Research Article GU J Sci 31(4): 1246-1266 (2018)

Gazi University

Journal of Science

http://dergipark.gov.tr/gujs

A Stratified Hybrid Tripartite Randomized Response Technique

Olusegun S. EWEMOOJE[1,*], Femi B. ADEBOLA[1], Adedamola A. ADEDIRAN[1]

[1]Department of Statistics, Federal University of Technology, Akure, Nigeria.

Article Info

Received: 22/05/2018
Accepted: 16/08/2018

Keywords

Proportional allocation
Neyman allocation
Sensitive attribute
Stratification relative
Efficiency

Abstract

This paper proposes a new stratified technique to address the problem involving estimation of the population proportion of people with sensitive attribute(s). Studying the proposed technique under proportional and Neyman allocations shows that the proposed technique is more efficient than (outperforms) the Singh & Gorey [1] and Tarray & Singh [2] stratified randomized response models. Applying the proposed technique to a survey on drug use disorder also shows the applicability of the model.

1. INTRODUCTION

In acquiring information from people with sensitive attribute(s), most respondents refuse to either answer or deliberately give false information in order to protect themselves. To remove bias that results from this and obtain reliable information/data, Warner [3] developed an interviewing procedure called Randomized Response Technique (RRT). This technique has attracted so many modifications, which include the works of [4-12] among many others. In addition, stratified sampling is being apply to RRT in order to protect researchers from obtaining a poor sample Kim and Warde [13]. Some other works on stratified RRT include [2, 14, 15]. Research shows that Kim and Elam [15] performs better than Kim and Warde [13] while Tarray and Singh [2] outperforms the Kim and Elam [15] model. Also, Singh and Gorey [1] is shown to be more efficient than Hong et al. [14], Mangat et al. [16], and Kim and Warde [13].

Therefore, we develop a stratified technique based on Adebola et al. [4] Hybrid Tripartite Randomized Response Technique and studies its percentage relative efficiency in relation to Singh & Gorey [1] and Tarray & Singh [2] stratified randomized response models. In addition, we apply the developed stratified technique to a survey in order to estimate the proportion of people belonging to the sensitive attribute "drug use disorder" stratified by sex.

2. HYBRID TRIPARTITE RANDOMIZED RESPONSE TECHNIQUE

Adebola et al. [4] proposed a Randomized Response Technique called Hybrid Tripartite Randomized Response Technique (HTRRT). The technique uses three randomized devices R_1, R_2 and R_3, with each device consisting of two unrelated questions such that $q_1 = \frac{\alpha}{\alpha+\beta+\delta}, \alpha \neq \beta \neq \delta$ is the probability of using R_1; $q_2 = \frac{\beta}{\alpha+\beta+\delta}, \alpha \neq \beta \neq \delta$ is the probability of using R_2; $q_3 = \frac{\delta}{\alpha+\beta+\delta}, \alpha \neq \beta \neq \delta$ is the probability of using R_3; where $\alpha, \beta,$ and δ are positive real numbers. These devices are with preset probabilities P_1, P_2 and P_3, respectively for sensitive question in each of the devices. Three responses "yes, no and undecided" were considered for the two unrelated questions each.

*Corresponding author, e-mail: osewemooje@futa.edu.ng

If all respondents respond truthfully, their population proportion of "yes" answer will be:

$$\Theta = \frac{\alpha}{\alpha+\beta+\delta}[P_1\pi_A + (1-P_1)\pi_U] + \frac{\beta}{\alpha+\beta+\delta}[P_2\pi_A + (1-P_2)\pi_U] + \frac{\delta}{\alpha+\beta+\delta}[P_3\pi_A + (1-P_3)\pi_U] \quad (1)$$

where π_U is the true proportion of respondents with the non-sensitive attribute and π_A is the true proportion of respondents with the sensitive attribute.

Their proposed unbiased estimate of the population proportion, $\hat{\pi}_{HTRRT}$ is given as:

$$\hat{\pi}_{HTRRT} = \frac{\hat{\theta}(\alpha+\beta+\delta)+(\alpha P_1 - \delta P_1 + \beta P_2 - \delta P_2 - \alpha - \beta)\pi_U}{\alpha P_1 - \delta P_1 + \beta P_2 - \delta P_2 + \delta} \quad (2)$$

where $\hat{\theta} = {n_0}/{n}$ and n_0 is number of respondents that answered "yes" to sensitive attribute while n is the sample size.

The variance of the proposed unbiased estimator is:

$$V(\hat{\pi}_{HTRRT}) = \left[\frac{\pi_A(2\pi_U - \pi_A)}{n} - \frac{\pi_U^2}{n}\right] + \left[\frac{(\alpha+\beta+\delta)[(\pi_U - \pi_U^2)(\alpha+\beta+\delta) + (\pi_A - \pi_U)(1-2\pi_U)[(\alpha-\delta)P_1 + (\beta-\delta)P_2 + \delta]]}{n[(\alpha-\delta)P_1 + (\beta-\delta)P_2 + \delta]^2}\right] \quad (3)$$

3. TARRAY & SINGH [2] STRATIFIED RANDOMIZED RESPONSE MODEL

They developed a procedure for stratified randomized response by allowing each individual respondent to use randomization device R_i in stratum i which consists of three types of cards bearing; I belong to sensitive group A, I belong to non-sensitive group Y and blank card with probabilities P_{i1}, P_{i2} and P_{i3} respectively where $P_{i1} + P_{i2} + P_{i3} = 1$. If a blank card is drawn, the respondent is ask to give "No" as answer while a correct answer is given when any other card is drawn. The probability X_i of a "Yes" answer for the procedure is:

$$X_i = P_{i1}\pi_{Si} + P_{i2}\pi_{yi} \quad (4)$$

where the proportion of people with sensitive and non-sensitive traits in stratum i are π_{Si} and π_{yi} respectively. The unbiased estimator of π_{Si} is:

$$\hat{\pi}_{Si} = \frac{1}{P_{i1}}\left[\hat{X}_i - P_{i2}\pi_{yi}\right] \quad (5)$$

where the proportion of "Yes" answer in the sample from stratum i is $\hat{X}_i$.

Therefore, the unbiased estimator of their procedure π_S is:

$$\hat{\pi}_S = \sum_i^k w_i \frac{1}{P_{i1}}\left[\hat{X}_i - P_{i2}\pi_{yi}\right] \quad (6)$$

Their corresponding variance given π_{yi} is:

$$V(\hat{\pi}_S/\pi_{yi}) = \sum_i^k w_i^2 \frac{1}{n_i P_{i1}^2}[X_i(1-X_i)] \quad (7)$$

Their proposed variance under proportional allocation is:

$$V(\hat{\pi}_S/\pi_{yi})_p = \frac{1}{n}\sum_i^k w_i \frac{1}{P_{i1}^2}[X_i(1-X_i)] \quad (8)$$

while their proposed variance under Neyman allocation is:

$$V(\hat{\pi}_S/\pi_{yi})_N = \frac{1}{n}\left[\sum_i^k w_i \frac{1}{P_{i1}}\sqrt{X_i(1-X_i)}\right]^2 \quad (9)$$

4. SINGH & GOREY [1] STRATIFIED RANDOMIZED RESPONSE MODEL

Singh and Gorey [1] developed a stratified randomized response procedure based on Mangat et al. [16] randomized response model and shown that their resulting estimator out performed [13, 14, 16] randomized response models. In doing this, they assume that the population is partitioned into k strata, n_h sample drawn using simple random sampling with replacement from stratum h. They then proposed a randomized response device, R_h, consisting of a deck having three types of cards where in stratum h, P_{1h} is the proportion of cards with the statement "I belong to category A", $P_{2h}(P_{1h} \neq P_{2h})$ is the proportion of cards with the statement "I do not belong to category A" and P_{3h} is the proportion of blank cards such that $P_{1h} + P_{2h} + P_{3h} = 1$. The respondents respond to type of card picked as in Mangat et al. [16].

Singh and Gorey [1] probability of "yes" answer stratum h is given by:

$$\varphi_h = P_{1h}\pi_{sh} + P_{2h}(1 - \pi_{sh}) \tag{10}$$

π_{sh} is the true proportion of respondents with the sensitive character in stratum h.

Note; the true population proportion of the sensitive character, π_s, can be obtained as $\pi_s = \sum_{h=1}^{k} w_h \pi_{sh}$ where $w_h = {N_h}/{N}$ and $\sum_{h=1}^{k} w_h = 1$. N_h is the number of elements in stratum h while N is the whole population.

Hence, the Singh & Gorey [1] stratified unbiased estimator is given as:

$$\hat{\pi}_{Ms} = \sum_{h=1}^{k} w_h \left[\frac{\hat{\varphi}_h - P_{2h}}{P_{1h} - P_{2h}}\right] \tag{11}$$

where $\hat{\varphi}_h$ is the proportion of "yes" response obtained from the survey in stratum h. The corresponding variance was given as:

$$V(\hat{\pi}_{Ms}) = \sum_{h=1}^{k} \frac{W_h^2}{n_h}\left\{\pi_{sh}(1 - \pi_{sh}) + \frac{\pi_{sh}P_{3h}}{(P_{1h} - P_{2h})} + \frac{P_{2h}(1 - P_{2h})}{(P_{1h} - P_{2h})^2}\right\} \tag{12}$$

Their proposed variance under proportional allocation is:

$$V(\hat{\pi}_{Ms(p)}) = \frac{1}{n}\sum_{h=1}^{k} W_h \left\{\pi_{sh}(1 - \pi_{sh}) + \frac{\pi_{sh}P_{3h}}{(P_{1h} - P_{2h})} + \frac{P_{2h}(1 - P_{2h})}{(P_{1h} - P_{2h})^2}\right\} \tag{13}$$

while their proposed variance under Neyman allocation is:

$$V(\hat{\pi}_{Ms(N)}) = \frac{1}{n}\left(\sum_{h=1}^{k} W_h \left\{\pi_{sh}(1 - \pi_{sh}) + \frac{\pi_{sh}P_{3h}}{(P_{1h} - P_{2h})} + \frac{P_{2h}(1 - P_{2h})}{(P_{1h} - P_{2h})^2}\right\}^{1/2}\right)^2 \tag{14}$$

5. PROPOSED STRATIFIED HYBRID TRIPARTITE RANDOMIZED RESPONSE TECHNIQUE

Dividing a finite population into L strata and selecting a simple random sample with replacement from each stratum independently. We provide three randomized devices R_{1h}, R_{2h} and R_{3h} with each device consisting of two unrelated questions (the sensitive attribute question A in which the interviewer is interested in and non-sensitive attribute question U that is unrelated to the sensitive question A) to the respondents in order to estimate the proportion of respondents belonging to a sensitive attribute. In the h[th] stratum, each respondent is provided with three options "yes, no and undecided", these were considered for each of the two questions. Notwithstanding, with the use of unrelated question some respondents might decide not to give response and hence the inclusion of undecided option. The randomized devices R_{1h}, R_{2h} and R_{3h} are as defined in the Hybrid Tripartite Randomized Response Technique (HTRRT) where $\alpha_h, \beta_h,$ and δ_h are positive real numbers such that $q_{1h} = \frac{\alpha_h}{\alpha_h + \beta_h + \delta_h}, \alpha_h \neq \beta_h \neq \delta_h$ is the probability of using R_{1h}; $q_{2h} = \frac{\beta_h}{\alpha_h + \beta_h + \delta_h}, \alpha_h \neq \beta_h \neq \delta_h$ is the probability of using R_{2h}; $q_{3h} =$

$\frac{\delta_h}{\alpha_h+\beta_h+\delta_h}, \alpha_h \neq \beta_h \neq \delta_h$ is the probability of using R_{3h}. These devices are with preset probabilities P_{1h}, P_{2h} and P_{3h}, respectively for sensitive question in each of the devices.

The population proportion of "yes" answers in h^th^ stratum is given by:

$$\theta_h = \frac{\alpha_h}{\alpha_h+\beta_h+\delta_h}[P_{1h}\pi_{Ah} + (1-P_{1h})\pi_{Uh}] + \frac{\beta_h}{\alpha_h+\beta_h+\delta_h}[P_{2h}\pi_{Ah} + (1-P_{2h})\pi_{Uh}] + \frac{\delta_h}{\alpha_h+\beta_h+\delta_h}[P_{3h}\pi_{Ah} + (1-P_{3h})\pi_{Uh}] \quad (15)$$

where π_{Ah} is the true proportion of respondent belonging to the sensitive attribute in h^th^ stratum and π_{Uh} is the true proportion of respondent belonging to the non-sensitive attribute in h^th^ stratum.

Solving θ_h further and obtaining π_{Ah} gives:

$$\pi_{Ah} = \frac{\theta_h(\alpha_h+\beta_h+\delta_h) - (\alpha_h+\beta_h+\delta_h)\pi_{Uh} + (\alpha_h P_{1h} + \beta_h P_{2h} + \delta_h P_{3h})\pi_{Uh}}{\alpha_h P_{1h} + \beta_h P_{2h} + \delta_h P_{3h}}$$

where $P_{1h} + P_{2h} + P_{3h} = 1$, $P_{3h} = 1 - P_{1h} - P_{2h}$

The unbiased estimate of the population proportion, π_{Ah} in h^th^ stratum is given as:

$$\hat{\pi}_{Ah} = \frac{\hat{\theta}_h(\alpha_h+\beta_h+\delta_h) + [(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} - (\alpha_h+\beta_h)]\pi_{Uh}}{(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} + \delta_h} \quad (16)$$

Taking the expectation of both sides, we have:

$$E(\hat{\pi}_{Ah}) = \frac{(\alpha_h+\beta_h+\delta_h)E(\hat{\theta}_h) + [(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} - (\alpha_h+\beta_h)]\pi_{Uh}}{(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} + \delta_h}$$

$$E(\hat{\pi}_{Ah}) = \frac{\theta_h(\alpha_h+\beta_h+\delta_h) + [(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} - (\alpha_h+\beta_h)]\pi_{Uh}}{(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} + \delta_h}$$

Substituting θ_h, we have:

$$E(\hat{\pi}_{Ah}) = \frac{[(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} + \delta_h]\pi_{Ah} - [(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} - (\alpha_h+\beta_h)]\pi_{Uh}}{(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} + \delta_h} + \frac{[(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} - (\alpha_h+\beta_h)]\pi_{Uh}}{(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} + \delta_h}$$

$$E(\hat{\pi}_{Ah}) = \frac{[(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} + \delta_h]\pi_{Ah}}{(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} + \delta_h} = \pi_{Ah}$$

Hence, $\hat{\pi}_{Ah}$ is an unbiased estimate of the population proportion, π_{Ah}, in h^th^ stratum.
Obtaining the variance of $\hat{\pi}_{Ah}$, $v(\hat{\pi}_{Ah})$,

$$V(\hat{\pi}_{Ah}) = \frac{[\alpha_h+\beta_h+\delta_h]^2 V(\hat{\theta}_h)}{[(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} + \delta_h]^2}$$

$$V(\hat{\pi}_{Ah}) = \frac{[\alpha_h+\beta_h+\delta_h]^2 \theta_h(1-\theta_h)}{n_h[(\alpha_h-\delta_h)P_{1h} + (\beta_h-\delta_h)P_{2h} + \delta_h]^2}$$

Substituting for θ_h and solving further, we have:

$$V(\hat{\pi}_{Ah}) = \left[\frac{\pi_{Ah}(2\pi_{Uh}-\pi_{Ah})}{n_h} - \frac{\pi_{Uh}^2}{n_h}\right] + \left[\frac{(\alpha_h+\beta_h+\delta_h)[(\pi_{Uh}-\pi_{Uh}^2)(\alpha_h+\beta_h+\delta_h)+(\pi_{Ah}-\pi_{Uh})(1-2\pi_{Uh})[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]]}{n_h[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]^2}\right] \quad (17)$$

The proposed unbiased stratified estimate of the population proportion π_{st} is:

$$\hat{\pi}_{st} = \sum_{h=1}^{L} W_h \hat{\pi}_{Ah} \quad (18)$$

where $W_h = N_h/N$ is the h^th^ stratum weight. Substituting equation (16) in (18), we have:

$$\hat{\pi}_{st} = \sum_{h=1}^{L} W_h \left[\frac{\hat{\theta}_h(\alpha_h+\beta_h+\delta_h)+[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}-(\alpha_h+\beta_h)]\pi_{Uh}}{(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h}\right] \quad (19)$$

Taking the expectation of both sides, we have:

$$E(\hat{\pi}_{st}) = \sum_{h=1}^{L} W_h E(\hat{\pi}_{Ah}) = \sum_{h=1}^{L} W_h \pi_{Ah} = \pi_A$$

Obtaining the variance of the proposed unbiased stratified estimator, we have:

$$V(\hat{\pi}_{st}) = \sum_{h=1}^{L} W_h^2 V(\hat{\pi}_{Ah}) \quad (20)$$

Substituting equation (17) in (20), leads to:

$$V(\hat{\pi}_{st}) = \sum_{h=1}^{L} W_h^2 \left\{\left[\frac{\pi_{Ah}(2\pi_{Uh}-\pi_{Ah})}{n_h} - \frac{\pi_{Uh}^2}{n_h}\right] + \left[\frac{(\alpha_h+\beta_h+\delta_h)[(\pi_{Uh}-\pi_{Uh}^2)(\alpha_h+\beta_h+\delta_h)+(\pi_{Ah}-\pi_{Uh})(1-2\pi_{Uh})[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]]}{n_h[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]^2}\right]\right\}$$

Therefore, the variance of the proposed unbiased stratified estimator is given as:

$$V(\hat{\pi}_{st}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left\{\pi_{Ah}(2\pi_{Uh}-\pi_{Ah}) - \pi_{Uh}^2 + \frac{(\alpha_h+\beta_h+\delta_h)[(\pi_{Uh}-\pi_{Uh}^2)(\alpha_h+\beta_h+\delta_h)+(\pi_{Ah}-\pi_{Uh})(1-2\pi_{Uh})[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]]}{[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]^2}\right\} \quad (21)$$

The variance of the proposed unbiased stratified estimator can be estimated using:

$$\hat{V}(\hat{\pi}_{st}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h - 1} \left\{\hat{\pi}_{Ah}(2\pi_{Uh}-\hat{\pi}_{Ah}) - \pi_{Uh}^2 + \frac{(\alpha_h+\beta_h+\delta_h)[(\pi_{Uh}-\pi_{Uh}^2)(\alpha_h+\beta_h+\delta_h)+(\hat{\pi}_{Ah}-\pi_{Uh})(1-2\pi_{Uh})[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]]}{[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]^2}\right\}$$

Theorem 1: In stratified random sampling with the cost function $= c_0 + \sum_{h=1}^{L} c_h n_h$, the variance of the estimated proportion, $\hat{\pi}_{st}$, is minimum for a specified cost, C, when the optimum allocation of the sample size of the h^th^ stratum is given as:

$$n_h = \frac{nW_h[a_h + b_h/d_h]^{1/2}}{\sum_{h=1}^{L} W_h[a_h + b_h/d_h]^{1/2}} \tag{22}$$

Proof:

Let $a_h = \hat{\pi}_{Ah}(2\pi_{Uh} - \hat{\pi}_{Ah}) - \pi_{Uh}^2$

$$b_h = (\alpha_h + \beta_h + \delta_h)[(\pi_{Uh} - \pi_{Uh}^2)(\alpha_h + \beta_h + \delta_h) + (\hat{\pi}_{Ah} - \pi_{Uh})(1 - 2\pi_{Uh})[(\alpha_h - \delta_h)P_{1h} + (\beta_h - \delta_h)P_{2h} + \delta_h]]$$

$$d_h = n_h[(\alpha_h - \delta_h)P_{1h} + (\beta_h - \delta_h)P_{2h} + \delta_h]^2$$

Therefore, the variance of the proposed unbiased stratified estimator becomes:

$$V(\hat{\pi}_{st}) = \sum_{h=1}^{L} W_h^2 \left(\frac{a_h}{n_h} + \frac{b_h}{d_h n_h}\right)$$

$$V(\hat{\pi}_{st}) = \sum_{h=1}^{L} W_h^2 \frac{a_h}{n_h} + \sum_{h=1}^{L} W_h^2 \frac{b_h}{d_h n_h}$$

Minimizing $V(\hat{\pi}_{st})$ subject to the cost function, $C = c_0 + \sum_{h=1}^{L} c_h n_h$, by using Lagrange Multiplier, we have:

$$F(*) = V(\hat{\pi}_{st}) + \lambda\left(\sum_{h=1}^{L} c_h n_h - C + c_0\right)$$

$$F(*) = \sum_{h=1}^{L} W_h^2 \frac{a_h}{n_h} + \sum_{h=1}^{L} W_h^2 \frac{b_h}{d_h n_h} + \lambda\left(\sum_{h=1}^{L} c_h n_h - C + c_0\right)$$

Differentiating with respect to n_h and equating to zero

$$\frac{dF(*)}{dn_h} = -\frac{W_h^2 a_h}{n_h^2} - \frac{W_h^2 b_h}{d_h n_h^2} + \lambda c_h = 0 \qquad (h = 1,2,\dots,L)$$

$$\frac{W_h^2 a_h}{c_h} + \frac{W_h^2 b_h}{d_h c_h} = \lambda n_h^2 \qquad (h = 1,2,\dots,L)$$

Taking square root of both sides

$$n_h\sqrt{\lambda} = \frac{W_h\sqrt{a_h}}{\sqrt{c_h}} + \frac{W_h\sqrt{b_h}}{\sqrt{d_h c_h}} \qquad (h = 1,2,\dots,L) \tag{*}$$

Summing over the strata

$$n\sqrt{\lambda} = \sum_{h=1}^{L} \frac{W_h\sqrt{a_h}}{\sqrt{c_h}} + \sum_{h=1}^{L} \frac{W_h\sqrt{b_h}}{\sqrt{d_h c_h}} \tag{**}$$

Dividing equation (*) by (**)

$$\frac{n_h}{n} = \frac{W_h[a_h + b_h/d_h]^{1/2}}{\sum_{h=1}^{L} W_h[a_h + b_h/d_h]^{1/2}}$$

$$n_h = \frac{nW_h[a_h + b_h/d_h]^{1/2}}{\sum_{h=1}^{L} W_h[a_h + b_h/d_h]^{1/2}}$$

Corollary 1: If n_h is substituted into the general form of $V(\hat{\pi}_{st})$ in equation (21), the minimum variance for the Neyman Allocation, $V\left(\hat{\pi}_{st(N)}\right)$, is given as:

$$V(\hat{\pi}_{st(N)}) = \frac{1}{n}\left(\sum_{h=1}^{L} W_h[a_h + b_h/d_h]^{1/2}\right)^2$$

$$V(\hat{\pi}_{st(N)}) = \frac{1}{n}\left\{\sum_{h=1}^{L} W_h\left(\left[\pi_{Ah}(2\pi_{Uh} - \pi_{Ah}) - \pi_{Uh}^2\right] + \left[\frac{(\alpha_h+\beta_h+\delta_h)[(\pi_{Uh}-\pi_{Uh}^2)(\alpha_h+\beta_h+\delta_h)+(\pi_{Ah}-\pi_{Uh})(1-2\pi_{Uh})[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]]}{[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]^2}\right]\right)^{1/2}\right\}^2 \quad (23)$$

Corollary 2: With Proportional Allocation $(n_h = nN_h/N,\ (h = 1,2,\ldots,L)$, the minimum variance, $V\left(\hat{\pi}_{st(p)}\right)$, is given as:

$$V(\hat{\pi}_{st(p)}) = \frac{1}{n}\sum_{h=1}^{L} w_h^2\left\{\left[\pi_{Ah}(2\pi_{Uh} - \pi_{Ah}) - \pi_{Uh}^2\right] + \left[\frac{(\alpha_h+\beta_h+\delta_h)[(\pi_{Uh}-\pi_{Uh}^2)(\alpha_h+\beta_h+\delta_h)+(\pi_{Ah}-\pi_{Uh})(1-2\pi_{Uh})[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]]}{[(\alpha_h-\delta_h)P_{1h}+(\beta_h-\delta_h)P_{2h}+\delta_h]^2}\right]\right\} \quad (24)$$

6. EFFICIENCY COMPARISON BETWEEN SINGH & GOREY AND TARRAY & SINGH STRATIFIED RANDOMIZED RESPONSE MODELS AND PROPOSED STRATIFIED TECHNIQUE.

The Percentage Relative Efficiency (PRE) of the proposed technique with respect to Singh & Gorey [1] and Tarray & Singh [2] Stratified Randomized Response Models are obtained under:

i) Proportional allocation using:

$$PRE_p = \frac{V(\hat{\pi}_{S(p)})}{V(\hat{\pi}_{st(p)})} \times 100$$

ii) Neyman allocation using:

$$PRE_N = \frac{V(\hat{\pi}_{S(N)})}{V(\hat{\pi}_{st(N)})} \times 100$$

Numerical investigation were carried out to obtain the PRE by setting the number of strata to two. $\pi_{A1} = \pi_{s1} = \pi_1$ values were change from 0.08 to 0.88 with a step of 0.2. Also, $\pi_{A2} = \pi_{s2} = \pi_2$ values were change from 0.13 to 0.93 with a step of 0.2 while $\pi_{U1} = \pi_{U2} = 0.25$. The probabilities of cards carrying the sensitive statement P_1 were change from 0.5 to 0.7 with a step of 0.1 to ensure moderate confidentiality and gain reliable information from respondents [17] while P_2 & P_3were change from 0.1 to 0.3 with a step of 0.05. The stratum weight takes on either 0.3 or 0.7 while the real numbers $\alpha_1 = \alpha_2 = 95$; $\beta_1 = \beta_2 = 15$ and $\delta_1 = \delta_2 = 5$. The resulting PRE are as shown in Tables 1-12.

The values of the percentage relative efficiencies of the proposed stratified technique over Singh and Gorey [1] stratified randomized response model under proportional allocation, PRE_p, is greater than 100 at almost all the points considered while it is equal to 100 at few cases when $P_1 = 0.7$. The PRE_p ranges from 100.00 to 761.79 while it was observed that it decreases with increase in P_1. It was also noted from Tables 1-3 that the errors incurred (variances) decrease as the P_1 values increases from 0.5 to 0.7.

Table 1. *Percentage Relative Efficiency of the Proposed Stratified Technique over Singh & Gorey Stratified Randomized Response Model under Proportional Allocation when* $P_1 = 0.5$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(p)})$	$V(\hat{\pi}_{S(p)})$	PRE_p
0.08	0.13	0.7	0.3	0.1	0.5	0.2	0.3	0.0036	0.0098	271.561
					0.5	0.25	0.25	0.0035	0.0159	450.905
					0.5	0.3	0.2	0.0035	0.0272	786.959
		0.3	0.7	0.12	0.5	0.2	0.3	0.0037	0.0100	266.220
					0.5	0.25	0.25	0.0037	0.0161	438.747
					0.5	0.3	0.2	0.0036	0.0273	761.790
0.28	0.33	0.7	0.3	0.3	0.5	0.2	0.3	0.0048	0.0114	236.476
					0.5	0.25	0.25	0.0047	0.0175	370.083
					0.5	0.3	0.2	0.0046	0.0288	619.185
		0.3	0.7	0.32	0.5	0.2	0.3	0.0049	0.0115	234.515
					0.5	0.25	0.25	0.0048	0.0177	365.412
					0.5	0.3	0.2	0.0047	0.0289	609.388
0.48	0.53	0.7	0.3	0.5	0.5	0.2	0.3	0.0056	0.0126	223.719
					0.5	0.25	0.25	0.0055	0.0187	338.117
					0.5	0.3	0.2	0.0054	0.0300	550.952
		0.3	0.7	0.52	0.5	0.2	0.3	0.0057	0.0127	223.117
					0.5	0.25	0.25	0.0056	0.0188	336.350
					0.5	0.3	0.2	0.0055	0.0301	546.991
0.68	0.73	0.7	0.3	0.7	0.5	0.2	0.3	0.0061	0.0134	221.734
					0.5	0.25	0.25	0.0059	0.0195	328.690
					0.5	0.3	0.2	0.0058	0.0308	527.546
		0.3	0.7	0.72	0.5	0.2	0.3	0.0061	0.0135	221.991
					0.5	0.25	0.25	0.0060	0.0196	328.658
					0.5	0.3	0.2	0.0059	0.0308	526.977
0.88	0.93	0.7	0.3	0.9	0.5	0.2	0.3	0.0061	0.0138	227.914
					0.5	0.25	0.25	0.0059	0.0199	335.294
					0.5	0.3	0.2	0.0058	0.0312	535.039
		0.3	0.7	0.92	0.5	0.2	0.3	0.0060	0.0139	228.998
					0.5	0.25	0.25	0.0059	0.0200	336.830
					0.5	0.3	0.2	0.0058	0.0312	537.440

Table 2. *Percentage Relative Efficiency of the Proposed Stratified Technique over Singh & Gorey Stratified Randomized Response Model under Proportional Allocation when* $P_1 = 0.6$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(p)})$	$V(\hat{\pi}_{S(p)})$	PRE_p
0.08	0.13	0.7	0.3	0.095	0.6	0.15	0.25	0.0025	0.0038	151.871
					0.6	0.2	0.2	0.0025	0.0057	228.534
					0.6	0.25	0.15	0.0024	0.0083	340.800
		0.3	0.7	0.115	0.6	0.15	0.25	0.0027	0.0040	149.841
					0.6	0.2	0.2	0.0026	0.0058	222.666
					0.6	0.25	0.15	0.0026	0.0084	329.222
0.28	0.33	0.7	0.3	0.295	0.6	0.15	0.25	0.0036	0.0050	139.472
					0.6	0.2	0.2	0.0035	0.0068	191.845
					0.6	0.25	0.15	0.0035	0.0093	268.220
		0.3	0.7	0.315	0.6	0.15	0.25	0.0037	0.0051	138.855
					0.6	0.2	0.2	0.0036	0.0069	189.889
					0.6	0.25	0.15	0.0036	0.0094	264.302
0.48	0.53	0.7	0.3	0.495	0.6	0.15	0.25	0.0042	0.0058	135.812
					0.6	0.2	0.2	0.0042	0.0075	178.891
					0.6	0.25	0.15	0.0041	0.0100	241.735
		0.3	0.7	0.515	0.6	0.15	0.25	0.0043	0.0058	135.689
					0.6	0.2	0.2	0.0042	0.0075	178.219
					0.6	0.25	0.15	0.0042	0.0100	240.273
0.68	0.73	0.7	0.3	0.695	0.6	0.15	0.25	0.0045	0.0061	136.075
					0.6	0.2	0.2	0.0044	0.0078	175.692
					0.6	0.25	0.15	0.0044	0.0102	233.636
		0.3	0.7	0.715	0.6	0.15	0.25	0.0045	0.0062	136.276
					0.6	0.2	0.2	0.0044	0.0078	175.765
					0.6	0.25	0.15	0.0044	0.0102	233.545
0.88	0.93	0.7	0.3	0.895	0.6	0.15	0.25	0.0044	0.0061	139.634
					0.6	0.2	0.2	0.0043	0.0077	179.630
					0.6	0.25	0.15	0.0042	0.0100	238.405
		0.3	0.7	0.915	0.6	0.15	0.25	0.0043	0.0061	140.205
					0.6	0.2	0.2	0.0043	0.0077	180.453
					0.6	0.25	0.15	0.0042	0.0100	239.634

Table 3. *Percentage Relative Efficiency of the Proposed Stratified Technique over Singh & Gorey Stratified Randomized Response Model under Proportional Allocation when* $P_1 = 0.7$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(p)})$	$V(\hat{\pi}_{S(p)})$	PRE_p
0.08	0.13	0.7	0.3	0.1	0.7	0.1	0.2	0.0018	0.0018	100.000
					0.7	0.15	0.15	0.0018	0.0027	147.775
					0.7	0.2	0.1	0.0018	0.0037	210.185
		0.3	0.7	0.12	0.7	0.1	0.2	0.0019	0.0019	100.000
					0.7	0.15	0.15	0.0019	0.0028	144.702
					0.7	0.2	0.1	0.0019	0.0038	203.033
0.28	0.33	0.7	0.3	0.3	0.7	0.1	0.2	0.0028	0.0028	100.000
					0.7	0.15	0.15	0.0027	0.0035	129.440
					0.7	0.2	0.1	0.0027	0.0045	167.714
		0.3	0.7	0.32	0.7	0.1	0.2	0.0029	0.0029	100.000
					0.7	0.15	0.15	0.0028	0.0036	128.495
					0.7	0.2	0.1	0.0028	0.0046	165.539
0.48	0.53	0.7	0.3	0.5	0.7	0.1	0.2	0.0033	0.0033	100.000
					0.7	0.15	0.15	0.0033	0.0040	122.947
					0.7	0.2	0.1	0.0032	0.0049	152.831
		0.3	0.7	0.52	0.7	0.1	0.2	0.0034	0.0034	100.000
					0.7	0.15	0.15	0.0033	0.0041	122.562
					0.7	0.2	0.1	0.0033	0.0050	151.955
0.68	0.73	0.7	0.3	0.7	0.7	0.1	0.2	0.0035	0.0035	100.000
					0.7	0.15	0.15	0.0034	0.0041	120.425
					0.7	0.2	0.1	0.0034	0.0050	147.160
		0.3	0.7	0.72	0.7	0.1	0.2	0.0035	0.0035	100.000
					0.7	0.15	0.15	0.0034	0.0041	120.315
					0.7	0.2	0.1	0.0034	0.0049	146.922
0.88	0.93	0.7	0.3	0.9	0.7	0.1	0.2	0.0032	0.0032	100.000
					0.7	0.15	0.15	0.0032	0.0038	120.395
					0.7	0.2	0.1	0.0031	0.0046	147.329
		0.3	0.7	0.92	0.7	0.1	0.2	0.0032	0.0032	100.000
					0.7	0.15	0.15	0.0031	0.0037	120.536
					0.7	0.2	0.1	0.0030	0.0045	147.689

In Tables 4-6, the values of the percentage relative efficiencies of the proposed stratified technique over Singh and Gorey [1] stratified randomized response model under Neyman allocation, PRE_N, is greater than 100 at almost all the points considered as well while it is equal to 100 at few cases when $P_1 = 0.7$. The PRE_N ranges from 100.00 to 753.15 while the errors incurred decreases as the P_1 values increases from 0.5 to 0.7. The variances due to the Neyman allocation were smaller than that due to proportional allocation at all levels.

Table 4. *Percentage Relative Efficiency of the Proposed Stratified Technique over Singh & Gorey Stratified Randomized Response Model under Neyman Allocation when* $P_1 = 0.5$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(N)})$	$V(\hat{\pi}_{S(N)})$	PRE_N
0.08	0.13	0.7	0.3	0.1	0.5	0.2	0.3	0.0021	0.0056	273.599
					0.5	0.25	0.25	0.0020	0.0092	455.546
					0.5	0.3	0.2	0.0020	0.0157	796.574
		0.3	0.7	0.12	0.5	0.2	0.3	0.0022	0.0058	264.383
					0.5	0.25	0.25	0.0022	0.0094	434.569
					0.5	0.3	0.2	0.0021	0.0159	753.147
0.28	0.33	0.7	0.3	0.3	0.5	0.2	0.3	0.0028	0.0066	237.206
					0.5	0.25	0.25	0.0027	0.0101	371.823
					0.5	0.3	0.2	0.0027	0.0167	622.834
		0.3	0.7	0.32	0.5	0.2	0.3	0.0029	0.0067	233.825
					0.5	0.25	0.25	0.0028	0.0103	363.768
					0.5	0.3	0.2	0.0028	0.0168	605.939
0.48	0.53	0.7	0.3	0.5	0.5	0.2	0.3	0.0033	0.0073	223.940
					0.5	0.25	0.25	0.0032	0.0108	338.766
					0.5	0.3	0.2	0.0031	0.0174	552.407
		0.3	0.7	0.52	0.5	0.2	0.3	0.0033	0.0074	222.903
					0.5	0.25	0.25	0.0033	0.0109	335.719
					0.5	0.3	0.2	0.0032	0.0175	545.577
0.68	0.73	0.7	0.3	0.7	0.5	0.2	0.3	0.0035	0.0078	221.641
					0.5	0.25	0.25	0.0034	0.0113	328.701
					0.5	0.3	0.2	0.0034	0.0178	527.752
		0.3	0.7	0.72	0.5	0.2	0.3	0.0035	0.0078	222.083
					0.5	0.25	0.25	0.0035	0.0114	328.647
					0.5	0.3	0.2	0.0034	0.0179	526.772
0.88	0.93	0.7	0.3	0.9	0.5	0.2	0.3	0.0035	0.0080	227.523
					0.5	0.25	0.25	0.0035	0.0116	334.741
					0.5	0.3	0.2	0.0034	0.0181	534.175
		0.3	0.7	0.92	0.5	0.2	0.3	0.0035	0.0080	229.392
					0.5	0.25	0.25	0.0034	0.0116	337.388
					0.5	0.3	0.2	0.0034	0.0181	538.314

Table 5. *Percentage Relative Efficiency of the Proposed Stratified Technique over Singh & Gorey Stratified Randomized Response Model under Neyman Allocation when* $P_1 = 0.6$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(N)})$	$V(\hat{\pi}_{S(N)})$	PRE_N
0.08	0.13	0.7	0.3	0.1	0.6	0.15	0.25	0.0014	0.0022	152.655
					0.6	0.2	0.2	0.0014	0.0033	230.805
					0.6	0.25	0.15	0.0014	0.0048	345.285
		0.3	0.7	0.12	0.6	0.15	0.25	0.0016	0.0023	149.152
					0.6	0.2	0.2	0.0015	0.0034	220.676
					0.6	0.25	0.15	0.0015	0.0049	325.300
0.28	0.33	0.7	0.3	0.3	0.6	0.15	0.25	0.0021	0.0029	139.703
					0.6	0.2	0.2	0.0020	0.0039	192.577
					0.6	0.25	0.15	0.0020	0.0054	269.685
		0.3	0.7	0.32	0.6	0.15	0.25	0.0021	0.0030	138.639
					0.6	0.2	0.2	0.0021	0.0040	189.202
					0.6	0.25	0.15	0.0021	0.0055	262.928
0.48	0.53	0.7	0.3	0.5	0.6	0.15	0.25	0.0025	0.0033	135.857
					0.6	0.2	0.2	0.0024	0.0043	179.137
					0.6	0.25	0.15	0.0024	0.0058	242.272
		0.3	0.7	0.52	0.6	0.15	0.25	0.0025	0.0034	135.645
					0.6	0.2	0.2	0.0025	0.0044	177.979
					0.6	0.25	0.15	0.0024	0.0058	239.751
0.68	0.73	0.7	0.3	0.7	0.6	0.15	0.25	0.0026	0.0036	136.002
					0.6	0.2	0.2	0.0026	0.0045	175.665
					0.6	0.25	0.15	0.0025	0.0059	233.669
		0.3	0.7	0.72	0.6	0.15	0.25	0.0026	0.0036	136.348
					0.6	0.2	0.2	0.0026	0.0045	175.792
					0.6	0.25	0.15	0.0025	0.0059	233.512
0.88	0.93	0.7	0.3	0.9	0.6	0.15	0.25	0.0025	0.0035	139.429
					0.6	0.2	0.2	0.0025	0.0045	179.336
					0.6	0.25	0.15	0.0024	0.0058	237.966
		0.3	0.7	0.92	0.6	0.15	0.25	0.0025	0.0035	140.414
					0.6	0.2	0.2	0.0025	0.0044	180.755
					0.6	0.25	0.15	0.0024	0.0058	240.084

***Table* 6.** *Percentage Relative Efficiency of the Proposed Stratified Technique over Singh & Gorey Stratified Randomized Response Model under Neyman Allocation when* $P_1 = 0.7$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(N)})$	$V(\hat{\pi}_{S(N)})$	PRE_N
0.08	0.13	0.7	0.3	0.1	0.7	0.1	0.2	0.0010	0.0010	100.000
					0.7	0.15	0.15	0.0010	0.0015	148.984
					0.7	0.2	0.1	0.0010	0.0021	213.001
		0.3	0.7	0.12	0.7	0.1	0.2	0.0012	0.0012	100.000
					0.7	0.15	0.15	0.0011	0.0016	143.676
					0.7	0.2	0.1	0.0011	0.0022	200.648
0.28	0.33	0.7	0.3	0.3	0.7	0.1	0.2	0.0016	0.0016	100.000
					0.7	0.15	0.15	0.0016	0.0020	129.795
					0.7	0.2	0.1	0.0016	0.0026	168.530
		0.3	0.7	0.32	0.7	0.1	0.2	0.0017	0.0017	100.000
					0.7	0.15	0.15	0.0016	0.0021	128.164
					0.7	0.2	0.1	0.0016	0.0027	164.779
0.48	0.53	0.7	0.3	0.5	0.7	0.1	0.2	0.0019	0.0019	100.000
					0.7	0.15	0.15	0.0019	0.0023	123.089
					0.7	0.2	0.1	0.0019	0.0029	153.153
		0.3	0.7	0.52	0.7	0.1	0.2	0.0020	0.0020	100.000
					0.7	0.15	0.15	0.0019	0.0024	122.425
					0.7	0.2	0.1	0.0019	0.0029	151.642
0.68	0.73	0.7	0.3	0.7	0.7	0.1	0.2	0.0020	0.0020	100.000
					0.7	0.15	0.15	0.0020	0.0024	120.465
					0.7	0.2	0.1	0.0020	0.0029	147.245
		0.3	0.7	0.72	0.7	0.1	0.2	0.0020	0.0020	100.000
					0.7	0.15	0.15	0.0020	0.0024	120.274
					0.7	0.2	0.1	0.0019	0.0029	146.836
0.88	0.93	0.7	0.3	0.9	0.7	0.1	0.2	0.0019	0.0019	100.000
					0.7	0.15	0.15	0.0018	0.0022	120.345
					0.7	0.2	0.1	0.0018	0.0027	147.201
		0.3	0.7	0.92	0.7	0.1	0.2	0.0018	0.0018	100.000
					0.7	0.15	0.15	0.0018	0.0022	120.588
					0.7	0.2	0.1	0.0018	0.0026	147.822

We also consider the values of the percentage relative efficiencies of the proposed stratified technique over Tarray and Singh [2] stratified randomized response model under proportional allocation (PRE_p). The, PRE_p, is greater than 100 at all the points and it ranges from 105.25 to 2539.44 while it was observed that it increases with increase in P_1 has seen in Tables 7-9. Also, the variances decrease while the PRE_p increases as the P_1 values increases from 0.5 to 0.7.

Table 7. *Percentage Relative Efficiency of the Proposed Stratified Technique over Tarray & Singh Stratified Randomized Response Model under Proportional Allocation when* $P_1 = 0.5$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(p)})$	$V(\hat{\pi}_{S(p)})$	PRE_p
0.08	0.13	0.7	0.3	0.1	0.5	0.2	0.3	0.0036	0.005	137.655
					0.5	0.25	0.25	0.0035	0.004	112.222
					0.5	0.3	0.2	0.0035	0.0036	105.253
		0.3	0.7	0.12	0.5	0.2	0.3	0.0037	0.0094	250.908
					0.5	0.25	0.25	0.0037	0.0068	185.024
					0.5	0.3	0.2	0.0036	0.0053	147.526
0.28	0.33	0.7	0.3	0.3	0.5	0.2	0.3	0.0048	0.0085	175.963
					0.5	0.25	0.25	0.0047	0.0065	136.914
					0.5	0.3	0.2	0.0046	0.0054	116.311
		0.3	0.7	0.32	0.5	0.2	0.3	0.0049	0.0157	318.872
					0.5	0.25	0.25	0.0048	0.0108	223.799
					0.5	0.3	0.2	0.0047	0.0081	170.916
0.48	0.53	0.7	0.3	0.5	0.5	0.2	0.3	0.0056	0.011	194.68
					0.5	0.25	0.25	0.0055	0.0082	148.807
					0.5	0.3	0.2	0.0054	0.0068	124.278
		0.3	0.7	0.52	0.5	0.2	0.3	0.0057	0.0201	353.117
					0.5	0.25	0.25	0.0056	0.0136	243.035
					0.5	0.3	0.2	0.0055	0.01	182.286
0.68	0.73	0.7	0.3	0.7	0.5	0.2	0.3	0.0061	0.0124	205.436
					0.5	0.25	0.25	0.0059	0.0092	155.504
					0.5	0.3	0.2	0.0058	0.0075	128.629
		0.3	0.7	0.72	0.5	0.2	0.3	0.0061	0.0227	373.333
					0.5	0.25	0.25	0.0060	0.0151	254.13
					0.5	0.3	0.2	0.0059	0.011	188.619
0.88	0.93	0.7	0.3	0.9	0.5	0.2	0.3	0.0061	0.0129	211.996
					0.5	0.25	0.25	0.0059	0.0095	159.419
					0.5	0.3	0.2	0.0058	0.0076	130.995
		0.3	0.7	0.92	0.5	0.2	0.3	0.0060	0.0234	386.146
					0.5	0.25	0.25	0.0059	0.0155	260.834
					0.5	0.3	0.2	0.0058	0.0112	192.146

Table 8. *Percentage Relative Efficiency of the Proposed Stratified Technique over Tarray & Singh Stratified Randomized Response Model under Proportional Allocation when* $P_1 = 0.6$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(p)})$	$V(\hat{\pi}_{S(p)})$	PRE_p
0.08	0.13	0.7	0.3	0.095	0.6	0.15	0.25	0.0025	0.0076	299.479
					0.6	0.2	0.2	0.0025	0.005	203.531
					0.6	0.25	0.15	0.0024	0.0039	158.547
		0.3	0.7	0.115	0.6	0.15	0.25	0.0027	0.0162	611.503
					0.6	0.2	0.2	0.0026	0.0101	389.498
					0.6	0.25	0.15	0.0026	0.0072	280.905
0.28	0.33	0.7	0.3	0.295	0.6	0.15	0.25	0.0036	0.0136	378.719
					0.6	0.2	0.2	0.0035	0.0087	244.977
					0.6	0.25	0.15	0.0035	0.0063	182.635
		0.3	0.7	0.315	0.6	0.15	0.25	0.0037	0.0287	781.125
					0.6	0.2	0.2	0.0036	0.017	471.107
					0.6	0.25	0.15	0.0036	0.0115	323.955
0.48	0.53	0.7	0.3	0.495	0.6	0.15	0.25	0.0042	0.0174	409.741
					0.6	0.2	0.2	0.0042	0.0109	260.446
					0.6	0.25	0.15	0.0041	0.0079	191.003
		0.3	0.7	0.515	0.6	0.15	0.25	0.0043	0.0366	851.544
					0.6	0.2	0.2	0.0042	0.0213	503.466
					0.6	0.25	0.15	0.0042	0.0141	339.861
0.68	0.73	0.7	0.3	0.695	0.6	0.15	0.25	0.0045	0.019	421.985
					0.6	0.2	0.2	0.0044	0.0118	265.586
					0.6	0.25	0.15	0.0044	0.0084	192.92
		0.3	0.7	0.715	0.6	0.15	0.25	0.0045	0.0398	882.392
					0.6	0.2	0.2	0.0044	0.0229	515.707
					0.6	0.25	0.15	0.0044	0.015	344.27
0.88	0.93	0.7	0.3	0.895	0.6	0.15	0.25	0.0044	0.0184	422.173
					0.6	0.2	0.2	0.0043	0.0113	263.661
					0.6	0.25	0.15	0.0042	0.008	190.057
		0.3	0.7	0.915	0.6	0.15	0.25	0.0043	0.0385	888.249
					0.6	0.2	0.2	0.0043	0.0219	514.304
					0.6	0.25	0.15	0.0042	0.0142	340.145

Table 9. *Percentage Relative Efficiency of the Proposed Stratified Technique over Tarray & Singh Stratified Randomized Response Model under Proportional Allocation when* $P_1 = 0.7$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(p)})$	$V(\hat{\pi}_{S(p)})$	PRE_p
0.08	0.13	0.7	0.3	0.1	0.7	0.1	0.2	0.0018	0.01591	866.935
					0.7	0.15	0.15	0.0018	0.00807	447.684
					0.7	0.2	0.1	0.0018	0.00522	294.694
		0.3	0.7	0.12	0.7	0.1	0.2	0.0019	0.03612	1854.2
					0.7	0.15	0.15	0.0019	0.01768	923.405
					0.7	0.2	0.1	0.0019	0.01089	578.51
0.28	0.33	0.7	0.3	0.3	0.7	0.1	0.2	0.0028	0.0298	1072.34
					0.7	0.15	0.15	0.0027	0.01437	524.491
					0.7	0.2	0.1	0.0027	0.0089	329.391
		0.3	0.7	0.32	0.7	0.1	0.2	0.0029	0.06719	2356.49
					0.7	0.15	0.15	0.0028	0.0311	1106
					0.7	0.2	0.1	0.0028	0.01825	657.875
0.48	0.53	0.7	0.3	0.5	0.7	0.1	0.2	0.0033	0.03753	1129.51
					0.7	0.15	0.15	0.0033	0.01778	542.473
					0.7	0.2	0.1	0.0032	0.01083	335.02
		0.3	0.7	0.52	0.7	0.1	0.2	0.0034	0.08442	2516.57
					0.7	0.15	0.15	0.0033	0.0383	1157.43
					0.7	0.2	0.1	0.0033	0.02205	675.494
0.68	0.73	0.7	0.3	0.7	0.7	0.1	0.2	0.0035	0.03909	1127.99
					0.7	0.15	0.15	0.0034	0.01829	535.717
					0.7	0.2	0.1	0.0034	0.01102	327.359
		0.3	0.7	0.72	0.7	0.1	0.2	0.0035	0.08781	2539.44
					0.7	0.15	0.15	0.0034	0.03929	1153.27
					0.7	0.2	0.1	0.0034	0.02231	664.786
0.88	0.93	0.7	0.3	0.9	0.7	0.1	0.2	0.0032	0.0345	1075.05
					0.7	0.15	0.15	0.0032	0.01592	504.886
					0.7	0.2	0.1	0.0031	0.00945	305.131
		0.3	0.7	0.92	0.7	0.1	0.2	0.0032	0.07736	2447.22
					0.7	0.15	0.15	0.0031	0.03405	1097.13
					0.7	0.2	0.1	0.0030	0.01902	624.086

Furthermore, Tables 10-12 show the values of the percentage relative efficiencies of the proposed stratified technique over Tarray and Singh [2] stratified randomized response model under Neyman allocation, (PRE_N). This also shows that PRE_N is greater than 100 at all the points considered and ranges from 159.67 to 3425.31 while the errors incurred decreases as the P_1 values increases from 0.5 to 0.7. The PRE_N due to the Neyman allocation is greater than that due to proportional allocation at all levels while variance is smaller at all levels.

Table 10. *Percentage Relative Efficiency of the Proposed Stratified Technique over Tarray & Singh Stratified Randomized Response Model under Neyman Allocation when* $P_1 = 0.5$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(N)})$	$V(\hat{\pi}_{S(N)})$	PRE_N
0.08	0.13	0.7	0.3	0.1	0.5	0.2	0.3	0.0021	0.0039	187.443
					0.5	0.25	0.25	0.0020	0.0034	168.716
					0.5	0.3	0.2	0.0020	0.0031	159.67
		0.3	0.7	0.12	0.5	0.2	0.3	0.0022	0.0083	376.951
					0.5	0.25	0.25	0.0022	0.0062	288.911
					0.5	0.3	0.2	0.0021	0.005	237.49
0.28	0.33	0.7	0.3	0.3	0.5	0.2	0.3	0.0028	0.0068	244.699
					0.5	0.25	0.25	0.0027	0.0057	208.464
					0.5	0.3	0.2	0.0027	0.005	187.841
		0.3	0.7	0.32	0.5	0.2	0.3	0.0029	0.014	486.921
					0.5	0.25	0.25	0.0028	0.01	354.669
					0.5	0.3	0.2	0.0028	0.0077	278.776
0.48	0.53	0.7	0.3	0.5	0.5	0.2	0.3	0.0033	0.0089	272.39
					0.5	0.25	0.25	0.0032	0.0073	227.396
					0.5	0.3	0.2	0.0031	0.0063	200.982
		0.3	0.7	0.52	0.5	0.2	0.3	0.0033	0.018	543.131
					0.5	0.25	0.25	0.0033	0.0126	387.833
					0.5	0.3	0.2	0.0032	0.0096	299.241
0.68	0.73	0.7	0.3	0.7	0.5	0.2	0.3	0.0035	0.0101	288.305
					0.5	0.25	0.25	0.0034	0.0082	238.033
					0.5	0.3	0.2	0.0034	0.007	208.107
		0.3	0.7	0.72	0.5	0.2	0.3	0.0035	0.0203	576.985
					0.5	0.25	0.25	0.0035	0.0141	407.444
					0.5	0.3	0.2	0.0034	0.0106	311.016
0.88	0.93	0.7	0.3	0.9	0.5	0.2	0.3	0.0035	0.0105	298.11
					0.5	0.25	0.25	0.0035	0.0084	244.287
					0.5	0.3	0.2	0.0034	0.0072	211.957
		0.3	0.7	0.92	0.5	0.2	0.3	0.0035	0.021	599.237
					0.5	0.25	0.25	0.0034	0.0144	419.901
					0.5	0.3	0.2	0.0034	0.0107	318.08

Table 11. *Percentage Relative Efficiency of the Proposed Stratified Technique over Tarray & Singh Stratified Randomized Response Model under Neyman Allocation when* $P_1 = 0.6$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(N)})$	$V(\hat{\pi}_{S(N)})$	PRE_N
0.08	0.13	0.7	0.3	0.1	0.6	0.15	0.25	0.0014	0.0047	323.619
					0.6	0.2	0.2	0.0014	0.0036	254.681
					0.6	0.25	0.15	0.0014	0.0031	221.347
		0.3	0.7	0.12	0.6	0.15	0.25	0.0016	0.0133	850.718
					0.6	0.2	0.2	0.0015	0.0087	565.798
					0.6	0.25	0.15	0.0015	0.0064	423.6
0.28	0.33	0.7	0.3	0.3	0.6	0.15	0.25	0.0021	0.0087	422.19
					0.6	0.2	0.2	0.002	0.0064	314.109
					0.6	0.25	0.15	0.002	0.0052	259.284
		0.3	0.7	0.32	0.6	0.15	0.25	0.0021	0.0238	1108.59
					0.6	0.2	0.2	0.0021	0.0148	698.041
					0.6	0.25	0.15	0.0021	0.0104	497.873
0.48	0.53	0.7	0.3	0.5	0.6	0.15	0.25	0.0025	0.0113	460.673
					0.6	0.2	0.2	0.0024	0.0081	336.339
					0.6	0.25	0.15	0.0024	0.0065	272.551
		0.3	0.7	0.52	0.6	0.15	0.25	0.0025	0.0305	1218.69
					0.6	0.2	0.2	0.0025	0.0185	752.458
					0.6	0.25	0.15	0.0024	0.0128	526.827
0.68	0.73	0.7	0.3	0.7	0.6	0.15	0.25	0.0026	0.0125	476.601
					0.6	0.2	0.2	0.0026	0.0089	344.349
					0.6	0.25	0.15	0.0025	0.007	276.094
		0.3	0.7	0.72	0.6	0.15	0.25	0.0026	0.0333	1270.06
					0.6	0.2	0.2	0.0026	0.0200	775.379
					0.6	0.25	0.15	0.0025	0.0136	536.907
0.88	0.93	0.7	0.3	0.9	0.6	0.15	0.25	0.0025	0.0122	478.624
					0.6	0.2	0.2	0.0025	0.0086	343.05
					0.6	0.25	0.15	0.0024	0.0067	272.728
		0.3	0.7	0.92	0.6	0.15	0.25	0.0025	0.0322	1285.43
					0.6	0.2	0.2	0.0025	0.0191	777.735
					0.6	0.25	0.15	0.0024	0.0129	533.631

Table 12. *Percentage Relative Efficiency of the Proposed Stratified Technique over Tarray & Singh Stratified Randomized Response Model under Neyman Allocation when* $P_1 = 0.7$

π_1	π_2	w_1	w_2	π	P_1	P_2	P_3	$V(\hat{\pi}_{st(N)})$	$V(\hat{\pi}_{S(N)})$	PRE_N
0.08	0.13	0.7	0.3	0.1	0.7	0.1	0.2	0.0010	0.00761	730.776
					0.7	0.15	0.15	0.0010	0.00461	451.184
					0.7	0.2	0.1	0.0010	0.00344	343.037
		0.3	0.7	0.12	0.7	0.1	0.2	0.0012	0.02781	2411.4
					0.7	0.15	0.15	0.0011	0.01422	1254.08
					0.7	0.2	0.1	0.0011	0.00911	817.311
0.28	0.33	0.7	0.3	0.3	0.7	0.1	0.2	0.0016	0.01486	930.994
					0.7	0.15	0.15	0.0016	0.00857	544.56
					0.7	0.2	0.1	0.0016	0.0061	393.198
		0.3	0.7	0.32	0.7	0.1	0.2	0.0017	0.05225	3131.03
					0.7	0.15	0.15	0.0016	0.0253	1537.1
					0.7	0.2	0.1	0.0016	0.01545	951.542
0.48	0.53	0.7	0.3	0.5	0.7	0.1	0.2	0.0019	0.01896	987.621
					0.7	0.15	0.15	0.0019	0.01076	567.916
					0.7	0.2	0.1	0.0019	0.00753	403.019
		0.3	0.7	0.52	0.7	0.1	0.2	0.0020	0.06586	3373.14
					0.7	0.15	0.15	0.0019	0.03128	1624.1
					0.7	0.2	0.1	0.0019	0.01875	986.908
0.68	0.73	0.7	0.3	0.7	0.7	0.1	0.2	0.0020	0.01992	990.342
					0.7	0.15	0.15	0.0020	0.01118	563.799
					0.7	0.2	0.1	0.0020	0.00773	395.877
		0.3	0.7	0.72	0.7	0.1	0.2	0.0020	0.06864	3425.31
					0.7	0.15	0.15	0.0020	0.03217	1629.60
					0.7	0.2	0.1	0.0019	0.01903	978.517
0.88	0.93	0.7	0.3	0.9	0.7	0.1	0.2	0.0019	0.01774	948.158
					0.7	0.15	0.15	0.0018	0.00983	534.637
					0.7	0.2	0.1	0.0018	0.00671	371.424
		0.3	0.7	0.92	0.7	0.1	0.2	0.0018	0.06061	3323.65
					0.7	0.15	0.15	0.0018	0.02797	1562.34
					0.7	0.2	0.1	0.0018	0.01628	926.400

7. APPLICATION

In applying the stratified HTRRT, we conduct a survey in Akure, Ondo state, Nigeria between April and June 2016 in order to estimate the proportion of people belonging to the sensitive attribute "drug use disorder" stratified by sex. Three decks of cards were used in each hth stratum containing both the sensitive question "*are you addicted to any drug?*" and unrelated question "*were you born before 1990*?" with sample size of 200 ($n_1 = 138,\ n_2 = 62$) where n_1 is the number of male respondents and n_2 is the number of female respondents. Real numbers $\alpha_h = 39$; $\beta_h = 34$; $\delta_h = 36$ so that the probabilities of choosing the device will be so close to increase respondents confidence in the process. The preset probabilities of the sensitive question in the three decks of cards are $P_{11} = P_{12} = 0.7$; $P_{21} = P_{22} = 0.2$; $P_{31} = P_{32} = 0.1$.

Therefore, the model estimated the proportion of male respondent that belongs to the sensitive attribute "*drug use disorder?"* as 0.199 while proportion of female respondent was estimated as 0.049 with variances 0.0140 and 0.0137, respectively. The stratified estimate of the population belonging to the sensitive attribute was 0.1234 with variance 0.0069 and coefficient of variation 5.6%. The summary is presented in Table 13.

Table 13. *Summary of estimates obtained from survey of people belonging to the sensitive attribute "drug use disorder"*

Stratum	π_{Uh}	$\hat{\theta}_h$	w_h	π_h	$w_h\pi_h$	$V(\pi_h)$	$w_h^2V(\pi_h)$
Male	0.445	0.36	0.496	0.199	0.0988	0.01396	0.00343
Female	0.15	0.115	0.504	0.049	0.0246	0.01372	0.00349
Total	0.595	0.475	1		0.1234		0.00692

8. CONCLUSION

The stratified hybrid tripartite randomized response technique has been developed and shown to be an efficient estimator in estimating proportion of people belonging to sensitive attribute in a population. The proposed stratified technique is shown to be more efficient than Singh & Gorey [1] and Tarray & Singh [2] stratified randomized response models under proportional and Neyman allocations. The percentage relative efficiency of the proposed estimator over the Singh and Gorey [1] estimator under proportional allocation was between 100.0 and 761.79 while under Neyman allocation it was between 100.0 and 753.5. Also, the percentage relative efficiency of the proposed estimator over the Tarray and Singh [2] estimator under proportional allocation was between 105.25 to 2539.44 while under Neyman allocation it was between 159.67 to 3425.31. The error incurred decreases with increase in probability of the sensitive question in the cards used. Applying the proposed technique to a survey on drug use disorder shows the applicability of the model. The proportion of drug use disorder among the respondents was estimated as 12.34% with coefficient of variation 5.6% showing that the proposed estimator was able to estimate the sensitive attribute with minimal error.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] Singh, H.P., Gorey, S.M., "An efficient stratified randomized response model", Journal of Statistical Theory and Practice, 11(4): 790-809, (2017).

[2] Tarray, T.A., Singh, H.P., "A Sinuous Stratified Unrelated Question Randomized Response Model", Communications in Statistics - Theory and Methods, 45(22): 6510-6520, (2016).

[3] Warner, S., "Randomized response: a survey technique for eliminating evasive answer bias", Journal of the American Statistical Association, 60: 63-69, (1965).

[4] Adebola, F.B., Adediran, A.A., Ewemooje, O.S., "Hybrid tripartite randomized response technique", Communications in Statistics-Theory and Methods, 46(23): 11756-11763, (2017).

[5] Adebola, F.B., Adepetun, A.O., "A new Tripartite Randomized Response Technique", Journal of the Nigerian Association of Mathematical Physics, 19: 119-122, (2011).

[6] Chaudhuri, A., "Estimating sensitive proportions from unequal probability sample using randomized responses", Pakistan Journal of Statistics, 17(3): 259–270, (2001).

[7] Ewemooje, O.S., "Estimating two sensitive characters with equal probabilities of protection", Cogent Mathematics, 4: 1-14, (2017).

[8] Ewemooje, O.S., Amahia, G.N., "Improving the Efficiency of Randomized Response Technique for Two Sensitive Characters", FUTA Journal of Research in Sciences, 12 (1): 65 – 72, (2016).

[9] Ewemooje, O.S., Amahia, G.N., Adebola, F.B., "Estimating prevalence of induced abortion and multiple sexual partners using improved randomized response technique for two sensitive attributes", Communications in Statistics: Case Studies, Data Analysis and Applications, 3(1-2): 21-28, (2018).

[10] Greenberg, B., Abul-Ela, A, Simmons, W., Horvitz, D., "The Unrelated question Randomized Response Model: Theoretical framework", Journal of American Statistical Association, 64: 529-539, (1969).

[11] Mangat, N. S., "An improved randomized response strategy", Journal of Royal Statistical Society, Series B, 56: 93 – 95, (1994).

[12] Ewemooje, O.S., Adebola, F.B., Amahia, G.N., "Alternative Estimator in Dichotomous Randomized Response Technique", Communications in Mathematics and Statistics, DOI:10.1007/s40304-018-0145-x, (2018).

[13] Kim, J-M., Warde, W.D., "A stratified Warner's randomized response model", Journal of Statistical Planning and Inference, 120 (1-2): 155-165, (2004).

[14] Hong, K., Yum, J., Lee, H., "A stratified randomized response technique", Korean Journal of Applied Statistics, 7:141–147, (1994).

[15] Kim, J.M., Elam, M.E., "A stratified unrelated randomized response model", Statistical Papers, 48: 215-233, (2007).

[16] Mangat, N.S., Singh, S., Singh, R., "On use of modified randomization device in Warner's Model", Journal of Indian Society of Operation Research, 14:1-4, (1995).

[17] Ewemooje, O.S., Amahia, G.N., "Improved Randomized Response Technique for Two Sensitive Attributes", Afrika Statistika, 10(2): 839-852, (2015).