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# A Fuzzy Model of Nervous System

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**Abstract:** *Still there is no model of mathematics that is able to represent the true biological process, which can transform the signals, obtained from the nervous system to logical reasoning. However, some mathematical models may approximate to the process of logical reasoning. This paper assumes the set of inputs as fuzzy propositions and uses a LCA (locally compact abelian) group structure to characterize knowledges as Fourier series. The representation used in the study enables us to introduce the notions of logical bases and logical dependencies of knowledges. Also, this makes possible to compare the truthfulness of propositions and define maximal truthfulness that corresponds to the true value in the Boolean logic. Finally, the axiom False ⇒ True of the Boolean logic results.*

**Keywords:** *Nervous system, neurons, Fuzzy Systems, Locally Compact Abelian Group, Fourier Series*

## Sinir Sistemi için Bir Fuzzy Modeli

**Özet:** *Sinir sistemi tarafından algılanan sinyallerin mantıksal akıl yürütmeye nasıl dönüştüğünü temsil eden bir matematiksel model yoktur. Ancak bu yönde önerilecek bazı modeller akıl yürütme sürecine yaklaşımı sağlayabilir. Bu yazı, sinir sistemine yapılan uyarıları birer fuzzy önermesi olarak alıyor ve onları LCA (yerel compact abelian) gruplar üzerindeki Fourier dönüşümleri ile karakterize ediyor. Bu temsil, bilginin mantıksal bir tabana dayalı olmasını sağlıyor ve bilginin, bool mantığına göre maximal doğruluğunu tanımlıyor. Son olarak bool mantığındaki False ⇒ True aksiyomu bir yan ürün olarak ispatlanıyor.*

**Anahtar kelimeler:** *Sinir sistemi, nöronlar, Fuzzy Sistemler, Lokal Kompakt Abel Grubu, Fourier Serileri*

## ASSUMPTIONS

The nervous system is a collection of cells, tissues, and organs through which an organism receives information from its surroundings and then coordinates its voluntary and involuntary actions and transmits signals to and from different parts of the organism.
The basic unit of the nervous system is a neuron. A neuron is a nerve cell capable of passing messages from one end to the other. The sensory (afferent) neurons transmit inputs into the CNS (Central Nerves System). It is assumed that they have the ability to learn from the inputs. So, the reservoir of inputs is a collection of knowledges. The efferent neurons bring instructions from the set of knowledges that tell the organs how to respond to the inputs.

In order to set out a mathematical model, we assume that the inputs transmitted by the afferent neurons are collected in a reservoir *G*, somewhere in the brain. During the transmission, the inputs are transformed into knowledges, which are fuzzy propositions.

*(This background section may be skipped by the reader if he or she wishes so.)*
We have assumed that the inputs transmitted by the neurons are collected in a reservoir somewhere in the brain. The problem is how knowledges interact with each other and produce logical reasoning.  To explain some of these relations on knowledges, we need to define an integral operation on the set *G* of knowledges. In order to achieve this, we use the simplest way by assuming that the set *G* of knowledges is a LCA (locally compact abelian) group and also use the Haar measure on *G*. It is worth reminding that a topological group is locally compact if and only if the identity *u* of the group has a compact neighborhood. This means that there is some open set *V*, containing *u* whose closure is compact in the topology of *G*.

### Mathematical Backgrounds

### Dual Group

If *G* is LCA group, a character of *G* is a continuous group homomorphism from *G* with values in 'Jr (the circle group in C). The set of all characters on *G* can be made into a LCA group, called the *dual* group of *G*, and usually denoted by *G*ˆ. The group operation on the dual group is given by point wise multiplication of characters, the inverse of a character is its complex conjugate. For simplicity of notation we use G = *G*ˆ.

The topology on the space of characters is that of uniform convergence on compact sets (i.e., the compact-open topology, viewing G as a subset of the space of all continuous functions from *G* to T. This topology, in general, is not metrizabale topology. However, if the group *G* is a separable locally compact abelian group, then the dual group G is metrizabale.

It is well known that the group *G* (= R) is isomorphic (as topological groups) to its dual group G (= R). Hence, it is self dual. While the reals and finite cyclic groups are self-dual, the group and its dual group are not naturally isomorphic and should be thought of as two different groups.

We use the simplest way to characterize the knowledges (characters)

$$\gamma : G \to \mathrm{T} \tag{1}$$

by assuming that $G$ = R. The assumption seems natural if we consider the map

$$\gamma : G \to \mathbb{T}, \quad \left(\gamma(x) = e^{ix\gamma}, \quad \gamma \in \mathfrak{G}, \quad x \in G\right) \tag{2}$$

Hence, from now on we assume that $G$ (= R) and G (= R) with G = $G$ˆ.

Accordingly, for a fixed γ ∈ G, while $x$ varies in $G$, the symbol ($x$, γ) will be interpreted as ”γ is a function from $G$ into T.” Likewise, for a fixed $x \in G$, while γ varies in G, the symbol ($x$, γ) will also be interpreted as ”$x$ is a function from $G$ into T.”

In view of this duality between G and $G$, we may interchangeably use the symbols $x$ (γ) = ($x$, γ) = γ($x$), γ ∈ G, $x \in G$ to make it adequate to its context.

**The Fourier Transform**

The *Haar* measure on $G$ allows us to define the notion of integral for (complex-valued) Borel functions, defined on the group $G$.

The dual group of a LCA group is used as the underlying space for an abstract version of the Fourier transform. If $f \in L1$ ($G$), then the *Fourier transform* is the function $f$ˆ on the dual G defined by

$$\hat{f}(\gamma) = \int_G \overline{\gamma(x)} f(x) d\mu(x) \tag{3}$$

where the integral is relative to Haar measure μ on $G$. Of course, the Fourier transform depends on the choice of *Haar* measure μ. The Fourier transform of an $L1$ function on $G$ is a bounded continuous function on G, which vanishes at infinity.

**Inner Product**

Having defined the integral on the LCA group $G$, we are now able to define the inner product of two functions $f$ and $g$ as usual

$$< f, g >= \int_G f(x)\overline{g(x)} d\mu(x) \tag{4}$$

In particular, the inner product of two knowledges (fuzzy propositions) accept the

$$< \gamma_1, \gamma_2 >= \int_G \gamma_1(x)\overline{\gamma_2(x)} dx \tag{5}$$

form.

**Knowledges as Vector Space**

The following is easy to show.

**Theorem 1.** *G and its dual group* G *are vector spaces over the field of reals or complex numbers.*

**Logical Dependence and Logical Basis**

**Definition 1.** *Let F be a finite family of knowledges in* G. *Any sum of the form*

$$f(x) = \sum c\gamma(-x, \gamma), (c\gamma \in \mathrm{C}, \gamma \in S)$$

*is said to be a finite combination of knowledges of F.*

**Definition 2.** *Let A be a family of knowledges in* G, *not necesserily finite. Then the set* A *of all combinations of all finite family of A is called the logical span of A.*

Note that A is the minimal linear space, which contains *A*. We now extend the notion of proposition as follows.

**Definition 3.** *Let A be any subset of* G. *Any element f* ∈ A *is said to be a proposition.*

At this point, assume that all functional connectivity between neurons takes place among the logical span A.
If D is orthonormal, we associate with each proposition *f*, a function *f*ˆ on the set D, defined by

$$\hat{f}(\gamma) = < f, \gamma >, \gamma \in \mathrm{D} \tag{6}$$

The following two lemmas follow from the definition. We have

**Lemma 1.**

$$U \subset V \Rightarrow \mathrm{U} \subset \mathrm{V}$$

*for all U, V* ⊂ G.

**Lemma 2 .** *The relation*

$$span(U \cap V) = \mathrm{U} \cap \mathrm{V}$$

*holds for all U, V* ⊂ G.

**Definition 4.** *The logical base of a proposition f, denoted by B f, is defined to be the set*

$$B f = \cap \{ A \mid f \in \mathrm{A} \} \tag{7}$$

Note that, the base *B f* of a proposition *f* is the subset of G with the property that

$$\hat{f}(\gamma) = \begin{cases} < f, \gamma > \neq 0 & if \quad \gamma \in B_f \\ 0 & if \quad \gamma \notin B_f \end{cases} \tag{8}$$

Let 0 be the falsity, i.e., the zero proposition on $G$ defined by $0(x) = 0$ for all $x \in G$. It is obvious that the logical base of 0 is empty, i.e., $B0 = \phi$ (empty set) and that the logical base of a proposition $f$ is the minimal set $A$ in G for which $f \in$ A.

**Definition 5.** *Two knowledges, f and g, are logically independent if* $B_f \cap B_g = \phi$.

**Definition 6.** *The logical dimension of a proposition f is defined to be the cardinal number of its logical base* $B_f$.

The cardinality $card\,(B_f)$ of $B_f$ satisfies the relation $0 \leq card B_f \leq +\infty$ for each proposition $f$, i.e., the logical base $B_f$ of $f$ is at most countably infinite. If $card\,(B_f) = n$ for some $n \in$ N, then we say that the logical dimension of $f$ is finite. This can be assumed for the majority of animals.

**Lemma 3.** $B\gamma = \{\gamma\}$ *for all* $\gamma \in$ G, *where* $\{\gamma\}$ *stands for the set whose sole element is* $\gamma$.
It follows that $B\gamma \cap B\lambda = \phi$ for all $\gamma, \lambda \in$ G with $\gamma$ -:/ $\lambda$, meaning that different propositions in G are logically independent.

An equivalence relation, denoted by ≡, on $\omega\,(G)$ of all knowledges on $G$ can be defined by

$$g \equiv f \quad \Leftrightarrow \qquad B_g = B_f \qquad (9)$$

Let $[f]$ denote the equivalence class of $f$ in $\omega(G)$, and $\Omega(G)$ stands for the set of equivalence classes $[\,f\,]$, $f \in \omega(G)$.

It is easy to see from (4) that $h \in [\,f\,]$ if there exists a number $a\gamma$ -:/ 0, that
$f\hat{}(\gamma) = a\gamma h\hat{}(\gamma)$ for each $\gamma \in$ G.

**Definition 7.** *We define the binary operation* ∧ *on* $\Omega\,(G)$ *by the relation*

$$[f] \wedge [g] = [f] \cap [g]$$

$[f] \wedge [g]$ is the equivalence class whose base is $B_f \cap B_g$. In particular, we define the operation $f \wedge g$ to be any element in $[f] \wedge [g]$.

**Definition 8.** *We define the binary operation* ∨ *on* $\Omega(G)$ *by the relation*

$$[f] \vee [g] = span\,([f] \cup [g])$$

## Comparisons of Propositions

**Definition 9.** *Let f and g be two knowledges. We say that f implies g, if and only if, the base of f is contained in the base of g. In symbols, we write*

$$f \Rightarrow g \Longleftrightarrow B_f \subseteq B_g \qquad (10)$$

This is an extension of the implications of the form $p \Rightarrow q$ in Boolean logic, where $p$ and $q$ are any two propositions. If we move from $\omega(G)$ to $\Omega(G)$, we may state a more general definition of implications as follows:

$$[f] \Rightarrow [g] \Leftrightarrow B_f \subseteq B_g \qquad (11)$$

**Theorem 2.** *The implication* $0 \Rightarrow f$ *holds for any proposition* $f$.

**Proof.** If $f = 0$, there is nothing to prove. So, let us assume that $f$ -:/ 0, in which case the logical base $B_f$ of $f$ cannot be empty. But, then the relation $B0 = \phi \subset B_f$ holds proving the required result. This simple result proves the important axiom $0 \Rightarrow 1$ in Boolean logic.

**Equivalent of truth in Boolean Logic**

**Definition 10.** *If the base* $B_f$ *of a proposition* $f$ *is equal to a complete orthonormal system* D ∈ G, *then* $f$ *is said to be of maximal truthfulness.*

Clearly, if the base $B_f$ of a proposition $f$ is equal to a complete orthonormal system, then its logical base is maximal. This corresponds fuzzy propositions with value 1. It turns out that they are the *true* values in Boolean logic.

If $< w, \gamma >= 0$ for all $\gamma \in D$ for all orthonormal system $D \subset G$, then the logical base $B_w$ is empty, i.e. $B_w = \emptyset$, implying that $w(x) = 0$ for all $x \in G$. The converse is also true. This corresponds fuzzy propositions with value 0. It turns out that they are the *false* values in Boolean logic.

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