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Scattering Analysis of Antenna by Using Ludwig Based Hybrid Method

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Abstract

Solving an electromagnetic problem can be handled in two phases. These are modelling the setup and carrying out the numeric evaluations. Throughout this study, the structure is modelled by Bézier surfaces and the antenna used is meshed with triangular patches. For the calculation part, the method of moments and physical optics (MoM-PO) hybrid method is implemented. While the calculations related with antenna are actualized by using MoM equations, the ones related with structure are obtained by using PO equations. Modified Ludwig's Algorithm is applied to calculate the current integral for the PO-region. This gives the ability to obtain successful results when the antenna is both close and far from the structure. Overall the stated modelling and calculation technique gives accurate results and saves time and memory in comparison with MoM.

1. INTRODUCTION

Over the years many studies and techniques have been performed to analyze antennas: exact solution techniques, high frequency asymptotic integration methods and sometimes hybrid methods that combine both. Hybrid approaches can be categorized as either current-based or ray-based. In [1, 2] method of moments-physical optics (MoM-PO) hybrid approach makes the combination of unknown MoM currents and surfaces currents obtained from physical optics (PO). Another application of hybrid MoM-PO method is [3], where MoM is used to solve for surface currents in the electric-field integral equation (EFIE) region, PO is applied for the calculation in the magnetic-field integral equation (MFIE) region.

PO is an efficient asymptotic method which calculates the scattered field by approximating the PO current. In order to calculate the oscillatory PO integral, different methods were employed over the past decades. The algorithms that are developed by Gordon [4] and Ludwig [5] are the most basic ones, where Ludwig approximated the phase and amplitude in the integrand by a linear form. Some other special calculation algorithms have been developed, which include the Filon method [6], Levin method [7], asymptotic expansion method [8] and numerical steepest descent path (NSDP) method [9]. Among them Levin method shows good results for complicated phase functions, but it tends to suffer from ill-conditioning. In [9] NSDP method is used to calculate PO integral on the parabolic patch for both monostatic and bistatic RCS calculations. Recently, stationary phase method (SPM) is another widely used technique [10-13] since its computation time is not dependent on frequency. To calculate the PO integral it only takes the effects of some critical points. On the other hand, SPM gives inaccurate results for near fields and additional modifications need to be performed for cases when two critical points are close to each other [10]. Compared to SPM, Ludwig's Algorithm [14-16] is less efficient but more stable, especially for structures including convex and concave parts.

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Non-uniform rational b-spline (NURBS) is an efficient modelling technique especially for complex bodies. In [12] NURBS modelling combined with SPM was performed for scattering analysis. But with only SPM based PO, it was not possible to analyze the antenna property. So, NURBS modelled SPM based hybrid MoM-PO method was recommended to be used to overcome the drawback [12]. Although this is proved to be an efficient combination, inaccurate results are obtained for cases when the antenna is located close ($L < 1.5\lambda$) to the structure.

In this article to overcome all the problems and drawbacks mentioned above, a hybrid method that combines MoM with PO is examined to analyze the scattering of antennas that are located near or far from perfectly electric conductor (PEC) platforms. NURBS surfaces are used to model these electrically large platforms and Ludwig Algorithm is implemented to calculate the PO current integral.

2. HYRBRID MOM-PO APPLICATION

In this study, hybrid MoM-PO method is used to analyze antenna radiation patterns around arbitrarily shaped structures. With the hybrid algorithm the mutual effect between antenna and structure is taken into account by considering the effect of both regions, as the impedance matrix is calculated. As seen in Figure 1, antenna is taken as the MoM-region where EFIE is employed, and the structure is taken as the PO-region where MFIE is employed. The surface current values in the PO-region and in the MoM-region are assumed to be $\vec{J}^{PO}$ and $\vec{J}^{MoM}$, respectively.

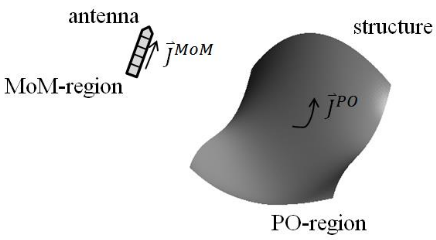


Figure 1. *Antenna around a NURBS surface*

For the presented method, MoM-region is modelled with triangular facets and PO-region with NURBS surfaces. A NURBS surface is a rational piecewise polynomial surface. It is defined with the help of some control points, each of which is effective by their own weighting value. Modelling with NURBS provides decrease in the number of patches and a more realistic modelling for curvature structures.

NURBS surfaces are commonly decomposed into rational Bézier surfaces to complete the numerical computations more practically. The transformation from NURBS into rational Bézier surface is done by applying Cox-De Boor algorithm. The position vector which is defined with the help of Bézier surface can be expressed as:

$$\vec{S}(x,y) = \frac{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{i,j}\vec{p}_{i,j}B_{i,m}(x)B_{j,n}(y)}{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{i,j}B_{i,m}(x)B_{j,n}(y)} \quad (1)$$

where $w_{i,j}$ are the weight values, $\vec{p}_{i,j}$ are the control points, *m* and *n* represent the control points numbers in *x* and *y* directions, with $(x,y) \in [0,1] \times [0,1]$, $B_{i,m}(x)$ and $B_{j,n}(y)$ are the Bernstein polynomials with a given formula [11]:

$$B_{i,n}(t) = \begin{cases} \frac{n!}{(n-i)!i!}(1-t)^{n-i}t^i, & 0 \le i \le n \\ 0, & others \end{cases} \quad (2)$$

When applying the conventional MoM, unknown currents are determined with the help of basis functions $\vec{f}_n$, which are generally selected as Rao-Wilton-Glisson (RWG) basis functions [17]:

$$\vec{J}^{MoM} = \sum_{n=1}^{N} \vec{f}_n \cdot a_n = [\vec{f}_1 \quad \dots \quad \vec{f}_N] \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix} = [\vec{f}][a] \tag{3}$$

where N represents the number of unknowns and a_n are the unknown coefficients $(n = 1,2, \dots, N)$ in MoM-region.

Presented hybrid MoM-PO is mainly based on the two equations given below. The first equation, which represents the EFIE, is applied over the antenna (MoM-region). The second one is implemented on the structure (PO-region) and it represents the MFIE.

$$L_e(\vec{J}^{MoM}) + L_e(\vec{J}^{PO}) = -\vec{E}_{tan}^{inc} \tag{4}$$

$$\vec{J}^{PO} = 2\delta_i \hat{n} \times \vec{H}_i + L_h(\vec{J}^{MoM}) = \vec{J}^{PO}{}_1 + \vec{J}^{PO}{}_2 \tag{5}$$

where $\vec{E}_{tan}^{inc}$ is the exciting source of the antenna, $\hat{n}$ is the surface outward normal vector and δ_i, which is set zero for shadowed regions, accounts for shadowing effect. As seen from (5), $\vec{J}^{PO}$ is divided into two parts. With respect to the incident magnetic field, a PO current indicated as $\vec{J}^{PO}{}_1$ is generated. The other term, $\vec{J}^{PO}{}_2$, indicates the PO current due to the currents over the MoM- region. L_e and L_h are the two linear integro-differential operators [3].

$$L_e(\vec{J}) = -\frac{j}{4\pi\varepsilon\omega} \vec{\nabla} \iint \left(\vec{\nabla} \cdot \vec{J}(\vec{r}')\right) G(\vec{r}, \vec{r}') dA' - j\omega \frac{\mu}{4\pi} \iint \vec{J}(\vec{r}') G(\vec{r}, \vec{r}') dA' \tag{6}$$

$$L_h(\vec{J}) = \frac{1}{4\pi} \vec{\nabla} \times \iint \vec{J}(\vec{r}') G(\vec{r}, \vec{r}') dA' \tag{7}$$

where ε is the permittivity and μ is the permeability of the medium, A' is the surface area and ω is the angular frequency.

$$G(\vec{r}, \vec{r}') = \frac{e^{-jkR}}{4\pi R} \tag{8}$$

is the Green's function, where $k = 2\pi/\lambda$, λ represents the wavelength and R is the distance between source and structure.

By using inner product process the EFIE equation (4) can be transformed into the following matrix equation

$$\left([Z_{MoM}^{MoM}] + [Z_{PO}^{MoM}]\right)[a] = [V_{MoM}] \tag{9}$$

where Z_{MoM}^{MoM} is the impedance matrix and V_{MoM} represents the source voltage vector in MoM-region. Z_{PO}^{MoM} is the mutual impedance matrix or the coupling matrix between PO and MoM-regions. Vector for source voltage (V_{MoM}) and impedance matrices $(Z_{MoM}^{MoM}, Z_{PO}^{MoM})$ are obtained from

$$(V_{MoM})_m = \langle w_m, -\vec{E}_{tan}^{inc} \rangle \tag{10}$$

$$(Z_{MoM}^{MoM})_{mn} = \langle w_m, L_e(f_n) \rangle, \; (Z_{PO}^{MoM})_{mn} = \langle w_m, \vec{E}_n^{PO} \rangle \tag{11}$$

where w_m is the weighting (or testing) function $(m = 1,2, \dots, N)$, $\vec{E}_n^{PO}$ is the scattered field from PO-region and n indicates which subdomain is active in MoM-region.

For the presented hybrid method once $\vec{E}_n^{PO}$ is calculated, the impedance matrices can be determined by using (11); then by using (9) a is calculated. After a is determined, with the help of (3) the current over the MoM-region can be calculated. Once $\vec{J}^{MoM}$ is known, the second term to determine the PO current in (5) can be determined and for the first term PO approximation is used.

Assuming that MoM-region is subdivided into N subdomains ($n = 1,2, \dots, N$), the impressed magnetic field to the PO-region from the nth subdomain of MoM-region is given as [13]:

$$\vec{H}_n(\vec{r}_s) = \iint -\frac{(1+jkR_{sd})}{4\pi R_{sd}^3}\left(\vec{R}_{sd} \times \vec{f}_n\right)e^{-jkR_{sd}}ds \tag{12}$$

where $\vec{R}_{sd} = \vec{r}_s - \vec{r}_d$, $\vec{r}_s$ and $\vec{r}_d$ are the points on the Bézier surface and over the MoM-region, respectively. s is the definition domain of the basis function. Below are the equations for basic PO approximation and the scattered electric field from Bézier surfaces [13], respectively.

$$\vec{J}^{PO} = \begin{cases} 2\hat{n} \times \vec{H}^i & lit \\ 0 & shadow \end{cases} \tag{13}$$

$$\vec{E}_S^{PO}(\vec{r}) = \frac{-1}{16j\omega\varepsilon\pi^2}\int_0^1\int_0^1 \vec{g}(u,v)e^{jkf(u,v)}dudv \tag{14}$$

where $\hat{n}$ is the surface outward normal vector, $\vec{H}^i$ is the incident magnetic field and

$$f(u,v) = -\left(R_{sd} - R_{fs}\right) \tag{15}$$

$$\vec{g}(u,v) = \left\{\frac{3-k^2R_{fs}^2+j3kR_{fs}}{R_{fs}^5}\vec{R}_{fs} \times \left(\vec{R}_{fs} \times 2\hat{n} \times \left(\vec{R}_{sd} \times \vec{f}_n\right)\right) + \frac{4+4jkR_{fs}}{R_{fs}^3}\left(\hat{n} \times \left(\vec{R}_{sd} \times \vec{f}_n\right)\right)\right\} \times \frac{1+jkR_{sd}}{R_{sd}^3}\left|\vec{r}_{su} \times \vec{r}_{sv}\right| \tag{16}$$

where $\vec{r}_f$ is the observation point and $\vec{R}_{fs} = \vec{r}_f - \vec{r}_s$ is the range between the observation point and the surface.

To determine the highly oscillatory surface integral in (14), modified Ludwig algorithm is performed. In Ludwig's method the integration domain is separated into Eq. (16), as shown in Figure 2. This may not seem very efficient, but for partially illuminated Bézier surfaces with the help of this subdivision very accurate results are obtained. The scattered field in (14) can be simplified to the calculation of the integral given in the form:

$$I = \int_{u=0}^{u=1}\int_{v=0}^{v=1} \vec{g}(u,v)e^{jkf(u,v)}dudv \tag{17}$$

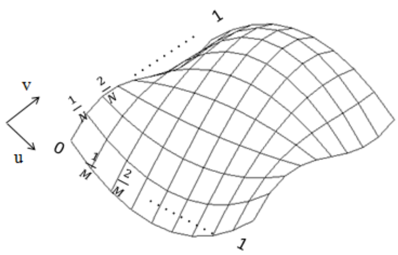


Figure 2. *Mosaic of integration cells on the structure*

Thus, the integral in (17) over a NURBS surface can be rewritten by using the subdomains:

$$I = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\int_{\frac{m}{M}}^{\frac{m+1}{M}}\int_{\frac{n}{N}}^{\frac{n+1}{N}} ge^{jkf}dudv \tag{18}$$

By Ludwig's Algorithm the double integral in (18) is redefined with ΔE_{mn} over a Bézier patch as

$$\Delta E_{mn} = \int_{\frac{m}{M}}^{\frac{m+1}{M}} \int_{\frac{n}{N}}^{\frac{n+1}{N}} g(u,v) e^{jkf(u,v)} du dv \qquad (19)$$

The functions g and f are approximated over the subdomain $\left[\frac{m}{M},\frac{m+1}{M}\right] \times \left[\frac{n}{N},\frac{n+1}{N}\right]$ by a simple form. and with the help of modified Ludwig's Algorithm ΔE_{mn} is presented for four different cases as the details are shown in [14, 16].

The accuracy for the results of ΔE_{mn} may be improved if the values of M and N increase. Beside this increase in the number of integral units causes an increment in the computation time. This makes the algorithm accurate but not efficient. An optimum value should be set depending on the shape of the structure. It is important to mention that; unlike SPM, no modifications are necessary for near-field calculations in Ludwig method.

3. NUMERICAL RESULTS

The simulation results obtained by asymptotic and numerical methods are compared under this title. The comparison is made for three different structures and when the antenna is positioned in different locations. For the simulations and mathematical calculations MATLAB R2016a is preferred, whereas for the modelling of structures a design based software program is used. For this purpose a PC with a 2.8 GHz processor is used.

Throughout the simulations performed in this study, three different methods are implemented: 1) the presented modified Ludwig's algorithm based hybrid MoM-PO method modelled by NURBS surfaces, 2) conventional MoM and 3) conventional MoM-PO method with triangular patch modelling. The size of the triangular facet is set as 0.13λ for triangular patch modelling.

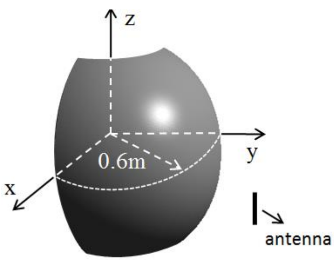


Figure 3. *Antenna around a curvy plate*

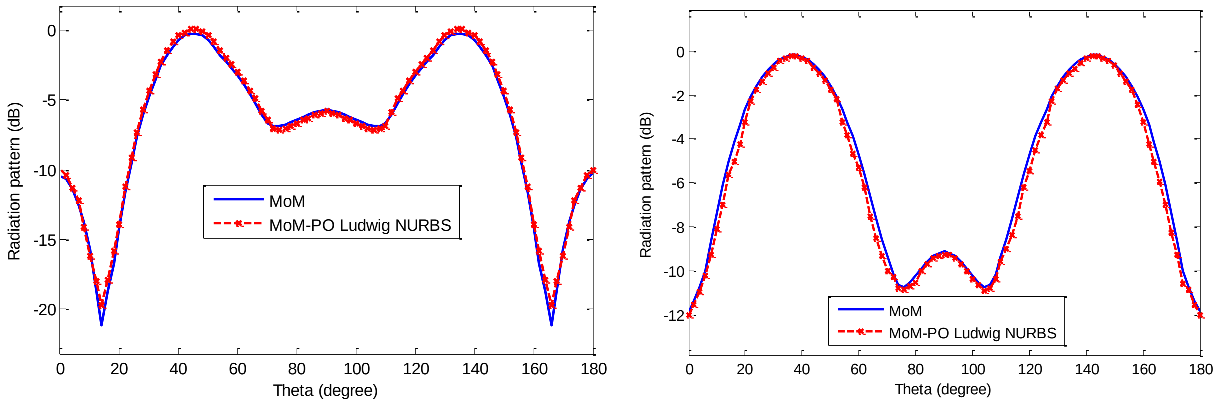
A curvy structure, which is a part of a sphere is analyzed in the first example, where the radius of the sphere is 0.6 m. The structure covers the angular region $0^o < \phi < 90^o$ and $45^o < \theta < 135^o$ as seen in Figure 3. An antenna with a length of 0.15 m and an operating frequency of 1 GHz is used. The structure is assigned as PO-region and the antenna as MoM-region. For this example two different cases are investigated where the position of the antenna is changing with respect to the structure.

First, a dipole is positioned at (0.7424, 0.7424, 0). The distance between antenna and surface is 0.45 m (1.5λ) and antenna is perpendicular to *xy*-plane. In Figure 4(a) the antenna radiation pattern is given for $\phi = 45^o$ cut. As seen from the results, presented method has nearly the same values obtained from the MoM technique which is an exact solution method. The maximum difference between the curves is 1.3 dB and it is obtained at $14°$ and $166°$. The computation time and number of unknowns for MoM algorithm are 41 min 10 s and 1224, respectively. Whereas, for the proposed hybrid method the computation time is 36 s and the number of unknowns in the MoM-region for the antenna is 8.

In the second case the antenna is positioned 0.15 m (0.5λ) away from the surface of the structure and located at (0.53033, 0.53033, 0). The radiation pattern of the given antenna is given in Figure 4(b) for $\phi = 45^o$

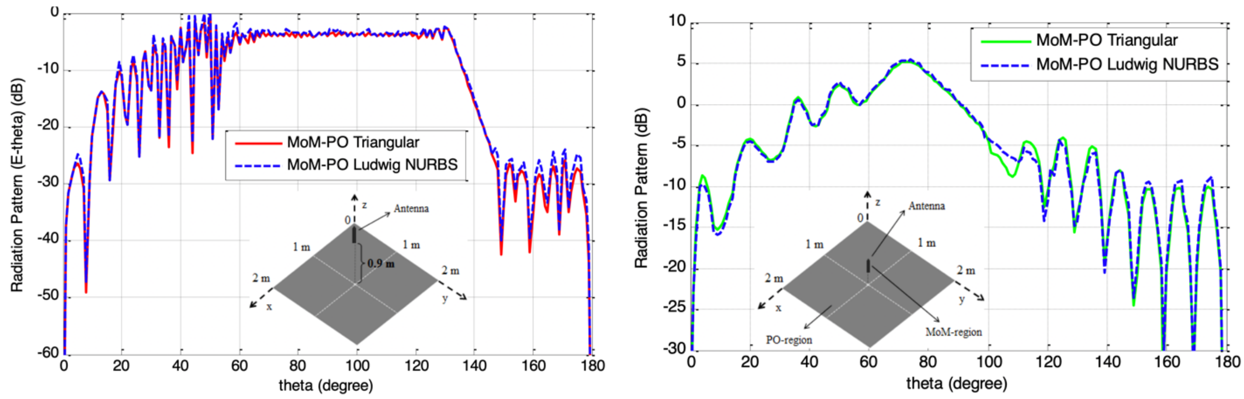
plane. Thanks to modified Ludwig algorithm, presented hybrid method gives accurate results for both cases when the antenna is far and near the platform.

The second example contains a square plate and a dipole antenna with an operating frequency of 1 GHz and a length of 2 m. One corner of the plate is positioned to the origin as seen from Figure 5(a) and Figure 5(b). In the first case the antenna is positioned far from the plate for 0.9 m (3λ), in the second case the distance is 0.15 m (0.5 λ). The results of the presented method for far and near cases are given in Figure 5(a) and Figure 5(b), respectively. In comparison to triangular facet modelled MoM-PO method accurate results are obtained for each case. The maximal errors for the first and second cases are 1.9 dB at 174° and 2.1 dB at 109°, respectively.



(a) (b)

Figure 4. *Radiation pattern; a) For first case b) For second case*



(a) (b)

Figure 5. *Radiation pattern; a) For the first case b) For the second case*

In the last example, a 0.15 meter long dipole antenna is located above and vertically to the middle of the fuselage of an aerial vehicle. As seen in Figure 6, the fuselage is parallel to *x-y* plane and the distance between antenna and fuselage is 0.15 m (0.5λ). The diameter and length of the fuselage is 1.4 m and 9 m, respectively. The width and length of each wing is 0.8 m and 3.6 m and the wings are connected on each side to the middle of the fuselage with 60° of angle. Antenna operating frequency is kept the same as the previous example as 1 GHz. Conventional MoM is not considered in the calculations of radiation pattern, since it takes hours to implement, due to the electrical size of the structure.

As seen in Figure 6, the structure is taken as the PO-region and the antenna is taken as the MoM-region. In the PO-region, PO integral is calculated with modified Ludwig Algorithm over Bézier surfaces. On the other hand in the MoM-region, moment method is applied over the surfaces that are modelled with triangular patches. Sub-domain basis functions are applied to model the expected behavior of the unknown function for the antenna. This is important, since it effects the way shadowing enters to simulation. For example as seen in Figure 6, the top part of the dipole antenna illumunates more parts of the fuselage in comparison to the parts that are close to the base of the dipole.

The antenna vertical radiation pattern for $\phi = 90°$ plane and $\phi = 45°$ plane are presented in polar form in Figure 7(a) and Figure 7(b), respectively. The waveforms are obtained with a step size of $2°$ with the angular range from $0°$ to $360°$. For the conventional triangular modelled MoM-PO, the whole surface of the structure is modelled with 52412 triangular patches, whereas for the proposed Ludwig based hybrid method the number of unknowns for the antenna is 8. In comparison to computation time, the conventional triangular based MoM-PO and the proposed Ludwig based hybrid method needs 7 min 38 s and 1 min 48 s, respectively. Although the waveforms obtained with both methods are nearly the same, a significant saving on time is achieved with the proposed algorithm, as seen in Table 1.

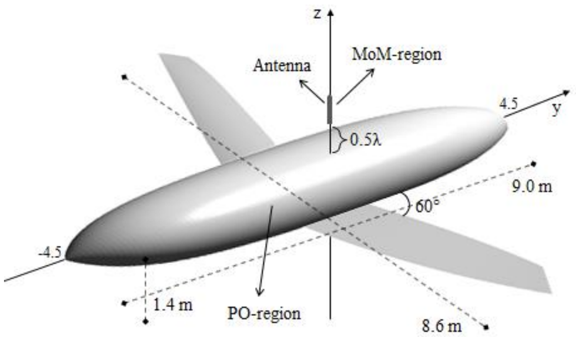


Figure 6. *MoM and PO-region distribution for aerial vehicle example*

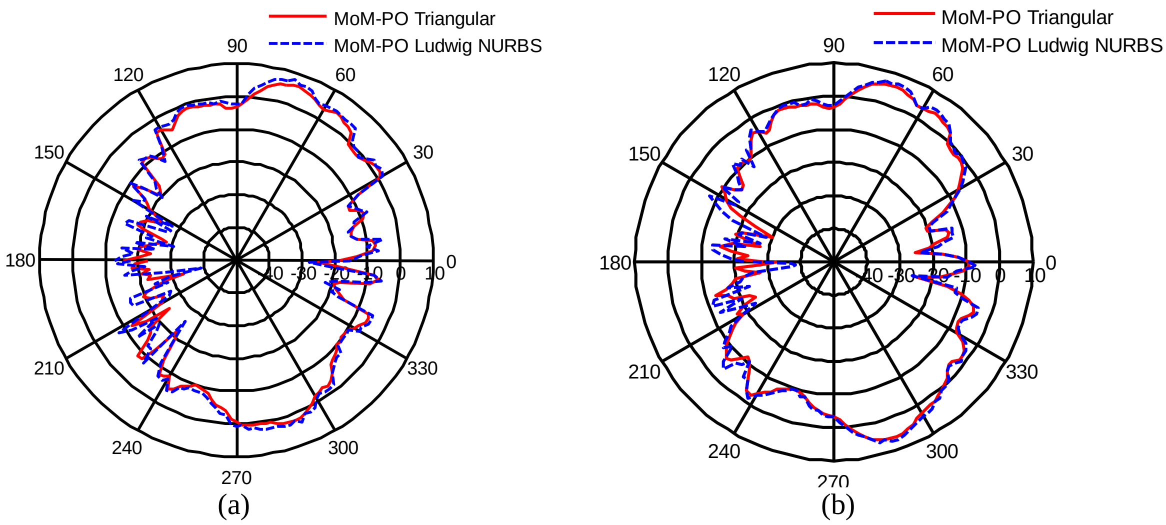


Figure 7. *Vertical radiation pattern of the dipole positioned over an aerial vehicle; a) For $\phi = 90°$ plane b) For $\phi = 45°$ plane*

Table 1. *Comparison of different methods*

Methods	CPU time	Number of Unknowns
MoM-PO Triangular	7 mins 38 sec	52412
MoM-PO Ludwig NURBS	1 mins 49 sec	8+NURBS

For second and third examples, it is not possible to identify which of the two methods applied is more accurate, as none of them are exact solution technique. But since both methods have close outcomes, one can say that with accurate results the presented hybrid method saves from computation time and memory. Comparing the results obtained by two different methods, shown in Figure 7(a) and Figure 7(b) individually, indicates little deviations. These are occurred since some surfaces for the presented Ludwig based-NURBS modelled hybrid MoM-PO method are partially illuminated. Too many patches may be necessary for the accuracy, depending on the shape of the structure. For the simulations in this study, it is important to emphasize that the sizes of the patches are checked for convergence.

4. CONCLUSION

Numerical examples illustrate the efficiency and accuracy of the presented hybrid method. On the other hand, shape of the structure and the problem type plays a major role on the efficiency of the proposed method. If the object is smooth and less curvy, fewer subdivisions are necessary. This provides a decrease in the CPU time and memory storage necessary for the calculation.

One of the drawbacks of PO approach is that only the illuminated regions have an effect on the calculation of the current, since the current is directly set as zero for shadowed regions. Besides that, edge diffractions and multiple reflections are not taken into account.

This hybrid algorithm provides great savings in time and memory especially when the structure shape is smooth and the unknowns in the MoM-region are significantly less in comparison to the unknowns in the PO-region.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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