boundary conditions:

$$\begin{aligned} \sigma_{21}^{(1),1,1} \Big|_{x_2^{(1)} = -h_1^{(1)}} - \sigma_{11}^{(1),1,0} \Big| \frac{df}{dx_1^{(1)}} &= 0, \\ \sigma_{22}^{(1),1,1} \Big|_{x_2^{(1)} = +h_1^{(1)}} &= 0, \\ \sigma_{23}^{(1),1,1} \Big|_{x_2^{(1)} = +h_1^{(1)}} - \sigma_{33}^{(1),1,0} \Big| \frac{df}{dx_3^{(1)}} &= 0, \end{aligned} \quad (12)$$

The complete cohesion conditions:

$$\begin{aligned} \sigma_{2i}^{(1),1,1} \Big|_{x_2^{(1)} = -h_1^{(1)}} - \sigma_{2i}^{(2),1,1} \Big|_{x_2^{(2)} = +h_1^{(2)}} &= (\sigma_{11}^{(1),1,0} - \sigma_{11}^{(2),1,0}) \\ &\frac{df_1}{dx_{11}^{(1)}} \delta_i^1 + (\sigma_{33}^{(1),1,0} - \sigma_{33}^{(2),1,0}) \frac{df_1}{dx_{31}^{(1)}} \delta_i^3 \end{aligned} \quad (13)$$

$$u_i^{(1),1,1} \Big|_{x_2^{(1)} = -h_1^{(1)}} - u_i^{(2),1,1} \Big|_{x_2^{(2)} = +h_1^{(2)}} = 0 \quad i=1, 2, 3$$

An advanced development of the method of solution is based upon the selection of the initial imperfection mode, i.e. on the selection of the function (5). In the present investigation, according to the undulation stability loss mode dealt with in the regarding works carried out for the time-independent materials and tabulated in Ref. (15), we presume that

$$f = f_1(x_1) \cdot f_3(x_3); \quad f = e^{-\left(\frac{x_1}{L_1}\right)^2 - \gamma^2 \left(\frac{x_3}{L_1}\right)^2} \quad (14)$$

$$\text{where } \gamma = \frac{l_1}{l_3}.$$

In the cause of $\gamma = \frac{l_1}{l_3} = 0$ ($l_3 \rightarrow \infty$) the results attained [10] in the two-dimensional problems.

We propose that $L \ll l_1$ and the dimensionless small parameter ε is identified by $\varepsilon=L/l_1$. Eventually, the expressions for, and in (11) and (12), according to (5) and(13), are defined as

$$\frac{df}{dx_1} = f_1'(x_1).f_3(x_3); \quad \frac{df}{dx_3} = f_1(x_1).f_3'(x_3) \quad (15)$$

Thus, for the considered case, the determination of the values of the first approximation is degraded to the solution of the integro-differential Eqs. (10) and (11) with boundary conditions (12) and contact conditions (13) .Note that the coefficients $\sigma_{11}^{(k)m,0}$ and $\sigma_{33}^{(k)m,0}$ in Eq. (10) are time-dependent, i.e. $\sigma_{11}^{(k)m,0} = \sigma_{11}^{(k)m,0}(t)$ and $\sigma_{33}^{(k)m,0} = \sigma_{33}^{(k)m,0}(t)$. Just this statement violates the application of the principle of correspondence in the solution to the problems (10), (11), (12), (13), (14) and (15). We propound the following method for the solution to the problem.

If one applies Laplace transform to the Eqs. (10), (11), (12) and (13), the following equations are obtained from (10)

$$\frac{\partial \sigma_{ji}^{(k)m,0}}{\partial x_{jm}^{(k)}} + \int_0^\infty \sigma_{11}^{(k)m,0}(t) \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2} \exp(-st) dt + \quad (16)$$

$$\int_0^\infty \sigma_{33}^{(k)m,0}(t) \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} \exp(-st) dt = 0 \quad i;j=1,2,3.$$

The other relations (11), (12) and (13) hold also for the Laplace transforms of the sought values with the changes

$$\left\{ \sigma_{ij}^{(k)m,0}, \varepsilon_{ij}^{(k)m,0}, u_i^{(k)m,0}, \lambda^{*(k)m}, \mu^{*(k)m} \right\} \Rightarrow \left\{ \bar{\sigma}_{ij}^{(k)m,0}, \bar{\varepsilon}_{ij}^{(k)m,0}, \bar{u}_i^{(k)m,0}, \bar{\lambda}^{*(k)m}, \bar{\mu}^{*(k)m} \right\} \quad (17)$$

Now we consider the integral terms in Eq.(16)

$$\int_0^\infty \sigma_{11}^{(k)m,0}(t) \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2} \exp(-st) dt \quad \text{and} \quad (18)$$

$$\int_0^\infty \sigma_{33}^{(k)m,0}(t) \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} \exp(-st) dt$$

For the mechanical reasons, the values of $\sigma_{11}^{(k)m,0}(t)$ and $\sigma_{33}^{(k)m,0}(t)$ must be limited by their values obtained at $t=0$ and ∞ . In connection with this, one may write

$$\int_0^{\infty} \sigma_{jj}^{(k)m,0}(t) \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} \exp(-st) dt \cong \sigma_{jj}^{(k)m,0}(t_*) \int_0^{\infty} \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} \exp(-st) dt =$$

$$\sigma_{jj}^{(k)m,0}(t_*) \frac{\partial^2 \bar{u}_i^{(k)m,0}}{\partial (x_{3m}^{(k)})^2} \quad j=1,3; i=1,2,3. \quad (19)$$

To finding a certain value of t_* under which Eq. (19) is satisfied exactly is generally impossible or very difficult. In the mean time the exact values of the critical parameters of the investigated stability problems must be limited with those attained under $t_* = 0$ and ∞ . We represent the critical time obtained at $t_* = 0$ ($t_* = \infty$) through $t_{cr,0}$ ($t_{cr,\infty}$).

From the discussions above, the determination of the values of t_{cr} , is reduced to the determinations of the values $t_{cr,0}$ and $t_{cr,\infty}$. In this case the Eq. (16) can be rewritten as follows

$$\frac{\partial \bar{\sigma}_{ji}^{(k)m,1}}{\partial x_{jm}^{(k)}} + \sigma_{11}^{(k)m,0} \frac{\partial^2 \bar{u}_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2} + \sigma_{33}^{(k)m,0} \frac{\partial^2 \bar{u}_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} = 0. \quad (20)$$

Now we try to find the solution to Eqs.(20), (11) and (17) which fulfills the boundary and contact conditions (12) and (13). Substituting (11) into in Eq. (20) we reach

$$\nabla^2 \bar{u}_i^{(k)m,1} + \left(1 + \frac{\bar{\lambda}^{(k)m}}{\bar{\mu}^{(k)m}}\right) \frac{\partial}{\partial x_{im}^{(k)}} \bar{\theta}^{(k)m,1} + \frac{\sigma_{11}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}} \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2} +$$

$$\frac{\sigma_{33}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}} \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} = 0. \quad (21)$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial (x_{1m}^{(k)})^2} + \frac{\partial^2}{\partial (x_{2m}^{(k)})^2} + \frac{\partial^2}{\partial (x_{3m}^{(k)})^2} \quad (22)$$

From Eq. (22) it can be seen that

$$\left(2 + \frac{\bar{\lambda}^{(k)m}}{\bar{\mu}^{(k)m}}\right) \nabla^2 \bar{\theta}^{(k)m,1} + \frac{\sigma_{11}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}} \frac{\partial^2}{\partial (x_{1m}^{(k)})^2} \bar{\theta}^{(k)m,1} +$$

$$\frac{\sigma_{33}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}} \frac{\partial^2}{\partial (x_{3m}^{(k)})^2} \bar{\theta}^{(k)m,1} = 0. \quad (23)$$

$$\nabla^2 \bar{u}_i^{(k)m,1} + \frac{\sigma_{11}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}} \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2} + \frac{\sigma_{33}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}} \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} = -\left(1 + \frac{\bar{\lambda}^{(k)m}}{\bar{\mu}^{(k)m}}\right) \frac{\partial}{\partial x_{im}^{(k)}} \bar{\theta}^{(k)m,1}. \quad (24)$$

If double Fourier transformation is applied, then we obtain to obtain the following ordinary differential equations.

$$\frac{d^2 \bar{\theta}_{13}^{(k)m,1}}{d^2 x_{2m}^{(k)}} - (S_1^2 + s_3^2 + \frac{\sigma_{11}^{(k)m,0}}{2\mu^{*(k)m} + \lambda^{*(k)m}} S_1^2 + \frac{\sigma_{33}^{(k)m,0}}{2\mu^{*(k)m} + \lambda^{*(k)m}} S_3^2) \bar{\theta}_{13}^{(k)m} = 0 \quad (25)$$

$$\frac{d^2 \bar{u}_i^{(k)m,1}}{d^2 x_{2m}^{(k)}} - (S_1^2 + s_3^2 + \frac{\sigma_{11}^{(k)m,0}}{\mu^{*(k)m}} S_1^2 + \frac{\sigma_{33}^{(k)m,0}}{\mu^{*(k)m}} S_3^2) \bar{u}_i^{(k)m} = -\left(1 + \frac{\lambda^{*(k)m}}{\mu^{*(k)m}}\right) \left(\frac{d \bar{\theta}_{13}^{(k)m}}{d x_{2m}^{(k)}} \delta_i^2 + \bar{\theta}_{13}^{(k)m,1} \delta_i^1 + \bar{\theta}_{13}^{(k)m,1} \delta_i^3\right) \quad (26)$$

Solutions of this equations can be given as follows:

$$\begin{aligned} \bar{\theta}_{13}^{(k)m,1} &= \bar{A}_1^{(k)m,1} e^{r_1^{(k)m} x_{2m}^{(k)}} + \bar{A}_2^{(k)m,1} e^{-r_1^{(k)m} x_{2m}^{(k)}} \\ \bar{u}_1^{(k)m,1} &= \bar{C}_{11}^{(k)m} e^{k_1^{(k)m} x_{2m}^{(k)}} + \bar{C}_{12}^{(k)m} e^{-k_1^{(k)m} x_{2m}^{(k)}} + \bar{A}_{11}^{(k)m} e^{r_1^{(k)m} x_{2m}^{(k)}} + \bar{C}_{A12}^{(k)m} e^{-r_1^{(k)m} x_{2m}^{(k)}} \\ \bar{u}_2^{(k)m,1} &= \bar{C}_{21}^{(k)m} e^{k_1^{(k)m} x_{2m}^{(k)}} + \bar{C}_{22}^{(k)m} e^{-k_1^{(k)m} x_{2m}^{(k)}} + \bar{A}_{21}^{(k)m} e^{r_1^{(k)m} x_{2m}^{(k)}} + \bar{C}_{A22}^{(k)m} e^{-r_1^{(k)m} x_{2m}^{(k)}} \\ \bar{u}_3^{(k)m,1} &= \bar{C}_{31}^{(k)m} e^{k_1^{(k)m} x_{2m}^{(k)}} + \bar{C}_{32}^{(k)m} e^{-k_1^{(k)m} x_{2m}^{(k)}} + \bar{A}_{31}^{(k)m} e^{r_1^{(k)m} x_{2m}^{(k)}} + \bar{C}_{A32}^{(k)m} e^{-r_1^{(k)m} x_{2m}^{(k)}} \end{aligned} \quad (27)$$

The equations for the unknown constants entering the expressions of these functions are obtained from the boundary and contact conditions (12) and (13). In way we, therefore determine completely the Laplace transform of the values the first approximation. These functions are found by employing Schapery's method [16] and the critical time is evaluated from the criterion

$$\left| u_2^{(1)1,1}(0) \right| \rightarrow \infty, \text{ as } t \rightarrow t_{cr} \quad (28)$$

At this point we restrict ourselves to consideration of the method of solution. This method with the corresponding changing can be applied for investigation of the other analogous problems. Moreover, the approach can also be carried out for the time-independent materials. In this case by introducing the notation $p_3 = np_1$ the criterion (28) must be relocated with the following one.

$$\left| u_2^{(1)1,1}(0) \right| \rightarrow \infty, \text{ as } p \rightarrow p_{1cr} \quad (29)$$

According to the problem statement and the expression (14), we employ the exponential double Fourier transformation $g_{13}(s_1, s_3) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(x_1, x_2) e^{-i(s_1 x_1 + s_3 x_3)} dx_1 dx_3$ to the Eqs. (16), (17), boundary and contact conditions (12), (13) with respect to the variable $x_1^{(k)}$.

4. NUMERICAL RESULTS AND DISCUSSIONS

We assume that the viscoelasticity of the material is described by the operators

$$E^{*(k)} = E_0^{(k)} [1 - \omega_0 R_\alpha^*(-\omega_0 - \omega_\infty)], \quad \nu^{*(k)} = \nu_0^{(k)} \left[1 + \frac{1 - 2\nu_0^{(k)}}{\nu_0^{(k)}} \omega_0 R_\alpha^*(-\omega_0 - \omega_\infty) \right], \quad (30)$$

$$\lambda^{*(k)} = \lambda_0^{(k)} \left[1 + \frac{1 - 2\nu_0^{(2)}}{2\nu_0^{(2)}(1 + \nu_0^{(2)})} \omega_0 R_\alpha^* \left(-\frac{3}{2(1 + \nu_0^{(2)})} \omega_0 - \omega_\infty \right) \right], \quad (31)$$

$$\mu^{*(k)} = \mu_0^{(k)} \left[1 - \frac{3\omega_0}{2\nu_0^{(2)}(1 + \nu_0^{(2)})} R_\alpha^* \left(-\frac{3}{2(1 + \nu_0^{(2)})} \omega_0 - \omega_\infty \right) \right]. \quad (31)$$

where $E_0^{(k)}$ and $\nu_0^{(k)}$ are the instantaneous values of Young's modulus and the Poisson ratio, respectively; $\lambda_0^{(k)}$ and $\mu_0^{(k)}$ are the instantaneous values of Lamé constants; α , ω_0 and ω_∞ are rheological parameters; R_α^* is the Rabotnov fractional-exponential operator [17], which is defined as

$$R_\alpha(\beta)\Psi(t) = \int_0^t R_\alpha(\beta, t - \tau)\Psi(\tau)d\tau, \quad R_\alpha(\beta) = t^\alpha \sum_{n=0}^{\infty} \frac{\beta^n t^{n(1+\alpha)}}{\Gamma[(n+1)(1+\alpha)]}, \quad -1 < \alpha \leq 0, \quad (32)$$

where $\Gamma(x)$ is the gamma function.

We introduce the dimensionless rheological parameter $\omega = \omega_\infty / \omega_0$ and dimensionless time $t' = \omega_0^{1/(1+\alpha)} t$. First, we consider the case where the materials of the covering layer and half-plane are also pure elastic and analyze the distribution of the normal stress $\sigma_{nn} (= \varepsilon \sigma_{22}^{(1),1})$ acting on the interface surface between covering and bond layers. We assume that $\varepsilon = 0.015$, $E^{(2)}/E^{(1)} = E^{(2)}/E^{(3)} = 50$, $\nu_0^{(3)} = \nu_0^{(2)} = \nu_0^{(1)} = 0.3$, $\gamma = 0; 20; 100; 200; 300$. The influence of the geometrical non-linearity on the mentioned distribution will be characterized through the parameter $e = p/E^{(2)} \times 10^3$. Thus, within the scope of the foregoing assumptions we analyze the numerical results and begin this analysis with those regarding to the dependence between $\sigma_{nn} / \sigma_{11}^{(1),0}$ (at $x_1/L = 0.0$) and $h^{(1)}/L$. The graphs of this dependence constructed for various values of e under $h^{(2)}/L = 0.3$, $m = 0.0$ are given in Fig. 2.

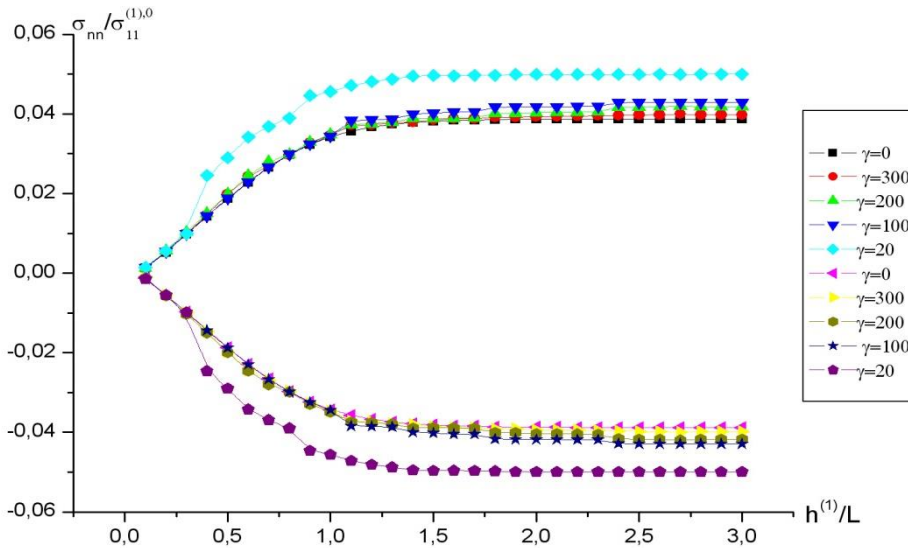


Figure 2. The relations between $\sigma_{nn}/\sigma_{11}^{(1),0}$ and $h^{(1)}/L$ for values of e, γ under $h^{(2)}/L=0.3$

According to the well-known mechanical consideration, the values of σ_{nn} must approach to its asymptotic (limit) values with $h^{(1)}/L$ and these limit values are the values of σ_{nn} attained for the case where layer 2 (Fig. 1) is contained in the infinite body from the material of the half-plane 3 (Fig. 1). This prediction is proved by the graphs given in Fig. 2. and the noted limit values under $|e|=1.0$ almost coincide with those obtained within the scope of the linear theory of elasticity and analyzed in the monograph [3]. At the same time, the graphs given in Fig. 2 show that as a result of the accounting of the geometrical non-linearity the absolute values of σ_{nn} decrease (increase) with $|e|$ under tension (compression) of the considered material. In the qualitative sense these results agree with the corresponding ones given in the monograph [3] and in the paper [22]. Consequently, the results illustrate the trustiness and validity of the employed algorithm and PC programs. From the Fig. 2. it can be easily seen that as γ is getting bigger values $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ values approaches to a limit in which γ is equal to zero, showing that the assumptions made here (in the work) is valuable.

Consider the graphs given in Fig. 3 which show the dependence between $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ (at $x_1/L=0.0$) and $h^{(2)}/L$ constructed for various values of e under $h^{(1)}/L=0.45$ $\gamma=0;100;200;300$. According to a lot of investigations detailed in the monograph [3] such type dependencies have a non-monotonic character. It follows from the graphs given in Fig. 3 that this character of the mentioned dependence occurs also for the considered case. Moreover, the graphs show that the value of $h^{(2)}/L$ under which $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ becomes maximum, increasing with e . It is clear from the Fig.3. that as γ gets higher values $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ approaches to a limit in which γ is equal to zero, which again justifies the assumptions made in this work.

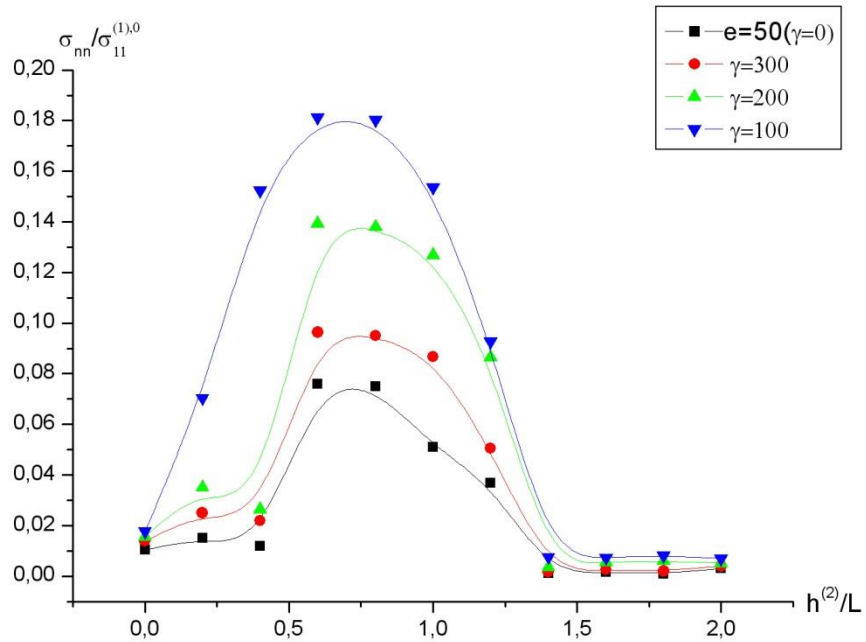


Figure 3. The relations between $\sigma_{nn}/\sigma_{11}^{(1),0}$ and $h^{(2)}/L$ for various e, γ under $h^{(1)}/L=0.45$.

Let us analyze the influence of the parameter m which enters the expression of the function (12), characterizing the local curving form of the reinforcing layer on the distribution of the $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ with respect to x_1/L . The graphs of this distribution attained in the case where $|e| = 5.0$, $h^{(1)}/L = 0.45$, $h^{(2)}/L = 0.3$, $\gamma = 0;100;200;300$ are given in Fig. 4. These results show that the values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ increase with m . Consequently, the oscillation character of the local curving form causes to increase values of the self-balanced normal stress. As can be seen from Fig.4. when γ gets higher values $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ approaches to a limit for which γ is equal to zero.

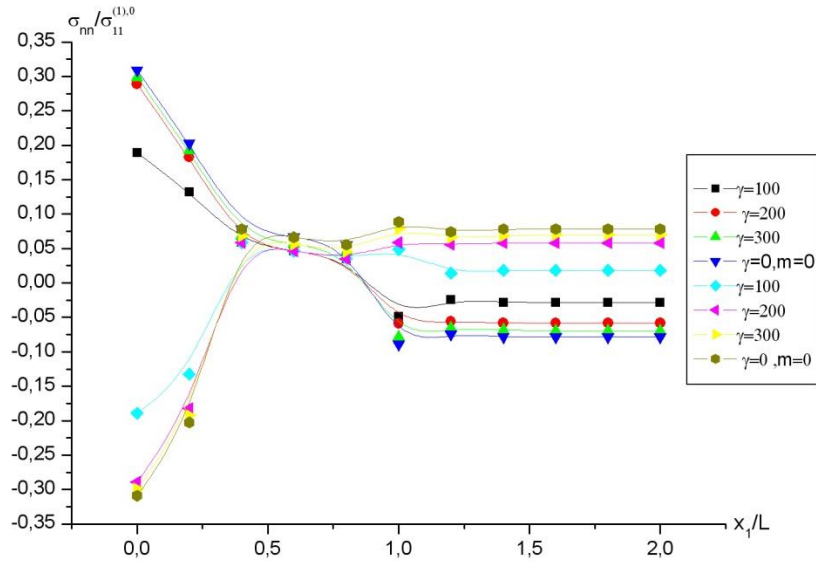


Figure 4. The relations between $\sigma_{nn}/\sigma_{11}^{(1),0}$ and x_1/L for values m, γ under $h^{(2)}/L=0.3, h^{(1)}/L=0.45$

Now we assume that the materials of the covering layer and of the half-plane are the viscoelastic ones with operators (17), (18). We investigate the influence of the rheological parameters ω and α on the values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ and assume that $t'_* = 0.0$ in equations (13) and (14). Fig. 5 shows the dependence between $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ (at $x_1/L = 0.0$) and dimensionless time t' for various values of ω (α) under $h^{(2)}/L = 0.3$, $h^{(1)}/L = 0.45$, $m = 0.0$, $\alpha = -0.5$ ($\omega = 3.0$), $\gamma = 0; 20; 100; 200; 300$. It follows from these results that the values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ increase with time and approach to the corresponding limit values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ attained in the case where the materials of the covering layer and half-plane are the elastic ones with elasticity constants $E_\infty^{(1)} = E_0^{(1)}(1 - 1/(1 + \omega))$, $\nu_\infty^{(1)} = \nu_0^{(1)}(1 + (1 + 2\nu_0^{(1)})/(\nu_0^{(1)}(1 + \omega)))$. At the same time, the results show that the values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ decrease with ω . As γ gets higher values the values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ approaches to a limit for which γ is equal to zero. This statement is explained with decreasing of the values of $E_\infty^{(1)}$ with ω . But the influence of the rheological parameter α on the values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ has more complicate character. For the certain values before (after) t' , the values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ increase (decrease) with $|\alpha|$. Such type results were also obtained in the papers listed [5, 6].

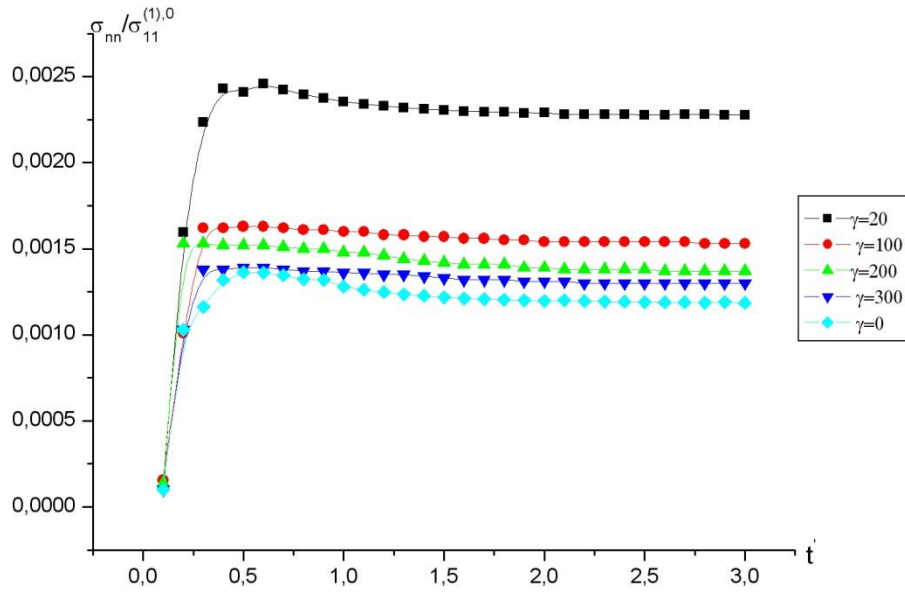


Figure 5. The relation between $\sigma_{nn}/\sigma_{11}^{(1),0}$ and t for various ω under $e=5.0$; $h^{(2)}/L=0.3$; $h^{(1)}/L=0.45$; $\alpha=-0.5$

With the foregoing investigations we restrict our-selves to consideration of the self-balanced normal stress distribution. Note that the near-surface adhesion strengths of this material depends mainly on the values of this stress. Now we consider the formulation of the failure criterion for the considered material. For this purpose we introduce the following notation: Π_1^\pm are the ultimate strengths, + and - representing respectively tension and compression along the Ox_1 axis, Π_2^+ is the ultimate strength in tension in the direction of the Ox_2 axis. Since the values of Π_2^+ are determined mainly by the adhesion strength or by the matrix material we have

$$\Pi_2^+/\Pi_1^+ \ll 1.0 \quad (33)$$

According to the experimental investigations given in the monograph [23], for the class-fiber reinforced plastics the relation $\Pi_2^+/\Pi_1^+ = 0.055 - 0.10$ occurs and this relation satisfies the inequality (19). Consequently, the near-surface failure of the considered material occurs when the relations

$$\sigma_{nn} = \Pi_2^+, \quad \sigma_{11}^{(1),0} < \Pi_1^+, \quad (34)$$

are hold. Foregoing numerical results show that in many considered cases the inequality $\sigma_{nn}/\sigma_{11}^{(1),0} > \Pi_2^+/\Pi_1^+$ satisfies. From this inequality it follows that for the investigated problem the criterion (20) is acceptable. Using the presented numerical results the change range of the values of the problem parameters under which the criterion (19), (20) can be satisfied can be easily determined. Consequently, the presented near-surface failure model and criterion is very real .

5. CONCLUSIONS

Within the framework of a piecewise homogeneous body model with the use of the exact equations of the geometrical non-linear theory for the viscoelastic body the near-surface self-balanced normal stress distribution in a body consisting of a viscoelastic half-plane, an elastic locally curved bond layer, and a viscoelastic covering layer has been investigated. A method for solving the problem considered by employing the Laplace and Fourier transformations was developed.

Numerical results on the self-balanced normal stress caused by the local curving (imperfection) of the elastic bond layer under stretching, as well as under compressing of the body mentioned along the free

face plane were presented and analyzed. The viscoelasticity of the materials was described by the Rabotnov fractional-exponential operators [17].

From the analyses performed the following conclusions can be drawn.

The absolute values of the self-balanced normal stress decrease with approaching of the local curving elastic layer to the free surface which bounds the body considered;

The absolute values of the self-balanced normal stress increase (decrease) with absolute values of the external compressing (stretching) forces. This statement is explained with the accounting of the geometrical non-linear effect;

The dependence between the self-balanced normal stress and ratio $h^{(2)}/L$ has non-monotone character;

The absolute values of the self-balanced normal stress grow with time and approach to its limit values which correspond the case where the materials of the covering layer and half-plane are pure elastic with material constants $E_{\infty}^{(1)} = E_0^{(1)}(1 - 1/(1 + \omega))$, $v_{\infty}^{(1)} = v_0^{(1)}(1 + (1 + 2v_0^{(1)})/(v_0^{(1)}(1 + \omega)))$;

An increase in the values of the rheological parameter ω causes to decreases in the self-balanced normal stress;

The macroscopic failure criterion (19), (20) is presented and according to the obtained numerical results it is established that this criterion is very real for the estimation of the near-surface failure of the considered type near-surface damage in the structure of the layered composite materials;

Taking the restrictions detailed in the papers [24, 25] into account, the approach used in solving the problem considered can also be applied to studying the near-surface failure problems for nano-composite materials in tension.

In the case of $\gamma = 0$ the results attained in [13] show that the three dimensional system problem reduces to the two-dimensional one.

As γ gets higher values the values of $|\sigma_{nn}/\sigma_{11}^{(1,0)}|$ approaches to a limit for which γ is equal to zero. Showing the assumptions made in this work is valuable.

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