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RESEARCH ARTICLE

General Helices in the 3-dimensional Dual Space

Ljiljana Primorac Gajčić*

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ABSTRACT

In this paper, we introduce general helices in the three-dimensional dual space $\mathbb{D}^3$ as dual curves of a real parameter, whose tangent vector makes a constant inner product with a fixed dual vector. Necessary and sufficient conditions for a curve to be a dual general helix are expressed in terms of the Frenet invariants of the real part. It is shown that there exists a family of general helices that share the same axis and have identical real parts, while their dual parts differ.

Keywords: *Dual space, dual curve, general helix.*

AMS Subject Classification (2020): *Primary: 53A04 ; Secondary: 53A17.*

1. Introduction

In Euclidean differential geometry, general helices, also called the curves of constant slope, are curves whose tangents make a constant angle with a fixed straight line, called the axis of a helix. It is known that such curves are characterized by the constant ratio of its torsion and curvature, [19]. General helices appear in many research, analyzed with some additional properties, for example [3, 11, 12, 16, 23] or their generalizations [17, 18, 27] with many others. They often appear in different scientific fields, such as biochemistry [7, 13, 26] or physics where they, for example, can be used for a curve design in kinematics [5, 21], as well as many other applications [9, 14, 28].

On the other side, the study of dual numbers and their geometric extensions has found applications in modern mechanics, kinematics, and differential geometry. The dual vector space D^3, defined as the set of all vectors of the form $v + \varepsilon w$, where $v, w \in \mathbb{R}^3$ and ε is a nilpotent element satisfying $\varepsilon^2 = 0$, provides an algebraic framework to simultaneously encode both, position and infinitesimal motion in three-dimensional space. One of the leading applications of $\mathbb{D}^3$ arises in screw theory, where dual vectors represent twists, combining angular and linear velocity into a single algebraic object $\omega + \varepsilon v$. This representation has become a foundational tool in robotics, mechanical design, and motion planning [2, 4]. Moreover, in line geometry, the dual space is deeply connected to Plücker coordinates, allowing a dual vector to represent an oriented line via its direction and moment, a dual formulation particularly useful in computational geometry and computer-aided design [15].

Recently, Lopez presented the theory of curves in three dimensional dual space in terms of their Frenet frame, curvature and torsion, [22]. Although there are a lot of papers concerning dual curves, starting from [30] and many following it, such as [1, 6, 20, 24, 25, 31], in [22] author clarified earlier misconceptions, particularly regarding the arc-length parameter and distinction between curves with a real parameter and curves with a dual parameter.

In previous studies on dual helices [20, 24], the curves are considered as curves of a dual parameter. In the present work, we consider dual curves as functions of a real parameter, which yields different results from those in [20]. We define dual general helices and present the consequence of the definition for a parametrization of a general helix (Theorem 3.1 and Theorem 3.2). From that follows that there exists a family of general helices that share the same axis and have identical real parts, while their dual parts differ (Corollary 3.1). We also give conditions for a curve to be a general helix (Theorem 3.3) and show that for the dual general helices the ratio of torsion and curvature does not need to be constant (Corollary 3.2).

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2. Preliminaries

In this section we recall some basic notions and notations concerning dual numbers, dual vectors, and dual curves in the three–dimensional dual space $\mathbb{D}^3$, see for example, [5, 8, 29, 30]. The presentation follows the notation used in [30].
A *dual number* has the form

$$\hat{a} = a + \varepsilon a^*,$$

where $a, a^* \in \mathbb{R}$ and ε is a dual unit satisfying $\varepsilon^2 = 0$ and $\varepsilon \neq 0$. The set of all dual numbers is denoted by

$$\mathbb{D} = \{\, a + \varepsilon a^* \mid a, a^* \in \mathbb{R} \,\}.$$

The *real part* of $\hat{a}$ is $\mathrm{Re}(\hat{a}) = a$, and the *dual part* is $\mathrm{Du}(\hat{a}) = a^*$. For two dual numbers $\hat{a} = a + \varepsilon a^*$ and $\hat{b} = b + \varepsilon b^*$, the basic operations are defined by

$$\hat{a} + \hat{b} = (a + b) + \varepsilon(a^* + b^*), \qquad \hat{a}\hat{b} = ab + \varepsilon(ab^* + a^*b).$$

Division is possible when $\hat{b} \neq 0$ and performed as

$$\frac{\hat{a}}{\hat{b}} = \frac{a + \varepsilon a^*}{b + \varepsilon b^*} = \frac{a}{b} + \varepsilon \frac{a^*b - ab^*}{b^2}.$$

The *three–dimensional dual vector space* $\mathbb{D}^3$ is defined as

$$\mathbb{D}^3 = \{\, \mathbf{v} + \varepsilon \mathbf{w} \mid \mathbf{v}, \mathbf{w} \in \mathbb{R}^3 \,\}.$$

If $\mathbf{V}_1 = \mathbf{v}_1 + \varepsilon \mathbf{w}_1$ and $\mathbf{V}_2 = \mathbf{v}_2 + \varepsilon \mathbf{w}_2$ are dual vectors, their scalar product is given by

$$\mathbf{V}_1 \cdot \mathbf{V}_2 = \mathbf{v}_1 \cdot \mathbf{v}_2 + \varepsilon\big(\mathbf{v}_1 \cdot \mathbf{w}_2 + \mathbf{w}_1 \cdot \mathbf{v}_2\big),$$

where $\cdot$ denotes the usual inner product in $\mathbb{R}^3$. The *norm* of a dual vector $\mathbf{V} = \mathbf{v} + \varepsilon \mathbf{w}$ is defined by

$$\|\mathbf{V}\| = \sqrt{\mathbf{V} \cdot \mathbf{V}} = \|\mathbf{v}\| + \varepsilon \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\|},$$

provided that $\mathbf{v} \neq 0$. A dual vector $\mathbf{U} = \mathbf{u} + \varepsilon \mathbf{u}^*$ is called *unit* if $\|\mathbf{U}\| = 1$. The cross product is given by

$$\mathbf{V}_1 \times \mathbf{V}_2 = \mathbf{v_1} \times \mathbf{v_2} + \varepsilon(\mathbf{v_1} \times \mathbf{w_2} + \mathbf{w_1} \times \mathbf{v_2}).$$

A *dual curve* in $\mathbb{D}^3$ is a mapping $\gamma : I \to \mathbb{D}^3$, $I \subseteq \mathbb{R}$,

$$\gamma(t) = \alpha(t) + \varepsilon\beta(t),$$

where $\alpha(t)$ and $\beta(t)$ are real vector–valued functions in $\mathbb{R}^3$. In [22] the author showed that the conditions of a dual curve γ to have reparametrizations by arc-length are that the curve α must be a regular curve and velocity vectors $\alpha'(t)$ and $\beta'(t)$ must be orthogonal for all $t \in I$. For now on we assume that γ is parametrized by the arc-length parameter s, that is also the arc–length parameter of the real curve α. For convenience, we drop the variable of γ when it is understood. The dual tangent, normal, and binormal vectors are respectively defined by

$$\begin{aligned} T_\gamma &= T_\alpha + \varepsilon\beta' \\ N_\gamma &= N_\alpha + \frac{\varepsilon}{\kappa_\alpha}(\beta'' - (\beta'' \cdot N_\alpha)N_\alpha), \\ B_\gamma &= B_\alpha + \frac{\varepsilon}{\kappa_\alpha}(T_\alpha \times \beta'' + \kappa_\alpha \beta' \times N_\alpha - (\beta'' \cdot N_\alpha)B_\alpha), \end{aligned}$$

where $\{T_\alpha, N_\alpha, B_\alpha\}$ is the Frenet field of the curve α and κ_α and τ_α its curvature and torsion, respectively. The dual curvature, respectively torsion are defined by, [22],

$$\begin{aligned} \kappa &= \kappa_\alpha + \varepsilon(\beta'' \cdot N_\alpha), \\ \tau &= \tau_\alpha + \frac{\varepsilon}{\kappa_\alpha}((\beta''' - \frac{\kappa_\alpha'}{\kappa_\alpha}\beta'' + \kappa_\alpha^2\beta') \cdot B_\alpha - \tau_\alpha(\beta'' \cdot N_\alpha)). \end{aligned}$$

3. General helices in $\mathbb{D}^3$

A regular curve $\alpha(s) \subset \mathbb{R}^3$ is called a *general helix* if its tangent vector makes a constant angle with a fixed direction $e_1 \in \mathbb{R}^3$, that is, $T_\alpha(s) \cdot e_1 = \text{const.}$, [10]. This property will play a key role in the definition of general helices in the dual space $\mathbb{D}^3$.
We now consider a dual curve $\gamma(s) = \alpha(s) + \varepsilon\beta(s)$, where $\alpha, \beta \colon I \to \mathbb{R}^3$, and define the corresponding notion of a dual general helix.

Definition 3.1. The dual curve $\gamma(s) = \alpha(s) + \varepsilon\beta(s)$ is called a dual general helix if there exists a nonzero constant dual vector $E = e_1 + \varepsilon e_2 \neq 0$, such that

$$T_\gamma \cdot E = \text{const.} \tag{3.1}$$

To analyze this condition, we expand the dual scalar product

$$T_\gamma \cdot E = (T_\alpha + \varepsilon\beta') \cdot (e_1 + \varepsilon e_2) = T_\alpha \cdot e_1 + \varepsilon(T_\alpha \cdot e_2 + \beta' \cdot e_1) = \text{const.}$$

Therefore, condition (3.1) is equivalent to the system

$$T_\alpha \cdot e_1 = \text{const.}, \tag{3.2}$$

$$T_\alpha \cdot e_2 + \beta' \cdot e_1 = \text{const.} \tag{3.3}$$

The first relation immediately shows that the real part α is a general helix in $\mathbb{R}^3$. Indeed, since $T_\alpha \cdot e_1 = const.$, differentiation with respect to the arc-length parameter s gives $N_\alpha \cdot e_1 = 0$. Hence, the fixed vector e_1 lies in the plane spanned by T_α and B_α, so it can be written as $e_1 = a(s)T_\alpha + b(s)B_\alpha$, where $a(s) = T_\alpha \cdot e_1$. Since $T_\alpha \cdot e_1$ is constant and e_1 is constant, giving $\|e_1\|^2 = a^2(s) + b^2(s) = const.$, it follows that both coefficients $a(s)$ and $b(s)$ are constant. Therefore we may write $e_1 = aT_\alpha + bB_\alpha$, with constants $a, b \in \mathbb{R}$. On the other hand, the dual curve γ is assumed to be arc-length parametrized. This implies $T_\alpha \cdot \beta' = 0$ meaning that $\beta'(s)$ has no tangential component. Consequently, $\beta'(s) = A(s)N_\alpha(s) + B(s)B_\alpha(s)$, for some differentiable functions $A, B : I \to \mathbb{R}$. We now compute the term $\beta'(s) \cdot e_1$:

$$\beta'(s) \cdot e_1 = (A(s)N_\alpha(s) + B(s)B_\alpha(s)) \cdot (aT_\alpha(s) + bB_\alpha(s)) = bB(s). \tag{3.4}$$

We point out that $\beta'(s) \cdot e_1$ does not need to be constant in order to obtain the constancy of (3.1). Instead, it is sufficient that it compensates the variation of the term $T_\alpha \cdot e_2$. More precisely, from equation (3.3) there exists a constant $\lambda \in \mathbb{R}$ such that $bB(s) = \lambda - T_\alpha \cdot e_2$. Therefore,

$$B(s) = \frac{1}{b}(\lambda - T_\alpha(s) \cdot e_2).$$

Theorem 3.1. *Let $\gamma(s) = \alpha(s) + \varepsilon\beta(s)$ be a dual general helix parametrized with arc–length parameter and with the axis $E = e_1 + \varepsilon e_2$, i.e. $T_\gamma(s) \cdot E = \lambda_1 + \varepsilon\lambda_2$, $\lambda_1, \lambda_2 \in \mathbb{R}$. Then the curve α is a general helix in $\mathbb{R}^3$ with the axis e_1 and the curve β is given by*

$$\beta(s) = \int (A(s)N_\alpha(s) + \frac{1}{b}(\lambda_2 - T_\alpha(s) \cdot e_2)B_\alpha(s))ds,$$

where $b = B_\alpha \cdot e_1$ and differentiable function $A : I \to \mathbb{R}$ is arbitrary.

The previous theorem may also be stated as follows, emphasizing the richer family of general helices in dual space.

Theorem 3.2. *Let α be a general helix in $\mathbb{R}^3$ with axis e_1 and Frenet frame $\{T_\alpha, N_\alpha, B_\alpha\}$. For any constant dual vector $E = e_1 + \varepsilon e_2$, $e_2 \in \mathbb{R}^3$ the curve $\beta \in \mathbb{R}^3$ given as*

$$\beta(s) = \int (A(s)N_\alpha(s) + \frac{1}{b}(\lambda_2 - T_\alpha(s) \cdot e_2)B_\alpha(s))ds,$$

where $\lambda_2 \in \mathbb{R}$ and A is an arbitrary real-valued function, determines a dual curve $\gamma = \alpha + \varepsilon\beta$ which is a dual general helix.

Next we present an example that verifies the previously stated theory.

Example 3.1. Let α be the helix parametrized by the arc-length parameter

$$\alpha(s) = \left(\frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\sin\frac{s}{\sqrt{2}}, \frac{s}{\sqrt{2}}\right)$$

with the axis $e_1 = (0, 0, 1)$. The Frenet frame of α is given by

$$T_\alpha(s) = \frac{1}{2}\left(-\sin\frac{s}{\sqrt{2}}, \cos\frac{s}{\sqrt{2}}, \sqrt{2}\right), \; N_\alpha(s) = \left(-\cos\frac{s}{\sqrt{2}}, -\sin\frac{s}{\sqrt{2}}, 0\right), \; B_\alpha(s) = \frac{1}{\sqrt{2}}\left(\sin\frac{s}{\sqrt{2}}, -\cos\frac{s}{\sqrt{2}}, 1\right).$$

We choose $E = (0, 0, 1) + \varepsilon(1, 0, 0)$ for dual axis and let $\lambda_2 = 1$. Since

$$B_\alpha \cdot e_1 = \frac{1}{\sqrt{2}}, \qquad T_\alpha \cdot e_2 = -\frac{1}{2}\sin\frac{s}{\sqrt{2}},$$

Theorem 3.1 implies

$$\beta_i'(s) = A_i(s)N_\alpha(s) + \sqrt{2}\left(1 + \frac{1}{2}\sin\frac{s}{\sqrt{2}}\right)B_\alpha(s),$$

where $A_i(s)$ is an arbitrary differentiable functions.
Hence, the corresponding dual curves $\gamma_i(s) = \alpha(s) + \varepsilon\beta_i(s)$, are dual general helices satisfying

$$T_{\gamma_i}(s) \cdot E = \frac{1}{\sqrt{2}} + \varepsilon.$$

Examples of the curves β_1 and β_2 are shown in Figure 1.

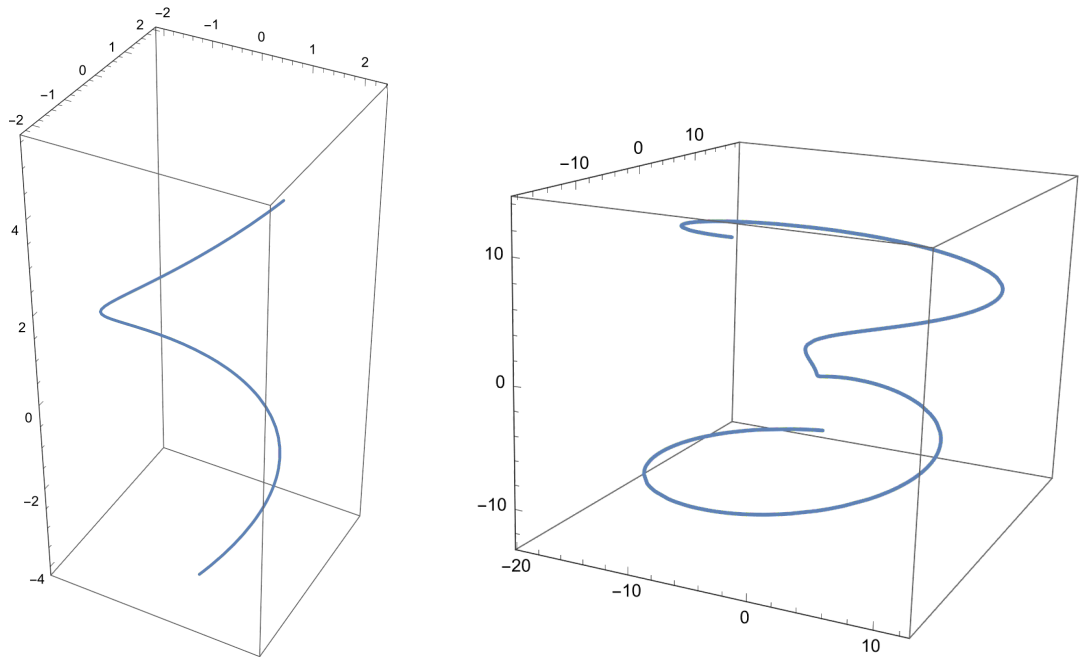

Figure 1. Curve β for $A(s) = 1$ (left), resp. $A(s) = s$(right)

As we show, the function A in the definition of the curve β is arbitrary, so we can conclude the following.

Corollary 3.1. *In the dual space, there exists a family of dual general helices that share the same axis and have identical real parts, while their dual components differ.*

However, the converse of Theorem 3.2 does not hold, unless additional conditions are imposed on the curve β.

Theorem 3.3. *Let $\gamma(s) = \alpha(s) + \varepsilon\beta(s)$ be a dual curve parametrized by an arc length, where α is a general helix in $\mathbb{R}^3$ and β has the form*

$$\beta(s) = \int (A(s)N_\alpha(s) + B(s)B_\alpha(s))ds,$$

where $A(s)$ is an arbitrary smooth function, $A : I \to \mathbb{R}$ and B is a solution of the ordinary differential equation

$$\frac{1}{\kappa_\alpha}B'' - \frac{\kappa_\alpha'}{\kappa_\alpha}B' + (\kappa_\alpha - \frac{\tau_\alpha^2}{\kappa_\alpha})B - \lambda\frac{\kappa_\alpha}{b} = 0, \tag{3.5}$$

for $\lambda \in \mathbb{R}$ and $b = \sin(\operatorname{arccot}\frac{\tau_\alpha}{\kappa_\alpha})$. Then there exists a constant dual vector E such that $T_\gamma \cdot E = const.$

Proof. Let $\gamma(s) = \alpha(s) + \varepsilon\beta(s)$ be a dual curve parametrized by an arc-length parameter where α is a general helix and β is a solution of (3.5). We construct constant dual vector $E = e_1 + \varepsilon e_2$ such that $T_\gamma \cdot E = const.$
Since α is a general helix in $\mathbb{R}^3$, there exists a constant vector $e_1 \in \mathbb{R}^3$ such that $T_\alpha \cdot e_1 = const.$ Thus, the real part of the condition for a dual general helix is already satisfied. We now need to determine a constant vector e_2 so that the dual part also becomes constant. We write e_2 in the Frenet frame of α:

$$e_2 = \overline{A}(s)T_\alpha + \overline{B}(s)N_\alpha + \overline{c}(s)B_\alpha \tag{3.6}$$

where for the functions $\overline{A}, \overline{B}, \overline{C}$ holds $\overline{A}(s) = T_\alpha \cdot e_2$, $\overline{B}(s) = N_\alpha \cdot e_2$, $\overline{C}(s) = B_\alpha \cdot e_2$. From $T_\alpha \cdot e_2 + \beta' \cdot e_1 = \lambda$, obtained earlier in equation (3.3), we have $\overline{A}(s) = \lambda - \beta' \cdot e_1$. Because α is a general helix, the constant vector e_1 can be written as $e_1 = aT_\alpha + bB_\alpha$ and by the theory of general helices in $\mathbb{R}^3$, [10], it holds $b = B_\alpha \cdot e_1 = \sin(\operatorname{arccot}(\tau_\alpha/\kappa_\alpha))$. Since $\beta'(s) = A(s)N_\alpha + B(s)B_\alpha$, it follows that $\beta' \cdot e_1 = bB(s)$. Hence, $\overline{A}(s) = \lambda - bB(s)$. Because e_2 must be constant, we require $e_2'(s) = 0$. Differentiating (3.6) and applying the Frenet formulas for curves in $\mathbb{R}^3$, [10],

$$T_\alpha' = \kappa_\alpha N_\alpha, \qquad N_\alpha' = -\kappa_\alpha T_\alpha + \tau_\alpha B_\alpha, \qquad B_\alpha' = -\tau_\alpha N_\alpha$$

we obtain

$$e_2' = \overline{A}'T_\alpha + \overline{A}\kappa_\alpha N_\alpha + \overline{B}'N_\alpha + \overline{B}(-\kappa_\alpha T_\alpha + \tau_\alpha B_\alpha) + \overline{C}'B_\alpha - \overline{C}\tau_\alpha N_\alpha.$$

Since T_α, N_α and B_α are linearly independent, we obtain the system

$$\overline{A}' - \kappa_\alpha\overline{B} = 0 \tag{3.7}$$
$$\overline{A}\kappa_\alpha + \overline{B}' - \tau_\alpha\overline{C} = 0 \tag{3.8}$$
$$\overline{B}\tau_\alpha + \overline{C}' = 0 \tag{3.9}$$

Therefore, the vector e_2 is constant precisely when these three equations are satisfied. Using equation $\overline{A}(s) = \lambda - bB(s)$, and substituting into (3.7), we get

$$-bB' - \kappa_\alpha\overline{B} = 0,$$

which gives

$$\overline{B} = -\frac{b}{\kappa_\alpha}B'.$$

Now by inserting the obtained expressions for $\overline{A}$ and $\overline{B}$ in (3.8), we obtain

$$\overline{C}(s) = \frac{1}{\tau_\alpha}[\kappa_\alpha(\lambda - b \cdot B) + b \cdot \frac{\kappa_\alpha'}{\kappa_\alpha}B' - \frac{b}{\kappa_\alpha}B''].$$

Finally, inserting these expressions into condition (3.9) produces exactly the differential equation

$$\frac{1}{\kappa_\alpha}B'' - \frac{\kappa_\alpha'}{\kappa_\alpha}B' + (\kappa_\alpha - \frac{\tau_\alpha^2}{\kappa_\alpha})B - \lambda\frac{\kappa_\alpha}{b} = 0.$$

Thus, whenever $B(s)$ is a solution of equation (3.5), all conditions required for $e_2' = 0$ are satisfied, meaning that e_2 is constant.

□

With the next example we confirm the previous theory.

Example 3.2. Let $\gamma(s) = \alpha(s) + \varepsilon\beta(s)$, be a dual curve where α is a helix parametrized by an arc-length parameter, $\alpha(s) = (1/\sqrt{2}\cos(s/\sqrt{2}, 1/\sqrt{2}\sin(s/\sqrt{2}), s/\sqrt{2})$ with the axis $(0,0,1)$ and curve β is given by $B(s) = A(s)N_\alpha + B(s)B_\alpha$, where function $B(s)$ is a solution of the equation

$$2\sqrt{2}B''(s) + \sqrt{2}B(s) - 2 = 0,$$

i.e. $B(s) = \sqrt{2} + a\cos(s/\sqrt{2}) + b\sin(s/\sqrt{2})$, $a, b \in \mathbb{R}$. According to Theorem 3.4, the dual vector E such that $T_\gamma \cdot E = const.$, has the form $E = (0,0,1) + \varepsilon(b, -a, 0)$. Examples for the curve β are shown on Figure 2.

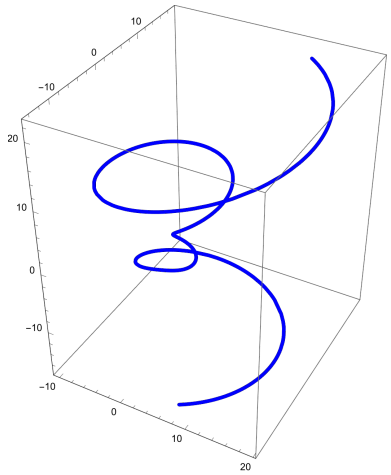

Figure 2. Curve β for $B(s) = \sqrt{2} + \cos(s/\sqrt{2}) + \sin(s/\sqrt{2})$ and $A(s) = -1/\sqrt{2}\cos(s/\sqrt{2})$(left), resp. $A(s) = -1/\sqrt{2}(s + \sin(s/\sqrt{2}) - \cos(s/\sqrt{2})$(right)

Contrary the Euclidean case, dual general helices are not characterized by the constant ratio of dual torsion and dual curvature what is easy to conclude from the next corollary.

Corollary 3.2. *Let $\gamma(s) = \alpha(s) + \varepsilon\beta(s)$ be a dual general helix. Then its dual curvature and dual torsion are given by*

$$\overline{\kappa}(s) = \kappa_\alpha + \varepsilon(\kappa_\alpha^2 A(s) + A''(s) - \tau_\alpha' B(s) - 2\tau_\alpha B''(s)),$$
$$\overline{\tau}(s) = \tau_\alpha + \varepsilon[A'(\tau_\alpha' - \frac{\kappa_\alpha'}{\kappa_\alpha}) + \tau_\alpha A'' + B(-2\tau_\alpha\tau_\alpha' + \frac{\kappa_\alpha'}{\kappa_\alpha}\tau_\alpha^2) + B'(3\tau_\alpha^2 + \kappa_\alpha^2) + 2\tau_\alpha^2 B''].$$

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Availability of data and materials

Not applicable.

Competing interests

The author declare no competing interests

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